数理解析研究所講究録
第 1777 巻 2012 年 107-109

PROBLEM SESSION

EDITED BY TERUAKI KITANO

1. HIROSHI GODA

Let L be a link in S^3 and $\tau = t_1 \cup \cdots \cup t_n$ arcs embedded in S^3 such that $\tau \cap L = \partial\tau$.

Definition 1.1. *τ is an unknotting tunnel system of L if $S^3 \setminus \mathring{N}(L \cup \tau)$ is a genus $(n+1)$ handlebody.*

We define the tunnel number of L by

$$t(L) := \min\{n \mid \exists t_1 \cup \cdots \cup t_n : \text{ unknotting tunnel system of } L\}$$

When $t(L) = 1$, $\tau (= t_1)$ is called an unknotting tunnel. Let S denote a minimal bridge sphere of a knot K or a link L.

Theorem 1.2 (Goda-Scharlemann-Thompson). *If K is a knot with $t(K) = 1$, then the unknotting tunnel τ may be isotoped into S.*

The next theorem is a key to prove Theorem 1.2.

Theorem 1.3 (Gordon-Reid). *If $t(K) = 1$, then K does not have an essential tangle decomposition.*

Conjecture 1.4 (Scharlemann). *If K does not have an essential tangle decomposition, then all arcs in the unknotting tunnel system τ may be isotoped into S.*

Problem 1.5. (1) *Prove this conjecture.*

(2) *Find a knot K with an essential tangle decomposition and its unknotting tunnel system τ such that τ cannot be isotoped to the minimal bridge sphere S.*

(3) *Consider the link case.*

REFERENCES

1. Goda, Hiroshi; Scharlemann, Martin; Thompson, Abigail, *Levelling an unknotting tunnel*, Geom. Topol. **4** (2000), 243–275.
2. Gordon, C. McA.; Reid, A. W., *Tangle decompositions of tunnel number one knots and links*, J. Knot Theory Ramifications **4** (1995), no. 3, 389–409.

2. Takayuki Morifuji

Recently Friedl and Vidussi showed in [3] that the twisted Alexander polynomials corresponding to all finite representations detect fibered 3-manifolds.

Let K be a knot in S^3 and $G(K)$ its knot group. We consider the twisted Alexander polynomial $\Delta_{K,\rho_0}(t)$ of a hyperbolic knot K associated with a discrete faithful representation $\rho_0 : G(K) \to SL(2,\mathbb{C})$.

Conjecture 2.1 (Dunfield-Friedl-Jackson [1]). *The twisted Alexander polynomial $\Delta_{K,\rho_0}(t)$ detects fibered knots. Moreover $\Delta_{K,\rho_0}(t)$ detects the genus of K. That is,* $\deg \Delta_{K,\rho_0}(t) = 4g(K) - 2$ *holds.*

Remark 2.2. *For any representation $\rho : G(K) \to SL(2,\mathbb{C})$, it is known that* $\deg \Delta_{K,\rho}(t) \leq 4g(K) - 2$ *holds. Dunfield-Friedl-Jackson checked in* [1] *that the conjecture is true for hyperbolic knots up to 15-crossings.*

Question 2.3 (Friedl [2]). *Is there an example of a hyperbolic knot $K \subset S^3$ with a faithful representation $\rho : G(K) \to SL(2,\mathbb{C})$ such that* $\deg \Delta_{K,\rho}(t) \neq 4g(K) - 2$*?*

Remark 2.4 (Friedl [2]). *There is a hyperbolic knot K with a faithful and irreducible representation $\rho : G(K) \to SL(3,\mathbb{C})$ such that $\Delta_{K,\rho}(t)$ does not detect the genus.*

References

1. N. Dunfield, S. Friedl and N. Jackson, *Twisted Alexander polynomials of hyperbolic knots*, arXiv:1108.3045.
2. S. Friedl, Private communication, 2011.
3. S. Friedl and S. Vidussi, *Twisted Alexander polynomials detect fibered 3-manifolds*, Ann. of Math. (2) **173** (2011), 1587–1643.

3. Masaaki Suzuki

Let K be a knot and $G(K)$ its knot group. Let $\Delta_{K,\rho}(t)$ be the twisted Alexander polynomial associated to a representation ρ of $G(K)$.

Theorem 3.1 (Kitano-Suzuki-Wada). *Let K_1, K_2 be knots. If there exist a prime number p and a representation $\rho_2 : G(K_2) \to SL(2; \mathbb{Z}/p\mathbb{Z})$ such that $\Delta_{K_1,\rho_1}(t)$ is not divisible by $\Delta_{K_2,\rho_2}(t)$ for any representation $\rho_1 : G(K_1) \to SL(2; \mathbb{Z}/p\mathbb{Z})$, then there does not exist an epimorphism $G(K_1) \to G(K_2)$.*

Problem 3.2. *Is the converse is true ?*

4. Takuya Sakasai

Let F be a free group of rank n and G be a compact Lie group. Consider a normal subgroup $\Gamma \lhd F$, for example, $[[F, F], F]$, $[[F, F], [F, F]]$, etc.

Problem 4.1. *Define a good measure in the space $R_\Gamma(G) := Hom(F/\Gamma, G)$.*

The background of this problem is as follows. Twisted invariants such as

- twisted Alexander polynomial,
- twisted Reidemeister torsion,
- Atiyah-Patodi-Singer's ρ-invariant,

are basically invariants of a manifold X (or link) *together with* a representation of some group F/Γ associated with X. Fixing the manifold X, we may regard these invariants as functions on the representation space $R_\Gamma(G)$.

To get an invariant of X itself, we would like to consider the "totality" of such a function, which often behaves well on $R_\Gamma(G)$. For example, Levine [1] defined G-concordance invariants of links after observing that Atiyah-Patodi-Singer's ρ-invariant gives bounded, continuous and homology cobordism invariant functions on representation spaces away from some singular loci.

To answer the above (somewhat abstract) problem derives us to define invariants of manifolds by integrations of twisted invariants on representation spaces.

References

[1] J. Levine, *Link invariants via the eta invariant*, Comment. Math. Helv. 69 (1994), 82–119.