Towards narrowing unexpected issues in future earthquakes: a review

Izuru Takewaki
Dept of Architecture and Architectural Engineering, Kyoto University, Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8540, Japan
E-mail: takewaki@archi.kyoto-u.ac.jp

Abstract

When we encounter a devastating earthquake disaster, we have upgraded the earthquake resistant design codes in the long history of earthquake structural engineering. However the repetition of this action does never resolve the essential problem. This is because building structures and input ground motions have various complex uncertainties and unexpected phenomena often occur. The 2011 off the Pacific coast of Tohoku earthquake also provided some unexpected phenomena. This review paper discusses how to narrow unexpected issues in future earthquakes by referring to several concepts. Critical excitation methods, info-gap theories for uncertainty representation and interval analysis methods are the principal concepts.

1. Background

The 2011 off the Pacific coast of Tohoku earthquake is believed to have changed the common sense in the community of earthquake structural engineering (AIJ 2011, 2012, NIED 2011, Takewaki et al. 2011, Takewaki 2011). We were faced with a real situation that a mega-earthquake repeats at the same place every more than 1000 years. It should be reminded at the same time that the action of building construction is a result of an economic activity. Therefore the cost should be taken into account in balance with the safety. This issue is one of the most difficult problems in earthquake structural engineering.

One of the remarkable aspects in the 2011 off the Pacific coast of Tohoku earthquake is the occurrence of multiple sequences of ground motions as shown in Fig.1. Although multiple sequences of ground motions had been considered mostly theoretically (Elnashai et al. 1998, Hatzigeorgiou and Beskos 2009, Moustafa and Takewaki 2010, 2011, 2012), it became actual one in the 2011 Tohoku earthquake. In these circumstances, it seems extremely important to make effort in narrowing unexpected issues for future earthquakes (Elishakoff and Ohsaki 2010, Takewaki et al. 2012). Some of the important subjects are (1) handling of uncertainties in earthquake ground motions and structural parameters, (2) advancement of worst case analysis (e.g.

critical excitation method), (3) incorporation of robustness, redundancy and resilience into the design of structures.

In this review paper, the concept of critical excitation is explained first in the context of advancement of worst case analysis. Then the issue of handling of various uncertainties is discussed. Finally some aspects for a new paradigm in earthquake resistant design is investigated, e.g. representation of uncertainty in selecting design ground motions, representation of uncertainty of long-period ground motion by critical excitation theory for earthquake input energy, resonance with earthquake input and improving earthquake resilience of buildings, scenario of increase of credible bound of input energy, incorporation of robustness, redundancy and resilience into the design of structures.

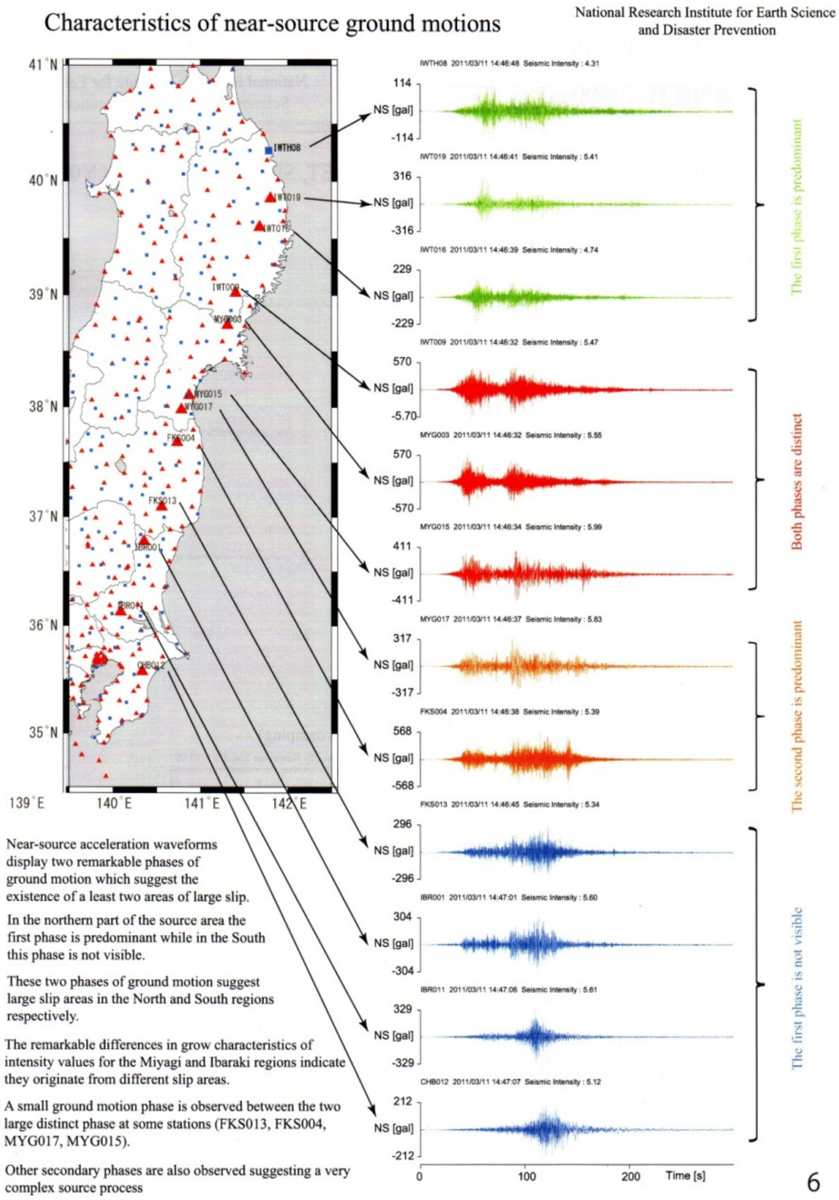

Fig.1 Characteristics of near-source ground motions along Pacific coast in East Japan during the 2011 Tohoku earthquake (NIED 2011)

2. Origin and subsequent development of critical excitation method

The worst case analysis is widely used in the field of engineering design. In the field of earthquake structural engineering, Drenick (Drenick 1970) introduced the concept of 'Model-free design' in 1970. The model-free means that the input motion model is not specified in the design stage and is determined based on some criteria. Shinozuka (Shinozuka 1970) formulated the same problem in the frequency domain and derived narrower bound. After Drenick's pioneering work in 1970, many researchers developed various methods.

Iyengar (Iyengar 1970) discussed the resonance and Iyengar and Manohar (Iyengar and Manohar 1987) introduced a stochastic concept in the critical excitation approach.

Regarding to the significance of critical excitation methods, Drenick (1977a) also pointed out that the combination of probabilistic approaches with worst-case analyses should be employed to make the seismic resistant design more robust. He explained that the data used in the calculation of failure probabilities, usually very small numbers, in the seismic reliability analysis are scarce and the reliable prediction of the failure probability is difficult only by the conventional reliability analysis which requires the tail shapes of probability density functions of disturbances. Practical application of critical excitation methods has then been proposed extensively.

Ahmadi (1979) formulated another critical excitation problem including the response acceleration as the objective function to be maximized. Acceleration is an important performance index together with deformation or displacement. He demonstrated that a rectangular wave in time domain is the critical one and recommended to introduce another constraint in order to make the solution more realistic.

Westermo (1985) considered the input energy from the ground motion to a structure divided by the mass as the objective function in a new critical excitation problem. In order to make the problem well posed, he also imposed another constraint on the time integral of squared input acceleration. He introduced a variational approach and demonstrated that the critical input acceleration is proportional to the response velocity. The damage of structures may be another measure of criticality. The corresponding problems have been tackled by some researchers.

Srinivasan et al. (1991) extended the basic approach due to Drenick (1970) to more general multi-degree-of-freedom (MDOF) models. They used a variational formulation and selected a quantity in terms of multiple responses as the objective function.

It was suggested that the critical excitation introduced by Drenick (1970) is conservative compared to the recorded ground motions. To resolve this problem, Drenick, Wang and their colleagues proposed a concept of "subcritical excitation" (Drenick 1973; Wang et al. 1976; Wang and Drenick 1977; Wang et al. 1978; Drenick and Yun 1979; Wang and Yun 1979; Abdelrahman et al. 1979; Bedrosian et al. 1980; Wang and Philippacopoulos 1980; Drenick et al. 1980; Drenick et al. 1984). They expressed an allowable set of input accelerations as a 'linear combination of recorded ground motions'.

Abdelrahman et al. (1979) extended the idea of subcritical excitation to the method in the frequency domain. An allowable set of Fourier spectra of accelerograms has been expressed as a linear combination of Fourier spectra of recorded accelerograms.

An optimization technique was used by Pirasteh et al. (1988) in one of the subcritical excitation problems. They superimposed some accelerograms recorded at similar sites to construct the candidate accelerograms, then used optimization and approximation techniques in order to find the most critical accelerogram.

Some other key issues are input energy measures, stochastic excitation, convex models, nonlinear or elastic-plastic problems, critical envelope functions and robust structural design. A more comprehensive review of critical excitation can be found in (Takewaki 2002, 2005, 2007, 2008).

