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"Managerial Incentives and the Role of
Advisors in the Continuous-Time Agency
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Managerial Incentives and the Role of Advisors in the Continuous-Time Agency Model[*]

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in the Continuous-Time Agency Model

Abstract

This paper explores a continuous-time agency model with double moral hazard. Using a venture capitalist–entrepreneur relationship where a manager provides unobservable effort while a venture capitalist (VC) both supplies unobservable effort and chooses the optimal timing of the initial public offering (IPO) with an irreversible investment, we show that optimal IPO timing is earlier under double moral hazard than under single moral hazard. Our results also indicate that the manager's compensation tends to be paid earlier under double moral hazard. We also derive several comparative static results for the IPO timing and managerial compensation profile, all of which provide new empirically testable implications. Usefully, even where the VC does not completely exit with the IPO, such that there is a requirement for a multiagent analysis after the IPO, most of our results remain unchanged. In addition, our model applies to not only the VC exit through the M&A (Mergers and Acquisitions) process but also the dissolution of joint ventures and corporate spin-offs.

1. Introduction

This paper examines the situation in which the joint provision of costly effort by a manager and an advisor initially improves the productivity of a project in a firm. However, after a substantial upward shift in productivity is achieved through irreversible investment, the role of the advisor ends, and only the manager remains to provide ongoing costly efforts to improve the productivity of the project. Because the timing of investment corresponds with the timing of the change in the role of the advisor, optimal timing of investment needs to consider the effect of this change. A difficulty may therefore arise if the manager's costly efforts are unobservable in that he cannot be provided with appropriate incentives. In this case, not only the advisor's costly efforts but also the manager's compensation may be controlled so as to provide the manager with appropriate incentives. In addition, the advisor's costly effort may be unobservable. Consequently, we must consider the provision of incentives for effort for both the manager and the advisor when solving the optimal timing problem of investment.

This particular situation typically arises when a venture capitalist (VC) exerts substantial effort in monitoring managerial activity, providing managerial advice to entrepreneurs, and choosing the optimal timing of both the initial public offering (IPO) and any investment financed by the IPO. Indeed, the monitoring and advisory role of the VC through ongoing long-term involvement can increase the value of its portfolio companies.[1] The VC often participates directly in management by holding one or more positions on the board of directors. The VC also specializes in a particular industry and uses those industry contacts to help entrepreneurs to perform various business activities.[2] Furthermore, some VCs employ consulting staff that are involved in the management of companies in their portfolios. However,

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[1] The monitoring and advising role of the VC is empirically documented by Barry et al. (1990), Gompers and Lerner (1999), Hellmann and Puri (2002), and Kaplan and Strömberg (2004).

[2] For example, the VC helps to shape and to recruit the management team, to shape the strategy and the business model before and after investing, to assist in production, to line up suppliers, and to develop customer relations.

it is difficult to verify the intensities of such monitoring and advising efforts provided by the VC.[3] The VC can harvest investments in private companies in one of two ways; namely, taking the firm public via an IPO or selling the firm to another company (mergers and acquisitions (M&A)). In fact, we can interpret the IPO process in this paper as the M&A process if the acquiring company's activity or investment has a synergetic effect on the acquired firm's productivity. Hence, we use the terms of the "IPO" and "M&A" processes for the VC exit route interchangeably.[4]

Another example is where a large established firm sells all of its claims in a joint venture with a small innovative firm or where a parent firm spins off one of its subsidiaries to its own shareholders and makes the subsidiary a new entity that is managed independently of the parent firm. With joint venture dissolution, as the large established firm provides resources in areas such as manufacturing, distribution and marketing, the large-established-firm–small-innovative-firm relationship acts like a VC–entrepreneur relationship. With corporate spin-offs, the parent firm's CEO–subsidiary manager relationship corresponds to a VC–entrepreneur relationship if the CEO's objective is to maximize the initial shareholders' payoff where the CEO is not involved in the management of the subsidiary following a spin-off.[5]

To capture this dynamic problem, we develop a continuous-time agency model with double moral hazard, in which an agent (manager) provides unobservable value-adding effort and a principal (advisor) also contributes unobservable value-adding effort as well as choosing the optimal timing of changing her role with irreversible investment. The basic question that we address is how the optimal timing of changing the advisor's role and the dynamic properties of optimal incentive provision are characterized under double moral hazard, compared with

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[3]The literature of optimal venture capital contracts makes a similar assumption. See Casamatta (2003), Schmidt (2003), Repullo and Suarez (2004), Inderst and Müller (2004), and Hellmann (2006).

[4]A similar situation occurs when buyout funds attempt to make private firms in their portfolio go public or when banks attempt to return bankrupt firms under their administration to public ownership.

[5]In both cases, restructuring costs need to be expended at the time of the dissolution or spin-off. Evidence of operating performance improvements following spin-offs is in Daley, Mehrotra, and Sivakumar (1997) and Desai and Jain (1999).

those under single moral hazard. We also discuss how the changes in the advisor's role and the manager's compensation are interrelated. We further explore the comparative statics regarding the timing of changing the advisor's role and the manager's compensation.

In our basic continuous-time agency model, the project generates cumulative cash flows that are affected by both the principal's and the manager's efforts. There are complementarities between the value-adding efforts of the principal and the manager. The principal also chooses the timing of exiting; that is, the timing of the IPO and investment for achieving an upward shift in the productivity of the project. Following the IPO, new outside investors own the firm and induce the manager to exert an unobservable effort to operate the project, although they cannot themselves expend any effort to operate the project.

Now, suppose as a benchmark the single moral hazard situation in which the principal's effort is observable and verifiable. Although the manager's effort is unobservable, he has an incentive to increase effort if his value depends more strongly on output. Hence, increasing volatility of the manager's value is required to attain a higher level of effort from the manager. However, this implies that poor results are met with penalties. As the manager's limited liability precludes negative wages, the project has to be terminated inefficiently once his future discounted payoff—that is, his continuation value—hits his outside option. Thus, there exists a trade-off between more incentive provision and inefficient termination. This also means that paying cash to the manager earlier makes future inefficient liquidation more likely, as it reduces the manager's continuation payoff. Hence, the possibility of inefficient liquidation causes deferred compensation. Furthermore, if costly investment is irreversible, the possibility of inefficient termination causes IPO execution to be more costly because the investment cost is less likely to be recovered. Hence, there exists another trade-off between earlier IPO timing and inefficient termination. This trade-off creates an option value of waiting for an IPO. In addition, the change in the governance mechanism with the IPO also needs to be considered in choosing the timing of the IPO, because this has a significant effect

on the manager's incentives and thereby substantially affects the merit of the IPO.

We next suppose the double moral hazard situation in which the principal's effort is unobservable. Then, the principal may reduce her effort ex post because she may not be able to commit to secure an appropriate ex post incentive for herself. The possibility of a decrease in the principal's effort may reduce the option value of waiting for the IPO (or the principal's expected profit obtained by waiting for the IPO), thereby advancing the IPO timing. This possibility may also have an adverse effect on the cost of compensating the manager for his effort and may make earlier compensation more costly under double moral hazard. However, we will show that this final intuition is incorrect.

The main results of the model are as follows. Our first main result is that optimal IPO timing is earlier under double moral hazard than under single moral hazard. Intuitively, the principal's effort supplied prior to the IPO is smaller under double moral hazard than under single moral hazard. The reason is that under double moral hazard, the principal has an ex post incentive to reduce her effort level, relative to that optimally obtained under single moral hazard, because she cannot internalize the externality effect of her effort on the manager's incentives when choosing the level of effort after the contract has been offered. This reduces the option value of waiting for the IPO, thus advancing the IPO timing.

The second main result is that the manager's compensation tends to be paid earlier under double moral hazard than under single moral hazard. Intuitively, the manager' compensation has never been paid before the IPO under either double or single moral hazard. Given that the cash payment threshold is located after the IPO and that only the single moral hazard situation arises after the IPO, the compensation payment timing is earlier under double moral hazard because the IPO timing is earlier under double moral hazard.

We carry out the comparative static results for IPO timing and the manager's compensation profile under double moral hazard. Then, the IPO is more likely to be brought forward when a need for monitoring by the VC is smaller, the increment in future expected cash flows

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is greater, and the volatility of cash flows is larger. Furthermore, the manager's compensation tends to be paid later the smaller the increment in future expected cash flows and the larger the volatility of cash flows. These comparative static results provide testable predictions about IPO timing and vesting provisions for the manager in early-stage start-up firms, management buyout firms, or firms reorganized following bankruptcy. These predictions can also be tested for the dissolution of joint ventures and for corporate spin-offs.

Practically, some VCs retain a fraction of the firm's shares and continue to hold their board positions even after the IPO. If the principal retains a portion of the equity claims in the firm and continues to provide effort after the IPO, a multiagent problem may arise. Then, the analysis needs to be carried out in a multiagent setting after the IPO. We extend our model into this case and show that most of our main results still hold. Hence, the multiagent problem existing after an IPO does not affect our main results.

The work in this paper is related to the growing literature on continuous-time agency models using the martingale techniques developed by Sannikov (2008).[6] Philippon and Sannikov (2007) extend the cash diversion model in DeMarzo and Sannikov (2006) by considering an endogenous shift in the timing in the mean of cash flows by irreversible investment or IPO. However, in Philippon and Sannikov (2007), the principal supplies no effort, and a standard Brownian motion represents a signal regarding the action of the manager, rather than the cumulative cash flow. The main difference between our model and the aforementioned continuous-time agency models is that our model deals with a double moral hazard situation in which the principal exerts unobservable effort to develop a project before making an irreversible investment and exiting the firm. This enables us to investigate how the possibility of the principal's moral hazard affects the timing of organizational change and the optimal properties of dynamic contract, and to explore the interplay between organizational

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[6]See DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin, and Rochet (2007), He (2009), Biais, Mariotti, Rochet, and Villeneuve (2010), Hoffmann and Pfeil (2010), Jovanovic and Prat (2010), Piskorski and Tchistyi (2010), and He (2011). The continuous-time agency model began with Holmstrom and Milgrom (1987). However, unlike the recent model, they assume that the agent can receive a lump-sum payment only at the end of an exogenous finite time interval (also see Sung (1997) and Ou-Yang (2003)).

change and the optimal properties of dynamic contract. In particular, unlike Philippon and Sannikov (2007), under double moral hazard, we can capture the effect of a change in the ownership structure or governance on the optimal IPO timing and compensation profile for the manager.[7] Furthermore, if the principal does not completely exit with the IPO, we are able to explore a continuous-time multiagent model following the IPO.

Several recent studies explore optimal venture capital contracts under double moral hazard.[8] However, they use a discrete-time agency model with two or three periods, which is not suitable for the analysis of the IPO timing or the dynamic optimal incentive provision.

Several existing studies have implications for firm characteristics during IPOs. Pagano and Röell (1998) examine the effect of ownership structure on the decision to go public. However, their model is static. Benninga, Helmantel, and Sarig (2005) and Pástor, Taylor, and Veronesi (2009) employ a dynamic IPO model by trading off the diversification benefits of going public against the benefits of private control. Both of these models suggest that going public is optimal when cash flows or the firm's expected future profitability is sufficiently high. However, they do not use the continuous-time agency model, nor do they explore the effect of the change in managerial control with the IPO (VC advice and monitoring vs. market discipline) on the IPO timing or the manager's compensation profile.

The paper is organized as follows. Section 2 presents a simple two-period model. Section 3 presents a continuous-time agency model. Sections 4 and 5 derive an optimal contract under single and double moral hazard. Section 6 considers the comparative statics. Section 7 allows the principal not to exit completely with the IPO. Section 8 provides empirical implications for our results, and Section 9 concludes.

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[7]Our result—that the more the need for the VC's monitoring role, the higher the IPO timing threshold—is novel. Furthermore, Philippon and Sannikov (2007) suggest that the IPO threshold is increasing in volatility. By contrast, our result shows that the IPO threshold may be decreasing in the risk of the project.

[8]See the literature mentioned in footnote 3.

2. A Simple Two-Period Model

We begin with a simple two-period model, which will form the basis for our continuous-time model and illustrate how our results depend on the continuous-time agency setting. Consider a two-period model, in which a principal hires an agent to operate a firm and has an opportunity to improve the firm's output process substantially by undertaking irreversible investment K when the principal exits. In the subsequent analysis, we take as an example an IPO for a private company presently under the control of a VC or buyout fund. The principal (the VC or buyout fund) hires a manager and then has an opportunity to exit with an IPO. The investment cost K is paid by the funds financed from new outside investors.

The cash flows produced by the firm at time t ($t = 1, 2$) are given by a random variable x_t, which is jointly independent in each period. More specifically, with probability $\xi \in (0, 1)$, the firm cannot generate any cash flows. However, with probability $1 - \xi$, the firm can generate positive cash flows.

When the positive cash flows can be generated, they depend not only on the manager's or principal's effort but also on the investment involved in the IPO. More specifically, before the IPO is undertaken, $x_t = \alpha a_{At} a_{Pt}$ for $\alpha > 0$, where a_{At} is the manager's effort, a_{Pt} is the principal's effort, and α is a constant positive complementarity parameter. As the principal makes an effort for monitoring and advising the manager to improve the firm's business, the efforts of the principal and the manager are interrelated before the IPO. On the other hand, after the IPO is undertaken, $x_t = \mu a_{At}$ for $\mu > 1$. If the IPO is undertaken, the principal exits and provides no effort, whereas the firm is owned by new outside investors. However, the scale of the firm expands because of the investment; thus, $\mu > 1$.

The principal and new outside investors are risk neutral, and they discount a cash flow stream at a riskless interest rate r. Before the IPO is undertaken, the principal chooses effort, a_{Pt}, from the set of feasible effort levels A_P, which is compact with the smallest element 0. The principal's effort cost function is given by $\eta \cdot (a_{Pt})^2$, where $\eta > 0$.

The manager is also risk neutral but faces limited liability and discounts a cash flow stream at a subjective discount rate γ. For simplicity, we assume that at each point in time, the manager can choose either to shirk ($a_{At} = 0$) or to work ($a_{At} = 1$). If the manager shirks, he receives a private benefit of shirking, h, because working is costly for the manager. The manager's best outside option is equal to zero in each period.

The manager can maintain a private savings account and choose how much to consume. Neither the principal nor new outside investors can observe the balance of the manager's saving account. However, if the manager's saving interest rate is lower than his discount rate, the standard argument can show that the optimal contract does not require private saving by the agent (see DeMarzo and Fishman (2007)). Hence, to simplify the analysis, we assume that private saving is impossible.

The cash flows x_t are publicly observable by all the agents. However, neither the principal nor new outside investors can observe a_{At}. On the other hand, the manager may observe and verify a_{Pt} or may not observe a_{Pt}.

The timing and the contract schedule are as follows.

(i) Suppose that the IPO has not been undertaken until the beginning of period t. At the beginning of period t, the principal determines whether to undertake the IPO.

(a) If the IPO is undertaken, the principal exits the firm via the IPO, and new outside investors can sign a contract with the manager that specifies payments w_t to the manager. After the contract is signed but before the cash flows x_t are realized, the manager exerts an effort.

(b) If the IPO is not undertaken, the principal can sign a contract with the manager that specifies payments w_t to the manager. If the manager can observe and verify a_{Pt}, the contract can also stipulate how a_{Pt} is provided. However, if the manager cannot observe a_{Pt}, the contract does not specify a_{Pt}. After the contract is signed but before the cash flows x_t are realized, the principal and the manager exert an effort.

(ii) Suppose that the IPO has been undertaken until the beginning of period t. Then, new outside investors can sign a contract with the manager that specifies payments w_t to the manager. After the contract is signed but before the cash flows x_t are realized, the manager exerts an effort.

We start our discussion with the single moral hazard situation in which the manager can observe and verify a_{Pt}. To simplify the analysis, we assume that $h < \min((1 - \xi)\mu - K, \frac{[(1-\xi)\alpha]^2}{4\eta})$ and that the liquidation values of the firm at $t = 1$ and $t = 2$ are equal to zero.

The game is solved through backward induction. At $t = 2$, the optimal contract depends on the decision as to whether the IPO has been undertaken. Suppose that the IPO has been undertaken. Because the realized cash flows are publicly observable, the payments w_2 may depend on x_2. In the present model, x_2 is always equal to zero if the manager shirks ($a_{A2} = 0$), whereas x_2 is equal to μ (or 0) with probability $1 - \xi$ (or ξ) if the manager works ($a_{A2} = 1$). Hence, we set $w_2 = w_{20} \geq 0$ if $x_2 = 0$, and $w_2 = w_{2+} \geq 0$ if $x_2 > 0$. The optimal contract now chooses (w_{20}, w_{2+}) so as to maximize the expected payoff of new outside investors at $t = 2$ by providing incentives for the manager to work.[9] This can be written as the following optimization problem:

$$V_2^I \equiv \max_{w_{20}, w_{2+}} \; -\xi w_{20} + (1 - \xi)(\mu - w_{2+}), \tag{1}$$

$$\text{s.t.} \quad \xi w_{20} + (1 - \xi)w_{2+} \geq h.$$

The objective function is the expected payoff of new outside investors at $t = 2$ when the manager works at $t = 2$. The constraint is the manager's incentive-compatibility constraint that induces the manager to work at $t = 2$. Note that the manager receives w_{20} (or w_{2+}) with probability ξ (or $(1 - \xi)$) if he works, but always receives h if he shirks. Let (w_{20}^I, w_{2+}^I)

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[9]If the manager shirks, the principal's revenue is always equal to zero. Hence, under our parametric assumption, the contract that induces the manager to shirk is not optimal. In addition, the manager's individual rationality constraint always holds because the manager's best outside option is equal to zero. This remark holds in all of the following optimization problems (1)–(4).

denote a solution to problem (1). Solving this problem, we obtain $w_{20}^I = 0$ and $w_{2+}^I = \frac{h}{1-\xi}$. Substituting (w_{20}^I, w_{2+}^I) into V_2^I yields $V_2^I = (1-\xi)\mu - h \ (> 0)$.

