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VORTEX and space-time distortion

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Abstract

The Universe is filled with "vortexes" (such as galaxies, accretion disks, stars and planetary systems, etc.) that clump and wind-up with magnetic fields. Strikingly absent in this rich narrative of growth and evolution of the cosmic systems, is a satisfactory "universal" mechanism that could have generated the original seed magnetic field. Because the explosive expansion of the universe must immensely dilute the magnetic field strength, very strong fields must have originated in the early universe. Exploiting the space-time distortion inherent in relativistic dynamics, we have unearthed just the mechanism that, by breaking the topological constraint forbidding the emergence of magnetic fields (vortexes), allows "general vorticities" — naturally coupled vortexes of matter motion and magnetic fields— to be created in an ideal fluid. The newly postulated relativistic mechanism, arising from the interaction between the inhomogeneous flow fields and inhomogeneous entropy, may be an attractive universal solution to the origin problem.

1 Introduction

Vortex is the most common appearance of existing, sustaining, and evolving "heterogeneity," at every scale hierarchy, in the Universe. Once a vortex is created, it behaves as if it were alive; vortex is basically a coherent, stable "object," while its motion is considerably complex; its interactions with other vortexes are not like those of particles, since interactions, in general, may penetrate into the *identity* of each vortex —*quantization* of vortex is not a strait-forward notion because of the essential nonlinearity of the kinematic description (while many different approaches have been proposed, and have made some interesting progresses).

Among rich narrative of various aspects of vortexes, the "origin problem" is one of the most challenging. The fact that the *circulation* must vanish for every ideal force (including the thermodynamic force as long as entropy is conserved) forbids the emergence of vorticity (or an axial vector field) in any *ideal* leading order model. This fundamental obstacle (a conservation law known as Kelvin's circulation theorem), anchored on the general Hamiltonian structure of ideal kinematics [1, 2, 3], seemingly inhibits the creation of the very fist vorticity in the Universe; since the vorticity of fluid motion is unified with the electromagnetic (EM) vorticity, that is magnetic field, the origin of the cosmological magnetic field is simultaneously questioned [4]. Invoking "non-ideal effects" has been the only known recourse to change the vortical state of a fluid; A typical example is the baroclinic mechanism [5], or Biermann battery [6], involving non-ideal thermodynamics in which the gradients of pressure and temperature have different directions [7, 8]. A velocity-space non-equilibrium distribution also provides a source of magnetic field via the so-called Weibel instability [9]. In early cosmology, inflation [10, 11], QCD phase transition [12, 13], or radiation effect [14] could create a source. While these mechanisms may, and likely will, play important roles in magnetic-field generation at some scales, none of these could be considered a universal mechanism operating at all scales [4].

In the present work, we demonstrate that a purely ideal mechanism, originating in the space-time distortion (shearing) caused by the demands of special relativity, can break the offending topological constraint. Vorticity, then, may be generated through an interaction between the inhomogeneous flow fields and inhomogeneous entropy. The new mechanism is universal, and is strong enough to overcome dissipation even for relatively weak flows [15, 16].

2 Generalized vorticity and circulation theorem

The mathematical and dynamical similarity between magnetic fields and fluid vorticity imparts both elegance and usefulness to the concept of generalized vorticity. Unless explicitly stated, generalized vorticity, denoted by $\boldsymbol{\Omega}$, will symbolize all physical quantities of this nature.