3. Handling of uncertainty

3.1 Info-gap theory

Structural engineers often encounter uncertainties in the planning and designing stage of building structures. Some uncertainties come from the definition of design earthquake ground motions and from the specification of temperature change in the life cycle of building structures (Boore 1983, Esteva 1988, Grigoriu et al. 1988, Heaton et al. 1995, Geller et al. 1997, Abrahamson et al. 1998, Stein 2003, Trifunac 2008, Aster 2012, Takewaki and Tsujimoto 2011). Some other uncertainties may result from variability in structural parameters. Those uncertainties can be tackled by probabilistic or possibilistic approaches (Ben-Haim and Elishakoff 1990, Ben-Haim 2002, 2006, Takewaki and Ben-Haim 2005, 2008, Kanno and Takewaki 2006, Takewaki 2009, IUTAM 2009, Takewaki and Fujita 2009, Elishakoff and Ohsaki 2010, Fujita and Takewaki 2010, 2011a, b, 2012a, b, Moustafa et al. 2010). Especially Ben-Haim introduced a new concept of ‘Info-Gap’ theory and formulated a robustness of structures in a logical manner.

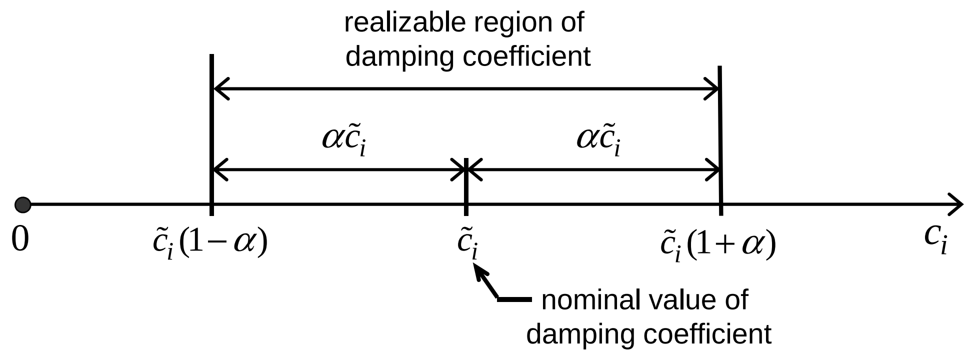

Fig.2 Uncertain damping coefficient with unknown horizon of uncertainty α

As a simple example, let us consider a vibration model with viscous damping systems in addition to masses and springs. It is well understood in the field of structural control and health monitoring that viscous damping coefficients $\mathbf{c} = \{c_i\}$ in a vibration model are very uncertain in comparison with masses and stiffnesses $\mathbf{k}$. By using a method for describing such uncertainty, the uncertain viscous damping coefficient can be expressed in terms of the nominal value $\tilde{c}_i$ and the unknown uncertainty level α as shown in Fig.2.

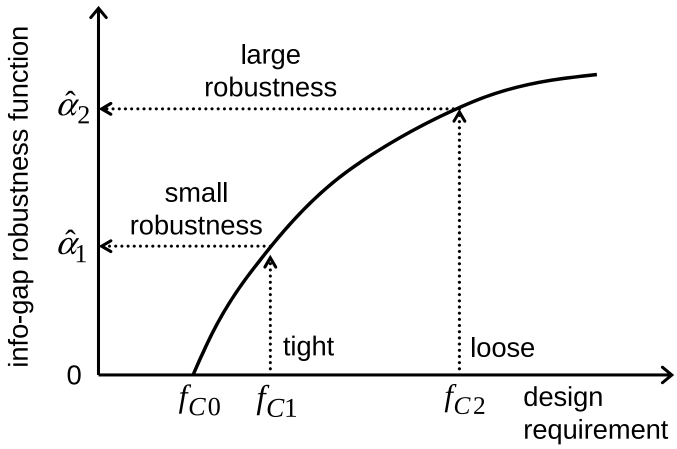

Fig.3 Info-gap robustness function $\hat{\alpha}$ with respect to design requirement f_{C}

The info-gap uncertainty analysis was introduced by Dr. Ben-Haim (Ben-Haim 2002, 2006) for measuring the robustness of a structure subjected to external loads. Simply speaking, the info-gap robustness is the greatest horizon of uncertainty, α, up to which the performance function $f(\mathbf{c},\mathbf{k})$ does not exceed a critical value, f_{C}. This info-gap robustness function is denoted by $\hat{\alpha}$. The performance function may be a peak displacement, peak stress or earthquake input energy, etc.

Let $f_{\mathrm{C0}} = f(\tilde{\mathbf{c}},\mathbf{k})$ denote the value of the performance function for the nominal damping coefficients $\tilde{\mathbf{c}}$. Since the info-gap robustness function $\hat{\alpha}$ is a function of $\mathbf{k}$ and f, it is denoted by $\hat{\alpha}(\mathbf{k}, f)$. Then one can show that $\hat{\alpha}(\mathbf{k}, f_{C0}) = 0$ for the specific value f_{C0}, as shown in Fig.3. Furthermore let us define $\hat{\alpha}(\mathbf{k}, f_{\mathrm{C}}) = 0$ if $f_{\mathrm{C}} \le f_{\mathrm{C0}}$ (see Fig.3). This means that, when the performance requirement is too small (tight), we cannot satisfy the performance requirement for any admissible damping coefficients. Fig.3 also shows examples of small and large robustness.

3.2 Interval analysis

Under uncertain circumstances, it is necessary to evaluate a peak response of an uncertain building structure subjected to an uncertain input. The interval analysis is one of the most reliable methods for evaluating the extreme responses under such uncertain circumstances (Chen and Wu 2004a, Liang et al. 2007, Henriques 2008, Moore 1966, Alefeld and Herzberger 1983).

There are many researches on the interval analysis. The concept of interval analysis was introduced by Moore in 1966. After almost two decades, Alefeld and Herzberger (1983) have done the pioneering work. They investigated linear interval equations, nonlinear interval equations and interval eigenvalue analysis by developing interval arithmetic. The interval arithmetic is a mathematical rule for the sets of intervals which appeared in 1924. Since their innovative achievements, the interval arithmetic algorithm has been used.

Recently Qiu et al. (1996) have applied the interval arithmetic algorithm to obtain the bounds of static response of a structure by using a convergent series expansion of the uncertain structural response. Qiu and Elishakoff (1998) have proposed an application of the interval arithmetic algorithm by taking advantage of Neumann series expansion of the inverse stiffness matrix. Mullen and Muhanna (1999) have introduced the bounds of the static structural response for all possible loading combinations using the interval arithmetic. The related works of the interval analysis for the static response or eigenvalue have been conducted by many researchers (Dong and Shah 1987, Koyluoglu and Elishakoff 1998, El-Gebeily et al. 1999, McWilliam 2001, Chen et al. 2003, Qiu 2003, Degrauwe et al. 2010, Hanss 2002, Moens and Vandepitte 2004, Donders et al. 2005). A comprehensive overview of the recent state of the art for the interval analysis has been provided by Moens and Hanss (2011).

In recent years, more practical and extensively applicable methods have been developed. The interval analysis using Taylor series expansion has been proposed by Chen et al. (2002), Chen and Wu (2004b) and Chen et al. (2009). In the early stage, first-order Taylor series expansion was introduced for the problems of static response and eigenvalue. Chen *et al.* (2009) devised an advanced matrix perturbation method using second-order Taylor series expansion and obtained approximate bounds of the objective function without interval arithmetic. They pointed out that the computational effort can be reduced from the number of calculation 2^N (N: number of interval parameters) to $2N$ by neglecting the non-diagonal elements of the Hessian matrix of the objective function with respect to interval parameters. Although neglecting the non-diagonal elements enhances the algorithm efficiency, the deterioration of accuracy should be checked carefully. Another efficient algorithm was also proposed by Donders et al. (2005).

Figs.4(a) and (c) present a monotonic contour map and its 3D view of the objective function for two design variables X_1, X_2. In this case, an exact solution for the peak response is included in the combination of end points of interval parameters. Therefore an exact solution can be obtained by conducting the response analysis for all the combinations of end points $\overline{X}_i = X_i^c + \Delta\overline{X}_i$ and $\underline{X}_i = X_i^c - \Delta\underline{X}_i$ of interval parameters. The symbol $(\ )^c$ denotes the nominal value of an interval parameter. When the number of uncertain parameters is given by N_x, the computational load (number of repetition) is given by 2^{N_x}. However this repetition becomes huge for a large number of interval parameters.

