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Kyoto University
A Vector Play Model for Finite-Element Eddy-Current Analysis Using the Newton-Raphson Method

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The differentiation of the vector hysteretic function represented by a vector play model is discussed for efficient nonlinear electromagnetic field computation using the Newton-Raphson method. The combination of the nonlinear finite-element method and the vector play model achieves accurate representation of the AC anisotropic magnetic property of non-oriented silicon steel sheet under rotational magnetic flux conditions. The proposed method is successfully applied to the eddy-current analysis of iron-cored inductors excited by a current or voltage source.

Index Terms— differentiation, finite element method, Newton-Raphson method, anisotropic vector hysteresis.

I. INTRODUCTION

Compact and high-powered electromagnetic machinery requires a high magnetic flux density reaching saturation in the iron-core, where electromagnetic field analysis has to handle a strong nonlinearity due to the anisotropic vector hysteretic property. To represent the vector hysteretic property of iron-core materials, a number of vector hysteresis models have been developed such as the vector Preisach model [1], the vector Jiles-Atherton model [2], and the E & S model [3]. Among them, the vector play model [4]-[6] is one of the most accurate and efficient models to represent the vector hysteretic property of silicon steel sheet and is expected to be applied in finite element electromagnetic field analysis.

However, generally the differentiation of the vector hysteretic function is required for the Newton-Raphson iteration, is not always easy. This paper discusses the differentiation of vector hysteretic function represented by the vector play model, where an equivalent representation is proposed for easy implementation of the differentiation procedure.

II. A VECTOR PLAY MODEL FOR FINITE-ELEMENT ANALYSIS

A. The differentiation of the isotropic vector hysteretic function represented by a play model

An isotropic vector play model [6] with an input of magnetic flux density \( B \) is given as:

\[
H = P(B) = \int_{\zeta_0}^{\zeta} f(\zeta', p_c(\zeta)) d\zeta'
\]

(1)

\[
f(\zeta', p) = f(\zeta', |p|) \frac{p}{|p|}
\]

(2)

where \( f(\zeta', p) \) is a shape function, \( B_s \) is the saturation magnetic flux density, and \( p_c \) is a vector play hysteron with radius \( \zeta \). The hysteron \( p_c \) is given as:

\[
p_c(\zeta') = \begin{cases} 
\frac{p^0}{B - p^0} & \text{if } |B - p^0| < \zeta \\
\frac{p^0}{B - p^0} & \text{if } |B - p^0| \geq \zeta
\end{cases}
\]

(3)

where \( p^0 \) is \( p_c \) at the previous time-point. The Newton-Raphson procedure requires the differentiation of (1). When \( |B - p^0| < \zeta \), \( \partial p_c / \partial B \) becomes 0 because \( p_c = p^0 \). When \( |B - p^0| > \zeta \), from (3) and:

\[
\frac{\partial B - p^0}{\partial B} = \frac{1}{|B - p^0|} (B - p^0)^\dagger,
\]

(4)

\[
\frac{\partial p_c}{\partial B} \text{ is obtained as:}
\]

\[
\frac{\partial p_c}{\partial B} = \begin{cases} 
1 & \text{if } |B - p^0| < \zeta \\
1 + \frac{\zeta}{|B - p^0|} (B - p^0)(B - p^0)^\dagger & \text{if } |B - p^0| \geq \zeta
\end{cases}
\]

(5)

where \( 1 \) is the unit matrix. The differentiation \( \partial p_c / \partial B \) is discontinuous when \( |B - p^0| = \zeta \).

From (2), (5) and:

\[
\frac{\partial}{\partial B} \left[ \frac{P_c^T}{P_c} \right] = \frac{p_c^T \partial p_c}{\partial B},
\]

(6)

the differentiation of \( f \) is given as:

\[
\frac{\partial f}{\partial B} = \left( \frac{\partial f}{\partial |p|} \right) \frac{1}{|P_c|} P_c^T + \frac{f}{|P_c|} \frac{1}{|P_c|} \frac{\partial p_c}{\partial B}.
\]

(7)

Because of the term \( 1/|P_c| \), the differentiation of \( f \) becomes numerically difficult around \( p = 0 \). This occurs in the demagnetization state. To avoid this difficulty, we propose an equivalent form described by:

\[
f(\zeta', p) = f_s(\zeta', |p|^2) p
\]

(8)

\[
f_s(\zeta', q) = f_s(\zeta', p^2) = f(\zeta', p)/p
\]

(9)

where \( q = p^2 \). The differentiation of \( f \) simply becomes:

\[
\frac{\partial f}{\partial B} = 2 \frac{\partial f_s}{\partial q} P_c^T + f_s \frac{\partial p_c}{\partial B}
\]

(10)

which does not require the term \( 1/|P_c| \).

