<table>
<thead>
<tr>
<th>Title</th>
<th>A definition of a field for Euclid's Elements without any set theories (Model Theory of Fields and its Applications)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>MURAKAMI, Masahiko</td>
</tr>
<tr>
<td>Citation</td>
<td>数理解析研究所講究録 (2012), 1794: 26-27</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2012-05</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/172873">http://hdl.handle.net/2433/172873</a></td>
</tr>
<tr>
<td>Right</td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
</tr>
<tr>
<td></td>
<td>Kyoto University</td>
</tr>
</tbody>
</table>
A definition of a field for Euclid’s Elements without any set theories

法政大学 村上 雅彦 (Masahiko MURAKAMI)
Hosei University

We introduce a definition of a field for Euclid’s Elements [E] without any set theories. Precisely, we define the non-negative part of a ordered field.

Since we never use any notion of set theory, we never say the language is \{+,-,0,1\}, but we say the symbols of binary operations “+” and “,” and the symbols of constants “0” and “1”.

Since we never use any notion of set theory, we never say \textit{infinitely} many variable symbols \(v_1, v_2, \ldots\), but we say variable symbols \(a, b, c, \ldots\) etc. as many as we need. because the words “infinite” and “finite” are notions of set theories. We introduce a unary predicate \(N(\bullet)\), and we say \(n\) is a natural number if \(N(n)\).

The symbol of equality is “=” and the logical connections are “\(\land\)”, “\(\lor\)”, “\(\Rightarrow\)” and “\(\neg\)”, and the quantifiers are “\(\forall\)” and “\(\exists\)”. By usual way of BNF, we define \textit{terms}, \textit{equations}, \textit{formulas}. They are not sets but they are on a paper, in our brain, in storages of computers, or etc.. We never say a \textit{set} of formulas against model theory.

By usual way we adopt the axiom of equality — replacing the terms which are connected by “\(=\)”. Finally, we work on the classical predicate logic. any proofs or Any deductions are never set.

Here are the definition. Every free variable is bound by universal quantifier.

\[ A0 \ N(0) \land N(1). \]
A1 \((a + b) + c = a + (b + c)\).

A3 \(a + b = b = 1\)

A4 \[P(0, a, b, c, d, e) \land \forall n\left[\mathbb{N}(n) \land P(n, a, b, c, d, e)\right] \Rightarrow P(n + 1, a, b, c, d, e)\] 
\[\Rightarrow \forall n[\mathbb{N}(n) \Rightarrow P(n, a, b, c, d, e)],\] where \(P(x, u, v, w, y, z)\) is a formula on a paper, in our brain, in storages of computers, or etc., and \(x, u, v, w, y, z\) are meta-symbols to replace terms.

A5 \(a + c = b + c \Rightarrow a = b\).

A6 \(\neg[a + 1 = 0]\).

A7 \(a \cdot 1 = a\).

A8 \((a \cdot b) \cdot c = a \cdot (b \cdot c)\).

A9 \(a \cdot b = b \cdot a\).

A10 \(a \cdot (b + c) = (a \cdot b) + (a \cdot c)\).

A11 \(\exists c [a = 0 \lor a \cdot c = b]\).

A12 \(\exists b a = b \cdot b\).

A13 \(\exists n \exists b [\mathbb{N}(n) \land a + b = n]\).

We denote \(\exists c a + c = b\) by \(a \leq b\) and denote \(\exists n [\mathbb{N}(n) \land a \cdot n = b]\) by \(a \mid b\).

**Reference**
