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A definition of a field for Euclid’s Elements without any set theories

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We introduce a definition of a field for Euclid’s Elements [E] without any set theories. Precisely, we define the non-negative part of a ordered field.

Since we never use any notion of set theory, we never say the language is \{+, \cdot, 0, 1\}, but we say the symbols of binary operations “+” and “\cdot” and the symbols of constants “0” and “1”.

Since we never use any notion of set theory, we never say *infinitely* many variable symbols $v_1, v_2, \ldots$, but we say variable symbols $a, b, c, \ldots$ etc. as many as we need. because the words “infinit"e” and “finite” are notions of set theories. We introduce a unary predicate $N(\bullet)$, and we say $n$ is a natural number if $N(n)$.

The symbol of equality is “=”, and the logical connections are “\&”, “\lor”, “\Rightarrow” and “\neg” and the quantifiers are “\forall” and “\exists”.

By usual way of BNF, we define *terms, equations, formulas*. They are not sets but they are on a paper, in our brain, in storages of computers, or etc. We never say a *set* of formulas against model theory.

By usual way we adopt the axiom of equality — replacing the terms which are connected by “\=”.

Finally, we work on the classical predicate logic. any proofs or Any deductions are never set.

Here are the definition. Every free variable is bound by universal quantifier.

$\textbf{A0} \ N(0) \land N(1)$. 
A1 \((a + b) + c = a + (b + c)\).

A3 \(a + b = b = 1\)

A4 \(P(0, a, b, c, d, e) \land \forall n [\mathbb{N}(n) \land P(n, a, b, c, d, e)] \Rightarrow P(n + 1, a, b, c, d, e)]\)
\(\Rightarrow \forall n [\mathbb{N}(n) \Rightarrow P(n, a, b, c, d, e)]\), where \(P(x, u, v, w, y, z)\) is a formula on a paper, in our brain, in storages of computers, or etc., and \(x, u, v, w, y, z\) are meta-symbols to replace terms.

A5 \(a + c = b + c \Rightarrow a = b\).

A6 \(\neg[a + 1 = 0]\).

A7 \(a \cdot 1 = a\).

A8 \((a \cdot b) \cdot c = a \cdot (b \cdot c)\).

A9 \(a \cdot b = b \cdot a\).

A10 \(a \cdot (b + c) = (a \cdot b) + (a \cdot c)\).

A11 \(\exists c [a = 0 \lor a \cdot c = b]\).

A12 \(\exists b a = b \cdot b\).

A13 \(\exists n \exists b [\mathbb{N}(n) \land a + b = n]\).

We denote \(\exists c a + c = b\) by \(a \leq b\) and denote \(\exists n [\mathbb{N}(n) \land a \cdot n = b]\) by \(a \mid b\).

**Reference**
