Trade Structure and Belief-Driven Fluctuations in a Global Economy*

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Abstract

This paper constructs a dynamic two-country model with country-specific production externalities and inspects the presence of equilibrium indeterminacy under alternative trade structures. It is shown that the presence of belief–driven economic fluctuations caused by equilibrium indeterminacy is closely related to the specified trade structure. If investment goods are not internationally traded and international lending and borrowing are allowed, then indeterminacy arises in a wider set of parameter space than in the corresponding closed economy. By contrast, either if both consumption and investment goods are traded in the absence of international lending and borrowing or if only investment goods are traded with financial transactions, then the indeterminacy conditions are the same as those for the closed economy counterpart.

Keywords: two-country model, non-traded goods, equilibrium indeterminacy, social constant returns

JEL classification: F43, O41

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1 Introduction

The central concern of this paper is to explore the relation between trade structure and belief-driven economic fluctuations. Using a dynamic two-country model with country-specific production externalities, we inspect conditions for equilibrium indeterminacy under alternative trade structures. In the presence of equilibrium indeterminacy, non-fundamental shocks (sunspots) affect expectations of agents, which gives rise to belief-driven business cycles. In this case not only shocks to the fundamentals but also extrinsic uncertainty can generate business fluctuations. We consider which trade structures may yield equilibrium indeterminacy in a wider parameter space than in the closed economy counterpart.

As for our question, the foregoing literature has provided us with two contrasting answers. On the one hand, Meng (2003), Meng and Velasco (2003 and 2004) and Weder (2001) show that small-open economies with production externalities hold indeterminacy under weaker conditions than in the corresponding closed economy models.[1] Hence, according to these studies, opening up international trade may enhance the possibility of economic fluctuations. Nishimura and Shimomura (2002), on the other hand, examine a dynamic Heckscher-Ohlin model of the two-country world in which there are country-specific production externalities. They show that the world economy has the same conditions for equilibrium indeterminacy as those for the closed economy counterpart. In addition, Sim and Ho (2007) find that if one of the two countries has no production externalities in Nishimura and Shimomura's model, then the equilibrium path of the world economy would be determinate even though the country with production externalities exhibits autarkic indeterminacy. These studies indicate that international trade does not necessarily enhance the possibility of belief-driven fluctuations.

At first sight, the opposite results mentioned above seem to stem from the difference in the modelling method used by the existing studies. The small-open economy models are based on partial equilibrium analysis in which behavior of the rest of the world is exogenously given. In contrast, the models of world economy employ the general equilibrium approach that treats the world economic system as a closed economy consisting of multiple countries. Thus one may think that the behavior of an integrated world economy is similar to the

[1] Lahiri (2001) also examines indeterminacy in a small-open economy model. Since he uses a framework different from the one used by Meng (2003) and others, his model needs a relatively high degree of external increasing returns to yield indeterminacy. Yong and Meng (2004) and Zhang (2008) also discuss equilibrium indeterminacy in small-open economies.

behavior of a closed economy. Such a conjecture is, however, misleading. We demonstrate that the key to the relation between international trade and belief-driven fluctuations is the specification of trade structure rather than the difference in modeling strategy, that is, partial versus general equilibrium analyses. In the foregoing investigations, the papers on small-open economies such as Meng and Velasco (2003, 2004) and Weder (2001) assume that investment goods are not internationally traded, while consumption goods are traded and international lending and borrowing are allowed. By contrast, Nishimura and Shimomura (2002) follow the Heckscher-Ohlin tradition where both consumption and investment goods are traded, while neither international lending nor borrowing are possible. We show that, as well as in the small-open economy models, if investment goods are traded in the domestic market alone, then the world economy model exhibits equilibrium indeterminacy under weaker conditions than those for the closed economy model.