Next, suppose that the IPO has not been undertaken. The optimal contract now chooses (w_{20}, w_{2+}, a_{P2}) to maximize the expected payoff of the principal at $t = 2$ by providing incentives for the manager to work. This can be written as the following optimization problem:

$$V_2^N \equiv \max_{w_{20}, w_{2+}, a_{P2}} -\xi w_{20} + (1-\xi)(\alpha a_{P2} - w_{2+}) - \eta \cdot (a_{P2})^2, \tag{2}$$

$$s.t. \quad \xi w_{20} + (1-\xi)w_{2+} \geq h.$$

The objective function is the expected payoff of the principal in this case. Note that the principal receives αa_{P2} with probability $1 - \xi$ but must always incur the effort cost. The constraint is the manager's incentive-compatibility constraint which is the same as problem (1). Let $(w_{20}^N, w_{2+}^N, a_{P2}^N)$ denote a solution to problem (2). Solving this problem, we obtain $w_{20}^N = 0$, $w_{2+}^N = \frac{h}{1-\xi}$ and $a_{P2}^N = \frac{(1-\xi)\alpha}{2\eta}$. Substituting $(w_{20}^N, w_{2+}^N, a_{P2}^N)$ into V_2^N leads to $V_2^N = \frac{(1-\xi)^2\alpha^2}{4\eta} - h \ (> 0)$.

We now consider whether or not the IPO is undertaken at $t = 2$. The principal sets the IPO price reasonably by anticipating an incentive for the manager to choose his effort level after the IPO. Because of arbitration, the profit of the principal from the IPO equals the profit of new outside investors, $V_2^I - K$. Thus, comparing $V_2^I - K$ and V_2^N, we show that the IPO is undertaken at $t = 2$ if $\mu \geq \frac{K}{1-\xi} + \frac{(1-\xi)\alpha^2}{4\eta}$; otherwise, the IPO is not undertaken at $t = 2$.

We proceed to the analysis at $t = 1$. In this case, we set $w_1 = w_{10} \geq 0$ if $x_1 = 0$, and $w_1 = w_{1+} \geq 0$ if $x_1 > 0$. Note that in the present model, neither the principal nor new outside investors have any incentive to vary the manager's continuation value in order to induce the manager to work at $t = 1$, because the contract termination date is exogenous under the stationary environment.[10]

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[10]Define U_{20} (U_{2+}) as the manager's expected payoff at $t = 2$ conditional on $x_1 = 0$ ($x_1 > 0$). Because the

Suppose that the IPO is undertaken at $t = 1$. Then, the principal exits at both $t = 1$ and 2. Thus, the optimal contract chooses (w_{10}, w_{1+}) so as to maximize the sum of the expected payoffs of new outside investors at $t = 1$ and 2 by providing incentives for the manager to work. Indeed, the period 2 optimal contract is given by the solution to problem (1) in this case because the IPO was undertaken at $t = 1$. Hence, the optimization problem is given by

$$V_1^I \equiv \max_{w_{10}, w_{1+}} - \xi w_{10} + (1 - \xi)(\mu - w_{1+}) + \frac{1}{1 + r} V_2^I, \tag{3}$$

$$s.t. \quad \xi w_{10} + (1 - \xi) w_{1+} \geq h.$$

Let w_{10}^I and w_{1+}^I denote a solution to problem (3). As V_2^I does not depend on (w_{10}, w_{1+}), we obtain $w_{10}^I = 0$, $w_{1+}^I = \frac{h}{1-\xi}$, and $V_1^I = (1 - \xi)\mu - h + \frac{1}{1+r} V_2^I$.

When the IPO is not undertaken at $t = 1$, the optimal contract chooses (w_{10}, w_{1+}, a_{P1}) so as to maximize the sum of the expected payoffs of the principal at $t = 1$ and $t = 2$ by providing incentives for the manager to work and considering the possibility of the IPO at $t = 2$. Then, the period 2 optimal contract is given by the solution to problem (1) or (2) according as the IPO is undertaken or not at $t = 2$. The optimization problem is then

$$V_1^N \equiv \max_{w_{10}, w_{1+}, a_{P1}} - \xi w_{10} + (1 - \xi)(\alpha a_{P1} - w_{1+}) - \eta \cdot (a_{P1})^2 + \frac{1}{1 + r} \max(V_2^I - K, V_2^N), \tag{4}$$

$$s.t. \quad \xi w_{10} + (1 - \xi) w_{1+} \geq h.$$

Note that the principal's profit from the IPO at $t = 2$ is equal to $V_2^I - K$. Let $(w_{10}^N, w_{1+}^N, a_{P1}^N)$ denote a solution to problem (4). As V_2^I and V_2^N do not depend on (w_{10}, w_{1+}, a_{P1}), solving this problem yields $w_{10}^N = 0$, $w_{1+}^N = \frac{h}{1-\xi}$, $a_{P1}^N = \frac{(1-\xi)\alpha}{2\eta}$, and $V_1^N = \frac{(1-\xi)^2\alpha^2}{4\eta} - h + \frac{1}{1+r} \max(V_2^I - K, V_2^N)$.

Comparing $V_1^I - K$ and V_1^N, we show that the IPO is undertaken at $t = 1$ if $\mu \geq \frac{(1+r)K}{(1-\xi)(2+r)}$

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manager needs to be induced to work at $t = 2$, in the present model, the optimal contract at $t = 2$ implies that $U_{20} = U_{2+} = h$, regardless of whether or not the IPO is undertaken at $t = 2$.

$+ \frac{(1-\xi)\alpha^2}{4\eta}$; otherwise, the IPO is not undertaken at $t = 1$. Given that the IPO is undertaken at $t = 2$ only if $\mu \geq \frac{K}{1-\xi} + \frac{(1-\xi)\alpha^2}{4\eta}$, this result implies that the IPO would be undertaken immediately at $t = 1$, or it would never be done at any time.

Under the double moral hazard situation in which the manager cannot observe a_{Pt}, the principal's maximization problems of (2) and (4) do not change because neither the principal's future expected value nor the manager's incentive-compatibility constraint depends on the principal's effort. Hence, the principal has no incentive to deviate from her effort level recommended by the optimal contract. Thus, the equilibrium contract under double moral hazard is the same as that under single moral hazard.

To summarize, in the discrete two-period model, we obtain the following results.

Results: *(i) There is no difference between the equilibria attained under the single and double moral hazard situations.*
(ii) In equilibrium, the IPO would be undertaken immediately at $t = 1$, or it would never be done at any time. Neither the payments to the manager nor the principal's effort varies over time.

The discrete two-period framework, however, restricts our analysis in several respects. First, even though the cash flows are independent and identically distributed, the dynamic incentive-compatibility constraint for the manager may depend on the principal's effort if the endogenous threat of contract termination makes the optimal contract depend on the manager's continuation value because such a contract gives stronger incentive for the manager to work hard in order to avoid the possibility of contract termination. Then, the principal needs to consider the effect of her effort on the future expected payoffs of the principal and the manager. This creates a difference between the equilibria under single and double moral hazard. In addition, the principal's effort may vary over time. Second, if the optimal contract depends on the manager's continuation value, the IPO would not be undertaken until the manager's future expected payoff is sufficiently high. This is because the possibility

of future contract termination should be sufficiently small to recover the IPO cost. Hence, it is very unlikely that the IPO would be undertaken immediately at the initial time or it would never be done at any time. Third, if the optimal contract depends on the manager's continuation value, an incentive for the manager can be given by varying his continuation payoff instead of paying him cash in the current period. Then, the manager's compensation may not be paid in order to induce him to make more effort. As we will show in the subsequent sections, moving in the continuous time framework with the endogenous threat of contract termination relaxes these restrictions of the discrete two-period framework and provides additional insights from the dynamic perspective.

3. Continuous-Time Agency Model

We now consider a continuous-time version of the principal–agent model. Again, we take as an example the IPO described in the preceding section. A principal (a VC or buyout fund) hires a manager and then has an opportunity to exit with an IPO. The irreversible investment cost K at the IPO is then financed by the funds from new outside investors in the IPO.

Let X_t denote the cumulative cash flows produced by the firm up to time t. The total output X_t evolves according to

$$dX_t = A_t dt + \sigma dZ_t, \tag{5}$$

where A_t is the drift of the cash flows, σ is the instantaneous volatility, and $Z = \{Z_t, \mathcal{F}_t; 0 \leq t < \infty\}$ is a standard Brownian motion on the complete probability space $(\Omega, \mathcal{F}, Q)$. More specifically, before the investment with the IPO, A_t is represented by

$$A_t = a_{At} + \zeta a_{Pt} + \alpha a_{At} a_{Pt},$$

where a_{At} is the manager's effort, a_{Pt} is the principal's effort, ζ is a constant nonnegative

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scale adjusted parameter, and α is a constant nonnegative complementarity parameter. As the principal makes an effort to monitor and advise the manager in order to improve the firm's business,[11] the efforts of the principal and the manager are interrelated before the IPO. However, after the investment with the IPO, for $\mu > 1$

$$A_t = \mu a_{At}.$$

If the IPO is undertaken, the principal exits, and the firm is now owned by new outside investors. Thus, $a_{Pt} = 0$. However, the scale of the firm expands because of the investment; thus, $\mu > 1$.

The principal and new outside investors are risk neutral and discount the flow of profit at rate r. The principal's effort is a stochastic process $\{a_{Pt} \in A_P, \, 0 \le t \le \tau_I\}$ progressively measurable with respect to $\mathcal{F}_t$, where the set of her feasible effort levels A_P is compact with the smallest element 0, and τ_I is the time of the IPO. The principal's effort cost function $g(a_{Pt})$—measured in the same units as the flow of profit—is an increasing, convex, and C^2 function. We normalize $g(0) = 0$.

The manager is risk neutral, but a negative wage is ruled out by limited liability. In addition, the manager also discounts his consumption at $\gamma \, (> r)$.[12] If the manager's saving interest rate is lower than the principal's discount rate and if the manager is risk neutral, DeMarzo and Sannikov (2006) show that there is an optimal contract in which the manager maintains zero savings. Hence, in this model, the manager can be restricted to consuming

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[11]In the monitoring–auditing model of Townsend (1979), the informed agent asks for costly state verification so that the realization of the project returns is made known to the uninformed agent. As the costly verification in equilibrium occurs only in the low-revenue states, the monitoring and auditing of his model naturally corresponds to bankruptcy proceedings. Hence, the monitoring and auditing cannot increase the productivity of the firm. By contrast, in our model, the principal, who is uninformed about the manager's effort, monitors and advises the manager, who is informed or uninformed about the principal's effort, as discussed in the introductory section. As a result, the monitoring and advising of our model can increase the productivity of the firm although it cannot make the manager's effort known to the principal.

[12]If the principal and the manager are equally patient when the manager is risk neutral, the principal can postpone payments to the manager indefinitely. This possibility precludes the existence of an optimal contract. See Biais, Mariotti, Rochet, and Villeneuve (2010).

what the principal pays him at any time. To simplify the analysis, we assume that at each point of time, the manager can choose either to shirk ($a_{At} = 0$) or to work ($a_{At} = 1$), where $A_A = \{0, 1\}$. Because working is costly for the manager or shirking results in a private benefit, we also suppose that the manager receives a flow of private benefit equal to hdt if he shirks.

The total output process $\{X_t, 0 \le t < \infty\}$ is publicly observable by all the agents. However, neither the principal nor new outside investors can observe $\{a_{At} \in A_A, 0 \le t < \infty\}$ or the flow of the manager's private benefit, whereas the manager may observe and verify $\{a_{Pt} \in A_P, 0 \le t \le \tau_I\}$ or may not observe $\{a_{Pt} \in A_P, 0 \le t \le \tau_I\}$.

For $0 \le t \le \tau_I$, the principal can sign a contract with the manager at time $t = 0$ that specifies how the manager's nondecreasing cumulative consumption, C_t, depends on the observation of X_t.[13] If the manager can observe and verify $\{a_{Pt} \in A_P, 0 \le t \le \tau_I\}$, the contract can also stipulate how a_{Pt} depends on the observation of X_t. Let $\Pi = \{C_t, a_{Pt} : 0 \le t \le \tau_I\}$ denote the contract prior to the IPO if the manager can observe and verify $\{a_{Pt} \in A_P, 0 \le t \le \tau_I\}$; and $\Pi_D = \{C_t : 0 \le t \le \tau_I\}$ denote the contract prior to the IPO if the manager cannot observe $\{a_{Pt} \in A_P, 0 \le t \le \tau_I\}$. On the other hand, for $\tau_I \le t < \infty$, new outside investors can sign a contract with the manager at time $t = \tau_I$ that specifies how C_t depends on the observation of X_t. Let $\overline{\Pi} = \{C_t : \tau_I \le t < \infty\}$ denote the contract after the IPO. The principal and new outside investors are able to commit to any such contract. In addition, the principal determines how the time of the IPO, τ_I, depends on the observation of X_t and how the time when the contract is terminated before the IPO, τ_{T_0}, depends on the observation of X_t. New outside investors also determine how the time when the contract is terminated after the IPO, τ_{T_1}, depends on the observation of X_t.

Now, for any contract Π (or Π_D) and $\overline{\Pi}$ and for any time $(\tau_I, \tau_{T_0}, \tau_{T_1})$, suppose that the manager chooses an effort-level process $\{a_{At} \in A_A, 0 \le t < \infty\}$. The manager's total

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[13]Because a negative wage is ruled out, note that C_t is nondecreasing.

expected payoff at $t = 0$ is then

$$E \left\{ \int_0^{\tau_{T_0 I}} e^{-\gamma t} \left[dC_t + h \cdot (1 - a_{At}) dt \right] + \mathbf{1}_{\tau_I \geq \tau_{T_0}} e^{-\gamma \tau_{T_0}} R \right.$$

$$\left. + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{-\gamma \tau_I} \left[\int_{\tau_I}^{\tau_{T_1}} e^{-\gamma(t - \tau_I)} \left[dC_t + h \cdot (1 - a_{At}) dt \right] \right] + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{-\gamma \tau_{T_1}} R \right\}, \tag{6}$$

where $\tau_{T_0 I} = \min(\tau_{T_0}, \tau_I)$; $\mathbf{1}_{\tau_I \geq \tau_{T_0}} = 1$ or 0 ($\mathbf{1}_{\tau_I < \tau_{T_0}} = 0$ or 1) according to $\tau_I \geq \tau_{T_0}$ or $\tau_I <$

τ_{T_0}; and the manager receives expected payoff R from an outside option when the contract

is terminated, irrespective of whether or not the IPO occurs. In addition, for any contract Π

(or Π_D) and $\overline{\Pi}$ and for any time $(\tau_I, \tau_{T_0}, \tau_{T_1})$, suppose that the principal and the manager

choose effort-level processes $\{a_{Pt} \in A_P, 0 \leq t \leq \tau_I\}$ and $\{a_{At} \in A_A, 0 \leq t < \infty\}$. Then, the

principal's total expected profit at $t = 0$ is

$$E \left\{ \int_0^{\tau_{T_0 I}} e^{-rt} \left[(a_{At} + \zeta a_{Pt} + \alpha a_{At} a_{Pt} - g(a_{Pt})) \, dt - dC_t \right] + \mathbf{1}_{\tau_I \geq \tau_{T_0}} e^{-r \tau_{T_0}} L_0 \right.$$

$$\left. + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{-r \tau_I} \left[\int_{\tau_I}^{\tau_{T_1}} e^{-r(t - \tau_I)} (\mu a_{At} dt - dC_t) - K \right] + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{-r \tau_{T_1}} L_1 \right\}, \tag{7}$$

where L_0 (L_1) is the expected liquidation payoff of the principal (new outside investors) when

the contract is terminated before (after) the IPO. Note that at $t = \tau_I$, new outside investors

buy the firm via the IPO. The principal anticipates an incentive for the manager to choose

his effort level after the IPO and sets the IPO price reasonably. Because of arbitration, the

principal's profit from the IPO equals the total expected payoff of new outside investors at t

$= \tau_I$ exclusive of the IPO cost, which is represented by the third and fourth terms in (7).[14]

To conclude this section, we show how the optimal contract is implemented with standard

compensation contracts and the IPO process. Up to the time of the IPO, τ_I, the principal

receives the net revenue produced by the firm, $dX_t - dC_t$, at each time t. At time τ_I,

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[14]In corporate spin-offs, the shares of the subsidiary are distributed on a pro rata basis to the parent firm's shareholders. If the principal is the parent firm's CEO whose preferences therefore coincide with those of the parent firm's shareholders, her objective function is still given by (7) because of not being involved in the management of the spun-off subsidiary.

the principal sells the firm's shares to new outside investors via the IPO and finances the investment cost K. After τ_I, new outside investors receive the net revenue produced by the firm, $dX_t - dC_t$, at each time t. The manager receives compensation dC_t at each time t, both before and after the IPO. Although the compensation contract prior to the IPO is separate from that after the IPO, the former can specify the manager's compensation dC_t at each time t prior to the IPO and the manager's continuation payoff W_I at the time of the IPO, as discussed in Philippon and Sannikov (2007). At time τ_I, the manager and new outside investors can commit to the post-IPO compensation contract dC_t at each time t, because they do not have any incentive to renegotiate at time τ_I.[15]

4. Optimal Contract in a Single Moral Hazard Situation

In this section, we assume that the manager can observe and verify $\{a_{Pt} \in A_P, 0 \leq t \leq \tau_I\}$. Thus, the principal need not design the contract to provide an appropriate ex post incentive for herself. We now derive the optimal contract in two steps. First, we obtain the optimal contract after the IPO, assuming that the IPO was made at $t = \tau_I$. Then, given the post-IPO value function, we characterize the optimal contract before the IPO, and the optimal timing of the IPO.