The "origin problem" has its genesis in the fact that the circulation associated with $\boldsymbol{\Omega}$ must vanish for every "ideal force" including the entropy conserving thermodynamic force. The reasons lie deep in the Hamiltonian structure governing the dynamics of an *ideal fluid*; the constrained dynamics implies the conservation of a "topological charge" that measures the generalized vorticity of the fluid —the invariance of the *generalized helicity*, which, for a non-relativistic charged flow, takes the familiar form $K = \int \boldsymbol{P}\cdot\boldsymbol{\Omega}\, dx$, where $\boldsymbol{P} = m\boldsymbol{V} + (q/c)\boldsymbol{A}$ is the canonical momentum and $\boldsymbol{\Omega} = \nabla \times \boldsymbol{P}$ is the generalized vorticity or generalized magnetic field (m: mass of a particle, q: charge of a particle, $\boldsymbol{V}$: fluid velocity, $\boldsymbol{A}$: vector potential, $\boldsymbol{B}$: magnetic field). Consequently, in any "ideal" leading order model, $\boldsymbol{\Omega}$ (consisting of both magnetic and kinematic components) cannot emerge from a zero initial value.

The problem of unearthing a primary generation mechanism for the magnetic field, found to be important in every scale hierarchy of universe, has defied a satisfactory solution to date [4]. Since the topological constraint on the ideal fluid forbids the vorticity to emerge, one resorts to "non-ideal dynamics" to affect a change. However, a satisfactorily strong and universal mechanism, operating at all scales, is not known. The search for such a universal mechanism provided the stimulus for this paper in which we make a clean break with the standard practice: Instead of relying on non-ideal effects we will show that $\boldsymbol{\Omega}$ can be generated in strictly ideal dynamics, as long as the dynamics is explicitly embedded in the space-time dictated by the demands of special relativity. The generalized vorticity is, then, generated through a source term born out of the special-relativistic "modifications" to the interaction of an inhomogeneous flow with inhomoge-

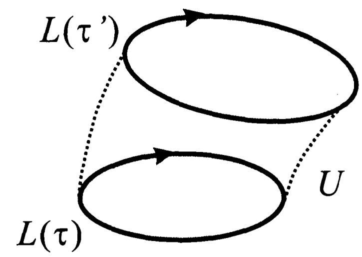

Figure 1: Transport of a loop and circulation [15]. Given a loop L in space, the circulation of a vector field $\boldsymbol{P}$ is the integral $\oint_L \boldsymbol{P} \cdot d\boldsymbol{x}$. Two loops $L(\tau)$ and $L(\tau')$, connected by the "flow" $d\boldsymbol{x}/d\tau = \boldsymbol{U}$ (the parameter τ may be regarded as time), are shown in the figure. A circulation theorem pertains to a "movement" of loops; the rate of change of circulation is calculated as (1). On a loop $L(\tau)$ carried by the fluid (i.e., $\tau = t$ and $\boldsymbol{U} = \boldsymbol{V}$), the circulation is conserved because $\oint_{L(t)} \nabla\mathcal{E} \cdot d\boldsymbol{x} \equiv 0$ (Kelvin's circulation law). To generalize the argument to the relativistic regime, we have to immerse the loop in the 4-d space-time and transport it by the 4-velocity $dx_\mu/d\tau = U_\mu$; see Fig. 2. The explicitly Lorentz covariant equality, written in terms of the proper time $\tau = s = ct/\gamma$ is found to be $d(\oint_{L(s)} P^\mu dx_\mu)/ds = \oint_{L(s)} (\partial^\mu P^\nu - \partial^\nu P^\mu) U_\nu dx_\mu$. Thus if the fluid equation could be cast in the form $(\partial^\mu P^\nu - \partial^\nu P^\mu) U_\nu = \partial^\mu \varphi$, the circulation would, indeed, be conserved. The relativistic space-time circulation conserves in ideal fluids; see (2).

neous entropy. To set the stage for a proper relativistic calculation we begin with some non-relativistic preliminaries and see how an "ideal" mechanics restricts the topology of fields.

The circulation $\oint_L \delta Q$, associated with a physical quantity δQ, calculated along the *loop L*, may be zero or finite depending on whether δQ equals an exact differential $d\varphi$ (φ being a state variable) or not. For example, if $\delta Q = T d\sigma$ (T: temperature, σ: entropy), the circulation is generally finite and measures the heat gained in a quasistatic thermodynamic cycle.