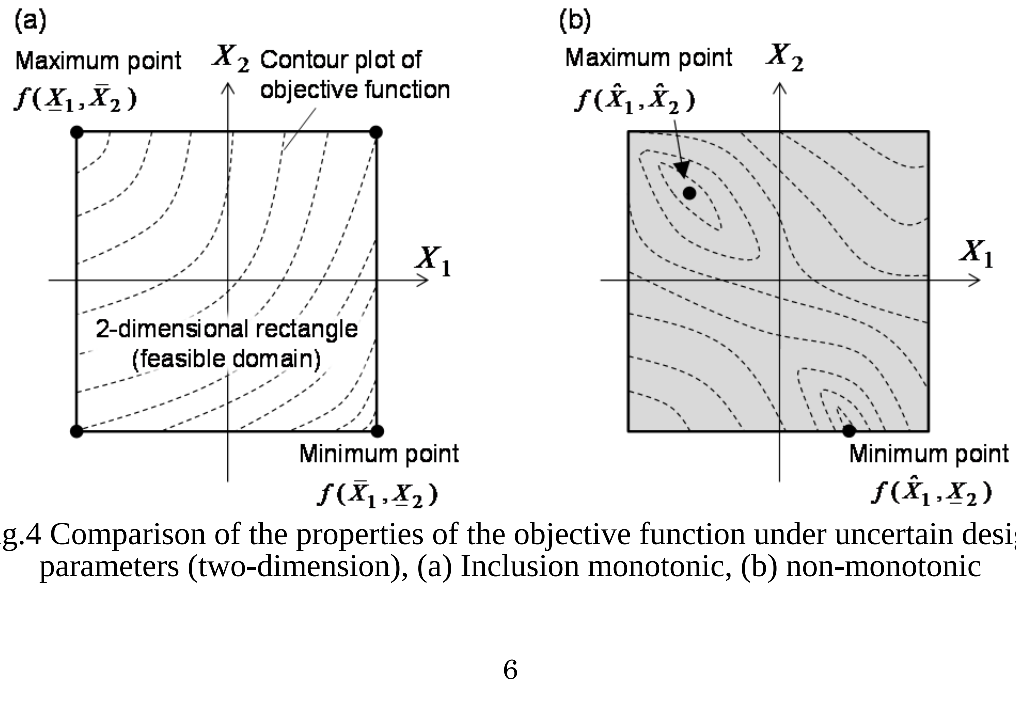

Fig.4 Comparison of the properties of the objective function under uncertain design parameters (two-dimension), (a) Inclusion monotonic, (b) non-monotonic

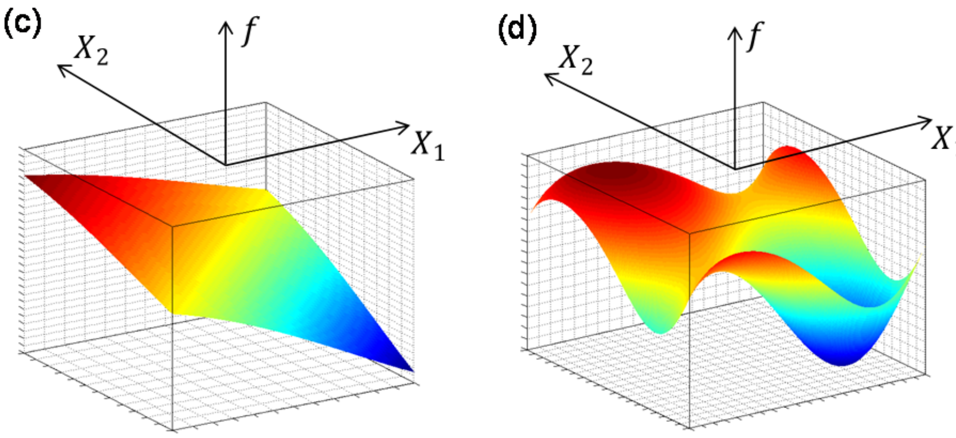

Fig.4 (continued) 3D view of the objective function under uncertain design parameters, (c) Inclusion monotonic, (d) non-monotonic

On the other hand, when the basic assumption "*inclusion monotonic*" cannot be applied to the objective function, it is not appropriate to evaluate the objective function only at the end points of interval parameters. Figs.4(b) and (d) show a contour map and its 3D view of the non-monotonic objective function. In this case, a sequential quadratic programming method or a response surface method has to be introduced in transforming the original problem of finding the upper and lower bounds of the objective function into an optimization problem. This procedure causes a large amount of computational work.

To overcome this problem, Fujita and Takewaki developed a new method called the URP (Updated Reference-Point) method (Fujita and Takewaki 2011a, b, 2012a, b). By using this method, the response of a structure under various uncertainties can be evaluated.

At the final stage of design, a decision has to be made in view of various design objectives.

4. Acceleration and velocity powers for scaling earthquake ground motions

The degree of uncertainty in earthquake ground motions is extremely large compared to structural parameters. The scaling of earthquake ground motions should be discussed in view of its physical realization and its influence on building structures. Fig.1 shows the characteristics of near-source ground motions along Pacific coast in East Japan during the 2011 off the Pacific coast of Tohoku earthquake. It can be found out that two or more series (or groups) of waves exist in some areas and most ground motions continue for over 2 minutes. This implies the repeated occurrence of the fault slips in wide areas. This phenomenon has been pointed out by many researchers (for example, Elnashai et al. 1998, Hatzigeorgiou and Beskos 2009, Moustafa and Takewaki 2010, 2011, 2012).

The acceleration and velocity powers (Drenick 1970, Arias 1970, Housner and Jennings 1975, Takewaki 2007, Takewaki and Tsujimoto 2011) of a ground acceleration $\ddot{u}_g(t)$ are defined by

$$\int_{-\infty}^{\infty} \ddot{u}_g(t)^2 dt = \bar{C}_A \tag{1a}$$

$$\int_{-\infty}^{\infty} \dot{u}_g(t)^2 dt = \bar{C}_V . \tag{1b}$$

It is well known that acceleration and velocity powers are closely related to the earthquake input energy (Housner and Jennings 1975, Takewaki 2007). It is further understood that the resonant sinusoidal motion can be an approximate critical excitation to elastic and inelastic structures under the constraint of acceleration power or velocity power (Drenick 1970, Takewaki 2007, Takewaki and Tsujimoto 2011). Therefore, a resonant sinusoidal motion will be used here as a class of input motions. In addition, the sinusoidal ground motion is used to model the long-period ground motion which is one of the principal topics after the 2011 off the Pacific coast Tohoku earthquake. It is well known that the velocity wave of the long-period ground motion can be well represented by a sinusoidal wave. The remaining issue is how to specify its amplitude and duration.

Let $\ddot{u}_{gs}(t) = a_{\max} \sin \omega_G t$ denote the sinusoidal ground motion acceleration. In this expression $a_{\max}$ and ω_G are the maximum ground acceleration and the circular frequency of the sinusoidal ground motion. The duration and natural period of the ground motion are denoted by t_0 and $T_G = 2\pi/\omega_G$, respectively. Denoting the duration of the ground motion by $t_0 = n \cdot T_G/4\ (n = 1, 2, \cdots)$, the acceleration power and velocity power can be expressed by

$$\bar{C}_A = \int_0^{t_0} \ddot{u}_{gs}(t)^2 dt = \frac{{a_{\max}}^2}{2} t_0 \tag{2a}$$

$$\bar{C}_V = \int_0^{t_0} \dot{u}_{gs}(t)^2 dt = \frac{{v_{\max}}^2}{2} t_0 , \tag{2b}$$

where $v_{\max} = a_{\max} / \omega_G$ is the maximum ground velocity.

Let t_A and t_V denote specific times before the ending time t_0 of input motion. The ratios $\bar{a}(t_A)$ and $\bar{v}(t_V)$ are defined by

$$\bar{a}(t_A) = \frac{\int_0^{t_A} \ddot{u}_g(t)^2 dt}{\int_0^{t_0} \ddot{u}_g(t)^2 dt} \tag{3}$$

$$\bar{v}(t_V) = \frac{\int_0^{t_V} \dot{u}_g(t)^2 dt}{\int_0^{t_0} \dot{u}_g(t)^2 dt} . \tag{4}$$

The times t_{A10} and t_{A90} denote the times corresponding to $\bar{a}(t_{A10}) = 0.1$ and $\bar{a}(t_{A90}) = 0.9$, respectively, and the times t_{V10} and t_{V90} denote the times corresponding to $\bar{v}(t_{V10}) = 0.1$ and $\bar{v}(t_{V90}) = 0.9$, respectively. The effective duration of primary (intensive) ground motion is defined by the acceleration point of view as ${}_e t_{A0} = t_{A90} - t_{A10}$ or the velocity point of view as ${}_e t_{V0} = t_{V90} - t_{V10}$. An example of the effective duration ${}_e t_{A0} = t_{A90} - t_{A10}$ based on the acceleration power is shown in Fig.5.