4.1. Optimal contract after the IPO.—

Consider the case where the IPO has already taken place. The contracting problem is then to find a combination of an incentive-compatible contract and termination timing, $(\overline{\Pi}, \tau_{T_1})$, and an incentive-compatible choice of the manager's effort process, $\{a_{At} \in A_A, \tau_I \leq t \leq \tau_{T_1}\}$, that maximize the expected profit of new outside investors subject to delivering the manager a required payoff W_I at the IPO, where W_I is defined by the manager's continuation value W_t at $t = \tau_I$ given below. The manager's effort process is incentive compatible with respect to $(\overline{\Pi}, \tau_{T_1})$ if it maximizes his total expected payoff from τ_I onward given $(\overline{\Pi}, \tau_{T_1})$; that is,

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[15]The proof is similar to that in Philippon and Sannikov (2007).

if it maximizes $E_{\tau_I}\left\{\int_{\tau_I}^{\tau_{T_1}} e^{-\gamma(t-\tau_I)}[dC_t + h \cdot (1 - a_{At})dt] + e^{-\gamma(\tau_{T_1} - \tau_I)}R\right\}$.

To characterize the optimal contract, we can write the contracting problem recursively with the manager's continuation utility as the single state variable. For any $(\overline{\Pi}, \tau_{T_1})$, the manager's continuation value W_t is his future expected discounted payoff at time t, given that he will follow $\{a_{At} \in A_A, \tau_I \leq t \leq \tau_{T_1}\}$:

$$W_t = E_t\left\{\int_t^{\tau_{T_1}} e^{-\gamma s}[dC_s + h \cdot (1 - a_{As})ds] + e^{-\gamma(\tau_{T_1} - t)}R\right\}. \tag{8}$$

In other words, W_t represents the manager's continuation value obtained under $(\overline{\Pi}, \tau_{T_1})$ when he plans to follow a_{At} from t onward. The manager's consumption and effort process is then specified by $\{C(W_t), a_A(W_t) : \tau_I \leq t \leq \tau_{T_1}\}$. Denote by $G(W)$ the value function of new outside investors before deducting the investment cost K (the highest expected present value of the profit to new outside investors that can be obtained from a contract that provides the manager with the continuation value W). To simplify our discussion, we assume that $G(W)$ is concave. The formal proof for the concavity of $G(W)$ will be provided in the Appendix.

The optimal contract is now derived using the technique in DeMarzo and Sannikov (2006) and Sannikov (2008). For the present, we assume that implementing the manager's high effort at any $t \in [\tau_I, \tau_{T_1}]$ is optimal for new outside investors. After presenting Proposition 2, we give a simple sufficient condition for the manager's high effort to remain optimal for all $W \in [R, W^{++}]$ by following the arguments of DeMarzo and Sannikov (2006).

First, Proposition 1 expresses the evolution of W_t and provides a necessary and sufficient condition for the manager's effort process to be incentive compatible.

Proposition 1: *For any $(\overline{\Pi}, \tau_{T_1})$, there exists a progressively measurable process $\{Y_t, \mathcal{F}_t :$*

$\tau_I \le t \le \tau_{T_1}\}$ *in* $\mathcal{L}^*$ *such that*[16]

$$dW_t = \{\gamma W_t - h \cdot [1 - a_A(W_t)]\}dt - dC(W_t) + Y(W_t)[dX_t - \mu a_A(W_t)dt], \qquad (9)$$

for every $t \in [\tau_I, \tau_{T_1}]$. *Implementing the manager's high effort* ($a_A(W_t) = 1$) *is incentive compatible with respect to* $\overline{\Pi}$ *if and only if*

$$Y(W_t)\mu \ge h, \qquad t \in [\tau_I, \tau_{T_1}]. \qquad (10)$$

Proof: Given that the manager is risk neutral, that $a_{At} = 1$ if he works, and that $a_{At} = 0$ and he receives hdt if he shirks, we can prove this statement by applying the proofs of Propositions 1 and 2 in the Appendix in Sannikov (2008). ∎

The evolution of W_t in (9) depends on a drift component that corresponds to promise keeping, and a diffusion component that links to the manager's effort choice and provides him with incentives. The drift component implies that W_t has to grow at the manager's discount rate γ, while it must decrease with $dC(W_t) + h \cdot [1 - a_A(W_t)]\, dt$. The diffusion component expressed by the final term in (9) is related to the manager's incentives. Here, $Y(W_t)$ is the sensitivity of the manager's continuation value to output. If the manager deviates from a recommended effort level, his actual effort affects only the flow of private benefit and the drift of X_t of (5) after the IPO, because the other parts are predetermined by the contract. Hence, taking the contract $\overline{\Pi}$ as given, the manager has an incentive to choose $a_A \in \{0, 1\}$ that maximizes the sum of the expected change of W_t and the flow of private benefit; that is, $\Gamma_1(a_A) \equiv Y(W_t)\mu a_A + h \cdot (1 - a_A)$. If implementing the agent's high effort is incentive compatible, this means that $\Gamma_1(1) \ge \Gamma_1(0)$, which is equivalent to (10).

Let

$$\beta_1 \equiv \min\{y : y\mu \ge h\} = \frac{h}{\mu}. \qquad (11)$$

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[16] A process Y is in $\mathcal{L}^*$ if $E\left[\int_0^t Y_s^2 ds\right] < \infty$ for all $t \in [0, \infty)$.

It is costly to expose the manager to risk. Hence, as argued in DeMarzo and Sannikov (2006) and Sannikov (2008), in the optimal contract, the principal must set $Y(W_t)$ at the minimal level that induces high effort level $a_A(W_t) = 1$, which satisfies the manager's incentive-compatibility constraint (10). It follows from (11) that β_1 is such a minimum level and satisfies (10).

As proved in DeMarzo and Sannikov (2006), the optimal compensation policy depends on $G'(W)$. Indeed, as new outside investors have the option to give dC units of consumption to the manager and move to the optimal contract with $W - dC$, the optimality of the contract implies $G(W) \geq G(W - dC) - dC$. Because the marginal cost of delivering the manager's continuation payoff can never exceed the cost of an immediate transfer in terms of the utility of new outside investors, we must have $G'(W) \geq -1$. Define W^{++} as the lowest value of W such that $G'(W) = -1$. Then, it is optimal to set the manager's compensation as

$$dC(W) = \max{(W - W^{++}, 0)}.$$

This compensation and the option to terminate keep the manager's continuation payoff between R and W^{++}.

Now, the following proposition summarizes the optimal contract after the IPO.

Proposition 2: *For the manager's starting value $W_I \in [R, W^{++}]$, the optimal contract is characterized by the unique concave function $G(W)$ that satisfies the Hamilton–Jacobi–Bellman (HJB) equation*

$$rG(W) = \max_{Y \geq \beta_1} \mu + G'(W)\gamma W + \frac{G''(W)}{2}Y^2\sigma^2, \tag{12}$$

with $\beta_1 = \frac{h}{\mu}$ and boundary conditions

$$G(R) = L_1, \ G'(W^{++}) = -1, \ \text{and} \ G''(W^{++}) = 0. \tag{13}$$

When $W_t \in [R, W^{++})$, $dC(W_t) = 0$. When $W_t = W^{++}$, payments dC_t cause W_t to reflect at W^{++}. If $W_t > W^{++}$, an immediate payment $W_t - W^{++}$ is made. The contract is terminated at time τ_{T_1} when W_t hits R for the first time. The optimal contract then attains profit $G(W_I)$ for new outside investors.[17]

Proof: See the Appendix. ∎

As $G''(W) < 0$, new outside investors dislike volatility in W and optimally choose the sensitivity of W to output; that is, $Y = \beta_1 = \frac{h}{\mu}$ in (12). The first boundary condition of (13) is the value-matching condition, which implies that the principal must terminate the contract to hold the agent's reservation value, R. The second boundary condition is the smooth-pasting condition that guarantees the optimal choice of W^{++}. The third boundary condition of (13) is the super contract condition for the optimal choice of W^{++}, which requires that the second derivatives match at the boundary. Using equations (12) and (13), this condition means that $rG(W^{++}) + \gamma W^{++} = \mu$; that is, payment to the manager is postponed until the new outside investors' and manager's required expected returns exhaust the available expected cash flows generated after the IPO. These boundary conditions fix the solution to the HJB equation in (12).

Finally, we provide a simple sufficient condition for the manager's high effort to be optimal at any $t \in [\tau_I, \tau_{T_1}]$. Let $W^S \equiv \frac{h}{\gamma}$ denote the manager's discounted payoff if the manager shirks forever, and let $W^{\max G}$ denote the value of W that achieves the greatest value of $G(W)$ in the range of $W \in [R, W^{++}]$. Then, we obtain the following lemma.

Lemma 1: *Implementing the manager's high effort at any $t \in [\tau_I, \tau_{T_1}]$ is optimal for new outside investors if*

$$\frac{\gamma}{r}G(W^S) + (1 - \frac{\gamma}{r})G(W^{\max G}) \geq 0. \tag{14}$$

Proof: See the Appendix. ∎

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[17]For any starting value of $W_I > W^{++}$, $G(W_I)$ is an upper bound on the total expected profit of new outside investors. However, this case can be excluded because $F'(W_I) = G'(W_I) > -1$, as will be shown in Proposition 4. If $W_I < R$, the manager never participates in the contract.

Intuitively, the condition of (14) ensures that the payoff rate of new outside investors from letting the manager shirk will be less than that under our existing contract. DeMarzo and Sannikov (2006, Proposition 8) derive a similar condition, but their condition is more stringent than (14) if the expected cash flow is larger than the agent's shirking private benefit. The condition of (14) implies a lower bound on W^S, or equivalently, h.

4.2. Optimal contract before the IPO.—

In this case, the contracting problem is to find a combination of an incentive-compatible contract and IPO and termination timing, $(\Pi, \tau_I, \tau_{T_0})$, and an incentive-compatible manager's effort process, $\{a_{At} \in A_A, 0 \le t < \infty\}$, that maximize the expected profit of the principal subject to delivering the manager an initial required payoff W_0. The manager's effort process is incentive compatible with respect to $(\Pi, \tau_I, \tau_{T_0})$ if it maximizes the manager's total expected payoff defined by (6), given $(\Pi, \tau_I, \tau_{T_0})$.

Given W_t as in (8), the processes of the manager's consumption and the manager's and principal's efforts can be specified by $\{C(W_t), a_A(W_t), a_P(W_t) : 0 \le t \le \tau_{T_0 I}\}$. Denote by $F(W)$ the value function of the principal. To facilitate our discussion, we assume that $F(W)$ is concave. The formal proof for the concavity of $F(W)$ will be provided in the Appendix.

For the present, we again assume that implementing the manager's high effort ($a_A(W_t)$ = 1) at any $t \in [0, \tau_{T_0 I}]$ is optimal for the principal. After we present Proposition 4, we provide a simple sufficient condition for the manager's high effort to remain optimal at any $t \in [0, \tau_{T_0 I}]$.

Now, as in Proposition 1, we obtain the following proposition.

Proposition 3: *For any $(\Pi, \tau_I, \tau_{T_0})$, there exists a progressively measurable process $\{Y_t, \mathcal{F}_t : 0 \le t \le \tau_{T_0 I}\}$ in $\mathcal{L}^*$ such that*

$$dW_t = \{\gamma W_t - h \cdot [1 - a_A(W_t)]\} \, dt - dC(W_t) + Y(W_t)\{dX_t - [a_A(W_t) + \zeta a_P(W_t) + \alpha a_A(W_t)a_P(W_t)]dt\},$$

$$(15)$$

for every $t \in [0, \tau_{T_0 I}]$. Implementing the manager's high effort ($a_A(W_t) = 1$) is incentive compatible with respect to Π if and only if

$$Y(W_t)\left[1 + \alpha a_P(W_t)\right] \geq h, \qquad t \in [0, \tau_{T_0 I}]. \tag{16}$$

Proof: Note that the manager is risk neutral, that $a_{At} = 1$ if he works, and that $a_{At} = 0$ and the manager receives hdt if he shirks. Then, we can prove this statement using a procedure similar to that in the proofs for Propositions 1 and 2 in the Appendix in Sannikov (2008). ∎

As in (9), the evolution of W_t in (15) depends on a predetermined drift part that corresponds to promise keeping and a diffusion part that links to the manager's effort choice and also provides him with incentives. Taking the contract Π as given, the manager has an incentive to choose $a_A \in \{0,1\}$ that maximizes the sum of the expected change of W_t and the flow of private benefit; that is, $\Gamma_0(a_A, a_P(W_t)) \equiv Y(W_t)[a_A + \alpha a_A a_P(W_t)] + h \cdot (1 - a_A)$. If implementing the manager's high effort is incentive compatible, this means that $\Gamma_0(1, a_P(W_t)) \geq \Gamma_0(0, a_P(W_t))$, which is equivalent to (16).

Before proceeding further, we make the following assumption according to Sannikov (2008).

Assumption 1: The sensitivity is bounded from below by $\underline{\beta}$ (> 0) such that $Y(W) \geq \underline{\beta}$.

Assumption 1 gives a positive lower bound of $Y(W)$, which ensures that the manager's incentives can be controlled. Note that $\underline{\beta}$ can be chosen as a sufficiently small positive number so that there is no restriction on the optimal solution.

We also make the following assumption, which ensures that the IPO does not take place at time 0 if W_0 is not sufficiently large.

Assumption 2: $K > \max(L_0, L_1)$.

Let

$$\beta_0(a_P) = \min\{y : y \cdot (1 + \alpha a_P) \geq h\} = \frac{h}{1 + \alpha a_P}. \tag{17}$$

Because it is costly to expose the manager to risk, in the optimal contract, the principal must set $Y(W_t)$ at the minimal level that induces the high effort level ($a_A = 1$), which satisfies the manager's incentive-compatibility constraint (16). It follows from (17) that $\beta_0(a_P)$ is such a minimum level and satisfies (16). Again, the optimal compensation policy depends on $F'(W)$ because we must have $F(W) \geq F(W - dC) - dC$ or $F'(W) \geq -1$. Define W^+ as the lowest value of W such that $F'(W) = -1$. Then, it is optimal to pay the manager according to

$$dC(W) = \max(W - W^+, 0).$$

In fact, as in Proposition 4, we will show that $W^+ \geq W_I$. Thus, we can indicate that the manager's compensation is zero before the IPO.

Now, the following proposition summarizes the optimal contract before the IPO.

Proposition 4: *Suppose that the manager can observe and verify $\{a_{Pt} \in A_P, 0 \leq t \leq \tau_{T_0I}\}$. For any starting value $W_0 \in [R, W_I]$, the optimal contract is characterized by the unique concave function $F(W)$ ($\geq G(W) - K$) that satisfies the HJB equation*

$$rF(W) = \max_{a_P \in A_P, Y \geq \beta_0(a_P)} 1 + (\zeta + \alpha)a_P - g(a_P) + F'(W)\gamma W + \frac{F''(W)}{2}Y^2\sigma^2, \qquad (18)$$

with $\beta_0(a_P) = \frac{h}{1+\alpha a_P}$ and boundary conditions

$$F(R) = L_0, \ F(W_I) = G(W_I) - K, \ \text{and} \ F'(W_I) = G'(W_I). \qquad (19)$$

When $W_t \in [R, W_I]$, $dC(W_t) = 0$. This means $W_I \leq W^+$. The IPO occurs when W_t reaches W_I ($> R$), or the contract is terminated when W_t hits R, whichever happens sooner. After the IPO, the continuation contract is given by Proposition 2 at the starting value W_I. The optimal contract then provides profit $F(W_0)$ to the principal.[18]

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[18] For the starting value of $W_0 > W_I$, the IPO takes place immediately or contracts with positive profit do not exist. If $W_0 < R$, the manager never participates in the contract.

Proof: See the Appendix. ∎

As $F''(W) < 0$, the principal dislikes volatility in W and optimally chooses the sensitivity of W to output; that is, $\beta_0(a_P) = \frac{h}{1+\alpha a_P}$ in (18). The first and second boundary conditions in (19) are the value-matching conditions, while the third boundary condition in (19) is the smooth-pasting condition that guarantees the optimal choice of W_I. Note that the IPO cost is K, which is deducted from $G(W_I)$. These three boundary conditions pin down the solution to the HJB equation in (18).

As in the optimal contract after the IPO, we give a simple sufficient condition for the manager's high effort to be optimal at any $t \in [0, \tau_{T_0 I}]$. Let $W^{\max F}$ denote the value of W that achieves the greatest value of $F(W)$ in the range of $[R, W_I]$.