More specifically, we construct a $2\times2\times2$ model of the world economy in which each country produces both investment and consumption goods under social constant returns. It is assumed that both countries have identical technologies and preferences. If we assume that both investment and consumption goods are tradable and international lending and borrowing are not allowed, then our model is identical to Nishimura and Shimomura (2002), so that opening up international trade does not affect the indeterminacy conditions. If investment goods are nontradables and international financial transactions are possible, then the world economy exhibits indeterminacy in a wider range of parameter space than in the corresponding closed economy. Finally, if consumption goods are not traded but investment goods are tradable in the presence of international lending and borrowing, then it is shown that the indeterminacy conditions are the same as those for the closed economy.

As suggested above, this paper is closely related to Meng and Velasco (2004) and Nishimura and Shimomura (2002). Both papers are based on Benhabib and Nishimura (1998) who investigate indeterminacy conditions in a closed, two-sector growth model with sector-specific production externalities and social constant returns. The main finding of Benhabib and Nishimura (1998) is that (i) if the consumption good sector is more capital intensive than the investment good sector from the private perspective but it is less capital intensive from the social perspective; and (ii) if the elasticity of intertemporal substitution in consumption of the representative family is sufficiently large, then there is a continuum of converging equilibrium paths around the steady sate. Since the integrated world economy discussed by Nishimura and Shimomura (2002) behaves like a single, closed economy, the indeterminacy conditions for their model is the same as those shown by Benhabib and Nishimura (1998). Meng and Velasco (2004) find that in a small-open economy model in which investment goods are non-traded and there are international lending and borrowing, only condition (i) is necessary for establishing indeterminacy: the shape of utility function has no relation to the indeterminacy conditions.[2] Our paper uses Nishimura and Shimomura's setting as the base model and introduces nontraded goods and intertemporal trade. The case where investment goods are not traded is, therefore, a two-(large) country version of Meng and Velasco (2004).[3]

The roles of nontraded goods have been extensively discussed in the literature. The static trade theory has focused on the effects of nontraded goods on trade patterns, terms of trade and resource allocation: see, for example, Komiya (1967), Either (1972) and Jones (1974). Also, there is a vast literature on this topic in international macroeconomics and finance. Those macroeconomic studies have been concerned with how the presence of nontraded goods affects real exchange rates, current accounts, asset positions, policy impacts and international business cycles caused by the fundamental shocks.[4] Turnovsky (1997, Chapter 4), among others, points out that the analytical outcomes may critically depend on which goods are not internationally traded. The foregoing contributions in most cases explore models with equilibrium determinacy. Therefore, the relation between trade structure and belief-driven business cycles has not been explored well in the foregoing studies. Our paper demonstrates that nontraded goods and trade structure play pivotal roles as to the destabilizing effect of international trade caused by indeterminacy and sunspots. We also confirm that in the presence of equilibrium indeterminacy, the long-run distribution of wealth in the world market

[2] In the two-sector endogenous growth model of a closed economy where each sector employes physical and human capital under social constant returns, the condition (ii) is not needed for holding indeterminacy: see Benhabib et al. (2000) and Mino (2001).

[3] Weder (2001) examines an open economy version of a two-sector closed economy model studied by Benhabib and Farmer (1996). In Weder's model the production technology of each sector exhibits constant returns from the private perspective, while it satisfies increasing (or decreasing) returns from the social perspective. It is also assumed that labor supply is endogenous and private factor intensity is identical in both sectors. Weder (2001) also considers the case where the home country is not small so that the world interest rate depends on the asset holding of the home country. Despite those differences from Meng and Velasco (2003), Weder (2001) also finds that the open economy yields indeterminacy under weaker restrictions than the closed economy.

[4] A small sample includes Baxter et al, (1998), Brock (1988), Engel and Kletzer (1989) and Turnovsky and Sen (1995). Obstfeld and Rogoff (1996, Chapter 4) and Turnovsky (1997, Chapter 4) present lucid expositions of open-macroeconomic models with nontraded goods.

and the steady-state level of asset position of each country become indeterminate: not only the initial holding of asset of each country but also sunspot shocks affect these long-run values. Therefore, if belief-driven economic fluctuations exist, we obtain outcomes and implications that are quite different from those obtained when the equilibrium path of the world economy is determinate.