The differentiation of \( P \) is given as:
\[ \frac{\partial P}{\partial B} = \int_0^{\zeta} \frac{\partial f}{\partial B} d\zeta. \]  

Ref. [1] proposed a modified vector play model as:

\[ P' = P_r + r(B) P_p \]  

(12)

where \( r(B) \) is the ratio of the measured hysteresis loss to the simulated loss given by \( P \) for the rotational input with amplitude \( B \), and \( P_r \) and \( P_p \) are the parallel and vertical components of \( P \) to \( B \), which are:

\[ P_r = \frac{(P \cdot B)B}{|B|^2} \]  

(13)

\[ P_p = P - P_r = P - \frac{(P \cdot B)B}{|B|^2}. \]  

(14)

Consequently, the differentiations of \( P_r \) and \( P_p \) are given by:

\[ \frac{\partial P_r}{\partial B} = \frac{1}{|B|^2} \left( B^T \frac{\partial P}{\partial B} + P^* \right) + \frac{(P \cdot B)B}{|B|^2} \]  

\[ \frac{\partial P_p}{\partial B} = \frac{\partial P}{\partial B} - \frac{\partial P_r}{\partial B}. \]  

(15)

The differentiation of \( P' \) is obtained as:

\[ \frac{\partial P'}{\partial B} = (1-r) \frac{\partial P}{\partial B} + r \frac{\partial P}{\partial B} + \frac{\partial r}{\partial B} \frac{\partial P}{\partial B} B^*. \]  

(17)

Because of the term \( 1/|B| \), the differentiation of \( P' \) becomes numerically difficult around \( B = 0 \). However, this difficulty is removed by fixing \( r(B) \) to 1 around \( B = 0 \) so that \( P' \) becomes \( P \).

The Jacobian matrices \( \frac{\partial P}{\partial B} \) and \( \frac{\partial P}{\partial B} \) are not symmetric. Consequently, the Newton-Raphson iteration requires the solution of a non-symmetric linear system.

\[ P_{at}(B) = W_B(B)P^* \]  

where \( W_B \) is an anisotropy matrix for which the components are single-valued functions of \( B \). The differentiation of \( P_{at} \) is given as follows. The anisotropy matrix \( W_B(B) \) is given as:

\[ W_B(B) = \text{diag}(w_s(B, \varphi_B), w_v(B, \varphi_B)) \]  

(20)

where \( \varphi_B = \tan^{-1}(B_y/B_x) \). The differentiation of \( \varphi_B \) is:

\[ (B_y \frac{\partial w_s}{\partial B} + B_x \frac{\partial w_v}{\partial B}) \]  

(21)

Consequently, the differentiation of \( W_B \) is obtained as:

\[ \frac{\partial w_s}{\partial B} = \frac{\partial w_s}{\partial B} - \frac{\partial w_v}{\partial B} \]  

(22)

\[ \frac{\partial w_v}{\partial B} = \frac{\partial w_v}{\partial B} + \frac{\partial w_s}{\partial B}. \]  

(23)

Similarly, \( w_s \) and \( w_v \) are fixed to 1 around \( B = 0 \) to avoid using the term \( 1/|B| \).

\[ \frac{\partial P}{\partial B}, \frac{\partial P}{\partial B} \]  

(18)

where \( \Delta \zeta = B_0/N_p \) and \( N_p \) is the number of hysteron. Because of the limitation of magnetic property measurement, \( B_0 \) and \( N_p \) are set to 1.7 T and 34. Fig. 1 shows the value of shape function \( f(\zeta, p) \Delta \zeta \) and \( f(\zeta, p) \Delta \zeta \) with \( \zeta = 0, B_0/2 \), where \( f(\zeta, 0) \Delta \zeta \) is obtained by extrapolation. They have negative values when \( \zeta > 0 \) because the input of this play model is not \( H \) but \( B \). Figs. 2 and 3 show the \( BH \) loops and \( \partial P/\partial B \) for the alternating sinusoidal input of \( B \) with amplitudes of 0.5, 1.0 T. The proposed form (8) gives almost the equivalent property to that represented by the conventional form (2). The shape function \( f \) gives step-wise waveform of \( \partial P/\partial B \), because \( f \) is piece-wise linear, which almost agrees with \( \partial P/\partial B \) given by the formulation using \( f_0 \).

\[ \frac{\partial P}{\partial B}, \frac{\partial P}{\partial B} \]  

(19)

C. Anisotropic vector hysteresis model

An anisotropic vector model has been proposed as [6]:

\[ \frac{\partial P}{\partial B} , \frac{\partial P}{\partial B} \]  