Lemma 2: *Implementing the manager's high effort at any $t \in [0, \tau_{T_0 I}]$ is optimal for the principal if*

$$\frac{\gamma}{r}F(W^S) + (1 - \frac{\gamma}{r})F(W^{\max F}) \geq 0. \tag{20}$$

Proof: See the Appendix. ∎

Again, this condition is weaker than the corresponding condition of DeMarzo and Sannikov (2006), and implies a lower bound on W^S, or equivalently, h.

To compare the optimal solution before the IPO with that after the IPO, let $\{C^*(W_t), \tau_I^*, \tau_{T_0}^*, a_A^*(W_t), a_P^*(W_t) : 0 \leq t \leq \tau_{T_0 I}^*\}$ denote the optimal choice of $(C, \tau_I, \tau_{T_0}, a_A, a_P)$ before the IPO, and let $\{C^{**}(W_t), \tau_{T_1}^{**}, a_A^{**}(W_t) : \tau_I^* \leq t < \infty\}$ denote the optimal choice of (C, τ_{T_1}, a_A) after the IPO.[19] Define W_I^* and W^{++*} as the corresponding value-maximizing IPO and cash payment thresholds.

We first examine the optimal choices of C and a_A by inspecting the results of Propositions 2 and 4. The optimal choice of C implies that $dC^*(W) = dC^{**}(W') = 0$ for any $W \in [R, W_I^*]$ and any $W' \in [R, W^{++*}]$. In addition, when $W_t \in (W^{++*}, \infty)$, an immediate payment

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[19]More precisely, $\tau_I^*, \tau_{T_0}^*$ and $\tau_{T_1}^{**}$ depend on W_t because these values are determined by the dates that satisfy (13) and (19).

$W_t - W^{+++*}$ is made. The intuition behind the result of $dC^*(W) = 0$ for any $W \in [R, W_I^*]$ is that if the manager receives compensation before the IPO, the principal must make the immediate payment $W_t - W^{++}$ to the manager. As this immediate payment causes W_t to be brought back to W^{++} for any $W_t \in (W^{+++*}, \infty)$, the principal's profit from the IPO always becomes smaller than her expected profit obtained by waiting for the IPO. Hence, the IPO would never be done at any time. The optimal choice of a_A means that $a_A^*(W) = a_A^{**}(W')$ $= 1$ for any $W \in [R, W_I^*]$ and any $W' \in [R, \infty)$.

We next discuss the optimal choice of a_P. The optimality implies that a_P maximizes $(\zeta + \alpha)a_P - g(a_P) + \frac{F''(W)}{2}[\beta_0(a_P)]^2 \sigma^2$, where the first and second terms are the expected flow of output from the principal's effort minus her disutility cost, and the third term is the cost of the principal exposing the manager to income uncertainty to create an incentive. It follows from (17) that if $a_P > 0$, the first-order condition leads to

$$\zeta + \alpha = g'(a_P^*(W)) - F''(W)\sigma^2\beta_0(a_P^*(W))\beta_0'(a_P^*(W))$$

$$\leq g'(a_P^*(W)) \qquad \text{for all } W \leq W_I^*, \text{ with strict inequality only if } \alpha > 0.$$

This implies that the marginal productivity of the principal's effort is smaller than its marginal cost when $\alpha > 0$. Intuitively, an increase in the principal's effort relaxes the manager's incentive-compatibility constraint because of the complementarity effect of the principal's effort $(\beta_0'(a_P^*(W)) < 0$ for $\alpha > 0)$. Hence, it reduces the cost of the principal's exposing the manager to income uncertainty to provide incentive. Thus, to provide the manager with appropriate less costly incentives using the complementarity effect, the principal increases her effort by a level at which the marginal productivity of her effort is smaller than its marginal cost. Note that if $\alpha = 0$, the principal's effort is fixed at the level where the marginal productivity of her effort is equal to its marginal cost.

These discussions are summarized as follows.

Proposition 5: *Suppose that the manager can observe and verify $\{a_{Pt} \in A_P, 0 \leq t \leq \tau_{T_0I}\}$.*

(i) $dC^*(W) = dC^{**}(W') = 0$ for any $W \in [R, W_I^*]$ and any $W' \in [R, W^{+++*}]$.

(ii) If the principal's effort is positive, the marginal productivity of her effort is smaller than its marginal cost when $\alpha > 0$, whereas the marginal productivity of her effort is equal to its marginal cost when $\alpha = 0$.

To conclude this section, we comment on the IPO timing. If the IPO is implemented, the investment cost arises at the time of the IPO. However, there is a risk of losing value if the contract is terminated. Because $L_1 < K$ implies that the liquidation value cannot compensate for the investment cost, it is inefficient to plan the IPO when there is a higher probability of liquidation; that is, when W_t is close to R. Indeed, a sufficiently small W_t raises the risk of losing value upon termination and reduces the IPO price, thereby making it impossible for the principal to recover the total cost of the IPO.[20] Hence, it is optimal to execute the IPO only when the manager accumulates a sufficient continued or promised payoff. In addition, the optimal IPO timing is affected by both the agency problem and the change in the governance mechanism. If a large amount of compensation has been paid to the manager before the IPO, W_t must be sufficiently small. Thus, the principal cannot plan the IPO until W_t is sufficiently large. Hence, the manager's compensation is not paid under the optimal contract before the IPO. Similarly, if the change in the governance mechanism with the IPO adversely affects the manager's incentives after the IPO, the principal will not undertake the IPO unless a high level of managerial incentives can be attained. This means that the change in management control with the IPO strongly affects the timing of the IPO.

5. Optimal Contract in a Double Moral Hazard Situation

In this section, we assume that the manager cannot observe $\{a_{Pt} \in A_P, 0 \leq t \leq \tau_{T_0 I}\}$.[21] This assumption can be justified because it is difficult to verify the intensities of the monitor-

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[20] On the other hand, if W_0 is sufficiently large, and if contracts with positive profit exist, it is optimal for the principal to execute the IPO at $t = 0$ immediately because she can recover the cost of the IPO.

[21] Although Zhao (2007) investigates optimal risk sharing in a dynamic model with double moral hazard, he exploits a discrete-time setting and does not discuss the IPO timing or consider liquidation or other retirement options.

ing and advising efforts provided by the VC. As the principal cannot commit to provide the predetermined level of a_P, she needs to take into account her own ex post incentives when designing the manager's compensation contract. Even in this case, a different formulation of the contract problem is required only before the IPO because the principal does not provide any effort after the IPO. Hence, Propositions 1 and 2 and Lemma 1 still hold.

Before the IPO, the principal's design for the incentive scheme must address two incentive problems: the manager's incentive problem and her own incentive problem.[22] Thus, the optimal contracting problem prior to the IPO is to find a combination of incentive-compatible contract and IPO and termination timing, $(\Pi_D, \tau_I, \tau_{T_0})$, an incentive-compatible manager's effort process, $\{a_{At} \in A_A, \ 0 \leq t \leq \tau_{T_0 I}\}$, and an incentive-compatible principal's effort process, $\{a_{Pt} \in A_P, \ 0 \leq t \leq \tau_{T_0 I}\}$, that maximize the expected profit of the principal subject to delivering the manager an initial required payoff W_0. The manager's effort process is incentive compatible with respect to $(\Pi_D, \tau_I, \tau_{T_0})$ and $\{a_{Pt} \in A_P, \ 0 \leq t \leq \tau_{T_0 I}\}$ if it maximizes his total expected utility defined by (6), given $(\Pi_D, \tau_I, \tau_{T_0}, \{a_{Pt} \in A_P, \ 0 \leq t \leq \tau_{T_0 I}\})$, while the principal's effort process is incentive compatible with respect to $(\Pi_D, \tau_I, \tau_{T_0})$ and $\{a_{At} \in A_A, \ 0 \leq t \leq \tau_{T_0 I}\}$ if it maximizes her total expected payoff defined by (7), given $(\Pi_D, \tau_I, \tau_{T_0}, \{a_{At} \in A_A, \ 0 \leq t \leq \tau_{T_0 I}\})$.

We still assume that implementing the manager's high effort ($a_A(W_t) = 1$) at any $t \in [0, \ \tau_{T_0 I}]$ is optimal for the principal, and later derive a simple sufficient condition for the manager's high effort to remain optimal at any $t \in [0, \ \tau_{T_0 I}]$.

Indeed, W_t and the incentive-compatibility constraint for the manager are still represented by Proposition 3. However, the incentive-compatible principal's effort process is determined by a maximizer to the following maximization problem, given the optimal level of Π_D, τ_I,

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[22]In our model, if the principal makes dC_t sufficiently large at some t' to penalize herself severely when the cash flows are very low, then W_t for $t > t'$ will be smaller than R. Thus, the possibility of contract termination implies that the principal cannot arbitrarily reduce the low outcome range in which she should take the penalty by increasing the penalty amount. Hence, in our dynamic setting, in contrast to the static double moral hazard model such as that in Kim and Wang (1998), the double moral hazard case cannot arbitrarily closely approach the single moral hazard case, even though there is no upper bound for the wage contract.

τ_{T_0}, $\{a_{At} \in A_A, 0 \le t \le \tau_{T_0 I}\}$, and $\widetilde{F}(W)$:

$$a_P = \underset{a_P \in A_P}{\arg\max} \, 1 + (\zeta + \alpha)a_P - g(a_P) + \widetilde{F}'(W)\gamma W + \frac{\widetilde{F}''(W)}{2}[Y(W)]^2 \sigma^2. \qquad (21)$$

Here, $\widetilde{F}(W)$ is the value function of the principal at each point of W given by Proposition 4' below, and $Y(W)$ is determined by the recommended principal's effort process at each point of W; that is, $Y(W) = \beta_0(a_P(W))$, where $a_P(W)$ is given by the recommended principal's effort at each point of W.[23] Note that after offering the contract, the principal can optimally choose her effort at each point of W without considering the manager's incentive-compatibility constraint, if the manager cannot observe $\{a_{Pt} \in A_P, 0 \le t \le \tau_{T_0 I}\}$.

To ensure that $a_P > 0$ under the optimal contract, we assume that $g'(0) < \zeta + \alpha$. Then, as the right-hand side of (21) is concave with respect to a_P, (21) is rewritten as

$$\zeta + \alpha = g'(a_P), \text{ or } a_P = g'^{-1}(\zeta + \alpha) = \psi(\zeta + \alpha). \qquad (21')$$

Now, repeating a procedure similar to that of Proposition 4 and Lemma 2, we can obtain the following proposition and lemma.

Proposition 4': *Suppose that the manager cannot observe $\{a_{Pt} \in A_P, 0 \le t \le \tau_{T_0 I}\}$. For any starting value $W_0 \in [R, W_I]$, the optimal contract is characterized by the unique concave function $\widetilde{F}(W)$ $(\ge G(W) - K)$ that satisfies the HJB equation*

$$r\widetilde{F}(W) = \underset{a_P = \psi(\zeta+\alpha), Y \ge \beta_0(a_P)}{\max} 1 + (\zeta + \alpha)a_P - g(a_P) + \widetilde{F}'(W)\gamma W + \frac{\widetilde{F}''(W)}{2}Y^2\sigma^2, \qquad (22)$$

with $\beta_0(a_P) = \frac{h}{1+\alpha a_P}$ and boundary conditions

$$\widetilde{F}(R) = L_0, \ \widetilde{F}(W_I) = G(W_I) - K, \text{ and } \widetilde{F}'(W_I) = G'(W_I). \qquad (23)$$

[23] In other words, the recommended $Y(W)$ is set equal to $\beta_0(a_P(W))$ under the optimal contract.

When $W_t \in [R, W_I]$, $dC(W_t) = 0$. This means that $W_I < W^+$. The IPO takes place when W_t reaches W_I, or the contract is terminated when W_t hits R, whichever happens sooner. After the IPO, the continuation contract is given by Proposition 2 at the starting value W_I. Then, the optimal contract attains profit $\widetilde{F}(W_0)$ for the principal.[24]

Lemma 2′: *Implementing the manager's high effort at any $t \in [0, \tau_{T_0 I}]$ is optimal for the principal if*

$$\frac{\gamma}{r}\widetilde{F}(W^S) + (1 - \frac{\gamma}{r})\widetilde{F}(W^{\max \widetilde{F}}) \geq 0, \tag{24}$$

where $W^{\max \widetilde{F}}$ denotes the value of W that achieves the greatest value of $\widetilde{F}(W)$ in the range of $[R, W_I]$.

We next compare the optimal choices of C, a_P, W^{++} and W_I under double moral hazard with those under single moral hazard. Under double moral hazard, let $\widetilde{C}^*(W_t)$, $\widetilde{C}^{**}(W_t)$ and $\widetilde{a}_P^*(W_t)$ denote the optimal consumption before the IPO, the optimal consumption after the IPO, and the optimal principal's effort before the IPO, respectively. In addition, let $\widetilde{W}_I^*$ and $\widetilde{W}^{++*}$ denote the corresponding value-maximizing IPO and cash payment thresholds, respectively.

We begin by discussing the optimal choices of C, a_P, and W^{++}. First, the choice rule of C before and after the IPO under double moral hazard is the same as that under single moral hazard. In addition, as in the single moral hazard situation, the IPO is undertaken before the principal makes a cash payment to the agent. Furthermore, even under double moral hazard, the choice rule of W^{++} is given by (12) and (13). Because $G(W)$ under double moral hazard is the same as that under single moral hazard, we see $\widetilde{W}^{++*} = W^{++*}$. Second, the first-order condition for a_P prior to the IPO under double moral hazard is represented by (21′). Given $g'' > 0$, this implies that a_P is smaller under double moral hazard than under single moral hazard.

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[24] For the starting value of $W_0 > W_I$, the IPO is immediately executed or contracts with positive profit do not exist.

Thus, Proposition 5 can be changed as follows.

Proposition 5′: *Suppose that the manager cannot observe* $\{a_{Pt} \in A_P, \, 0 \le t \le \tau_{T_0 I}\}$.

(i) $\widetilde{C}^*(W) = \widetilde{C}^{**}(W') = 0$ *for any* $W \in [R, \widetilde{W}_I^*]$ *and any* $W' \in [R, \widetilde{W}^{++*}]$. *Furthermore,*
$\widetilde{W}^{++*} = W^{++*}$.

*(ii) Double moral hazard induces the principal to undersupply her own effort relative to the
case of single moral hazard.*

The intuition behind Proposition 5′ is as follows. The choice rule of C before and after
the IPO depends on the functional forms of $F(W)$ and $G(W)$ (or $\widetilde{F}(W)$ and $G(W)$). The
reason is that in our continuous-time agency model, the manager's incentive is generated
only through a variation in his continuation payoff. This implies that the problem of how
the manager is given incentives can be separated from the problem of when and how much
compensation he receives according to the level of W_t. Given that the double moral hazard
setting does not affect the functional form of $G(W)$ and that the manager has never been paid
before the IPO even under double moral hazard for the same reason as that discussed before
Proposition 5, the statement of Proposition 5′(i) is self-evident. Next, for the choice of a_P, if
the manager cannot observe a_P, the principal cannot commit to considering the manager's
incentive-compatibility constraint in choosing her own effort after the compensation contract
has been offered. Hence, the principal cannot internalize the external effect of her own effort
on the manager's incentives. As a result, the noncontractibility of the principal's effort level
leads to an undersupply of the principal's effort.

Philippon and Sannikov (2007) suggest that under a single moral hazard model, the man-
ager is not paid until a certain period after the IPO. Proposition 5′(i) confirms their finding
even under a double moral hazard model.

This finding also suggests that the optimal contract is not linear under the continuous-time
agency model with double moral hazard. By contrast, in the static agency model with double
moral hazard, Romano (1994) and Bhattacharyya and Lafontaine (1995) show that a simple

linear contract with a fixed fee implements the second-best outcome when the agent is risk neutral. Our result depends on the possibility of contract termination with the manager's limited liability, which is not considered in their model.[25]

We next explore how the difference in the moral hazard situation affects the IPO threshold. Then, we establish the following proposition.

Proposition 6 *For all* $W \geq R$, $\widetilde{F}(W) < F(W)$. *In addition,*

$$\widetilde{W}_I^* < W_I^*.$$

Proof: See the Appendix. ∎

Proposition 6 shows that the optimal IPO timing is earlier under double moral hazard than under single moral hazard.

Intuitively, the double moral hazard situation lowers the expected present value of the principal's profit at any point of W more than the single moral hazard situation because the principal cannot provide an appropriate ex post incentive for herself. However, the expected present value of the profit of new outside investors after deducting K is the same under both situations. This implies that the net expected payoff of the principal obtained when postponing the IPO under double moral hazard is always smaller than that under single moral hazard ($\widetilde{F}(W) - [G(W) - K] < F(W) - [G(W) - K]$ for all $W \geq R$). Because the IPO does not take place until the expected present value of the principal's profit ($\widetilde{F}(W)$ or $F(W)$) touches her profit from the IPO ($G(W) - K$), it follows from the concavity of $\widetilde{F}(W)$, $F(W)$, and $G(W)$ that the IPO threshold is lower under double moral hazard than under single moral hazard.

Combining Propositions 5′(i) and 6, we obtain the following proposition.

Proposition 7: *The manager's compensation tends to be paid earlier under double moral*

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[25]Kim and Wang (1995) also find that even in the static contract setting, the optimality of the linear contract is not robust in the sense that the optimal contract obtained under double moral hazard with the risk-averse agent does not approach the linear contract as the agent's risk aversion approaches zero.

hazard than under single moral hazard.