An *ideal fluid* can be viewed as a realization of an infinite number of ideal isolated cycles covering space. Along the time dependent loop $L(t)$, convected by the fluid motion (see Fig. 1), the rate of change of circulation associated with the canonical momentum $\oint_{L(t)} \boldsymbol{P} \cdot d\boldsymbol{x}$ is identically zero: connecting two loops $L(t)$ and $L(t')$ by a "flow" $d\boldsymbol{x}/dt = \boldsymbol{U}$, the rate of change of circulation is calculated as

$$\frac{d}{dt} \oint_{L(t)} \boldsymbol{P} \cdot d\boldsymbol{x} = \oint_{L(\tau)} [\partial_\tau \boldsymbol{P} + (\nabla \times \boldsymbol{P}) \times \boldsymbol{U}] \cdot d\boldsymbol{x}. \tag{1}$$

The ideal equation of motion for the momentum $\boldsymbol{P} = m\boldsymbol{V}$ can be written in the form $\partial_t \boldsymbol{P} + (\nabla \times \boldsymbol{P}) \times \boldsymbol{V} = -\nabla\mathcal{E}$ with the energy density $\mathcal{E} = P^2/2m + \phi + H$ (ϕ: potential energy, H: enthalpy). Hence, the rate of change of circulation equals the circulation of an exact fluid-dynamic force derived from the energy density, i.e., $\oint_{L(t)} \nabla\mathcal{E} \cdot d\boldsymbol{x} = \oint_{L(t)} d\mathcal{E} = 0$. In the standard non-relativistic description of an ideal fluid, therefore, if the initial state has no circulation (vorticity), the later state will also be vorticity-free (Kelvin's circulation theorem). For the vorticity to be created, the "force" on the fluid must not be an *exact differential.*

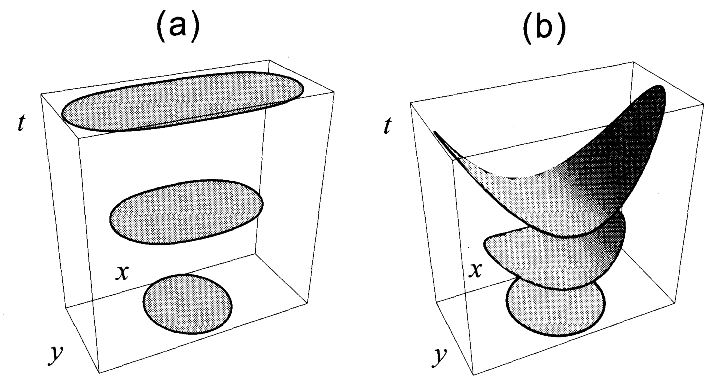

Figure 2: Transport of a surface (and its boundary) in space-time. Two figures compare the evolution of a surface and its boundary (loop) moved, respectively, by (A) the non-relativistic velocity ($dx_j/dt = V_j$: 3-vector) and (B) the relativistic 4-velocity ($dx_\mu/ds = U_\mu$). The figures are drawn in the space-time x-y-t with $\boldsymbol{V}/c = (\tanh x, 0, 0)$ (thus $\gamma = \mathrm{sech}^{-1} x$). In the Lorentz-covariant theory, the circulation theorem applies to a loop $L(s)$ that is moved by the 4-velocity U_μ in the 4-dimensional space-time. However, the vorticity (or magnetic field) is a reference-dependent quantity defined on the synchronic cycle $L(t)$, requiring a mapping from the naturally (relativistically) distorted $L(s)$to $L(t)$; this map multiplies the thermodynamic force by a Jacobian weight γ^{-1} breaking the exactness of the differential form.

In a plasma, the momentum must be generalized to the canonical momentum that includes the EM part, i.e., $\boldsymbol{P} = m\boldsymbol{V} + (q/c)\boldsymbol{A}$. The generation of a canonical circulation (or vorticity), then, implies the emergence of magnetic field.