In what follows, we first sets up an analytical basis of our discussion. Then we examine three types of trade structures: (i) both investment and consumption goods are tradables; (ii) only consumption goods are traded and; (iii) only investment goods are traded. In case (i) international lending and borrowing are not allowed. Cases (ii) and (iii) assume the presence of lending and borrowing between the two countries. The next section presents the base model. Section 3 examines case (i). Section 4, the main part of our paper, investigates cases (ii) and (iii). Section 5 gives intuitive implication of our findings. This section also discusses empirical plausibility of the assumptions made for establishing our main results.

2 Baseline Setting

Consider a world economy consisting of two countries, home and foreign. Both countries have the same production technologies. In each country there is the representative household. Households in both countries have an identical time discount rate and the same form of instantaneous felicity function. The only difference between the two countries is the initial stock of wealth held by the households in each country. In this section we concentrate on modelling the home country. Since taste and technology are symmetric between the two countries, the following formulations are applied to the foreign country as well.

2.1 Production

The production side of our model is the same as that used by Nishimura and Shimomura (2002). The home country has two production sectors. The first sector ($i = 1$) produces investment goods and the second sector ($i = 2$) produces pure consumption goods. The production function of i-th sector is specified as

$$Y_i = A_i K_i^{a_i} L_i^{b_i} \bar{X}_i, \qquad a_i > 0,\ b_i > 0,\ \ 0 < a_i + b_i < 1, \quad i = 1, 2,$$

where Y_i, K_i and L_i are i-th sector's output, capital and labor input, respectively. Here, $\bar{X}_i$ denotes the sector and country-specific production externalities.[5] We define:

$$\bar{X}_i = \bar{K}_i^{\alpha_i - a_i} \bar{L}_i^{1-\alpha_i - b_i}, \quad a_i < \alpha_i < 1, \quad \alpha_i + b_i < 1 \quad i = 1, 2.$$

Normalizing the number of producers to one, then it holds that $\bar{K}_i = K_i$ and $\bar{L}_i = L_i$ $(i = 1, 2)$ in equilibrium. This means that the i-th sector's social production technology that internalizes the external effects is:

$$Y_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i}, \quad i = 1, 2. \tag{1}$$

Hence, the social technology satisfies constant returns to scale, while the private technology exhibits decreasing returns to scale.[6]

The factor and product markets are competitive, so that the private marginal product of each production factor equals its real factor price. These conditions are given by the following:

$$r = pa_1 \frac{Y_1}{K_1} = a_2 \frac{Y_2}{K_2}, \tag{2a}$$

$$w = pb_1 \frac{Y_1}{L_1} = b_2 \frac{Y_2}{L_2}, \tag{2b}$$

where w is the real wage rate, r is the rental rate of capital and p denotes the price of investment good in terms of the consumption good.

Considering that $\bar{K}_i = K_i$,and $\bar{L}_i = L_i$, we find that $(2a)$ and $(2b)$ yield:

$$r = pa_1 A_1 k_1^{\alpha_1 - 1} = a_2 A_2 k_2^{\alpha_2 - 1}, \tag{3a}$$

$$w = pb_1 A_1 k_1^{\alpha_1} = b_2 A_2 k_2^{\alpha_2}, \tag{3b}$$

where $k_i = K_i / L_i . (i = 1, 2)$. By use of $(3a)$ and $(3b)$, we can express the optimal factor

[5] We shall omit time argument in each endogenous variable unless necessary.

[6] This specification of production technology was first introduced by Benhabib and Nishimura (1998) who demonstrate that equilibrium indterminacy may hold even in the absence of social increasing returns. Benhabib et al. (2000), Meng (2003), Meng and Velasco (2003, 2004), Mino (2001) and Nishimura and Shimomura (2002) use the same production functions.

intensity in each production sector as a function of relative price:

$$k_1 = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_2}{\alpha_2-\alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}} p^{\frac{1}{\alpha_2-\alpha_1}} \equiv k_1(p),$$

$$k_2 = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_1}{\alpha_2-\alpha_1}} \left(\frac{b_1}{b_2}\right)^{\frac{\alpha_1-1}{\alpha_1-\alpha_2}} p^{\frac{1}{\alpha_2-\alpha_1}} \equiv k_2(p),$$