Hence, the manager's compensation as a function of performance history depends on whether the principal's effort is observable, even though the manager's compensation as a function of W is not.[26]

6. Comparative Statics under Double Moral Hazard

Using (12), (13), (22), and (23), we compute the comparative statics on the IPO strategy and on the manager's compensation profile in the optimal contract under double moral hazard. To avoid complicating the notation, we drop the tilde from all variables in this section. The key parameters are the governance role of the VC and IPO (the degree of complementarity between the principal's and the manager's efforts, α, and the post-IPO scale of the firm, μ) and the risk of the project, σ^2. Table 1 summarizes our results.

We first discuss the case after the IPO. For each parameter θ and a given W_t, let W_θ^{++*} denote the optimal cash payment threshold, let $W_{I\theta}^*$ denote the optimal IPO threshold, and let $G_\theta(W_t)$ denote the expected present value of the profit of new outside investors before deducting K, respectively.

The following proposition shows the comparative static results on $G_\theta(W_t)$ and W_θ^{++*}.

Proposition 8: *Consider $\mu_1 < \mu_2$ and $\sigma_1^2 < \sigma_2^2$.*

(i) For all $W \geq R$, $G_{\mu_1}(W) < G_{\mu_2}(W)$ and $G_{\sigma_1^2}(W) > G_{\sigma_2^2}(W)$.

(ii) $W_{\sigma_1^2}^{++*} < W_{\sigma_2^2}^{++*}$. If $\frac{\partial G_\mu(W)}{\partial \mu} > \frac{1}{r}$ for all $W \geq R$, then $W_{\mu_1}^{++*} > W_{\mu_2}^{++*}$.

Proof: See the Appendix. ∎

Proposition 8 shows that after the IPO, $G_\theta(W)$ is increasing in the post-IPO scale of the firm, μ, but is decreasing in the risk of the project, σ^2. It also means that after the IPO, the manager's compensation is paid earlier when σ^2 is smaller. Furthermore, if the sensitivity of $G_\mu(W)$ with respect to μ is larger than the inverse of the discount rate of new outside

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[26]This point is suggested by an anonymous referee.

investors, the manager's compensation is paid earlier after the IPO when μ is larger.

The intuition is as follows. First, the manager's impatience ($\gamma > r$) implies that the optimal contract pays cash to the manager as early as possible. However, paying cash earlier to the manager reduces his continuation payoff (see (9)). Under limited liability of the manager, new outside investors are forced to terminate the contract when the manager's continuation payoff hits R. Thus, paying cash earlier to the manager might cause future inefficient termination of contract to be more likely. Hence, as discussed below Proposition 2, the payment to the manager is postponed until the new outside investors' and the manager's required expected returns exhaust the available expected cash flows. Now, in the present model, the manager's incentive is generated through a variation in his continuation payoff. However, it is costly for the principal to expose the manager to more income uncertainty because $G(W)$ is concave. Hence, an increase in μ increases $G_\mu(W)$ because it not only raises the stream of the expected cash flows (μ) but also reduces the variation in the manager's continuation payoff by relaxing the manager's incentive-compatibility constraint and providing the manager with more incentive to work. Thus, when μ increases, the new outside investors' required expected returns ($rG_\mu(W)$) increase. On the other hand, an increase in μ raises the available expected cash flows minus the manager's required expected returns ($\mu - \gamma W$). If $\frac{\partial G_\mu(W)}{\partial \mu}$ is greater than $\frac{1}{r}$, an increase in μ raises the new outside investors' required expected returns ($rG_\mu(W)$) more than the available expected cash flows minus the manager's required expected returns ($\mu - \gamma W$). As the former effect dominates the latter effect, the cash payment threshold needs to be reduced under the assumption of $\gamma > r$ because the marginal cost of delivering the continuation payoff to the manager ($-G'_\mu(W)$) is smaller than 1 when $W < W^{++}$. Hence, an increase in μ induces new outside investors to pay cash earlier to the manager.

Second, an increase in σ^2 raises the cost of exposing the manager to income uncertainty because it directly increases the variation in the manager's continuation value. This reduces

$G_{\sigma^2}(W)$ and induces new outside investors to pay cash later to the manager under the possibility of future inefficient liquidation.

We now investigate the case before the IPO, using the results obtained after the IPO. For each parameter θ and a given W_t, let $a_{P\theta}^*(W_t)$ denote the optimal principal's effort before the IPO, and let $F_\theta(W_t)$ denote the expected present value of the principal's profit.

Then, we establish the following proposition.

Proposition 9: *Consider $\alpha_1 < \alpha_2$, $\mu_1 < \mu_2$, and $\sigma_1^2 < \sigma_2^2$.*

(i) For all $W \geq R$, $F_{\theta_1}(W) < F_{\theta_2}(W)$, $\theta = \alpha$, μ; and $F_{\sigma_1^2}(W) > F_{\sigma_2^2}(W)$.

(ii) Furthermore,

$$W_{I\alpha_1}^* < W_{I\alpha_2}^*,$$

$$W_{I\mu_1}^* > W_{I\mu_2}^* \ \ \text{if} \ \ |\mu_1 - \mu_2| \ \ \text{is sufficiently small,}$$

$$W_{I\sigma_1^2}^* > W_{I\sigma_2^2}^* \ \ \text{if} \ r \ \text{and} \ \left|\sigma_1^2 - \sigma_2^2\right| \ \text{are sufficiently small.}$$

(iii) The larger is α (or the larger is μ), the later the manager's compensation tends to be paid (or earlier if $\frac{\partial G(W)}{\partial \mu} > \frac{1}{r}$ for all $W \geq R$ and if $|\mu_1 - \mu_2|$ is sufficiently small).

Proof: See the Appendix. ∎

Proposition 9 implies that before the IPO, $F_\theta(W)$ is increasing in the degree of complementarity, α, and the post-IPO scale of the firm, μ, but is decreasing in the risk of the project, σ^2. On the other hand, the IPO threshold $W_{I\theta}^*$ is increasing in α, it is decreasing in μ if the variation of μ is sufficiently small, and it is decreasing in σ^2 if the interest rate and the variation in σ^2 are sufficiently small. In addition, As in Proposition 7, combining the results of W_θ^{+++*} and $W_{I\theta}^*$, we show that the manager's compensation tends to be paid later (or earlier) the larger α (or the larger μ if $\frac{\partial G(W)}{\partial \mu} > \frac{1}{r}$ for all $W \geq R$ and if $|\mu_1 - \mu_2|$ is sufficiently small).

The intuition is as follows. First, an increase in α directly increases $F_\alpha(W)$ because it raises the stream of expected cash flows $(1 + (\zeta + \alpha)a_P)$ but also reduces the variation in the

manager's continuation payoff by relaxing the manager's incentive-compatibility constraint and providing the manager with more incentives to work. However, an increase in α does not affect $G_\alpha(W)$ because the principal's effort has no effects on the cash flows realized after the IPO. As the principal's profit obtained when postponing the IPO becomes larger relative to her profit generated from the IPO, $W_{I\alpha}^*$ increases with α.

Second, as shown in Proposition 8, an increase in μ raises $G_\mu(W)$. This effect also increases $F_\mu(W)$ because the principal's profit from the IPO is equal to $G_\mu(W) - K$. In fact, if the variation in μ is sufficiently small, $G_\mu(W)$ increases with μ more rapidly than $F_\mu(W)$ near the IPO threshold. The reason is that an increase in the scale of the firm immediately increases the expected cash flows of the firm after the IPO and raises the expected revenue of new outside investors, whereas it does not increase the expected cash flows of the firm before the IPO; that is, the expected revenue of the principal until the IPO is undertaken. This implies that near the IPO threshold, the principal's profit generated from the IPO $(G_\mu(W) - K)$ is more sensitive to μ than her profit obtained when postponing the IPO $(F_\mu(W))$. Hence, if the variation in μ is sufficiently small, the principal is more likely to speed up the IPO timing as μ increases: $W_{I\mu}^*$ is decreasing in μ.

Third, as shown in Proposition 8, an increase in σ^2 reduces $G_{\sigma^2}(W)$. This effect also decreases $F_{\sigma^2}(W)$ because the principal's profit from the IPO decreases. On the other hand, an increase in σ^2 directly reduces $F_{\sigma^2}(W)$ because it raises the cost of exposing the manager to income uncertainty by increasing the variation in the manager's continuation value. Note that the first effect on $G_{\sigma^2}(W)$ increases $W_{I\sigma^2}^*$, whereas the second and third effects on $F_{\sigma^2}(W)$ decrease $W_{I\sigma^2}^*$. In fact, if the interest rate and the variation in σ^2 are sufficiently small, the first and second effects cancel out each other. Thus, only the third effect remains near the IPO threshold. As the principal's profit obtained when postponing the IPO becomes smaller relative to her profit generated from the IPO, $W_{I\sigma^2}^*$ is decreasing in σ^2.

A small literature explores comparative static analysis of optimal IPO timing in dynamic

models. Clementi (2002) builds a discrete-time model of the optimal IPO decision and suggests that private firms with a larger positive productivity shock are more likely to go public. Benninga, Helmantel, and Sarig (2005) consider a simple value function model of optimal IPO timing. They predict that firms go public when cash flow is high. Pástor, Taylor, and Veronesi (2009) develop a continuous-time model of the optimal IPO decision in the presence of learning about average profitability but in the absence of unobservable action problems. They assume that both investors and the entrepreneur are risk averse, and they show that an IPO is more likely at some prespecified time when the expected profitability or the volatility of profitability is higher. However, their qualitative results on the likelihood of an IPO depend on the assumption that IPO timing is exogenously given.

Philippon and Sannikov (2007) extend the continuous-time cash diversion model in De-Marzo and Sannikov (2006), in which all of the contracting parties are risk neutral. They assume that a standard Brownian motion represents a signal regarding the action of the manager rather than the cumulative cash flow. They suggest that the IPO threshold is decreasing in the increment of the expected productivity after the IPO. This result is consistent with our corresponding result in Proposition 9, although Philippon and Sannikov (2007) do not consider the double moral hazard situation. However, they indicate that the IPO threshold is increasing in the volatility of the signal. By contrast, we show that the IPO threshold is decreasing in the risk of the project (equal to the volatility of the cumulative cash flow) under certain conditions. Intuitively, in the cash diversion model, there is no difference between the incentive-compatibility constraints for the manager before and after the IPO. On the other hand, in our double moral hazard model, differences exist between these constraints before and after the IPO. The differences result not only from the complementarity between the principal's and the manager's efforts before the IPO but also from the upward shift in productivity after the IPO. As a result, and in contrast to the model in Philippon and Sannikov (2007), the net expected profit of the principal obtained when postponing the IPO is

decreasing in σ^2 for all $W \geq R$ in our model, thus generating the different result. Our result concerning the degree of complementarity is also novel.

In addition, the IPO timing literature referred to above does not elaborate on the role of advisors such as the VC or buyout funds, nor does it examine the dynamic compensation profile for the manager. Of course, studies of the optimal venture capital contract often discuss the role of advisors and examine the properties of the optimal compensation contract for the manager. However, they are restricted in that they rely upon a static framework.

7. Model of the VC Not Exiting the Firm following the IPO

In preceding sections, we considered the case where the principal (the VC) exits completely with the IPO. In this section, we assume that for some reasons not modeled in this analysis, the VC retains a fraction of ω $(< \overline{\omega})$ of the firm's shares and provides effort to increase the productivity of the firm even after the IPO. However, the firm is under the control of new outside investors because the VC must set ω sufficiently small in order to recover the funds invested for future investment opportunities or in order to finance the investment cost K for expanding the scale of the firm. Furthermore, as the new outside investors cannot observe the manager's effort nor the principal's effort, they need to design an optimal incentive scheme in a multiagent environment following the IPO.

Here, we assume that the drift of the cash flows after the IPO is given by $A_t^N = \mu(a_{At} + \zeta a_{Pt} + \alpha a_{At} a_{Pt})$. Then, the manager's total expected payoff at $t = 0$ is still written by (6), whereas the principal's total expected payoff at $t = 0$ is

$$
E \left\{ \int_0^{\tau_{T_0 I}} e^{-rt} \left[(A_t - g(a_{Pt})) \, dt - dC_t \right] + \mathbf{1}_{\tau_I \geq \tau_{T_0}} e^{-r\tau_{T_0}} L_0 \right.
$$
$$
+ \mathbf{1}_{\tau_I < \tau_{T_0}} \left[e^{-r\tau_I} \left[\int_{\tau_I}^{\tau_{T_1}} e^{-r(t-\tau_I)} \left[\left(\omega A_t^N - g(a_{Pt}) \right) dt - \omega dC_t \right] - \omega K \right] + e^{-r\tau_{T_1}} \omega L_1 \right]
$$
$$
\left. + \mathbf{1}_{\tau_I < \tau_{T_0}} \left[e^{-r\tau_I} (1 - \omega) \left[\int_{\tau_I}^{\tau_{T_1}} e^{-r(t-\tau_I)} \left(A_t^N dt - dC_t \right) - K \right] + e^{-r\tau_{T_1}} (1 - \omega) L_1 \right] \right\}. \quad (25)
$$

Note that the profit of the principal from the IPO equals the total expected payoff for new outside investors at $t = \tau_I$, which is shown by the terms in the third curly bracket in (25).

In designing the optimal incentive scheme after the IPO, for simplicity we assume that there is no colluding agreement between the principal and the manager, and that if there are multiple effort processes of $\{(a_{At}, a_{Pt}) \in A_A \times A_P, \tau_I \leq t < \infty\}$ as Nash equilibrium for the principal and the manager, new outside investors can implement their most preferred effort processes from among those effort processes. In addition, new outside investors cannot offer the principal or the manager a contract contingent on the manager's report of the principal's effort even when the manager can observe the principal's effort. This assumption can be justified because such contracts are not found in practice. Assumptions 1 and 2 are still assumed to hold. Again, we assume that implementing the manager's high effort $(a_A(W_t) = 1)$ is always optimal. In the Appendix, we derive a sufficient condition for the manager's high effort to remain optimal at all times, regardless of the situation of single or double moral hazard.

Now, regardless of whether we assume a single or double moral hazard situation before the IPO, we can prove the proposition that corresponds to Proposition 1 after the IPO. Hence, there exists a progressively measurable process $\{Y_t, \mathcal{F}_t : \tau_I \leq t \leq \tau_{T_1}\}$ in $\mathcal{L}^*$ such that

$$dW_t = \{\gamma W_t - h \cdot [1 - a_A(W_t)]\} \, dt - dC(W_t) + Y(W_t)\{dX_t - \mu[a_A(W_t) + \zeta a_P(W_t) + \alpha a_A(W_t)a_P(W_t)]dt\},$$

for every $t \in [\tau_I, \tau_{T_1}]$. Then, the incentive-compatibility constraint for the manager implementing high effort after the IPO is summarized by $Y = \eta_1(a_P)$, where

$$\eta_1(a_P) \equiv \min \{y : y\mu(1 + \alpha a_P) \geq h\} = \frac{h}{\mu(1 + \alpha a_P)}. \tag{26}$$

Let $G_N(W)$ and $G_P(W)$ denote the value functions of new outside investors and the principal after the IPO, respectively. Applying the same logic as that described before

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Proposition 2, we can show that the optimal compensation policy is determined by

$$dC(W) = \max(W - W_N^{++}, 0), \tag{27}$$

where W_N^{++} is the lowest value of W such that $G_N'(W) = -1$. As $G_N(W)$ is independent of whether we assume a single or double moral hazard situation before the IPO, W_N^{++} does not depend on the situation of single or double moral hazard before the IPO.

Then, irrespective of whether we assume single or double moral hazard before the IPO, we show in the Appendix that $G_N(W)$ and $G_P(W)$ are given by (A18) and (A19), respectively. Hence, the principal optimally chooses her effort after the IPO as

$$a_P = \arg\max_{a_P \in A_P} \omega\mu\left[1 + (\zeta + \alpha)a_P\right] - g(a_P) + G_P'(W)\gamma W + \frac{G_P''(W)}{2}\left[Y(W)\right]^2 \sigma^2.$$

Note that $Y(W)$ is determined by the recommended principal's effort process at each point of W after the IPO; that is, $Y(W) = \eta_1(a_P(W)) = \frac{h}{\mu[1+\alpha a_P(W)]}$, where $a_P(W)$ is given by the recommended principal's effort at each point of W after the IPO. If we assume that $\omega\mu(\zeta + \alpha) > g'(0)$, the incentive-compatibility constraint for the principal after the IPO is represented by

$$\omega\mu(\zeta + \alpha) = g'(a_P) \text{ or } a_P = g'^{-1}(\omega\mu(\zeta + \alpha)) = \psi(\omega\mu(\zeta + \alpha)). \tag{28}$$

Given $0 < \omega < 1$, (28) implies that the principal's effort is undersupplied because the marginal productivity of the principal's effort, $\mu(\zeta + \alpha)$, is larger than its marginal cost, $g'(a_P)$.

Now, using the HJB equations for this model derived in the Appendix, we can obtain the following results. First, we can rephrase Propositions 5 and 5' as follows.

Proposition 10: (i) Suppose that the manager can observe and verify $\{a_{Pt} \in A_P, 0 \leq t \leq \tau_{T_0I}\}$.

(a) The manager does not receive any compensation until W_t hits W_N^{++*} after the IPO.