3 Relativistic circulation theorem in space-time

Interestingly enough, the space-time *unity* imposed by special relativity provides a pathway to create vorticity. This purely kinematic relativistic effect acts by imposing a *Jacobian weight* $\gamma^{-1} = \sqrt{1 - (V/c)^2}$ that destroys the *exactness* of the ideal thermodynamic force; relativity transforms the loop integral $\oint_{L(t)} dH$, to $\oint_{L(t)} \gamma^{-1} dH$ which is no longer zero. Thus vorticity could be created within purely ideal dynamics.

For a geometric visualization of the new creation mechanism, let us see how relativity brings about a fundamental reconstruction of the notion of *circulation*. In the relativistic space-time, the loop $L(t)$ pertaining to a "synchronic space" (t = constant cross section of space-time in a reference frame) ceases to be the appropriate geometric object along which the circulation must be evaluated (see Fig. 1). The loop moves in space-time with a 4-velocity $U^\mu = (\gamma, \gamma V^j/c)$ (V^j: the reference-frame velocity) and the *relativistic circulation* must be described as a function of the proper time s. In Fig. 2, the respective evolutions of the "synchronic loop" $L(t)$ and the "relativistic loop" $L(s)$ are compared. The synchronicity of the loop $L(s)$ is broken by the nonuniformity of the proper time. The *relativistic Kelvin's theorem* applies to the relativistic loop; the circulation of a 4-

vector $\wp^\mu$ along $L(s)$ obeys

$$\frac{d}{ds}\left(\oint_{L(s)} \wp^\mu dx_\mu\right) = \oint_{L(s)} (\partial^\mu \wp^\nu - \partial^\nu \wp^\mu)\, U_\nu dx_\mu. \tag{2}$$

If $\wp^\mu$ is an appropriate momentum, the relativistic equation of motion relates the integrand $(\partial^\mu \wp^\nu - \partial^\nu \wp^\mu)\, U_\nu$ with an effective force. If the force is exact, the relativistic circulation will be conserved; the ideal fluid, indeed, obeys this relativistic circulation theorem. However, vorticity (or magnetic field) is defined on synchronic space (hence, it is reference-dependent); its *circulation* still pertains to the synchronic loop $L(t)$. The field must be mapped from the naturally distorted $L(s)$ back to $L(t)$—this reciprocal distortion, represented by a *Jacobian* γ^{-1}, imparts a *shear* to the thermodynamic force (i.e, changes dH to $\gamma^{-1}dH$) destroying its exactness.

These formal considerations will, now, be translated into an explicit calculation showing how relativity helps us to circumvent the "no-circulation" theorem. A covariant theory of vorticity generation follows from the recently formulated unified theory of relativistic, hot magneto-fluids [17]. The central construction of this theory is the relativistic generalized 4-momentum $\wp^\mu = mcfU^\mu + (q/c)A^\mu$ (A^μ: 4-vector potential) and the anti-symmetric tensor

$$M^{\mu\nu} = \partial^\mu \wp^\nu - \partial^\nu \wp^\mu = mcS^{\mu\nu} + (q/c)F^{\mu\nu}, \tag{3}$$

where $S^{\mu\nu} = \partial^\mu(fU^\nu) - \partial^\nu(fU^\mu)$ is the flow-field tensor representing both the inertial and thermal forces, and $F^{\mu\nu} = \partial^\mu A\nu - \partial^\nu A^\mu$ is the electromagnetic tensor. The factor f represents the thermally induced increase in effective mass ($h \equiv fmc^2$ relates f to the molar enthalpy h; h is an increasing function of temperature T, $f \approx 1$ for non-relativistic rising to $f \approx 6.66$ for $T = 1$ MeV) [1, 18]. In standard text books and papers $h = (\rho + p)/n$ with ρ and p being the proper energy density and pressure, respectively. The generalized vorticity $\hat{\Omega}$ (or the generalized magnetic field $\hat{\boldsymbol{B}}$) is defined by $\nabla \times \wp$ (or $(c/q)\nabla \times \wp$), where $\wp$ is the vector part of $\wp^\mu$. It must be emphasized that the *flow-EM field tensor* $M^{\mu\nu}$ contains both the inertial and the thermal forces.