Table 1 Acceleration power, velocity power and effective duration of representative recorded ground motions (Takewaki and Tsujimoto 2011)

Earthquake	Site and component	C_A [m^2/s^3]	C_v [m^2/s]	${}_e t_{A0}$ [s]	${}_e t_{V0}$ [s]
Near fault motion/rock records					
Loma Prieta 1989	Los Gatos NS	49.5	1.49	9.1	5.9
	Los Gatos EW	19.4	0.26	6.1	5.7
Hyogoken-Nanbu 1995	JMA Kobe NS	52.4	0.79	5.8	7.9
	JMA Kobe EW	34.0	0.52	7.5	8.5
Near fault motion/soil records					
Cape Mendocino 1992	Petrolia NS	21.5	0.25	16.0	14.8
	Petrolia EW	23.9	0.51	13.9	5.6
Northridge 1994	Rinaldi NS	25.0	0.62	5.5	4.2
	Rinaldi EW	46.3	1.13	7.0	6.5
	Sylmar NS	31.3	0.86	4.4	3.9
	Sylmar EW	16.3	0.45	5.2	4.6
Imperial Valley 1979	Meloland NS	5.4	0.36	5.5	16.6
	Meloland EW	6.9	1.06	4.8	23.3
Long duration motion/rock records					
Michoacan 1985	Caleta de Campos NS	4.0	0.08	18.9	14.7
	Caleta de Campos EW	2.9	0.04	23.3	23.5
Miyagiken-oki 1978	Ofunato NS	2.4	0.01	11.8	12.1
	Ofunato EW	4.2	0.03	11.8	25.7
Long duration motion/soil records					
Chile 1985	Vina del Mar NS	34.3	0.46	41.5	43.1
	Vina del Mar EW	18.7	0.20	40.7	43.4
Olympia 1949	Seattle Army Base NS	1.3	2.29	28.0	39.6
	Seattle Army Base EW	0.9	0.02	31.8	40.3

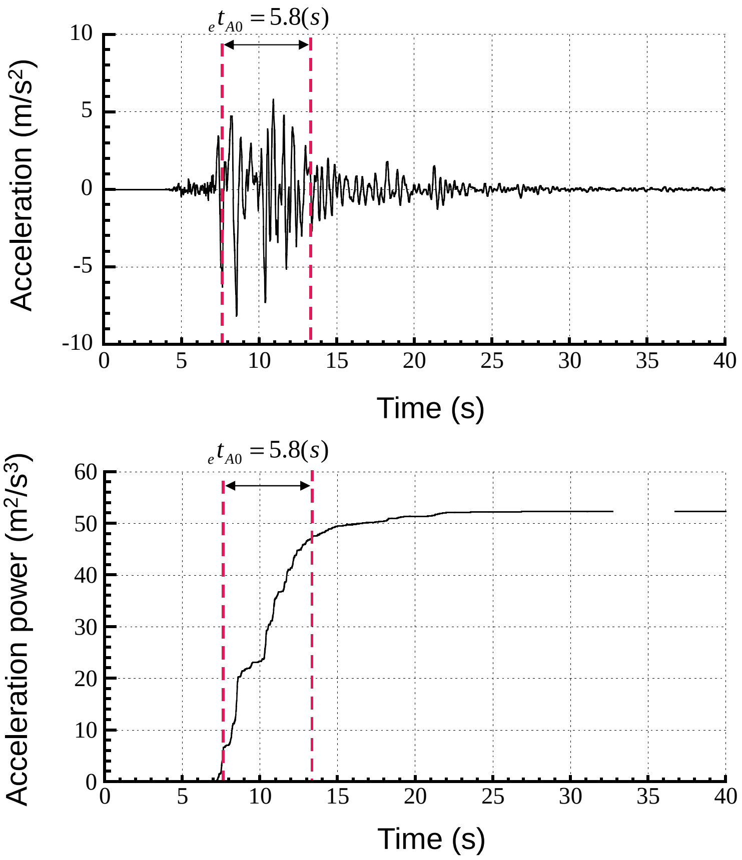

Fig.5 Example of effective duration ${}_e t_{A0} = t_{A90} - t_{A10}$ based on acceleration power (Takewaki and Tsujimoto 2011)

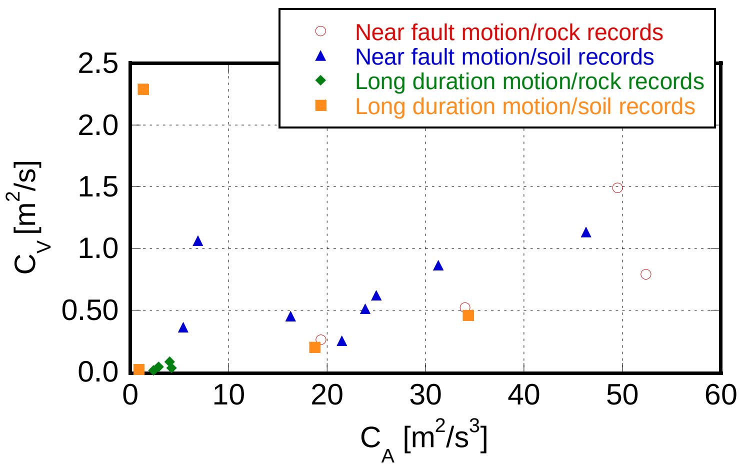

Fig.6 Plot of velocity power versus acceleration power of four classes of recorded ground motions (Takewaki and Tsujimoto 2011)

The duration of the sinusoidal ground motion is determined from the natural period of the objective building structure for treating resonant cases and the data of the effective durations of actual ground motions. The near-field ground motion may be characterized by the period of 0.5s and the duration of 4s (this is critical to the 5-story building model) and the far-field ground motion may be characterized by the period of 2.0s and the duration of 36s (this is critical to the 20-story building model). The acceleration power, velocity power and the effective duration of the representative actual ground motions (Abrahamson et al. 1998) are shown in Table 1 for reference. The corresponding velocity power versus acceleration power of four classes of recorded ground motions is plotted in Fig.6.

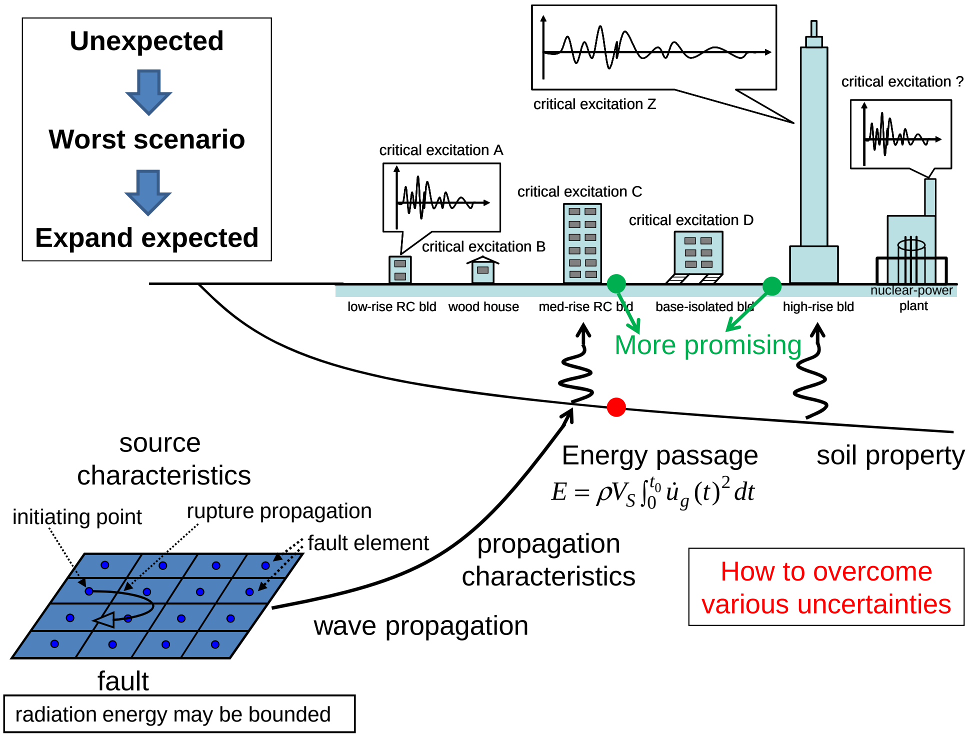

Fig.7 Scenario to overcome various uncertainties in modeling design earthquake ground motions

5. New paradigm in earthquake resistant design

5.1 Representation of uncertainty in selecting design ground motions

Earthquake inputs are uncertain both in epistemic and aleatory sense and it does not appear easy to predict forthcoming events precisely (Geller et al. 1997, Stein 2003, Aster 2012). Near-field ground motions (Northridge in 1994, Kobe in 1995, Turkey in 1999 and Chi-Chi in 1999) and the far-field motions (Mexico in 1985, Tohoku in 2011) have some peculiar, unpredictable characteristics.

In the history of earthquake resistant design of building structures, we learned a lot of lessons from actual earthquake disasters after Nobi earthquake in 1891 (Japan) and San Francisco earthquake in 1906 (USA). After we encounter a major earthquake disaster, we upgraded the earthquake resistant design codes. However the repetition of this action does never resolve the essential problem. To overcome this problem, the concept of critical excitation was introduced as explained in Section 2. Approaches based on the concept of "critical excitation" seem to be promising. Drenick (1970) formulated this problem in a mathematical framework and many researchers followed him. The detailed history can be found in the reference (Takewaki 2007).

In order to take into account highly uncertain long-period ground motions, a new paradigm should be devised. There are various buildings in a city (Fig.7). Each building has its own natural period of amplitude-dependency and its original structural properties. When an earthquake occurs, a variety of ground motions are induced in the city, e.g. combination of body waves (including pulse wave) and surface waves, long-period ground motions. The relation of the building natural period with the predominant period of the induced ground motion may lead to disastrous phenomena in the city (see Fig.7). In other words, the most critical issue in the seismic resistant design is the resonance. Many past earthquake observations demonstrated such phenomena repeatedly, e.g. Mexico in 1985, Northridge in 1994, Kobe in 1995. One of the promising approaches to this is to shift the natural period of the building through seismic control (Takewaki 2009) and to add damping in the building. However it is also true that the seismic control is under development and more sufficient time is necessary to respond to uncertain ground motions.

It is believed that earthquake has a bound on its magnitude and the earthquake energy radiated from the fault has a bound (Trifunac 2008). The problem is to find the most unfavorable ground motion for a building or a group of buildings (see Fig.7). There are two possibilities. One is to define a velocity power at the bottom of the basin based on the fault rupture mechanism and wave propagation characteristics. The other is to set the velocity power at the ground surface level. In the case of definition at the bottom of the basin, the surface ground wave propagation has to be considered properly. However this procedure may include another uncertainty. In this sense, the setting of the velocity power at the ground surface level may be preferable.