These expressions show that

$$\text{sign}\,[k_1(p) - k_2(p)] = \text{sign}\left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \tag{5}$$

$$\text{sign}\; k_i'(p) = \text{sign}\;\; (\alpha_2 - \alpha_1), \quad i = 1, 2. \tag{6}$$

In the above, the sign of $a_1/b_1 - a_2/b_2$ represents the factor intensity ranking from the private perspective, while sign $(\alpha_1 - \alpha_2)$ expresses the factor intensity ranking from the social perspective.

We assume that production factors shiftable between the sectors, but they cannot cross the borders. Thus the full employment conditions for capital and labor in the home country are respectively given by

$$K_1 + K_2 = K, \quad L_1 + L_2 = 1.$$

where K denotes the aggregate capital in the home country. The labor supply is assumed to be constant and normalized to one. These full-employment conditions are summarized as

$$k_1(p)\,L_1 + (1 - L_1)\,k_2(p) = K. \tag{7}$$

In this paper we restrict our attention to the interior equilibrium in which the two countries produce both consumption and investment goods.[7] Thus we focus on the situation where the labor allocation to the first sector given by (7) satisfies the following:

$$L_1 = \frac{K - k_2(p)}{k_1(p) - k_2(p)} \in (0, 1). \tag{8}$$

[7] See Footnote 10 on this restriction.

The supply functions of investment and consumption goods are respectively given by

$$y^1(K,p) \equiv L_1 A_1 k_1(p)^{\alpha_1} = \frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1}, \tag{9a}$$

$$y^2(K,p) = (1 - L_1) A_2 k_2(p)^{\alpha_2} \equiv \frac{k_1(p) - K}{k_1(p) - k_2(p)} A_2 k_2(p)^{\alpha_2}. \tag{9b}$$

It is easy to see that these supply functions satisfy:

$$\text{sign } y^1_K(K,p) = \text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \quad \text{sign } y^1_p(K,p) = \text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2), \tag{10a}$$

$$\text{sign } y^2_K(K,p) = -\text{ sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b2}\right), \quad \text{sign } y^2_p(K,p) = -\text{ sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2). \tag{10b}$$

Note that if the private and social factor-intensity rankings have opposite signs, that is, $\left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2) < 0$, then the duality between the Rybczynski and Stolper–Samuelson effects fails to hold.

2.2 Households

There is a continuum of identical households with a unit mass. Each household supply one unit of labor in each moment. The objective functional of the representative household is given by

$$U = \int_0^\infty \frac{C^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \sigma > 0, \quad \rho > 0,$$

where C is consumption and ρ denotes a given time discount rate. When $\sigma = 1$, then the instantaneous felicity function is $\log C$.

(i) Financial Autarky

If the households cannot access to the international financial market, then the aggregate asset of the home country equals the aggregate capital stock held by the domestic households. Thus their flow budget constraint is

$$\dot{K} = \left(\frac{r}{p} - \delta\right) K + \frac{1}{p}(w + \pi_1 + \pi_2 - C), \tag{11}$$

where $\delta \in [0, 1)$ is the rate of capital depreciation and π_i denotes the excess profits in the

i-th sector[8]. The households maximize U subject to (11) and the initial holding of capital, K_0. When solving the optimization problem, the households take the sequences of $\{r_t, w_t, \ \pi_{1,t}, \pi_{2,t}, p_t\}_{t=0}^{\infty}$ as given. Letting q be the implicit price of capital, the necessary conditions for an optimum include the following:

$$C^{-\sigma} = q/p, \tag{12a}$$

$$\dot{q} = q\left(\rho + \delta - r/p\right), \tag{12b}$$

together with the transversality condition; $\lim_{t\to\infty} e^{-\rho t} qK = 0$.