Following an explicitly covariant procedure, the equation of motion

$$\partial_\mu T^{\mu\nu} + qnU_\mu F^{\mu\nu} = 0, \tag{4}$$

where $T^{\mu\nu} = nhU^\mu U^\nu - Pg^{\mu\nu}$ is perfect fluid energy momentum tensor and the right hand side is the Lorentz force, can be displayed as [17]

$$cU_\mu M^{\mu\nu} = T\partial^\nu \sigma, \tag{5}$$

where we have written as $\partial^\nu h - n^{-1}\partial^\nu P = T\partial^\nu\sigma$ invoking a "thermodynamic relation," with a temperature T and a molar entropy σ, to represent the non-exact residual of the left-had side (T and σ are assumed to be numbers independent of the choice of coordinate).

The vector part of (5)

$$q\left(\hat{\boldsymbol{E}} + \frac{\boldsymbol{V}}{c} \times \hat{\boldsymbol{B}}\right) = \frac{T\nabla\sigma}{\gamma}, \tag{6}$$

with the *generalized electric and magnetic fields* given by

$$\hat{\boldsymbol{E}} = \boldsymbol{E} - (1/q)[(\partial_t(\gamma f m V^j) + \nabla(\gamma f m c^2)], \tag{7}$$

$$\hat{\boldsymbol{B}} = \boldsymbol{B} + (c/q)\nabla \times (\gamma f m V^j), \tag{8}$$

is a concise way in which the equation of motion of a hot relativistic fluid is expressed in a form reminiscent of the non-relativistic version. By construction ($S^{\mu\nu}$ was defined to have the exact form of $F^{\mu\nu}$), the generalized fields satisfy Faraday's law $\partial_t \hat{\boldsymbol{B}} = -c\nabla \times \hat{\boldsymbol{E}}$ [19].

The appearance of γ^{-1} on the right-hand side of (6) is due to the mapping back of the relativistic space-time onto the synchronic space in which the conventional circulation and the vorticity are to be calculated. To evaluate the rate of change of $\hat{\boldsymbol{B}}$ (with respect to the reference time t), we must go back to (6) whose curl reveals the source for magnetic field generation:

$$\partial_t \tilde{\boldsymbol{B}} - \nabla \times (\boldsymbol{V} \times \tilde{\boldsymbol{B}}) = -\nabla \left(\frac{cT}{q\gamma} \right) \times \nabla\sigma \equiv \mathfrak{S}, \tag{9}$$

where the right-hand-side generation term is broken into the familiar baroclinic term $\mathfrak{S}_B = -(c/q\gamma)\nabla T \times \nabla\sigma$ and the relativistically induced new term

$$\mathfrak{S}_R = -\left(\frac{cT}{q} \right) \nabla\gamma^{-1} \times \nabla\sigma = -\left(\frac{c\gamma}{2qn} \right) \nabla \left(\frac{V}{c} \right)^2 \times \nabla p. \tag{10}$$

4 Relativistic source of magnetic field

The discovery of $\mathfrak{S}_R$ is the principal result of this paper. Following conclusions are readily deducible:

1) For homogeneous entropy, there is no vorticity drive—either baroclinic or relativistic.

2) As long as the kinetic energy is inhomogeneous, its interaction with inhomogeneous entropy keeps $\mathfrak{S}_R$ non-zero, even in a barotropic fluid.

