

**RIMS Kôkyûroku Bessatsu B5**

*Algebraic Analysis and  
the Exact WKB Analysis for Systems of  
Differential Equations*

January, 2008

Research Institute for Mathematical Sciences  
Kyoto University

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for Systems of Differential Equations*

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*Kyoto University, Kyoto, Japan*

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## PREFACE

This volume presents a collection of research and survey articles which were contributed by invited speakers in the RIMS workshop “Algebraic Analysis and the Exact WKB Analysis for Systems of Differential Equations” held at Research Institute for Mathematical Sciences (RIMS), Kyoto University from 11 through 14 December, 2006, in which experts from many areas of mathematics participated.

Algebraic analysis and the exact WKB analysis have been extensively used to explore structure of solutions of differential equations. The aim of the workshop was to give an overview of present stage of researches in these fields and obtain prospects for the future research. We are quite encouraged by the fact that not only specialists, but many young researchers and students participated in the workshop and enjoyed discussions of exchanging research ideas.

The workshop was supported by RIMS and it was held in the period of the chief organizer (T. A.) being a Specially Appointed Professor of RIMS. We would like to express our sincere gratitude to RIMS and, in particular, to the secretary staffs of RIMS.

Takashi AOKI

Naoto KUMANO-GO

Susumu YAMAZAKI

# PROGRAM

RIMS Workshop on

## Algebraic Analysis and the Exact WKB Analysis for Systems of Differential Equations

Organizers: Takashi AOKI (Kinki, Osaka/RIMS, Kyoto)

Susumu YAMAZAKI (Nihon, Tokyo)

Naoto KUMANO-GO (Kogakuin, Tokyo)

December 11 (Mon)–December 14 (Thu), 2006

Lecture Hall (Room No. 115) of RIMS, Kyoto University

### December 11, Monday

14:00–14:50 Hideshi YAMANE (Kwansei, Hyogo)

Nonlinear wave equations and Fuchsian equations

15:00–15:50 Yayoi NAKAMURA (Kinki, Osaka)

On annihilators of an algebraic local cohomology

16:00–16:50 Naofumi HONDA (Hokkaido)

On the determination of normal or dotted lines of Stokes geometries

### December 12, Tuesday

10:00–10:50 Yoshishige HARAOKA (Kumamoto)

Studies on deformation of Fuchsian systems from the viewpoint of rigidity

11:00–11:50 Kouichi TAKEMURA (Yokohama)

Finite-gap potential and Heun's differential equation

14:00–14:50 Takuya WATANABE (Tohoku, Sendai)

Adiabatic transition probability for an avoided crossing with interaction parameters

15:00–15:50 Takashi AOKI (Kinki, Osaka/RIMS, Kyoto)

Multiple scale analysis for higher order Painlevé equations

16:00–16:50 Takahiro KAWAI (RIMS, Kyoto)

Yoshitsugu TAKEI (RIMS, Kyoto)

Structure of instanton-type solutions of higher order Painlevé equations near a simple turning point of the first kind

**December 13, Wednesday**

- 10:00–10:50 Takayuki IIZUKA (Kinki, Osaka)  
Stokes graphs of second order Fuchsian differential equations with five regular singular points
- 11:00–11:50 Masafumi YOSHINO (Hiroshima)  
Divergence and a resummation in normal form theory of vector fields
- 14:00–14:50 Katsunori IWASAKI (Kyushu, Hakata)  
Algebraic analysis of the sixth Painlevé equation
- 15:00–15:50 Yusuke SASANO (Tokyo)  
Coupled Painlevé systems with affine Weyl group of type  $D_n^{(1)}$
- 16:00–16:50 Tatsuya Koike (Kyoto)  
Higher order Painlevé equations and the degenerate Garnier systems

**December 14, Thursday**

- 10:00–10:50 Yasunori OKADA (Chiba)  
Remarks on the kernel theorems in hyperfunctions (joint work with O. LIESS (Bologna))
- 11:00–11:50 Susumu YAMAZAKI (Nihon, Tokyo)  
Remark on division theorem of ultradistributions by Fuchsian differential operator
- 14:00–14:50 Yutaka MATSUI (Tokyo)  
Topological Radon transforms and their applications (joint work with Kiyoshi TAKEUCHI (Tsukuba))
- 15:00–15:50 Naoto KUMANO-GO (Kogakuin, Tokyo)  
Path integrals and semi-classical approximation (joint work with Daisuke FUJIWARA (Gakushuin, Tokyo))
- 16:00–16:50 Hiroshi YAMAZAWA (Caritas, Yokohama)  
On true solutions with Gevery type Asymptotic expansion of some nonlinear partial differential equations in the complex domain

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