(ii) International Lending and Borrowing

If the households in the home country can lend to or borrow from the foreign households, then their flow budget constraint is given by

$$\dot{\Omega} = R\Omega + w + \pi_1 + \pi_2 - C, \tag{13}$$

where Ω denotes the net wealth (evaluated in terms of consumption good):

$$\Omega = B + pK.$$

where B is the stock of bonds (IOUs). We assume that bond and capital are perfect substitutes and, hence, the non-arbitrage condition between the two assets requires that the rate of return to bond equal to the net rate return to capital plus capital gain:

$$R = \frac{r}{p} - \delta + \frac{\dot{p}}{p}. \tag{14}$$

Using (14) and $\dot{\Omega} = \dot{B} + p\dot{K} + \dot{p}K$, the flow budget constraint (13) is rewritten as

$$\dot{B} = RB + rK + w + \pi_1 + \pi_2 - C - pI, \tag{15}$$

[8] Remember that the private technology of each production sector exhibits decreasing returns to scale with respect to capital and labor.

where I denote gross investment, so that

$$\dot{K} = I - \delta K. \tag{16}$$

The representative household maximizes U subject (15), (16) and the non-Ponzi-game scheme given by

$$\lim_{t\to\infty} \exp\left(-\int_0^t R_s ds\right) B_t \geq 0.$$

Set up the Hamiltonian function for the optimization problem:

$$H = \frac{C^{1-\sigma}-1}{1-\sigma} + \lambda\left[RB + rK + w + \pi_1 + \pi_2 - C - pI\right] + q\left(I - \delta K\right),$$

where λ and q respectively denote the implicit prices of bonds and domestic capital. Focusing on an interior solution, we see that the necessary conditions for an optimum are:

$$C^{-\sigma} = \lambda \tag{17a}$$

$$p\lambda = q, \tag{17b}$$

$$\dot{\lambda} = \lambda\left(\rho - R\right), \tag{17c}$$

$$\dot{q} = q\left(\rho + \delta\right) - \lambda r = q\left(\rho + \delta - \frac{r}{p}\right). \tag{17d}$$

The optimization conditions also involve the transversality conditions on holding bond and capital: $\lim_{t\to\infty} \lambda e^{-\rho t} B = 0$ and $\lim_{t\to\infty} q e^{-\rho t} K = 0$.

3 The Model with Financial Autarky

We first assume that there is only intratemporal trade: both investment and consumption goods are freely traded but households in each country neither lend to nor borrow from the foreign households. This is the Heckscher-Ohlin setting employed by Nishimura and Shimomura (2002).[9] This section summarizes the main results of their contribution in order

[9] If international lending and borrowing are possible in the Heckscher-Ohlin setting, the instantaneous equilibrium itself becomes indeterminate. This is a reconfirmation of Mundell's (1957) results shown in the static Heckscher-Ohlin model. See also Cremers (1997) on this point. Note that we may introduce international lending and borrowing into the Heckscher-Ohlin model, if the model economy involves financial frictions or

to clarify the effects of introducing nontraded goods and financial transactions into the base model.

Under free trade of both goods, the world market equilibrium conditions for investment and consumption goods are repetitively given by

$$Y_1 + Y_1^* = \dot{K} + \dot{K}^* + \delta K + \delta K^*, \tag{18}$$

$$Y_2 + Y_2^* = C + C^*, \tag{19}$$

where an asterisk indicates the corresponding foreign variable. When both countries produce both goods, all the firms in the world economy face the common world price, p. [10] Hence, given the assumption of symmetric technologies between the two countries, both home and foreign firms in each production sector select the same capital intensity, and thus it holds that $k_i(p) = k_i^*(p) \quad (i = 1, 2)$ for all $t \geq 0$. As a result, from (9a) and (18) the aggregate capital in the world market changes according to

$$\dot{K}_w = \frac{K_w - 2k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1} - \delta K_w, \tag{20}$$

where $K_W = K + K^*$. In addition, since the factor prices are equalized between the two countries, (12b) gives

$$\dot{q}/q = \dot{q}^*/q^* = \rho + \delta - r(p)/p. \tag{21}$$

This means that q^*/q stays constant over time, so that from (12a) C^*/C is also constant even out of the steady state. Thus the aggregate consumption demand in the world market is written as $C + C^* = (1 + \bar{n}) C$, where $\bar{n}$ is a positive constant.