3) When baroclinic drive is nonzero, and, in addition, the kinematic and thermal gradients are comparable, we can estimate

$$\frac{|\mathfrak{S}_R|}{|\mathfrak{S}_B|} \approx \frac{(V/c)^2}{1-(V/c)^2}. \tag{11}$$

For highly relativistic flows (cosmic particle-antiparticle plasmas, electron-positron plasmas in the magnetosphere of neutron stars, relativistic jets, etc.), $\mathfrak{S}_R$ will be evidently dominant, and can be far larger than the conventional estimates for the baroclinic mechanism. One must also remember that most long lived plasmas will tend to have $\nabla T \times \nabla\sigma = 0$ because of the thermodynamic coupling of temperature and entropy. In this large majority of physical situations, $\mathfrak{S}_R$ may be the only vorticity generation mechanism; no physical constraints will force the alignment of the gradients of kinematic γ and statistical σ. Thus, the relativistic drive is truly universal.

5 Separation of kinetic vorticity and magnetic field

After having shown that the new drive $\mathfrak{S}_R$ will always dominate the traditional baroclinic drive $\mathfrak{S}_B$ for relativistic plasmas, we will now attempt to estimate its strength in a few representative cases. Since the basic theory pertains to the generation of the generalized vorticity $\hat{\Omega}$, the eventual apportioning of $\hat{\Omega}$ into the magnetic part and the thermal-kinetic part will be a difficult system-dependent exercise —for example, if the plasma consists of relativistic electrons in a neutralizing ion background or it is an electron-positron pair plasma where both species are dynamic.

Here we consider a typical pair plasma, neutral in its rest frame, with density $n_+ = n_- = n = \text{constant}$ [16]. The suffix + (−) labels the positive (negative) particles. We also assume that the particles have the same homogeneous temperature so that their temperature modified effective masses $m^*_\pm \equiv f_\pm m$ (m: rest mass) are also the same ($m^*_+ = m^*_- = m^* = \text{const.}$). The generalized canonical momenta are

$$\wp^j_\pm = m^* c U^j_\pm \pm (e/c) A^j,$$

and the associated *generalized vorticities* are

$$\begin{aligned} \frac{1}{m^*}\nabla\times\wp_\pm &= \nabla\times(c\boldsymbol{U}_\pm) \pm \frac{e}{m^* c}\boldsymbol{B} \\ &\equiv \boldsymbol{\omega}_\pm \pm \boldsymbol{\omega}_c, \end{aligned} \tag{12}$$

in terms of which, the induction equation (9) takes the form

$$\partial_t(\boldsymbol{\omega}_\pm \pm \boldsymbol{\omega}_c) - \nabla\times[\boldsymbol{V}_\pm\times(\boldsymbol{\omega}_\pm \pm \boldsymbol{\omega}_c)] = -\nabla\times\left(\frac{cT\nabla\sigma_\pm}{\gamma_\pm m^*}\right). \tag{13}$$

To close the system, we need a determining equation for $\boldsymbol{\omega}_c = e\boldsymbol{B}/(m^* c)$ (the normalized magnetic field). When the large-scale slowly evolving EM is decoupled from the photons, the displacement current may be neglected [20] and the resulting Ampere's law

$$\nabla\times\boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} = 4\pi e n\,(\boldsymbol{U}_+ - \boldsymbol{U}_-) \tag{14}$$

may be written as

$$\delta^2\nabla\times\boldsymbol{\omega}_c = c\hat{n}\,(\boldsymbol{U}_+ - \boldsymbol{U}_-)\,. \tag{15}$$

Here $\delta = c/\omega_{pe}$ (electron inertia length) with $\omega_{pe}^2 = 4\pi e^2\bar{n}/m^*$ (plasma frequency), $\bar{n}$ is the average density, and $\hat{n} = n/\bar{n}$ is the normalized density. The curl of (15):

$$\delta^2\nabla\times\left(\hat{n}^{-1}\nabla\times\boldsymbol{\omega}_c\right) = \boldsymbol{\omega}_+ - \boldsymbol{\omega}_- \tag{16}$$

shows that the magnetic field is related to the difference in the normal vorticities of the two fluids.