The Fourier spectrum of a ground motion acceleration has been proposed at the rock surface depending on the seismic moment M_0, distance R from the fault, etc. (for example Boore 1983).

$$\left|A(\omega)\right| = CM_0 S(\omega,\omega_C) P(\omega,\omega_{\max}) \exp(-\omega R/(2\beta Q))/R \tag{5}$$

In Eq.(5), C: constant, $S(\omega,\omega_C)$: source spectrum $S(\omega,\omega_C) = \omega^2/\{1+(\omega/\omega_C)^2\}$, $P(\omega,\omega_{\max})$: high-cut filter, β: velocity of shear wave at rock, Q: Q-value. Such spectrum may contain uncertainties. One possibility or approach is to specify the acceleration or velocity power (Takewaki 2007) as a global measure and allow the variability of the spectrum. Yazdani and Komachi (2009) and Tokmechi (2011) derived a relation between timeinvariant mean squared value of linear response of a single degree freedom system and seismological variables using Eq.(5). As for the Great East Japan Earthquake, $\left|A(\omega)\right|$ is reported to be about 0.5(m/s) near the fault region. However this treatment has a difficulty in confirming the reliability of the theory and of specification of the fault site. The change of ground motion by surface soil conditions is another difficulty. Based on this observation, a concept of critical excitation is introduced.

A significance of critical excitation is supported by its broad perspective. In general there are two classes of buildings in a city. One is the important building which plays an important role during and after disastrous earthquakes. The other is ordinary building. The former one should not be damaged during an earthquake and the latter one may be damaged to some extent especially for critical excitation larger than code-specified design earthquakes. Just as the investigation on limit states of structures plays an important role in the specification of response limits and performance levels of structures during disturbances, the clarification of critical excitations for a given structure or a group of structures appears to provide structural designers with useful information in determining excitation parameters in a risk-based reasonable way. It is expected that the concept of critical excitation enables structural designers to make ordinary buildings more seismic-resistant and seismic-resilient (Takewaki et al. 2012).

5.2 Representation of uncertainty of long-period ground motion by critical excitation theory for earthquake input energy

The total input energy is an appropriate quantity for evaluating the demand of earthquake ground motions (Housner 1959, Akiyama 1985, Takewaki 2007). It is appropriate from the viewpoint of quantification of uncertainties of ground motions and structures to introduce the credible bounds of the input energy per unit mass E_I / m to a single-degree-of-freedom (SDOF) model for acceleration and velocity constraints. In order to explain the credible bounds of the input energy, let us introduce the energy transfer function $F(\omega)$ defined by

$$F(\omega) = \frac{2h\Omega\omega^2}{\pi\{(\Omega^2 - \omega^2)^2 + (2h\Omega\omega)^2\}} \tag{6}$$

where Ω: natural circular frequency of the SDOF model, h: damping ratio and ω: the excitation frequency. This energy transfer function can be derived as $-\mathrm{Re}[H_V(\omega;\Omega,h)]/\pi$ where $H_V(\omega;\Omega,h)$ is the velocity transfer function defined by $\dot{X}(\omega) = H_V(\omega;\Omega,h)A(\omega)$ ($\dot{X}(\omega)$: Fourier transform of response velocity of the SDOF model, $A(\omega)$: Fourier transform of ground acceleration). The input energy per unit mass E_I / m to the SDOF model can then be expressed by

$$E_I / m = \int_0^\infty |A(\omega)|^2 F(\omega)\mathrm{d}\omega \tag{7a}$$

or

$$E_I / m = \int_0^\infty |V(\omega)|^2 \omega^2 F(\omega)\mathrm{d}\omega \tag{7b}$$

where $V(\omega)$ is the Fourier transform of the ground velocity. This expression has been derived based on the assumption that the base ground movement becomes zero at the end. The credible bounds of the input energy per unit mass E_I / m to an SDOF model for acceleration and velocity constraints can be obtained by modeling the Fourier amplitude of the worst input acceleration and velocity as rectangular ones centered at the natural frequency of the SDOF model (Takewaki 2007). The maximum value of actual Fourier amplitude of a ground motion is used as the height of the rectangular Fourier amplitude. This point could be discussed in more detail for deeper understanding of 'credible bound'. The introduction of Eq.(5) may be promising and an effective strategy for advanced and sophisticated treatment of uncertain ground motions.

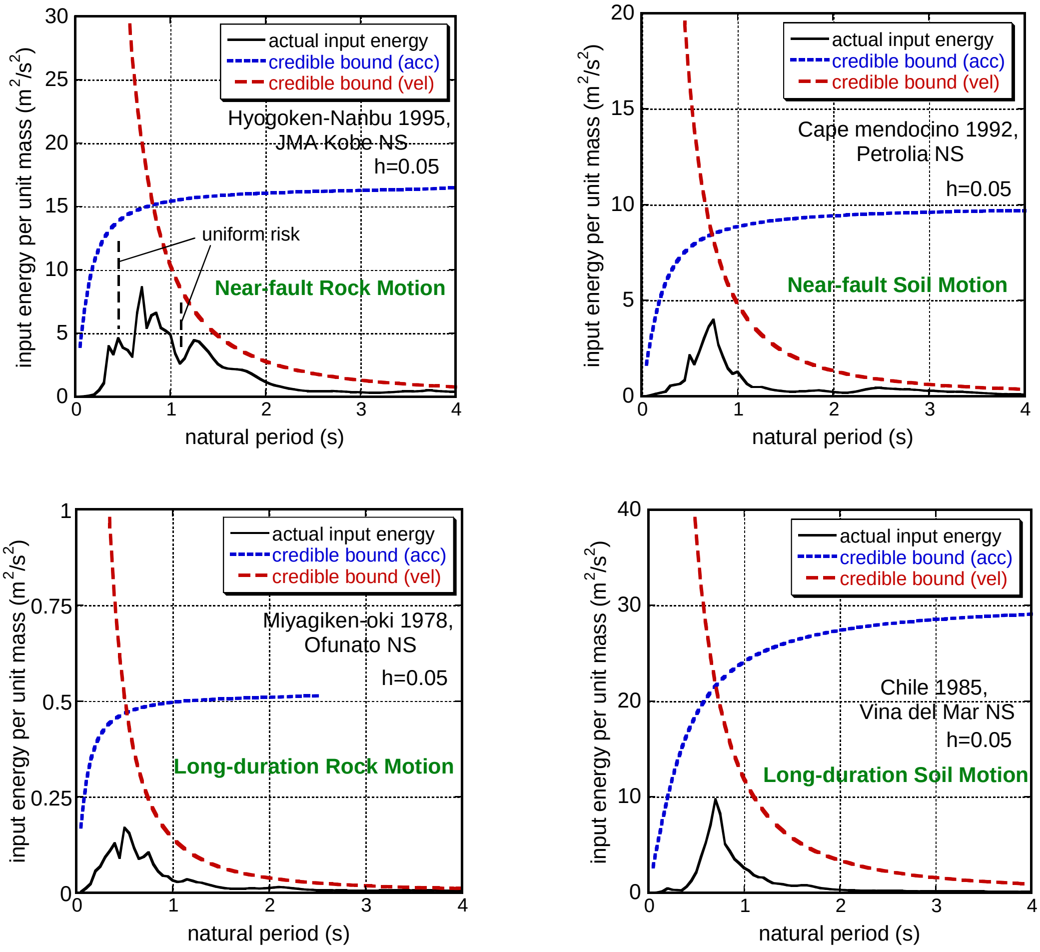

Fig.8(a) Credible bound of input energy for various types of ground motions (near-fault rock motion, near-fault soil motion, long-duration rock motion, long-duration soil motion) (Takewaki 2007)

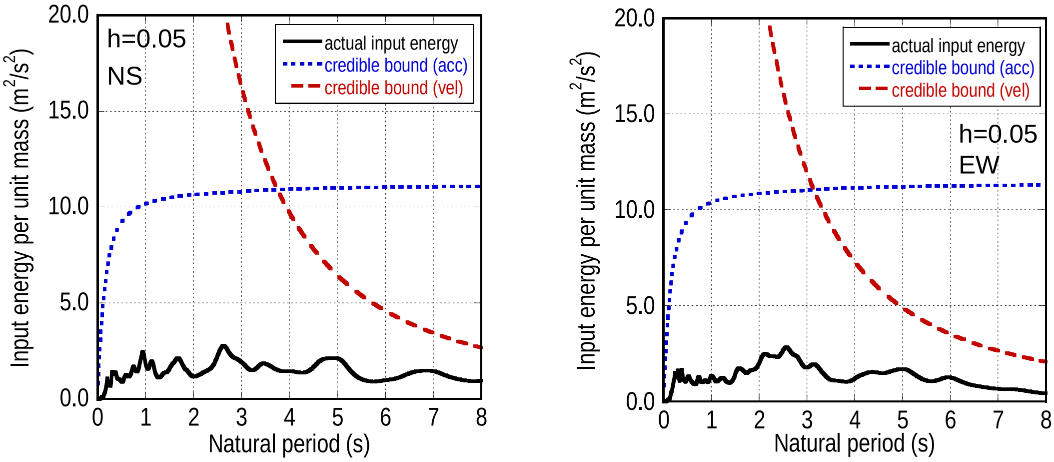

Fig.8(b) Credible bound of input energy for ground motion recorded at K-NET, Shinjuku station (TKY007)

Fig.8(a) presents the credible bounds of input energy for JMA Kobe NS during Hyogoken-Nanbu earthquake 1995, Petrolia NS during Cape Mendocino earthquake 1992, Ofunato NS during Miyagiken-oki earthquake 1978 and Vina del Mar NS during Chile earthquake 1985. It is seen that the property of the uniform risk holds. In other words, the ratio of the actual input energy to the credible bound is almost constant in a wide range of natural period.