As is well known, the conditions mentioned above enable us to treat the world economy as if it were a closed economy with two types of households holding different levels of capital

investment adjustment costs: see Antras and Caballero (2009) and Ono and Shibata (2010).

[10] Our discussion depends on this assumption. If at least one country completely specializes, dynamic systems of the world economy examined in Sections 3 and 4 are different from those displayed in this paper. However, as shown by Appendix 1 of the paper, provided that both countries have identical taste and technology, the steady-state equilibrium of the world economy is inside the diversification cone where both countries produce both goods. Therefore, our assumption is justified as long as we focus on the local dynamics of the world economy around the steady state equilibrium. To analyze the global behavior of the model, we need to treat the model out of the diversification cone. Atkeson and Kehoe (2000) explore the dynamic behavior of a small country that specializes in producing one of the two goods. Caliendo (2011) presents a detailed analysis of dynamic behavior of a $2 \times 2 \times 2$ model outside the diversification cone.

stocks. Nishimura and Shimomura (2002) first show that the steady-state levels of K_w and p are uniquely determined, while the steady-state conditions of the world economy do not pin down the steady-state levels of K and K^*. Then they present the following:

Proposition 1 *(Nishimura and Shimomura 2002) The steady-state equilibrium of the world economy is locally indeterminate, if* $\frac{a_1}{b_1} - \frac{a_2}{b_2} < 0$ *and* $\alpha_1 - \alpha_2 > 0$ *and* (ii) $1/\sigma > \max\{1,\ 1/\bar{\sigma}\}$, *where* $\bar{\sigma}$ *is a function of parameters involved in the model.*

The first condition in Proposition 1 means that the investment good sector employes less capital intensive technology than the consumption good sector from the private persecutive, while it uses more capital intensive technology from the social perspective. The second condition requires that the elasticity of intertemporal substitution in consumption is high enough.[11] Since the aggregate dynamics of the world economy is identical to dynamics of the closed economy, the indeterminacy conditions given above are the same as those found by Benhabib and Nishimura (1998).

If the steady state of the world economy satisfies determinacy, the initial values of q and q^* (so the value of $\bar{n}$) are uniquely specified under a given set of initial levels of K and K^*. This means that the steady-state levels of K and K^* are uniquely given by the initial distribution of capital stocks. In this case, if the home country initially holds a larger amount of capital than the foreign country, then the home country can keep her comparative advantage in producing the capital-intensive goods during the transition towards the steady-state equilibrium.[12] Such a dynamic version of the Heckscher-Ohlin theorem, however, fails to hold when the world economy exhibits equilibrium indeterminacy. When indeterminacy exists, there is a continuum of converging paths around the steady state so that the steady-state distribution of capital depends on which path is actually selected. Consequently, not only the initial holdings of factor endowments but also belief-driven fluctuations may affect

[11] The precise expression of $1/\bar{\sigma}$ in Proposition 1 is

$$\frac{1}{\bar{\sigma}} = \frac{(1-\alpha_1)a_2b_1(\rho+\delta) + \alpha_1 a_1 \left[\rho b_2 + \delta b_1 a_2 + (1-a_1)b_2\delta\right]}{(a_2b_1 - a_1b_2)(\alpha_1 - \alpha_2)\left[\rho + \delta(1-a_1)\right]}$$

[12] This conclusion depends on the functional forms of production and utility functions we use as well as on the fact that we restrict our attention to the model behavior near the steady state. As for more general analyses on income and wealth distribution among the countries in the Heckscher-Ohlin world, see Atkson and Kehoe (2000) and Bajona and Kehoe (2010). Atkeson and Kehoe (2000) treat a small-country model, while Bajona and Kehoe (2010) explore a two-country model.

long-term trade patterns of the world economy.