We denote the generation drives as

$$\boldsymbol{G}_\pm = -\nabla\times\left[\frac{cT\nabla\sigma_\pm}{\gamma_\pm m^*}\right].$$

Assuming $\boldsymbol{V}_+ \approx \boldsymbol{V}_- \approx \overline{\boldsymbol{V}}$, and defining $\overline{\boldsymbol{\omega}} = (\boldsymbol{\omega}_+ + \boldsymbol{\omega}_-)/2$, we may rewrite (13) as

$$\partial_t\overline{\boldsymbol{\omega}} - \nabla\times\left(\overline{\boldsymbol{V}}\times\overline{\boldsymbol{\omega}}\right) = \frac{\boldsymbol{G}_+ + \boldsymbol{G}_-}{2}, \tag{17}$$

$$\partial_t\boldsymbol{\omega}_c - \nabla\times\left(\overline{\boldsymbol{V}}\times\boldsymbol{\omega}_c\right) = \frac{\boldsymbol{G}_+ - \boldsymbol{G}_-}{2}. \tag{18}$$

Here we have approximated, using (16) and assuming a large scale ($\gg \delta$),

$$\begin{aligned} \boldsymbol{\omega}_c + (\boldsymbol{\omega}_+ - \boldsymbol{\omega}_-)/2 &= \boldsymbol{\omega}_c + (\delta^2/2)\nabla\times(\hat{n}^{-1}\nabla\times\boldsymbol{\omega}_c) \\ &\approx \boldsymbol{\omega}_c. \end{aligned}$$

6 Cosmological application

One of the primary motivations to look for an ideal drive was to investigate if such a drive could generate a magnetic field in early universe (when the plasma is in strict thermal equilibrium) that is strong enough to leave its mark in an expanding universe. We present here a possible scenario that could emerge in the light of the current relativistic drive. The scenario is intertwined with the thermal history of the universe. Although there are earlier hotter eras, let us begin our considerations around 100 MeV.

(i) 100 MeV (10^{12} K) age ($\sim 10^{-4}$ s): At this time the muon-antimuon are beginning to annihilate, and primary constituents of the universe are electron-positron pairs, neutrionos-antineutrinos and photons, all in thermal equilibrium, with a very small amount of nucleons (protons and neutrons).

(ii) 10 MeV age ($\sim 10^{-2}$ s): The main constituents are electron-positron pairs, neutrinos and photons, and a very small amount of nucleons (protons getting considerably more than neutrons). At this stage neutrinos are decoupled and are freely expanding. The electron-positron pairs and photons are coupled and in thermal equilibrium.

(iii) 0.5 MeV age ($\sim$ 4 s): The neutrinos are in free expansion. The electron-positron pairs are beginning to annihilate.

(iv) 0.1 MeV age ($\sim$ 180 s): Almost all pairs are gone, and what we have now is an electron-proton plasma contaminated by lots of neutrons and gammas (gammas are still electromagnetically coupled).

(v) This plasma continues for a long time, but it is barely relativistic (protons are not). Nucleosynthesis converts some protons and neutrons to He (about 25% of the mass). But we continue with an electron-ion mildly relativistic and then essentially nonrelativistic electrons till 4000 K when atomic hydrogen forms by the absorption of electrons in the protons. At this time the universe is about 400000 years old.

Notice that till there is plasma whether electron-positron or electron-proton (He), the radiation keeps the particles in thermal equilibrium, so there is no baroclinic term. Hence the only thing that could generate seed vorticity is the relativistic source. Now we could envisage the "magnetic field" generation in several stages:

1) The universal "ideal" relativistic drive creates seed vorticity in the MeV era of the early universe when the electron-positron (e_-, e_+) plasma is the dominant component decoupled from neutrinos. In the context of this paper, this is the crucial element of the total scenario —the rest is cobbling together pieces of highly investigated phenomena.