(b) If the principal's effort before the IPO is positive, the marginal productivity of her effort is smaller than its marginal cost when $\alpha > 0$, whereas the marginal productivity of her effort is equal to its marginal cost when $\alpha = 0$. On the other hand, the marginal productivity of the principal's effort after the IPO is larger than its marginal cost.

(ii) Suppose that the manager cannot observe $\{a_{Pt} \in A_P, 0 \le t \le \tau_{T_0I}\}$.

(a) The manager does not receive any compensation until W_t hits W_N^{++*} after the IPO.

(b) Before the IPO, double moral hazard induces the principal to undersupply her own effort relative to the case of single moral hazard. After the IPO, the principal's effort is the same as that in the case of single moral hazard.

Second, we can prove that Propositions 6 and 7 regarding the differences between the IPO and compensation payment thresholds under the single and double moral hazard situations continue to hold.

Third, for the comparative statics under the double moral hazard situation, let $G_{N\theta}$ and $W_{N\theta}^{++*}$ denote the value function of new outside investors and the optimal cash payment threshold for each parameter θ $(= \alpha, \mu, \text{ and } \sigma^2)$ when the principal does not exit the firm completely with the IPO. Then, we obtain the following proposition.

Proposition 11: *Consider $\alpha_1 < \alpha_2$, $\mu_1 < \mu_2$, and $\sigma_1^2 < \sigma_2^2$.*

(i) For all $W \ge R$, $G_{N\alpha_1}(W) < G_{N\alpha_2}(W)$, $G_{N\mu_1}(W) < G_{N\mu_2}(W)$, and $G_{N\sigma_1^2}(W) > G_{N\sigma_2^2}(W)$.

(ii) $W_{N\sigma_1^2}^{++*} \le W_{N\sigma_2^2}^{++*}$. In addition, $W_{N\alpha_1}^{++*} \ge W_{N\alpha_2}^{++*}$ if $\frac{\partial G_{N\alpha}(W)}{\partial \alpha} > (1 - \omega)\frac{\partial A^{N**}(W)}{\partial \alpha}$ for all W $\ge R$; and $W_{N\mu_1}^{++*} \ge W_{N\mu_2}^{++*}$ if $\frac{\partial G_{N\mu}(W)}{\partial \mu} > (1 - \omega)\frac{\partial A^{N**}(W)}{\partial \mu}$ for all $W \ge R$. Here, $A^{N**}(W)$ $= \mu[1 + (\zeta + \alpha)a_P^{**}(W)]$ is the expected productivity of the firm under the optimal contract after the IPO.

The intuition for the results of μ and σ^2 is similar to that given below Proposition 8. The intuition for the results of α can be explained in a way similar to that in the case of μ.

Similarly, we can derive the comparative static results prior to the IPO under double moral

hazard. Again, to avoid complicating the notation, we drop the tilde for all variables in the rest of this section. Then, we can show the following proposition.

Proposition 12: *Consider $\alpha_1 < \alpha_2$, $\mu_1 < \mu_2$, $\sigma_1^2 < \sigma_2^2$, and $\omega_1 < \omega_2$.*

(i) For all $W \geq R$, $F_{\alpha_1}(W) < F_{\alpha_2}(W)$, $F_{\mu_1}(W) < F_{\mu_2}(W)$, $F_{\sigma_1^2}(W) > F_{\sigma_2^2}(W)$, and $F_{\omega_1}(W) < F_{\omega_2}(W)$.

(ii) $W_{I\alpha_1}^* < W_{I\alpha_2}^*$ if r and $|\alpha_1 - \alpha_2|$ are sufficiently small, whereas $W_{I\sigma_1^2}^* > W_{I\sigma_2^2}^*$ if r and $|\sigma_1^2 - \sigma_2^2|$ are sufficiently small. For $\theta = \mu$ and ω, $W_{I\theta_1}^* > W_{I\theta_2}^*$ if $|\theta_1 - \theta_2|$ is sufficiently small.

(iii) The larger is μ, the earlier the manager's compensation tends to be paid if $\frac{\partial G_{N\mu}(W)}{\partial \mu} > (1 - \omega)\frac{\partial A^{N**}(W)}{\partial \mu}$ for all $W \geq R$ and if $|\mu_1 - \mu_2|$ is sufficiently small.

Intuitively, the results of μ and σ^2 can be explained in a way similar to that discussed below Proposition 9. For the results of α, we should notice that an increase in α raises $G_{N\alpha}(W) + G_{P\alpha}(W)$, where $G_{P\theta}(W)$ denotes the value function of the principal after the IPO for each parameter θ ($= \alpha$, μ, σ^2, and ω). However, if the interest rate r and the variation in α are sufficiently small, the increasing effect of α on $F_\alpha(W)$ dominates the increasing effect of α on $G_\alpha(W)$. Thus, under this condition, $W_{I\alpha}^*$ increases with α. The intuition for the results of ω can be explained as follows. An increase in ω directly increases $G_{N\omega}(W) + G_{P\omega}(W)$ because it not only raises the stream of expected cash flows by inducing the greater principal's effort after the IPO but also reduces the variation in the manager's continuation payoff by providing the manager with more incentive to work through the complementarity effect of the increase in the principal's effort. In fact, an increase in ω also increases $F_\omega(W)$ because the sum of the principal's profits at the IPO and after the IPO is equal to $G_{N\omega}(W) + G_{P\omega}(W) - K$. However, if the variation in ω is sufficiently small, the principal's profit generated from the IPO ($G_{N\omega}(W) + G_{P\omega}(W) - K$) is more sensitive to μ than that obtained when postponing the IPO ($F_\omega(W)$) near the IPO threshold. Hence, $W_{I\omega}^*$ is decreasing in ω.

To summarize, the possibility of the VC not exiting the firm completely with the IPO

invokes a multiagent setting when new outside investors cannot observe either the VC's or the manager's efforts. However, most of our main conclusions continue to hold, even in this case. Therefore, our main results do not depend on the assumption that the principal (VC) completely exits with the IPO.

8. Empirical Implications

In this section, we derive several empirical implications supported by our analysis, discuss their relevance to actual empirical findings, and propose new testable predictions. We begin by proposing empirical implications for the IPO timing, using the comparative static results regarding the IPO threshold.

(A) An IPO is likely to be earlier the less the need for the VC's monitoring role, the greater the increment in future expected cash flows, the higher the volatility of cash flows, and the larger the equity claim of the VC after the IPO.

Using US manufacturing firm data, Chemmanur, He, and Nandy (2010) report that firms with higher sales growth and higher total factor productivity are more likely to go public. This finding is consistent with our implication that the IPO is more likely to be earlier when the increment in future expected cash flows increases. Chemmanur, He, and Nandy (2010) also suggest that firms in industries characterized by riskier cash flows are more likely to go public. This is also consistent with our implication that the IPO is more likely to be earlier the higher the volatility of cash flows. Our implications for the IPO timing concerning the need for the VC's monitoring role and the post-IPO equity position of the VC provide new testable predictions for empirical research. Indeed, the need for the VC's monitoring role can be measured by several indexes. For example, Lee and Wahal (2004) suggest that venture financing is disproportionately provided to firms in technology-intensive industries, in particular software and commercial biological research. They also indicate that venture capitalists generally take smaller and younger firms public. These findings imply that the

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VC's monitoring role is needed more in firms in technology-intensive industries, smaller firms, and younger firms. Hence, our theory predicts that an IPO is likely to be later in these kinds of firms in which the effort of the VC would be more important.

A large number of IPOs do not necessarily have any VC contract relationships. Although our present model cannot suitably deal with the decision as to whether an entrepreneur and a VC make a contract relationship, our theory implies that firms with a high initial continuation value of the manager W_0 ($> W_I$) undertake the IPO immediately. If larger and mature firms have a higher W_0, this suggests that larger and mature firms need not receive the VC's support for undertaking the IPO, which is consistent with the empirical findings of Lee and Wahal (2004).[27]

Our theory also derives new implications for the managerial compensation profile, using the comparative static results regarding the cash payment threshold.

(B) Managerial compensation will tend to be paid later, the smaller the increment in future expected cash flows, if the discounted expected profit of new outside investors to equity is more sensitive to this increment than the firm's expected productivity. In addition, managerial compensation will tend to be paid later after the IPO, the higher the volatility of cash flows and the less acute the need for the VC's monitoring role if the discounted expected profit of new outside investors to equity is more sensitive to this need than the firm's expected productivity.

It is common for VC–entrepreneur relationships to include vesting provisions (see Kaplan and Strömberg (2003)). Our implications are novel in providing testable predictions that time vesting is used more, the smaller the increment in future expected cash flows; and that

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[27]There are several empirical findings of long-run post-IPO underperformance (for example, see Ritter (1991) and Loughran and Ritter (1995) for the negative abnormal returns after the IPO, and Pástor, Taylor, and Veronesi (2009) for the lower profitability after the IPO). Even in our model, we show that $\widetilde{F}(W) \geq G(W) - K$ (or $\widetilde{F}(W) \geq G_N(W) + G_P(W) - K$) for all $W \geq R$ under the optimal contract. Hence, if the firm must promise to expend K at the time of the IPO but real expenditure evolves only gradually after the IPO and if the manager's continuation value W can be fully controlled under the estimating equation, our model suggests that the post-IPO performance of the firm is not better than the pre-IPO one from the point of view of the negative abnormal returns after the IPO.

it is also used more after the IPO the larger the volatility of cash flows and the less the need for the VC's monitoring role.

Under our optimal compensation contract, the manager cannot receive any payment before the IPO nor any lump-sum payment at the IPO. Instead, the manager can receive lump-sum payment only after the IPO. Practically, this delay of payment can be interpreted as a lockup period during which the manager may not sell their shares for a period of time after the IPO. In any case, our model indicates that the contract can specify a large payment for the manager around the IPO. Furthermore, the VC has recently been more likely to exit through the sale of its portfolio firm to another company (M&A) instead of the IPO.[28] As has been mentioned in the introductory section, our model can be applied to the case where the VC exits through M&A. In this case, the manager is less likely to receive a large amount of lump-sum payment at the exit of the VC.[29] Thus, the result that the manager does not receive lump-sum payment at the exit of the VC does not restrict the explanatory power of our VC model.

Because the role of the VC can also be played by buyout funds or banks, our statements (A) and (B) with the associated arguments apply not only to IPOs or M&A for early-stage start-up firms but also to IPOs or M&A involving the relisting of management buyout firms or firms reorganized following bankruptcy.[30] Our model also allows us to derive new implications for the dissolution of joint ventures or corporate spin-offs. In the case of corporate spin-offs, parent companies separate one of their subsidiaries from themselves, sell the equity of the subsidiary to their own shareholders, and make the subsidiary a new entity that is managed independently of the parent companies. Thus, if the parent company's CEO (principal)

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[28] See Ball, Chiu, and Smith (2011) and Yearbook 2012, National Venture Capital Association.

[29] A venture capitalist states that the case in which the manager receives a large amount of lump-sum payment at M&A is usually the case in which he quits the firm at M&A. Hence, except for this golden-parachute case, the manager is less likely to receive a large amount of lump-sum payment at the exit of the VC via M&A. As our model focuses on the case in which the manager remains in the firm even after the VC exits, we can neglect such a golden-parachute case.

[30] Note that M&A is the most common route for the exit of buyout funds. See Kaplan and Strömberg (2009).

attempts to maximize the expected payoff of the existing shareholders, the process is well characterized by the model in which the principal completely exits upon the spin-off timing. Daley, Mehrotra, and Sivakumar (1997) and Desai and Jain (1999) find that the number of focus-increasing spin-offs is more than twice that of non-focus-increasing spin-offs. Ahn and Walker (2007) also report that diversified firms with more effective corporate governance are more likely to conduct spin-offs. These findings are consistent with our hypothesis that a spin-off is likely to be earlier the lesser the need for the parent company's involvement.

9. Conclusion

In this paper, we explore a continuous-time agency model with double moral hazard in which an agent provides unobservable effort whereas a principal (advisor) supplies unobservable effort and chooses an optimal exit timing with irreversible investment. To formalize the model, we take an example in which a VC or buyout fund and a manager jointly exert an unobservable level of effort to develop a project before the firm goes public to finance irreversible investment and the VC or buyout fund exits. We show that the optimal IPO timing is earlier under double moral hazard than under single moral hazard. Our result also indicates that the manager's compensation tends to be paid earlier under double moral hazard. We derive several comparative static results for the IPO timing and the manager's compensation profile with respect to the governance role parameters of the VC (or buyout fund) and the IPO as well as the risk parameter of the project.

Furthermore, even where the VC or buyout fund does not completely exit with the IPO, such that we require a multiagent analysis, we argue that most of our main results remain unaffected. In addition, our model can be applied to not only the dissolution of joint ventures but also corporate spin-offs.

Appendix

Proof of Proposition 2: Our contract model after the IPO is essentially similar to the hidden effort model of DeMarzo and Sannikov (2006, Section III). The difference is that the manager's action increases cash flows in our model, whereas it decreases cash flows in De-Marzo and Sannikov (2006). Hence, we prove the statement of this proposition by applying a procedure similar to that of DeMarzo and Sannikov (2006). ∎

Proof of Lemma 1: When the contract induces the manager to shirk, his promised payoff would evolve according to

$$dW_t = (\gamma W_t - h)\, dt - dC(W_t) + \beta_1 \sigma dZ_t.$$

For the manager's high effort to be optimal at any $t \in [\tau_I,\ \tau_{T1}]$, the expected payoff rate of new outside investors from letting the manager shirk would be less than that under our existing contract for all $W \in [R,\ W^{++}]$:

$$rG(W) \geq G'(W)(\gamma W - h). \tag{A1}$$

The manager's and the principal's expected payoffs if the manager shirks forever are given by $W^S = \frac{h}{\gamma}$ and $G^S = 0$. Using this, we rewrite (A1) as

$$G(W) + \frac{\gamma}{r} G'(W)(W^S - W) \geq 0. \tag{A2}$$

To prove that the condition of (14) guarantees (A2), we need to show that for all $W \in [R,\ W^{++}]$,

$$G(W) + \frac{\gamma}{r} G'(W)(W^S - W) \geq G(W^S) - \frac{\gamma - r}{r}[G(W^{\max G}) - G(W^S)].$$

Applying a procedure similar to that of Proposition 8 in DeMarzo and Sannikov (2006), we can prove that the above inequality holds for all $W \in [R,\ W^{++}]$. ∎

Proof of Proposition 4: We first consider the following HJB equation:

$$rF(W) = \max_{a_P \in A_P, Y \geq \beta_0(a_P), dC} 1 + (\zeta + \alpha)a_P - g(a_P) - dC + F'(W)\gamma W + \frac{F''(W)}{2}Y^2\sigma^2. \quad \text{(A3)}$$

Then, we prove the regulatory properties of $F(W)$ in (A3).

Lemma A1: *The solutions to (A3) exist and are unique and continuous in initial conditions $F(W)$ and $F'(W)$. In addition, initial conditions with $F''(W) < 0$ result in a concave function.*

Proof: Using a procedure similar to that in the proof of Lemma 1 in the Appendix in Sannikov (2008), we can show the first statement of this lemma under Assumption 1.

To prove the second statement, let us define

$$H(F, F', W) = \min_{a_P \in A_P, Y \geq \beta_0(a_P), dC} \frac{rF - 1 - (\zeta + \alpha)a_P + g(a_P) + dC - F'\gamma W}{\frac{1}{2}Y^2\sigma^2}.$$

Then, $F''(W) = H(F(W), F'(W), W)$. We can show that if ever $H(F, F', W) = 0$ on the path of a solution, the corresponding solution must be $F(W') = F(W) + \frac{\gamma}{r}(W' - W)F'(W)$. To see this, we need to verify that $H(F + \frac{\gamma}{r}(W' - W)F', F', W') = 0$ for all W, $W' \geq R$. Indeed,

$$
\begin{aligned}
&H(F + \frac{\gamma}{r}(W' - W)F', F', W')\\
&= \min_{a_P \in A_P, Y \geq \beta_0(a_P), dC} \frac{rF + \gamma(W' - W)F' - 1 - (\zeta + \alpha)a_P + g(a_P) + dC - F'\gamma W'}{\frac{1}{2}Y^2\sigma^2}\\
&= \min_{a_P \in A_P, Y \geq \beta_0(a_P), dC} \frac{rF - 1 - (\zeta + \alpha)a_P + g(a_P) + dC - F'\gamma W}{\frac{1}{2}Y^2\sigma^2}\\
&= H(F, F', W) = 0.
\end{aligned}
$$

However, if $F(W') = F(W) + \frac{\gamma}{r}(W' - W)F'(W)$ for all W, $W' \geq R$, rearranging this relation yields

$$\frac{F'(W) - F(W)}{W' - W} = \frac{\gamma}{r}F'(W), \qquad \text{for all } W, W' \geq R.$$

As $W' \to W$, this implies that $F'(W) = \frac{\gamma}{r}F'(W)$, which contradicts $\gamma > r$. Hence, the second derivative of the solution can never reach zero. We therefore verify that a solution with a negative second derivative at one point must be concave everywhere. $\|$

Next, we examine the existence and uniqueness of the solution that solves (A3) with (19).