4 The Model with Lending and Borrowing

The main part of this section examines the case where investment goods are not internationally traded, while there are international lending and borrowing. We also briefly consider the opposite case where consumption goods are nontradable.

4.1 Nontradable Investment Goods

We now assume that consumption goods are internationally traded, and international lending and borrowing are allowed, but investment goods are non-tradables.[13] Although such an assumption is restrictive one, it elucidates the role of trade structure in a dynamic world economy. In Section 5.2 we discuss the empirical plausibility of alternative trade structures used in this paper. Since investment goods are traded in the domestic market alone and consumption goods are internationally traded, the market equilibrium conditions for investment and consumption goods are respectively given by

$$Y_1 = \dot{K} + \delta K, \qquad Y_1^* = \dot{K}^* + \delta K^*, \tag{22}$$

$$Y_2 + Y_2^* = C + C^*, \tag{23}$$

The equilibrium condition for the bond market is

$$B + B^* = 0, \tag{24}$$

which means that $\Omega + \Omega^* = pK + p^*K^*$. Bonds are IOUs between the home and foreign households and, hence, the aggregate value of bonds is zero in the world financial market at large.

[13] In the small-country setting, the trade structure assumed here is a kind of dependent economy models discussed in open-economy macroeconomics literature. Meng and Velasco (2003 and 2004) and Weder (2001) employ such a formulation. In the forgoing studies on models without externalities, Turnovsky and Sen (1995) treat a small-open economy model with non-tradable capital and Turnovsky (1997, Chapter 7) studies a neoclassical two-country, two-sector model in which capital goods are not traded. Mino (2008) also discusses the similar two-country model with external increasing returns. See also Chapter 5 in Turnovsky (2009) for a brief literature review.

4.2 Dynamic System

Investment goods are traded in the domestic market alone, so that the price of investment goods in each country may differ from each other. Using (22), we find that capital stock in each country changes according to

$$\dot{K} = \frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1} - \delta K. \tag{25a}$$

$$\dot{K}^* = \frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_2 k_1(p^*)^{\alpha_1} - \delta K^*. \tag{25b}$$

Dynamics of the shadow values of capital are:

$$\dot{q} = q[\rho + \delta - \widetilde{r}(p)], \tag{26a}$$

$$\dot{q}^* = q^*[\rho + \delta - \widetilde{r}(p^*)], \tag{26b}$$

Here, p does not necessarily equal p^* during the transition. Therefore, unlike the model in the previous section, the relative shadow value of capital, q/q^*, does not stay constant out of the steady state. Dynamic equations $(25a)$, $(25b)$, $(26a)$ and $(26b)$ depict behaviors of capital stocks and their implicit prices in the home and foreign countries.

To obtain a complete dynamic system, we should find the relations between p and p^* and the state variables, K, K^*, q and q^*. The foreign country's optimization conditions corresponding to $(17a)$ and $(17c)$ are respectively given by $C^{*-\sigma} = \lambda^*$ and $\dot{\lambda}^*/\lambda^* = \rho - R$. Therefore, both λ^*/λ and C^*/C stay constant over time. Let us denote $C^*/C = (\lambda^*/\lambda)^{-1/\sigma} = \bar{m}\,(>0)$. Then the world market equilibrium condition for consumption (23) is expressed as

$$(1 + \bar{m})\,\lambda^{-\frac{1}{\sigma}} = y^2(K, p) + y^2(K^*, p^*), \tag{27}$$

where $y^2(K, p)$ is defined by $(9b)$ and $y^2(K^*, p^*)$ is given by

$$y^2(K^*, p^*) = \frac{k_1(p^*) - K^*}{k_1(p^*) - k_2(p^*)} A_2 k_2(p^*)^{\alpha_2}.$$

In view of (27), we see that λ is expressed as a function of capital stocks, prices and $\bar{m}$:

$$\begin{aligned}\lambda &= (1+\bar{m})^{\sigma}\left[y^{2}(K,p)+y^{2}(K^{*},p^{*})\right]^{-\sigma} \\ &\equiv \lambda(K,K^{*},p,p^{*};\bar{m}). \end{aligned} \tag{28}$$