2) Below 1 MeV, as the (e_-, e_+) pairs begin to annihilate, the electron-proton plasma tends to be the dominant component. This era lasts for 400,000 years till the temperature falls to 4000 K when the plasma disappears and the radiation decouples from matter. During this relatively long era, the seed field, created in the MeV era, is vastly magnified by what we call the Early Universe Dynamo (EUD).

3) At the hydrogen formation time, this magnetic field (however big it is) is decoupled from matter —there is no plasma left, and like the photons, the macroscopic magnetic field becomes a relic and red shifts (goes down in intensity) conserving flux. The relic field manifests in later eras appropriately diluted (conserving flux) by the cosmic expansion.

4) This diluted field would, then, provide the seed for an intergalactic or a galactic dynamo.

7 Concluding remarks

We have found that a recourse to special relativity uncovers an ideal, ubiquitous, fundamental vorticity generation mechanism. The exploration of this mechanism is likely to help us understand, *inter alia*, the origin of the magnetic fields in astrophysical and cosmic settings.

We end this paper by making a few comments about the finer points concerning vorticity, the generalized vorticity, and the relativistic generalized vorticity. As the physical system becomes more and more complicated (from an uncharged fluid to a charged fluid to a relativistic charged fluid), one must invent more and more sophisticated physical variables so that the fundamental dynamical structure (vortical form), epitomized in (5) is maintained. We do this because the very beautiful vortical structure is so thoroughly studied that reducing a more complicated system to this form immediately advances our understanding of new larger physical systems or, possibly, of more advanced space-time geometries.

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[18] In the relativistic regime, the definition of vorticity must be appropriately generalized: Based on the Lorentz covariant magneto-fluid unified theory [17], the canonical momentum $\boldsymbol{P} = mf\gamma\boldsymbol{V} + (q/c)\boldsymbol{A}$ modifies through the relativistic factors γ and f. The latter represents the increase of the effective mass by large random motions; we may write $f = f(T) = K_3(mc^2/T)/K_2(mc^2/T)$ for a Maxwellian fluid, where K_j is the modified Bessel function. The factor f is related to enthalpy through $H = mn_R f$ where n_R is the fluid density in the rest frame; see D. I. Dzhavakhrishvili, N. L. Tsintsadze, Sov. Phys. JETP **37**, 666 (1973); V. I. Berezhiani, S. M. Mahajan, Phys. Rev E **52**, 1968 (1995).

[19] In the definition of the generalized electric field, notice the terms $\nabla(\gamma fmc^2) \equiv \nabla(\gamma h)$, and $(\partial_t(\gamma fmV^j)$ that contain manifestations of the thermal effects. Combining the first of these with the right-hand side entropy term yields an effective pressure force which reproduces the standard pressure term in the nonrelativistic limit ($\gamma \to 1$): $-\nabla(\gamma h)+\gamma^{-1}T\nabla\sigma \to -n^{-1}\nabla P$. The second term $(\partial_t(\gamma fmV^j)$ contains time derivatives of f. This term with a little manipulation, and help from the zero component of (5), can be converted, if one so wished, into a term proportional to $\partial_t p$. For example, eq. (9) of T. Katsoulous and W. Mori, Phys. Rev. Lett. **61**, 90 (1988) will be the one dimensional limit of such an equation derived from (5).

[20] The displacement-current term $\partial_t\boldsymbol{E}/c$ in Ampere's law will appear as $-\omega_{pe}^{-2}\partial_t^2\boldsymbol{\omega}_c$ on the left-hand side of (16). In the time scale $\tau \gg \ell/c$ (ℓ is the length scale of the structures), this term may be neglected with respect to $\delta^2\nabla \times \left(\hat{n}^{-1}\nabla \times \boldsymbol{\omega}_c\right)$, and the D'Alembert operator collapses to the elliptic operator, eliminating the EM waves. The displacement-current term may not be neglected when one compares the divergence of Ampere's law with the mass conservation law. But this is not pertinent to the present calculations.