Lemma A2: *There exists a unique concave function $F(W) \geq G(W) - K$ that solves (A3) with (19) for some $W_I > R$.*

Proof: Let us consider the solutions of (A3) with $F(R) = L_0$ and various slopes $G'(R) > F'(R)$. By Lemma A1, all of these solutions are unique and continuous in $F'(R)$. It also follows from Lemma A1 and $G''(W) < 0$ for $W \in [R, W^{++})$ that all of these solutions are concave if they satisfy (19).[31] Because $F(R) = L_0 > L_1 - K = G(R) - K$ by Assumption 2, it is evident that $W_I > R$. Now, as $G'(R) > F'(R)$, the resulting solution $F(W)$ must reach $G(W) - K$ at some point $W_I \in (R, \infty)$, as typically shown in Figure 1. $\|$

We next prove that $dC_t = 0$ for any $t \in [0, \tau_{T_0 I}]$.

Lemma A3: $W_I \leq W^+$. Thus, $dC(W_t) = 0$ for all $W_t \in [R, W_I]$

Proof: If $W_I > W^+$, the immediate payment $W_I - W^+$ must be made as soon as the IPO is completed. Thus, the principal's profit from the IPO is reduced to $G(W^+) - K$. However, it follows from $W_I > W^+$ and $F(W) > G(W) - K$ for all $W \in [R, W_I)$ that $F(W^+) > G(W^+) - K$, which means that the IPO should not be done. If $W_I \leq W^+$, it follows from $F''(W) < 0$ for all $W \in [R, W_I]$ and $F'(W_I) = G'(W_I) \geq -1$ that $dC(W_t) = 0$ for all $W_t \in [R, W_I]$. $\|$

Now, we conjecture an optimal contract from the solution of (18) that we have just constituted.

Lemma A4: *Consider the unique solution $F(W) \geq G(W) - K$ that solves (18) and satisfies (19) for some $W_I > R$. Let $a_P : [R, W_I] \rightarrow A_P$ and $Y : [R, W_I] \rightarrow [\beta_0, \infty]$ be the maximizers in (18); in particular, $Y(W_t) = \beta_0(a_P(W_t)) = \frac{h}{1 + \alpha a_P(W_t)}$. For any starting condition $W_0 \in [R, W_I]$, there is a unique in the sense of probability law weak solution to the following equation:*

$$dW_t = \gamma W_t dt + \beta_0(a_P(W_t))\{dX_t - [1 + (\zeta + \alpha)a_P(W_t)]dt\}, \tag{A4}$$

until the time $\tau_{T_0 I}$. The process of the manager's and the principal's efforts $\{a_A(\cdot), a_P(\cdot)\}$

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[31] If $F''(W) \geq 0$ for all $W \geq R$, then $F(W) > G(W) - K$ for all $W \in (W_I, W^{++}]$, which is a contradiction.

$\equiv \{a_A(W), a_P(W) : W \in [R, W_I]\}$ *is defined by*

$$
\begin{cases}
a_{At} = 1, \ a_{Pt} = a_P(W_t), \ \text{and} \ Y_t = \beta_0(a_P(W_t)), \\
\qquad \text{for} \ t \in [0, \ \tau_{T_0 I}) \ \text{and for} \ t = \tau_{T_0 I} \ \text{and} \ W_{\tau_{T_0 I}} = W_I, \\
a_{At} = 0, \ a_{Pt} = 0, \ \text{and} \ Y_t = 0, \quad \text{for} \ t \geq \tau_{T_0 I} \ \text{and} \ W_{\tau_{T_0 I}} = R,
\end{cases}
$$

is incentive compatible, and has value W_0 to the manager and the expected present value of the profit $F(W_0)$ to the principal.

Proof: There is a weak solution of (A4) that is unique in the sense of probability law because the drift of W_t is bounded on $[R, W_I]$ and the volatility is bounded above 0 by $\frac{h}{1+\alpha a_P(W_t)}\sigma$. Note that $a_P(W_t) \in A_P$ and A_P is a compact set.

Now, using a procedure similar to that in the proof of Proposition 3 in the Appendix in Sannikov (2008), we can verify that the principal obtains the profit $F(W_0)$. ‖

We finally show the following lemma.

Lemma A5: *Consider a concave solution $F(W)$ of the HJB equation that satisfies (18) with (19). Any incentive-compatible contract Π achieves the expected present value of the profit of at most $F(W_0)$.*

Proof: Let W_t evolve according to (15) in Proposition 3. Because implementing $a_{At} = 1$ for all $t \in [0, \tau_{T_0 I}]$ is optimal for the principal, define

$$
J_t = \int_0^t e^{-rs} \left[(1 + (\zeta + \alpha)a_{Ps} - g(a_{Ps}))\, ds - dC_s\right] + e^{-rt}F(W_t), \tag{A5}
$$

for any $t \in [0, \tau_{T_0 I}]$ and any incentive-compatible contract Π implementing $a_{At} = 1$ at any $t \in [0, \tau_{T_0 I}]$. Note that the process $\{J_t, 0 \leq t \leq \tau_{T_0 I}\}$ is such that J_t is $\mathcal{F}_t$-measurable. It follows from Ito's lemma and $a_{At} = 1$ at any $t \in [0, \tau_{T_0 I}]$ that

$$
\begin{aligned}
e^{rt}dJ_t &= \left[1 + (\zeta + \alpha)a_{Pt} - g(a_{Pt}) + F'(W_t)\gamma W_t + \frac{F''(W_t)}{2}Y_t^2\sigma^2 - rF(W_t)\right] dt \\
&\quad - [1 + F'(W_t)]dC_t + Y_t F'(W_t)\sigma dZ_t\}. \tag{A6}
\end{aligned}
$$

Thus, for any $t \in [0, \tau_{T_0 I}]$, it is found from (18) and (A6) that

$$
\begin{aligned}
e^{rt} dJ_t \;=\; & \left\{ 1 + (\zeta + \alpha)a_{Pt} - g(a_{Pt}) + F'(W_t)\gamma W_t + \frac{F''(W_t)}{2}Y_t^2 \sigma^2 \right. \\
& \left. - [1 + (\zeta + \alpha)a_P(W_t) - g(a_P(W_t)) + F'(W_t)\gamma W_t + \frac{F''(W_t)}{2}\beta_0(a_P(W_t))^2 \sigma^2] \right\} dt \\
& - [1 + F'(W_t)]dC_t + Y_t F'(W_t)\sigma dZ_t.
\end{aligned} \tag{A7}
$$

The first component of the right-hand side of (A7) is less than or equal to zero for any $t \in [0, \tau_{T_0 I}]$ because $a_P(W_t)$ and $Y(W_t) = \beta_0(a_P(W_t))$ are the maximizers in (18). The second component is also less than or equal to zero for any $t \in [0, \tau_{T_0 I}]$ because $F'(W_t) \geq -1$. Hence, the process $\{J_t, 0 \leq t \leq \tau_{T_0 I}\}$ is an $\mathcal{F}_t$-supermartingale at any $t \in [0, \tau_{T_0 I}]$. In addition, the process $\{J_t, 0 \leq t \leq \tau_{T_0 I}\}$ is an $\mathcal{F}_t$-martingale at any $t \in [0, \tau_{T_0 I}]$ for the optimal contract.

We now evaluate the principal's expected payoff for an arbitrary incentive-compatible contract Π implementing $a_{At} = 1$ at any $t \in [0, \tau_{T_0 I}]$, which equals

$$
\begin{aligned}
J_\Pi \;=\; & E\left\{ \int_0^{\tau_{T_0 I}} e^{-rs}\left[(1 + (\zeta + \alpha)a_{Ps} - g(a_{Ps}))\,ds - dC_s\right] \right. \\
& \left. + \mathbf{1}_{\tau_I \geq \tau_{T_0}} e^{-r\tau_{T_0}} L_0 + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{-r\tau_I}\left[G(W_I) - K\right] \right\}.
\end{aligned}
$$

Using (A5), we show that under any $t \in [0, \infty)$ and any arbitrary incentive-compatible contract Π implementing $a_{At} = 1$ at any $t \in [0, \tau_{T_0 I}]$,

$$
\begin{aligned}
J_\Pi = & \; E(J_{t \wedge \tau_{T_0 I}}) + E\left\{ \mathbf{1}_{t < \tau_{T_0 I}} \left[\int_t^{\tau_{T_0 I}} e^{-rs}\left[(1 + (\zeta + \alpha)a_{Ps} - g(a_{Ps}))\,ds - dC_s\right] + \mathbf{1}_{\tau_I \geq \tau_{T_0}} e^{-r\tau_{T_0}} L_0 \right.\right. \\
& \left.\left. + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{-r\tau_I}\left[G(W_I) - K\right] - e^{-rt} F(W_t) \right] \right\} \\
\leq & \; F(W_0) + E\left\{ \mathbf{1}_{t < \tau_{T_0 I}} \left[\int_t^{\tau_{T_0 I}} e^{r(t-s)}\left[(1 + (\zeta + \alpha)a_{Ps} - g(a_{Ps}))\,ds - dC_s\right] + \mathbf{1}_{\tau_I \geq \tau_{T_0}} e^{r(t - \tau_{T_0})} L_0 \right.\right. \\
& \left.\left. + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{r(t-\tau_I)}\left[G(W_I) - K\right] - F(W_t) \right] \,\middle|\, \mathcal{F}_t \right\} e^{-rt},
\end{aligned} \tag{A8}
$$

where the inequality follows from the fact that $J_{t \wedge \tau_{T_0 I}}$ is a supermartingale and $J_0 = F(W_0)$.

In addition,

$$E\left\{\mathbf{1}_{t<\tau_{T_0 I}}\left[\int_t^{\tau_{T_0 I}} e^{r(t-s)}\left[(1+(\zeta+\alpha)a_{Ps} - g(a_{Ps}))\,ds - dC_s\right] + \mathbf{1}_{\tau_I \geq \tau_{T_0}} e^{r(t-\tau_{T_0})} L_0\right]\bigg|\,\mathcal{F}_t\right\}$$

$$\leq\ E\left\{\mathbf{1}_{t<\tau_{T_0 I}}\left[\frac{1}{r}\max_{a_P}\left[1+(\zeta+\alpha)a_P - g(a_P)\right] - W_t\right]\bigg|\,\mathcal{F}_t\right\}. \tag{A9}$$

This is because the right-hand side of (A9) is the upper bound on the principal's expected profit under the first-best contract when $t < \tau_{T_0 I}$. Combining (A8) and (A9), we obtain

$$J_{\Pi}\ \leq\ F(W_0) + E\left\{\mathbf{1}_{t<\tau_{T_0 I}}\left[\frac{1}{r}\max_{a_P}\left[1+(\zeta+\alpha)a_P - g(a_P)\right] - W_t\right.\right.$$
$$\left.\left. +\,\mathbf{1}_{\tau_I<\tau_{T_0}} e^{r(t-\tau_I)}\left[G(W_I) - K\right] - F(W_t)\right]\bigg|\,\mathcal{F}_t\right\} e^{-rt}. \tag{A10}$$

Using $F'(W_t) \geq -1$ and $F(R) = L_0$, we have $W_t + F(W_t) \geq L_0$ for any $W_t \geq R$. Hence, applying this to (A10), we see

$$J_{\Pi}\ \leq\ F(W_0)$$
$$+e^{-rt} E\left\{\mathbf{1}_{t<\tau_{T_0 I}}\left[\frac{1}{r}\max_{a_P}\left[1+(\zeta+\alpha)a_P - g(a_P)\right] - L_0 + \mathbf{1}_{\tau_I<\tau_{T_0}} e^{r(t-\tau_I)}\left[G(W_I) - K\right]\right]\bigg|\,\mathcal{F}_t\right\}.$$

Taking $t \to \infty$ yields

$$J_{\Pi} \leq F(W_0).$$

Let Π^* be a contract that satisfies the conditions of the proposition. This contract is incentive compatible because $Y_t = \beta_0(a_P(W_t)) = \frac{h}{1+\alpha a_P(W_t)}$. Furthermore, under this contract, the process $\{J_t, 0 \leq t \leq \tau_{T_0 I}\}$ is a martingale until time $\tau_{T_0 I}$ because $F'(W_t)$ stays bounded. Therefore, the payoff $F(W_0)$ is achieved with equality under Π^*. $\|$

Combining Lemmas A1–A5, we establish Proposition 4. ∎

Proof of Lemma 2: When the contract induces the manager to shirk, his promised payoff would evolve according to

$$dW_t = (\gamma W_t - h)\,dt - dC(W_t) + \beta_0 \sigma dZ_t.$$

For the manager's high effort to be optimal at any $t \in [0, \tau_{T_0 I}]$, the expected payoff rate

of the principal from having the manager shirk would be less than that under our existing contract for all $W \in [R, W_I]$:

$$rF(W) \geq F'(W)(\gamma W - h).$$

Now, repeating a procedure similar to that of Lemma 1, we can prove the statement of Lemma 2. For simplicity, we assume that $\zeta < g'(0)$. Hence, if the manager shirks forever, it is not optimal for the principal to supply any of her own effort. $\blacksquare$

Proof of Proposition 6: Comparing (22) and (23) with (18) and (19), we see that the only difference between the HJB equations and boundary conditions of the double and single moral hazard situations is that the right-hand side of (22) is maximized subject to the principal's moral hazard constraint at each level of W, whereas the right-hand side of (18) is not. Hence, $\widetilde{F}(W) < F(W)$ for all $W \geq R$. In addition, using (19), (23), $G''(W) < 0$, $F''(W) < 0$, $\widetilde{F}''(W) < 0$, $F(W) > G(W) - K$ for $W \in [R, W_I^*)$, and $\widetilde{F}(W) > G(W) - K$ for $W \in [R, \widetilde{W}_I^*)$, it follows that $\widetilde{W}_I^* < W_I^*$ if $F(W) - G(W) > \widetilde{F}(W) - G(W)$ for all $W \geq R$. Thus, it is evident from $\widetilde{F}(W) < F(W)$ that $\widetilde{W}_I^* < W_I^*$. $\blacksquare$

Proof of Proposition 8: (i) Applying procedures similar to those in the proofs for Lemmas 4 and 6 in DeMarzo and Sannikov (2006) or the proof for Lemma 3 in the Appendix in He (2009), we obtain

$$\frac{\partial G_\theta(W)}{\partial \theta} = E\left\{\int_{\tau_I}^{\tau_{T_1}} e^{-r(t-\tau_I)}\left[\frac{\partial \mu}{\partial \theta} + \frac{\partial \gamma W_t}{\partial \theta}G'_\theta(W_t) + \frac{1}{2}\frac{\partial \left(\frac{h\sigma}{\mu}\right)^2}{\partial \theta}G''_\theta(W_t)\right]dt \right.$$
$$\left. + e^{-r(\tau_{T_1}-\tau_I)}\frac{\partial L_1}{\partial \theta}\bigg|_{W_t=W_{\tau_{T_1}}}\bigg|W_{\tau_I} = W\right\}. \tag{A11}$$

Note that $\beta_1 = \frac{h}{\mu}$ from (11) and $G_\theta(R) = L_1$ at $W = R$. It follows from (A11) that

$$\frac{\partial G_\mu(W)}{\partial \mu} > 0 \text{ and } \frac{\partial G_{\sigma^2}(W)}{\partial \sigma^2} < 0. \tag{A12}$$

(ii) It follows from (12) and (13) that $rG(W^{++}) + \gamma W^{++} = \mu$. Hence, W^{+++*} is given by

the intersection of $y = G(W)$ and $y = \frac{\mu - \gamma W}{r}$. Because $G_{\sigma_1^2}(W) > G_{\sigma_2^2}(W)$ for all $W \geq R$, it is easily found from (13) and $\gamma > r$ that $W_{\sigma_1^2}^{+++} < W_{\sigma_2^2}^{+++}$. However, a rise in μ increases both $G_\mu(W)$ and $\frac{\mu - \gamma W}{r}$. In fact, if $\frac{\partial G_\mu(W)}{\partial \mu} > \frac{1}{r}$, the effect of μ on $G_\mu(W)$ dominates that on $\frac{\mu - \gamma W}{r}$. Hence, $W_{\mu_1}^{+++} > W_{\mu_2}^{+++}$ if $\frac{\partial G_\mu(W)}{\partial \mu} > \frac{1}{r}$. $\blacksquare$

Proof of Proposition 9: (i) Applying procedures similar to those in the proofs for Lemmas 4 and 6 in DeMarzo and Sannikov (2006) or the proof for Lemma 3 in the Appendix in He (2009), we obtain

$$\frac{\partial F_\theta(W)}{\partial \theta} = E\left\{ \int_0^{\tau_{T_0 I}} e^{-rt} \left[\frac{\partial \left[1 + (\zeta + \alpha)a_P(W_t) - g(a_P(W_t))\right]}{\partial \theta} + \frac{\partial (\gamma W_t)}{\partial \theta} F_\theta'(W_t) \right. \right.$$

$$+ \frac{1}{2} \frac{\partial \left[\frac{h\sigma}{1 + \alpha a_P(W_t)} \right]^2}{\partial \theta} F_\theta''(W_t) \, dt + \mathbf{1}_{\tau_I \geq \tau_{T_0}} e^{-r\tau_{T_0}} \frac{\partial L_0}{\partial \theta}$$

$$\left. \left. + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{-r\tau_I} \frac{\partial \left[G_\theta(W_t) - K\right]}{\partial \theta} \right|_{W_t = W_{\tau_I}} \right| W_0 = W \right\}. \tag{A13}$$