On the other hand, Fig.8(b) shows the credible bounds of input energy for ground motions recorded at K-NET, Shinjuku station (TKY007). The ratio of the bound to the corresponding actual input energy is large for these ground motions. This implies that the ground motion of March 11 includes wave components in a broad period range and this contradicts to the procedure of concentrating wave components to one frequency employed in the process of obtaining the critical input. When a ground motion has a remarkable predominant period, the procedure of concentrating wave components to that frequency is acceptable and the ratio of the credible bound to the corresponding actual input energy is rather small. On the other hand, when a ground motion does not have a remarkable predominant period, the procedure of concentrating wave components to that frequency is not acceptable and the ratio of the credible bound to the corresponding actual input energy becomes large.

The critical excitation in terms of the credible bound should be used for important structures, for example, nuclear power plants, super high-rise buildings, hospitals for post-earthquake bases. For these structures, there is no definite design criterion so far. The crisis experienced in March 11, 2011 Tohoku earthquake in nuclear power plants (Fukushima), super high-rise buildings (Tokyo and Osaka), hospitals (Miyagi Prefecture) strongly supports the need of introduction of such reliable methods, even though the safety factor is rather large. Furthermore, uncertainties due to many factors can be overcome by introducing the credible bound of input energy.

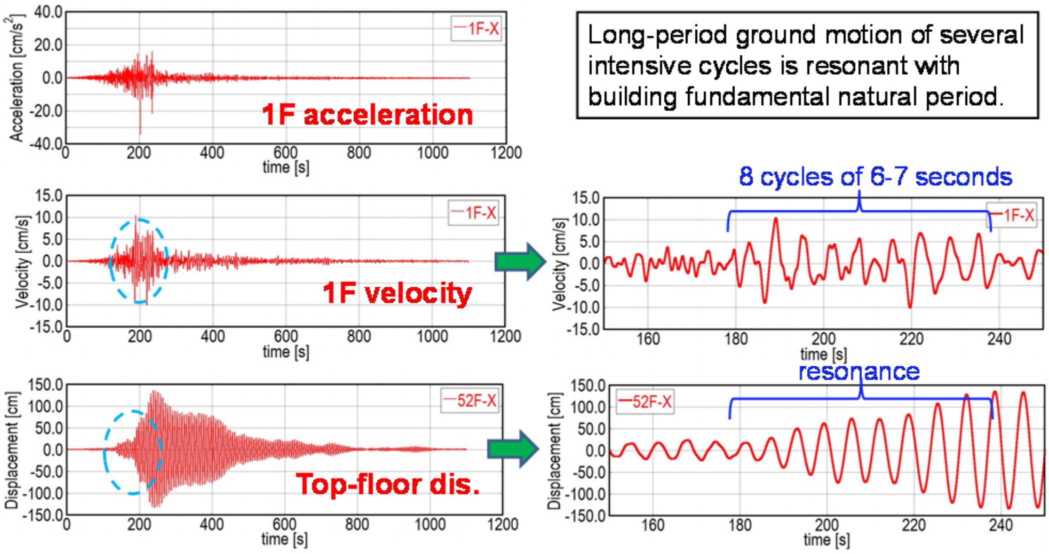

Fig.9 Ground acceleration recorded at the first floor, ground velocity and top-story displacement recorded (or numerically integrated) in the building at Osaka bay area

5.3 Resonance with earthquake input and improving earthquake resilience of buildings

Fig.9 shows the ground acceleration recorded at the first floor, ground velocity and top-story displacement recorded (or numerically integrated) in the building at Osaka bay area. It can be observed that a clear resonance occurs during eight cycles (ground fundamental natural period is $4H/V_s$=4x1.6/1.0=6.4s). It seems that such clear observation has never been reported so far in super high-rise buildings. This implies the need of consideration of long-period ground motions in the seismic resistant design of super high-rise buildings in mega cities even though the site is far from the epicenter. It is also becoming controversial that the expected Tokai, Tonankai and Nankai event is closer to this building (about 160km from the boundary of the fault region) and several times of the ground motion may be induced during that event based on the assumption that body waves are predominant outside of the Osaka basin. Further investigation will be inevitable for retrofit of the building. The seismic retrofit using hysteretic steel dampers, oil dampers and friction dampers is being planned.

5.4 Scenario of increase of credible bound of input energy

In the long-period ground motions, it is reported that those motions continue for more than 10 minutes and it may be possible for more than 20 minutes depending on the combination and order of the subsequent fault ruptures. It is also reported that the predominant period of those motions depends on the size of the fault ruptures. In these situations, it may be appropriate to consider those parameters as those with variability. As for structural parameters, the natural period and damping ratio of building structures have certain variability and the structural designers should take into account those variability.

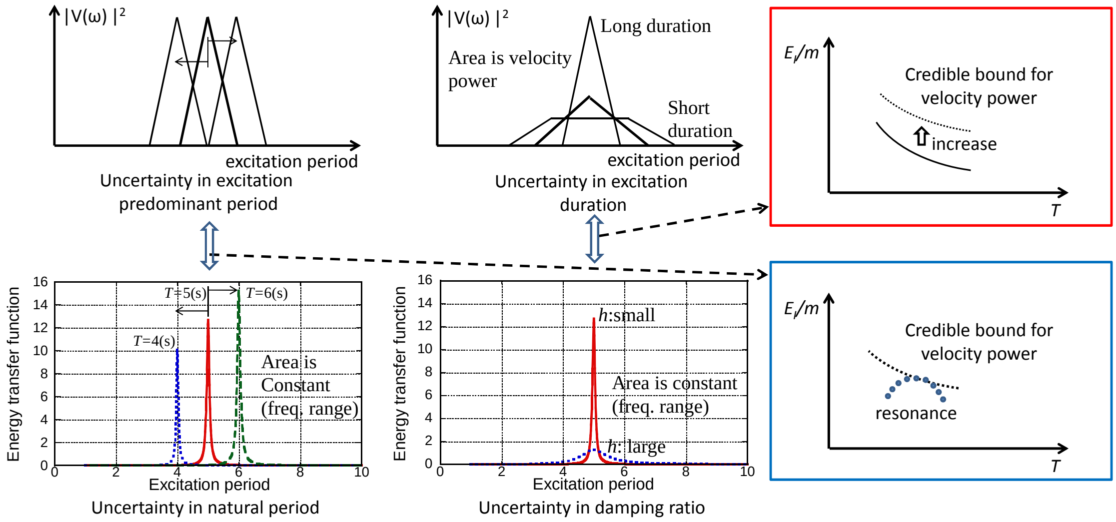

Fig.10 Scenario of increase of credible bound of input energy for the velocity power constraint due to uncertainties in input excitation duration (lengthening) and in structural damping ratio (decrease)

Fig.10 shows a scenario of increase of credible bound of input energy for the velocity power constraint due to uncertainties in input excitation duration (lengthening) and in structural damping ratio (decrease). As for uncertainties in excitation predominant period and in natural period of a structure, the resonant case is critical. It can be understood from Fig.9 that the lengthening of input motion duration and decrease of structural damping ratio may have caused large input in the super high-rise building in Osaka bay area. Especially the decrease of structural damping ratio induces unsmoothing of the energy spectrum and the period region to increase the energy spectrum happened to coincide with the fundamental natural period of the super high-rise building.

5.5 Incorporation of robustness and redundancy for improving earthquake resilience of building structures

It is well understood that ‘redundancy’ and ‘robustness’ are two major concepts playing a central role in the upgrade of earthquake resilience (Ben-Haim 2002, 2006, Takewaki and Ben-Haim 2005, 2008, Kanno and Takewaki 2006, Bertero RD and Bertero VV 1999). The concept of redundancy is defined and used in various situations. The fail-safe mechanism is a representative one and redundancy in statics is another key measure. The margin of a response to the limit value (safety factor) may be another important aspect of redundancy. To investigate the redundancy and robustness, various examinations are necessary. One of the effective examinations is the worst case approach (Drenick 1970, Shinozuka 1970, Takewaki 2002, 2007). The ‘stress test’ in nuclear power plant facilities may be one of the examinations. The stress test is used in Japan in order to check the safety factor or margin of nuclear power plants after March 11, 2011 Tohoku earthquake by increasing gradually the input level of earthquakes or other external forces. It may be important to employ the number of different types of earthquake ground motions or other external forces in addition to the input level.

6. Conclusions

In this review paper, it has been discussed how to narrow unexpected issues in future earthquakes. The principal contents can be summarized as follows.

(1) A set of recorded earthquake ground motions only is not a sufficient set of design earthquake ground motions. The concept of critical excitation can broaden the range of design earthquake ground motions and narrow the range of unexpected class of design earthquake ground motions.