(18)

where \( \Delta \zeta = B_0/N_p \) and \( N_p \) is the number of hysteron. Because of the limitation of magnetic property measurement, \( B_0 \) and \( N_p \) are set to 1.7 T and 34. Fig. 1 shows the value of shape function \( f(\zeta, p) \Delta \zeta \) and \( f(\zeta, p) \Delta \zeta \) with \( \zeta = 0, B_0/2 \), where \( f(\zeta, 0) \Delta \zeta \) is obtained by extrapolation. They have negative values when \( \zeta > 0 \) because the input of this play model is not \( H \) but \( B \). Figs. 2 and 3 show the \( BH \) loops and \( \partial P/\partial B \) for the alternating sinusoidal input of \( B \) with amplitudes of 0.5, 1.0 T. The proposed form (8) gives almost the equivalent property to that represented by the conventional form (2). The shape function \( f \) gives step-wise waveform of \( \partial P/\partial B \), because \( f \) is piece-wise linear, which almost agrees with \( \partial P/\partial B \) given by the formulation using \( f_0 \).
The differentiation of $P_m$ is obtained as:

$$\frac{\partial P_m}{\partial B} = \left( P_x^*, \frac{\partial \omega_x}{\partial B} \right) + W_0 \frac{\partial P^*}{\partial B}. \tag{24}$$

III. APPLICATION TO FINITE ELEMENT ANALYSIS

A. Simulation of AC anisotropic vector hysteretic property of non-oriented silicon steel sheet.

Using the vector play model, Ref. [7] developed an accurate and efficient AC anisotropic vector hysteresis model for non-oriented steel sheets up to a frequency of 200 Hz, where a dynamic term is added for the representation of the eddy-current field. However, its representation at higher frequencies is not very accurate because it ignores the phase lag of the eddy-current field with respect to $dB/dt$. To describe the influence of the eddy-current field accurately, several hysteresis models include one-dimensional analysis along the sheet thickness direction [8], [9].

In this subsection, the one-dimensional finite element analysis is combined with the vector play model to represent the AC anisotropic vector hysteretic property of non-oriented silicon steel sheet.

The vector field in the steel sheet is described by the vector potential $A = (A_x(z), A_y(z), 0)$, where $z$ is the coordinate along the direction of thickness. The governing equations are given as:

$$-\frac{\partial H_z(B_x, B_y)}{\partial z} = -\sigma \frac{\partial A_x}{\partial t},$$

$$\frac{\partial H_y(B_x, B_y)}{\partial z} = -\sigma \frac{\partial A_y}{\partial t}$$

$$B_x = -\frac{\partial A_y}{\partial z}, \quad B_y = \frac{\partial A_x}{\partial z} \tag{25}$$

where $\sigma$ is the conductivity. The anomalous eddy-current loss is ignored because it is small under large rotational flux conditions [7].

The rotational magnetic property of NO silicon steel sheet JIS: 35A230 is measured and simulated under the rotational magnetic flux condition. The sheet thickness and conductivity are 0.35 mm and 1.8 $\times 10^6$ S/m. Its circular sample having slits at the periphery is excited by a stator of a single-phase, two-pole induction motor [12]. A digital feedback waveform control achieves the circular loci of $B$ with an error of less than 1 %.

Fig. 4 portrays the measured and simulated loci of the surface magnetic field vector at 200 and 500 Hz with an amplitude of 1 - 1.4 T. The anisotropic rotational hysteretic property almost agrees with the measured one. A small discrepancy is observed between the measured and simulated loci mainly because the representation of anisotropy using (19) and (20) is too simple.

Combined with the line search method [10], the Newton-Raphson iteration converges successfully.

B. Two-dimensional Finite Element Analysis

The vector play model is implemented in a simple two-dimensional finite-element analysis to confirm the numerical feasibility of the proposed model.
amplitude of 1.0, 2.0 and 4.0 A. Fig. 7(b) shows the $\Phi/l_I-I$ relationship when the inductor is excited by the voltage source as shown in Fig. 5(b), where $E/l_I = 1, 1.3, 1.6 \text{kV/m}$ and $R/l_I = 40 \Omega/m$. The Newton-Raphson iteration assisted by the line search method converges successfully with 5 to 6 iterations on average, where a direct linear system solver is used.

IV. CONCLUSIONS

The differentiation scheme of a vector hysteretic function represented by the vector play model is improved for efficient nonlinear electromagnetic field computation using the Newton-Raphson method. The finite-element method is successfully applied to the eddy-current analysis of iron-cores with an anisotropic vector hysteretic property.

Fig. 7 Relationship between $I$ and $\Phi/I$, of iron-cored inductor: (a) excited by a sinusoidal current source, and (b) excited by a sinusoidal voltage source.

Fig. 8 Three-phase iron-cored Inductor.

Next, the three-phase iron-cored inductor shown in Fig. 8 is analyzed in a similar way to the single-phase case. The excitation frequency is 50 Hz and the number of turns is 100. Fig. 9 portrays the loci of $B$ and $H$ at points A, B and C in Fig. 8 when the inductor is excited by a three-phase current source with amplitude of 1 A. A rotational field is observed at point B. The Newton-Raphson iteration assisted by the line search method converges successfully with about 6 iterations on average. Fig. 10 shows the convergence of the Newton-Raphson iterations for the first five time-steps, where a nearly quadratic convergence is achieved.

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VI. REFERENCES