Note that $\beta_0(a_P) = \frac{h}{1 + \alpha a_P}$ from (17), $\zeta + \alpha = g'(a_P)$ from (21'), $F_\theta(R) = L_0$ at $W = R$, and $F_\theta(W_{\tau_I}) = G_\theta(W_{\tau_I}) - K$ at $W = W_{\tau_I}$. It follows from (A12) and (A13) that

$$\frac{\partial F_\mu(W)}{\partial \mu} = E\left[\mathbf{1}_{\tau_I < \tau_{T_0}} e^{-r\tau_I} \left. \frac{\partial G_\mu(W_t)}{\partial \mu} \right|_{W_t = W_{\tau_I}} \right| W_0 = W \right] > 0, \tag{A14}$$

$$\frac{\partial F_{\sigma^2}(W)}{\partial \sigma^2} = E\left\{ \int_0^{\tau_{T_0 I}} e^{-rt} \frac{1}{2} \left[\frac{h}{1 + \alpha a_P(W_t)} \right]^2 F_{\sigma^2}''(W_t) \, dt \right.$$

$$\left. + \mathbf{1}_{\tau_I < \tau_{T_0}} e^{-r\tau_I} \left. \frac{\partial G_{\sigma^2}(W_t)}{\partial \sigma^2} \right|_{W_t = W_{\tau_I}} \right| W_0 = W \right\} < 0. \tag{A15}$$

Furthermore, given $\zeta + \alpha = g'(a_P)$, we obtain

$$\frac{\partial F_\alpha(W)}{\partial \alpha} = E\left\{ \int_0^{\tau_{T_0 I}} e^{-rt} \left[a_P(W_t) - \frac{(h\sigma)^2 a_P(W_t)}{\left[1 + \alpha a_P(W_t)\right]^3} F_\alpha''(W_t) \right. \right.$$

$$\left. \left. - \frac{\alpha (h\sigma)^2}{\left[1 + \alpha a_P(W_t)\right]^3 g''(a_P(W_t))} F_\alpha''(W_t) \, dt \right] dt \right| W_0 = W \right\} > 0. \tag{A16}$$

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(ii) Using (23), $F_\theta''(W) < 0$, $G_\theta''(W) < 0$, and $F_\theta(W) > G_\theta(W) - K$ for all $W \in [R, W_{I\theta})$, we show that $W_{I\theta_1}^* \lesseqqgtr W_{I\theta_2}^*$ for $\theta = \alpha$, μ, and σ^2 if $F_{\theta_1}(W) - G_{\theta_1}(W) \lesseqqgtr F_{\theta_2}(W) - G_{\theta_2}(W)$ for all $W \geq R$. In particular, when $\theta = \alpha$, we obtain $\frac{\partial G_\alpha(W)}{\partial \alpha} = 0$. Thus, it follows from (A16) that $\frac{\partial[F_\alpha(W) - G_\alpha(W)]}{\partial \alpha} > 0$. As $\alpha_1 < \alpha_2$, this implies that $W_{I\alpha_1}^* < W_{I\alpha_2}^*$.

Regarding $\theta = \mu$ and σ^2, we focus on the perturbation in the small neighborhood of $W = W_{\tau_I}$ ($\equiv W_I$). We consider only the case of $\tau_I < \tau_{T_0}$ (that is, $\tau_{T_0 I} = \tau_I$ and $\mathbf{1}_{\tau_I < \tau_{T_0}} = 1$) without loss of generality because we study the comparative statics on W_I. Then, for $\theta = \mu$, it follows from (A12) and (A14) that $\frac{\partial[F_\mu(W) - G_\mu(W)]}{\partial \mu}\Big|_{W=W_{\tau_I}} \simeq (e^{-r\tau_I} - 1)\frac{\partial G_\mu(W)}{\partial \mu}\Big|_{W=W_{\tau_I}} < 0$ if $|\mu_1 - \mu_2|$ is sufficiently small. Thus, we obtain $W_{I\mu_1}^* > W_{I\mu_2}^*$ if $|\mu_1 - \mu_2|$ is sufficiently small. On the other hand, for $\theta = \sigma^2$, it follows from (A12), (A15), and $W = W_{\tau_I}$ that if r and $|\sigma_1^2 - \sigma_2^2|$ are sufficiently small,

$$\frac{\partial[F_{\sigma^2}(W) - G_{\sigma^2}(W)]}{\partial \sigma^2}\bigg|_{W=W_{\tau_I}} \simeq (e^{-r\tau_I} - 1)\frac{\partial G_{\sigma^2}(W)}{\partial \sigma^2}\bigg|_{W=W_{\tau_I}}$$

$$+ \frac{1 - e^{-r\tau_I}}{r}\frac{1}{2}\left[\frac{h}{1 + \alpha a_P(W_{\tau_I})}\right]^2 F_{\sigma^2}''(W_{\tau_I}) < 0. \tag{A17}$$

Although (A12) means $\frac{\partial G_{\sigma^2}(W)}{\partial \sigma^2}\Big|_{W=W_{\tau_I}} < 0$, the last inequality of (A17) is derived because on the right-hand side of (A17), the first term has a higher order of r than the second term. Hence, it follows that $W_{I\sigma_1^2}^* > W_{I\sigma_2^2}^*$ if r and $|\sigma_1^2 - \sigma_2^2|$ are sufficiently small. ∎

HJB equations for the model of the VC not exiting the firm completely with the IPO: Define $G_N(W)$ as the value function of new outside investors after the IPO. We can prove that for the manager's starting value $W_I \in [R, W_N^{++}]$, the optimal contract after the IPO is characterized by the unique concave function $G_N(W)$ that satisfies the HJB equation

$$rG_N(W) = \max_{a_P = \psi(\omega\mu(\zeta+\alpha)), Y \geq \eta_1(a_P)} (1-\omega)\mu\left[1 + (\zeta+\alpha)a_P\right] + G_N'(W)\gamma W + \frac{G_N''(W)}{2}Y^2\sigma^2, \tag{A18}$$

where $\eta_1(a_P)$ and $\psi(\omega\mu(\zeta + \alpha))$ are defined by (26) and (28), and the boundary conditions are given by $G_N(R) = (1 - \omega)L_1$, $G_N'(W_N^{++}) = -1$, and $G_N''(W_N^{++}) = 0$.[32] On the other

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[32] As the smooth-pasting and super contract conditions are the same as those of the model of DeMarzo and Sannikov (2006), we can prove the concavity of $G_S(W)$ as in Proposition 2.

hand, define $G_P(W)$ as the value function of the principal after the IPO. Then, for any $W \in [R, W_N^{++}]$, there is the unique concave function $G_P(W)$ that satisfies the HJB equation

$$rG_P(W) = \max_{a_P \in A_P} \omega\mu\left[1 + (\zeta + \alpha)a_P\right] - g(a_P) + G_P'(W)\gamma W + \frac{G_P''(W)}{2}Y^2\sigma^2, \quad (A19)$$

with the boundary conditions of $G_P(R) = \omega L_1$ and $G_P'(W_N^{++}) = 0$.[33,34] Note that Y is determined by the recommended principal's effort process at each point of W.

We now derive the HJB equation before the IPO. Suppose that the manager can observe and verify $\{a_{Pt} \in A_P, 0 \le t \le \tau_I\}$. As Proposition 3 still holds, the incentive-compatibility constraint for the manager prior to the IPO is again summarized by $Y = \beta_0(a_P)$, which is defined by (17). Thus, we can show that the optimal contract prior to the IPO is characterized by the unique concave function $F(W)$ ($\ge G_P(W) + G_S(W) - K$) that satisfies the HJB equation

$$rF(W) = \max_{a_P \in A_P, Y \ge \beta_0(a_P)} 1 + (\zeta + \alpha)a_P - g(a_P) + F'(W)\gamma W + \frac{F''(W)}{2}Y^2\sigma^2, \quad (A20)$$

with $\beta_0(a_P) = \frac{h}{1+\alpha a_P}$ and the boundary conditions of $F(R) = L_0$, $F(W_I) = G_N(W_I) + G_P(W_I) - K$, and $F'(W_I) = G_N'(W_I) + G_P'(W_I)$.

When the manager cannot observe $\{a_{Pt} \in A_P, 0 \le t \le \tau_I\}$, Proposition 3 again continues to hold. However, as in Section 5, we can show that the optimal contract prior to the IPO is characterized by the unique concave function $\widetilde{F}(W)$ ($\ge G_P(W) + G_S(W) - K$) that satisfies the HJB equation

$$r\widetilde{F}(W) = \max_{a_P = \psi(\zeta+\alpha), Y \ge \beta_0(a_P)} 1 + (\zeta + \alpha)a_P - g(a_P) + \widetilde{F}'(W)\gamma W + \frac{\widetilde{F}''(W)}{2}Y^2\sigma^2, \quad (A21)$$

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[33]The optimal compensation policy (27) implies that $G_N(W) + G_P(W) = G_N(W_N^{++}) + G_P(W_N^{++}) - (W - W_N^{++})$ for any $W > W_N^{++}$. Rearranging this equation yields $\frac{G_N(W)-G_N(W_N^{++})}{W-W_N^{++}} = -\frac{G_P(W)-G_P(W_N^{++})}{W-W_N^{++}} - 1$. As $W \to W_N^{++}$, we obtain $G_N'(W_N^{++}) = G_P'(W_N^{++}) - 1$. Hence, it follows from $G_N'(W_N^{++}) = -1$ that $G_P'(W_N^{++}) = 0$.

[34]The concavity of $G_P(W)$ can be derived if $\frac{\mu}{r} > L_1$. Indeed, using arguments similar to those in the proof of Lemma A1 in the proof of Proposition 4, we can prove that a solution with a negative second derivative at one point must be concave everywhere. Now, suppose that $G_P''(W) \ge 0$ for all $W \in [R, W_N^{++}]$. Then, it follows from (A19), $G_P'(W_N^{++}) = 0$, $g(0) > 0$, and $\frac{\mu}{r} > L_1$ that $G_P(W_N^{++}) = \frac{1}{r}\{\max_{a_P \in A_P} \omega\mu[1 + (\zeta + \alpha)a_P] - g(a_P) + \frac{G_P''(W_N^{++})}{2}Y^2\sigma^2\} > \omega L_1 = G_P(R)$. Hence, there is some $W' \in [R, W_N^{++})$ such that $G_P'(W') > 0$. However, as $G_P'(W_N^{++}) = 0$, this implies that there is some $W'' \in [W', W_N^{++})$ such that $G_P''(W'') < 0$. Thus, $G_P(W)$ is concave for all $W \in [R, W_N^{++}]$.

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where $\psi(\zeta + \alpha)$ is defined by (21′) and the boundary conditions are given by $\widetilde{F}(R) = L_0$, $\widetilde{F}(W_I) = G_N(W_I) + G_P(W_I) - K$, and $\widetilde{F}'(W_I) = G'_N(W_I) + G'_P(W_I)$.

Finally, using a procedure similar to that of Lemma 1, we can obtain the following sufficient conditions for the manager's high effort to remain optimal at all times.

Lemma A6: *(i) Suppose that the manager can observe and verify $\{a_{Pt} \in A_P, \ 0 \le t \le \tau_{T_0 I}\}$. Then, implementing the manager's high effort at all times is optimal for the principal if*

$$\frac{\gamma}{r} G_N(W^S) + (1 - \frac{\gamma}{r}) G_N(W^{\max G_N}) \ge 0, \tag{A22}$$

and (20) hold. Note that $W^{\max G_N}$ denotes the value of W that achieves the maximal value of $G_N(W)$ for any $W \in [R, W_N^{++}]$.

(ii) Suppose that the manager cannot observe $\{a_{Pt} \in A_P, 0 \le t \le \tau_{T_0 I}\}$. Then, implementing the manager's high effort at all times is optimal for the principal if (A22) and (24) hold. ∎

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Table 1: Comparative Statics under Double Moral Hazard When the VC Exits the Firm Following the IPO

θ	α	μ	σ^2
After	the	IPO	
$\partial G\left(W\right)/\partial\theta$	0	>0	<0
$\partial\widetilde{W}^{+++*}/\partial\theta$	0	$<0^a$	>0
Before	the	IPO	
$\partial\widetilde{F}\left(W\right)/\partial\theta$	>0	>0	<0
$\partial\widetilde{W}_I^*/\partial\theta$	>0	$<0^b$	$<0^c$

Notes:

[a] We assume that $\frac{\partial G(W)}{\partial\mu}>\frac{1}{r}$ for all $W\geq R$.

[b] We assume that $d\mu$ is sufficiently small.

[c] We assume that r and $d\sigma^2$ are sufficiently small.

Table 2: Comparative Statics under Double Moral Hazard When the VC Does Not Exit the Firm Following the IPO

θ	α	μ	σ^2	ω
After	the	IPO		
$\partial G_N\left(W\right)/\partial\theta$	>0	>0	<0	$?$
$\partial\widetilde{W}_N^{+++*}/\partial\theta$	$<0^a$	$<0^b$	>0	$?$
Before	the	IPO		
$\partial\widetilde{F}\left(W\right)/\partial\theta$	>0	>0	<0	>0
$\partial\widetilde{W}_I^*/\partial\theta$	$>0^c$	$<0^d$	$<0^c$	$<0^d$

Notes:

[a] We assume that $\frac{\partial G_N(W)}{\partial\alpha}>\frac{\partial\left[(1-\omega)A^{N**}(W)\right]}{\partial\alpha}$, where $A^{N**}\left(W\right)=\mu[1+(\zeta+\alpha)a_P^{**}\left(W\right)]$.

[b] We assume that $\frac{\partial G_N(W)}{\partial\mu}>\frac{\partial\left[(1-\omega)A^{N**}(W)\right]}{\partial\mu}$.

[c] We assume that r and $d\theta$ are sufficiently small for $\theta=\alpha$ and σ^2.

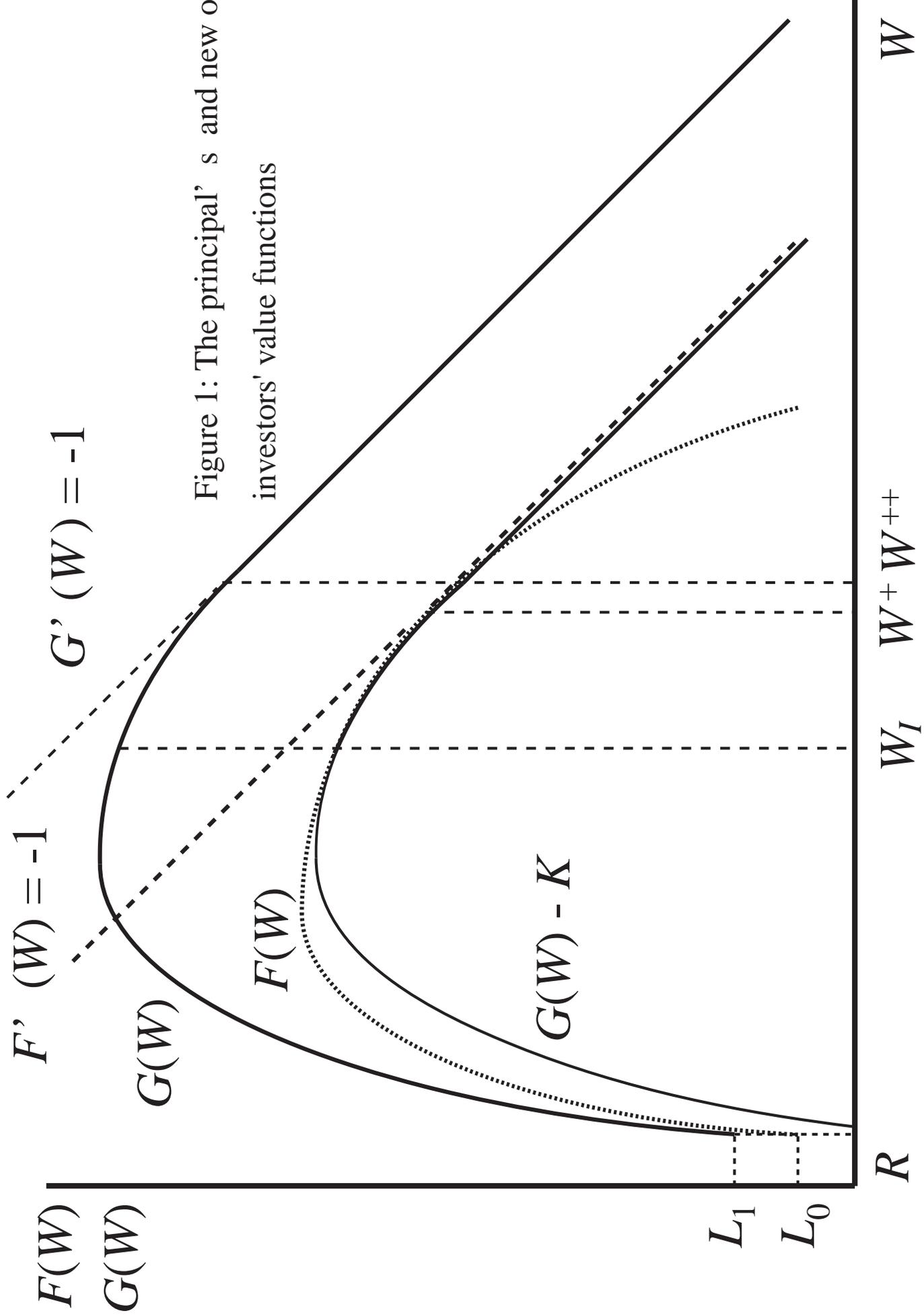[d] We assume that $d\theta$ is sufficiently small for $\theta=\mu$ and ω.

Figure 1: The principal's and new outside investors' value functions