(2) The definition of design earthquake ground motions at the surface ground is more direct to the consideration of resonance to building structures than that at the bottom of surface ground.

(3) The concept of critical excitation can take into account both the aleatory and epistemic uncertainties.

(4) Systematic and logical handling of uncertainties is essential for narrowing the range of unexpected issues in earthquake structural engineering.

(5) Incorporation of robustness and redundancy is a key action for improving earthquake resilience of building structures.

Acknowledgement

Part of the present work is supported by the Grant-in-Aid for Scientific Research of Japan Society for the Promotion of Science (No.24246095). This support is greatly appreciated. The use of ground motion records at K-NET and Building Research Institute of Japan is also highly appreciated. The ground motions provided by PEER (Pacific Earthquake Engineering Research) Center, USA are also appreciated.

NOTATION

$a_{\max}$: acceleration amplitude of sinusoidal ground motion
$A(\omega)$: Fourier spectrum of ground motion acceleration at ground surface or rock surface
$\mathbf{c} = \{c_i\}$: set of damping coefficients
C : constant
C_A : acceleration power
C_V : velocity power
E_I : earthquake input energy
f : performance function or objective function
f_{C} : design requirement
$F(\omega)$: energy transfer function
h : damping ratio
H : thickness of surface ground
$\mathbf{k}$: set of stiffnesses
M_0 : seismic moment
m : mass
$P(\omega, \omega_{\max})$: high-cut filter
Q_β : Q-value
R : distance from the fault

$S(\omega, \omega_C)$: source spectrum, $S(\omega, \omega_C) = \omega^2 / \{1 + (\omega / \omega_C)^2\}$
t_A , t_V : specific times before the ending time t_0 characterizing the ratios of acceleration and velocity powers to the total ones
T_G : period of sinusoidal ground motion
t_0 : duration of ground motion
$\ddot{u}_g$: ground acceleration
$\ddot{u}_{gs}$: sinusoidal ground motion
$v_{\max}$: velocity amplitude of sinusoidal ground motion
V_S : velocity of shear wave
$V(\omega)$: Fourier spectrum of ground motion velocity at ground surface
X_i : uncertain parameter
$\overline{X}_i$, $\underline{X}_i$: upper and lower limit of uncertain parameter
α : uncertainty level
$\hat{\alpha}$: info-gap robustness function
β : velocity of shear wave at rock
ω_C : corner frequency
$\omega_{\max}$: cutoff frequency
Ω : natural circular frequency

References

Abdelrahman, A.M., Yun, C.B., and Wang, P.C. (1979). "Subcritical excitation and dynamic response of structures in frequency domain." *Comput. and Struct.*, **10**(2), 761-771.

Abrahamson N, Ashford S, Elgamal A, Kramer S, Seible F and Somerville P. (1998). *Proc. of 1st PEER Workshop on Characterization of Special Source Effects*, 1998.

Ahmadi, G. (1979). "On the application of the critical excitation method to aseismic design." *J. Struct. Mech.*, **7**, 55-63.

Akiyama H. (1985). *Earthquake Resistant Limit-State Design for Buildings*. University of Tokyo Press, Tokyo, Japan.

Alefeld G, Herzberger J. (1983). Introduction to interval computations. New York: Academic Press.

Architectural Institute of Japan (AIJ) (2012). Preliminary reconnaissance report of the 2011 Tohoku-Chiho Taiheiyo-Oki earthquake, Springer.

Arias A. (1970). A measure of earthquake intensity. In *Seismic Design for Nuclear Power Plants* (Ed. R.J.Hansen), The MIT Press, Cambridge, MA, 438-469.

Architectural Institute of Japan (AIJ) (2011). Preliminary reconnaissance report on the 2011 Off the Pacific coast of Tohoku earthquake, April 6, 2011 (in Japanese).

Aster R. (2012). Expecting the unexpected: black swans and seismology, *Seismological Research Letters*, **83**(1): 5-6.

Bedrosian, B., Barbela, M., Drenick, R. F., and Tsirk, A. (1980). *Critical excitation method for calculating earthquake effects on nuclear plant structures: An assessment study*, NUREG/CR-1673, RD, U.S. Nuclear Regulatory Commission, Burns and Roe, Inc., Oradell, N.J.

Ben-Haim Y. (2002, 2006). *Info-gap Decision Theory: Decisions Under Severe Uncertainty*, first and 2nd edition, Academic Press, London.

Ben-Haim Y. and Elishakoff I (1990). *Convex Models of Uncertainty in Applied Mechanics.* Amsterdam, Netherlands.

Bertero R.D. and Bertero V.V. (1999). Redundancy in earthquake-resistant design, *J.Struct. Eng,* ASCE, **125**(1), 81-88.

Boore DM. (1983). Stochastic simulation of high-frequency ground motions based on seismological models of the radiated spectra, *Bulletin of the Seismological Society of America*; **73**(6): 1865-1894.

Chen SH, Lian H, Yang X. (2002). Interval static displacement analysis for structures with interval parameters. Int J Numer Meth Engrg; **53**: 393-407.

Chen SH, Lian H., Yang X. (2003). Interval eigenvalue analysis for structures with interval parameters. Finite Element in Analysis and Design; **39**: 419-431.

Chen SH, Wu J. (2004a). Interval optimization of dynamic response for uncertain structures with natural frequency constraints. Eng Struct; **26**: 221-232.

Chen SH, Wu J. (2004b). Interval optimization of dynamic response for structures with interval parameters. Comp Struct; **82**: 1-11.

Chen SH, Ma L, Meng GW, Guo R. (2009). An efficient method for evaluating the natural frequency of structures with uncertain-but-bounded parameters. Comp Struct; **87**: 582-590.

Degrauwe D., Lombaert G., De Roeck G. (2010). Improving interval analysis in finite element calculations by means of affine arithmetic. Comp Struct; **88**: 247-254.

Donders S, Vandepitte D, van de Peer J, Desmet W. (2005). Assessment of uncertainty on structural dynamic responses with the short transformation method. J Sound & Vib; **288**(3): 523-543.

Dong W and Shah. H. (1987). Vertex method for computing functions of fuzzy variables. Fuzzy Sets and Systems; **24**(1): 65-78.

Drenick R.F. (1970). Model-free design of aseismic structures. *J. Engrg. Mech. Div.,* ASCE, **96**(EM4), 483-493.

Drenick, R. F. (1973). "Aseismic design by way of critical excitation." *J. Engrg. Mech. Div.,* ASCE, **99**(EM4), 649-667.

Drenick, R. F. (1977). "On a class of non-robust problems in stochastic dynamics." *Stochastic Problems in Dynamics*, B.L.Clarkson, Ed., Pitman, London, 237-255.

Drenick, R. F., and Yun, C-. B. (1979). "Reliability of seismic resistance predictions." *J. Struct.Div.,* ASCE, **105**(ST10), 1879-1891.

Drenick, R. F., Wang, P. C., Yun, C. B., and Philippacopoulos, A. J. (1980). "Critical seismic response of nuclear reactors." *J. Nuclear Engrg. and Design*, **59**, 425-435.

Drenick, R. F., Novmestky, F., and Bagchi, G. (1984). "Critical excitation of structures." *Wind and Seismic Effects, Proc. of the 12th Joint UJNR Panel Conference,* NBS Special Publication, 133-142.

Elishakoff I. and Ohsaki M (2010). *Optimization and Anti-optimization of Structures under Uncertainties*. Imperial College Press, Singapore.

Elnashai, A., Bommer, J. J., and Martinez-Pereira A (1998). Engineering implications of strong-motion records from recent earthquakes. Proceedings of 11th European Conference on Earthquake Engineering, Paris, CD-ROM.

El-Gebeily MA, Abu-Baker Y., Elgindi MB. (1999). The generalized eigenvalue problem for tridiagonal symmetric interval matrices. Int J Control; **72**: 531-535.

Esteva L (1988). The Mexico Earthquake of September 19, 1985—Consequences, Lessons, and Impact on Research and Practice, *Earthquake Spectra*; **4**(3), 413-426.

Fujita K. and Takewaki I. (2010). Critical correlation of bi-directional horizontal ground motions, *Engineering Structures***,** **32**(1), 261-272.

Fujita K. and Takewaki I. (2011a). Sustainable building design under uncertain structural- parameter environment in seismic-prone countries, *Sustainable Cities and Society*, **1**(3), 142-151.
Fujita K. and Takewaki I. (2011b). An efficient methodology for robustness evaluation by advanced interval analysis using updated second-order Taylor series expansion, *Engineering Structures*, **33**(12), 3299-3310.
Fujita K. and Takewaki I. (2012a). Robustness Evaluation on Earthquake Response of Base-Isolated Buildings with Uncertain Structural Properties under Long-Period Ground Motions, *Architectoni.ca* Journal, **1**(1), 46-59.
Fujita K. and Takewaki I. (2012b). Robust passive damper design for building structures under uncertain structural parameter environments, *Earthquakes and Structures* (accepted for publication).
Geller RJ, Jackson DD, Kagan YY and Mulargia F. (1997). Earthquakes cannot be predicted. *Science*, **275**: 1616.
Grigoriu M., Ruiz SE and Rosenblueth E. (1988). The Mexico Earthquake of September 19, 1985—Nonstationary models of seismic ground acceleration, *Earthquake Spectra*; **4**(3), 551-568.
Hanss M. (2002). The Transformation Method for the simulation and analysis of systems with uncertain parameters. Fuzzy Sets and Systems; **130**(3): 277-289.
Hatzigeorgiou, G. D., and Beskos, D. E. (2009). Inelastic displacement ratios for SDOF structures subjected to repeated earthquakes, *Engineering Structures* **31**, 2744-2755.
Heaton T, Hall J, Wald D and Halling M. (1995). Response of high-rise and base-isolated buildings to a hypothetical M 7.0 blind thrust earthquake, *Science*, **267**: 206-211.
Henriques AA. (2008). Efficient analysis of structural uncertainty using perturbation techniques. Eng Struct; **30**: 990-1001.
Housner GW. (1959). Behavior of structures during earthquakes. *J. Engrg Mech. Div.*, ASCE; **85**(4): 109-129.
Housner G.W. and Jennings P.C. (1975) The capacity of extreme earthquake motions to damage structures, *Structural and geotechnical mechanics*: A volume honoring N.M.Newmark edited by W.J. Hall, 102-116, Prentice-Hall Englewood Cliff, NJ.
IUTAM (2009). Proc. of the IUTAM Symposium on the Vibration Analysis of Structures with Uncertainties. held in St. Petersburg, Russia, July 5-9, 2009.
Iyengar RN. (1970). *Matched inputs*, Report 47, Series J, Center of Applied Stochastics, Purude University, West Lafayette, Ind.
Iyengar RN. and Manohar CS. (1987). Nonstationary random critical excitations. *J. Engrg Mech.*; ASCE, **133**, 529-541.
Jiang C, Han X, Guan FJ, Li YH. (2007). An uncertain structural optimization method based on nonlinear interval number programming and interval analysis method. Eng Struct 2007; **29**: 3168-3177.
Kanno Y. and Takewaki I. (2006). Sequential semidefinite program for maximum robustness design of structures under load uncertainties, *J. Optimization, Theory and Application*, **130**(2), 265-28.
Koyluoglu HU, Elishakoff I. (1998). A comparison of stochastic and interval finite elements applied to shear frames with uncertain stiffness properties. Comp Struct;

67: 91-98.
McWilliam S. (2001). Anti-optimization of uncertain structures using interval analysis. Comp Struct; **79**: 421-430.
Moens D, Hanns M. (2011). Non-probabilistic finite element analysis for parametric uncertainty treatment in applied mechanics: Recent advances. Finite Element in Analysis and Design; **47**(1):4-16.
Moens D, Vandepitte D. (2004). An interval finite element approach for the calculation of envelope frequency response functions. Int J Num Meth Engrg; **61**(14): 2480-2507.
Moore RE (1966). Interval analysis, Englewood Cliffs, New Jersey: Prentice-Hall.
Moustafa A. and Takewaki I. (2010). Critical ground-motion sequences for inelastic structures, *Advances in Structural Engineering,* **13**(4), 665-679. (Erratum Vol.14, No.1, pp121, 2011)
Moustafa A. and Takewaki I. (2011). Response of nonlinear single-degree- of-freedom structures to random acceleration sequences, *Engineering Structures*, **33**(4), 1251-1258.
Moustafa A. and Takewaki I. (2012). Earthquake ground motion of multiple sequences and associated structural response, *Earthquakes and Structures*, **3**(5) (in press).
Moustafa A., Ueno K. and Takewaki I. (2010). Critical earthquake loads for SDOF inelastic structures considering evolution of seismic waves, *Earthquakes and Structures,* **1**(2), 147-162.
Mullen RL, Muhanna RL. (1999). Bounds of structural response for all possible loading combinations. J Struct Engrg ASCE; **125**: 98-106.
NIED (National Research Institute for Earth Science and Disaster Prevention) (2011). 2011 Off the Pacific Coast of Tohoku earthquake, Strong Ground Motion, Emergency meeting of Headquarters for Earthquake Research Promotion, March 13, 2011. (Available from http://www.k-net.bosai.go.jp/k-net/ topics/ TohokuTaiheiyo_20110311/nied_kyoshin2e.pdf, [Accessed on April 20, 2011]).
Pirasteh, A. A., Cherry, J. L., and Balling, R. J. (1988). "The use of optimization to construct critical accelerograms for given structures and sites." *Earthquake Engrg. Struct. Dyn.*, **16**, 597-613.
Qiu ZP, Chen SH, Song D. (1996). The displacement bound estimation for structures with an interval description of uncertain parameters. C Numer Meth Engng; **12**: 1-11.
Qiu ZP, Elshakoff I. (1998). Antioptimization of structures with large uncertain-but-nonrandom parameters via interval analysis. Comp Meth Appl Mech Engrg; **152**: 361-372.
Qiu ZP. (2003). Comparison of static response of structures using convex models and interval analysis method. Int J Numer Meth Engng; **56**: 1735-1753.
Shinozuka M. (1970). Maximum structural response to seismic excitations. *J. Engrg Mech.*; ASCE, **96**, 729 –738.
Srinivasan, M., Ellingwood, B., and Corotis, R. (1991). "Critical base excitations of structural systems." *J. Engrg. Mech.*, ASCE, **117**(6), 1403-1422.
Stein RS. (2003). Earthquake conversations. *Scientific American*, **288**(1): 72-79.
Takewaki I. (2002). Seismic critical excitation method for robust design: A review, *J. Struct. Engrg*, ASCE, **128**(5), 665-672.
Takewaki I. (2005). A comprehensive review of seismic critical excitation methods for

robust design, *Advances in Structural Engineering*, **8**(4), 349-363.
Takewaki I. (2007). *Critical Excitation Methods in Earthquake Engineering*, Elsevier.
Takewaki I. (2008). Critical excitation methods for important structures, invited as a Semi-Plenary Speaker, EURODYN 2008, July 7-9, 2008, Southampton, England.
Takewaki I. (2009). *Building Control with Passive Dampers: -Optimal Performance-based Design for Earthquakes-*, John Wiley & Sons Ltd. (Asia), September.
Takewaki I (2011). The 2011 off the Pacific Coast of Tohoku Earthquake and Its Impact on Building Structural Design, Keynote paper (Plenary speaker) at The ASEM11+ Congress, September 18-23, 2011 in Seoul, Korea.
Takewaki I. and Ben-Haim Y. (2005). Info-gap robust design with load and model uncertainties, *J. Sound and Vibration*, **288**(3), 551-570.
Takewaki I. and Ben-Haim Y. (2008). *Info-gap Robust Design of Passively Controlled Structures with Load and Model Uncertainties*, Chapter 19 in "Structural Design Optimization Considering Uncertainties" (eds.) Yiannis Tsompanakis, Nikos D. Lagaros & Manolis Papadrakakis, Taylor & Francis, Chapter 19, pp531-548, 2008.
Takewaki I. and Fujita K. (2009). Earthquake input energy to tall and base-isolated buildings in time and frequency dual domains, *J. of The Structural Design of Tall and Special Buildings*, **18**(6), 589–606.
Takewaki I., Moustafa A. and Fujita K. (2012). *Improving the Earthquake Resilience of Buildings: The Worst Case Approach*, Springer (London).
Takewaki I. Murakami S. Fujita K. Yoshitomi S. and Tsuji M. (2011). The 2011 off the Pacific coast of Tohoku earthquake and response of high-rise buildings under long-period ground motions, *Soil Dyn. Earthq. Engrg*, **31**(11), 1511-1528.
Takewaki I. and Tsujimoto H. (2011). Scaling of design earthquake ground motions for tall buildings based on drift and input energy demands, *Earthquakes and Structures*, **2**(2), 171-187.
Tokmechi Z (2011). Earthquake Stochastic Response and Soil Type Variable Effects, Australian Journal of Basic and Applied Sciences, 5(8): 581-586.
Trifunac MD. (2008). Energy of strong motion at earthquake source. *Soil Dyn. Earthq. Engrg.* **28**: 1-6.
Wang, P. C., and Drenick, R. F. (1977). "Critical seismic excitation and response of structures." *Proc. of 6WCEE*, Vol.II, 1040-1045.
Wang, P. C., Drenick, R. F., and Wang, W. Y. L. (1978). "Seismic assessment of high-rise buildings." *J. Engrg. Mech. Div.*, ASCE, **104**(EM2), 441-456.
Wang, P. C., and Philippacopoulos, A. J. (1980). "Critical seismic assessment of life-line structures." *Proc. of 7WCEE*, Vol.8, 257-264.
Wang, P. C., Wang, W., Drenick, R., and Vellozzi, J. (1976). "Critical excitation and response of free standing chimneys." *Proc. of the Int. Symposium on Earthquake Struct. Engrg.*, Vol.I, St. Louis, Mo., Aug., 269-284.
Wang, P.C., and Yun, C. B. (1979). "Site-dependent critical design spectra." *Earthquake Engrg. Struct. Dyn.*, **7**, 569-578.
Westermo, B. D. (1985). "The critical excitation and response of simple dynamic systems." *J. Sound and Vibration*, **100**(2), 233-242.
Yazdani A and Komachi Y (2009). Computation of earthquake response via fourier amplitude spectra**,** *IJE Transactions B: Applications*, 22(2), 147-152.