Thus optimization conditions $(17b)$ and $q^{*}=\lambda^{*}p^{*}$ give

$$p=\frac{q}{\lambda(K,K^{*},p,p^{*};\bar{m})},\qquad p^{*}=\frac{q^{*}}{\bar{m}^{-\sigma}\lambda(K,K^{*},p,p^{*};\bar{m})}.$$

Solving these equations with respect to p and p^{*} presents the following expressions:

$$p=\pi(K,K^{*},q,q^{*};\bar{m}),\qquad p^{*}=\pi^{*}(K,K^{*},q,q^{*};\bar{m}). \tag{29}$$

Substituting (29) into $(25a)$, $(25b)$, $(26a)$ and $(26b)$, we obtain a dynamic system of K, K^{*}, q and q^{*} under a given level of $\bar{m}$. In Appendix 2 of the paper, we analyze this dynamic system to derive indeterminacy conditions.

Alternatively, we can obtain a dynamic system of K, K^{*}, p and p^{*} in the following manner. Differentiate both sides of (28) logarithmically with respect to time, which yields

$$\frac{\dot{\lambda}}{\lambda}=-\sigma\left[\frac{Y_{K}^{2}K}{Y^{2}}\frac{\dot{K}}{K}+\frac{Y_{K^{*}}^{2}K^{*}}{Y^{2}}\frac{\dot{K}^{*}}{K^{*}}+\frac{Y_{p}^{2}p}{Y^{2}}\frac{\dot{p}}{p}+\frac{Y^{2}p^{*}}{Y^{2}}\frac{\dot{p}^{*}}{p^{*}}\right], \tag{30}$$

where $Y^{2}\equiv y^{2}(K,p)+y^{2}(K^{*},p^{*})$ denotes the aggregate supply of consumption goods in the world market. Note that from $(17b)$, $(17c)$, $(17d)$ we obtain:

$$\frac{\dot{p}}{p}=\frac{\dot{q}}{q}-\frac{\dot{\lambda}}{\lambda}=R+\delta-\widetilde{r}(p), \tag{31a}$$

$$\frac{\dot{p}^{*}}{p^{*}}=\frac{\dot{q}^{*}}{q^{*}}-\frac{\dot{\lambda}^{*}}{\lambda^{*}}=R+\delta-\widetilde{r}(p^{*}). \tag{31b}$$

Substituting $(25a)$, $(25b)$, $(31a)$, and $(31b)$ into (30) yields the following:

$$\begin{aligned}\rho - R &= -\sigma\left[\frac{Y_K^2 K}{Y^2}\left(\frac{y^1(K,p)-\delta K}{K}\right)+\frac{Y_{K^*}^2 K^*}{Y^2}\left(\frac{y^2(K^*,p)-\delta K^*}{K^*}\right)\right.\\ &\left.+\frac{Y_p^2 p}{Y^2}\left(R+\delta-\widetilde{r}(p)\right)+\frac{Y_{p^*}^2 p^*}{Y^2}\left(R+\delta-\widetilde{r}(p^*)\right)\right].\end{aligned}$$

Observe that each side of the above equation does not involve $\bar{m}$. Solving the above with respect to R, we find that the equilibrium level of the world interest rate can be expressed as a function of K, K^*, p and p^* :

$$R = R(K, K^*, p, p^*). \tag{32}$$

Consequently, by use of $(25a)$, $(25b)$, $(31a)$, $(31b)$ and (32), we obtain the dynamic system with respect to (K, K^*, p, p^*) in such a way that

$$\left.\begin{aligned}\dot{K} &= y^1(K,p)-\delta K,\\ \dot{K}^* &= y^1(K^*,p^*)-\delta K^*,\\ \dot{p} &= p\left[R(K,K^*,p,p^*)+\delta-\widetilde{r}(p)\right],\\ \dot{p}^* &= p^*\left[R(K,K^*,p,p^*)+\delta-\widetilde{r}(p^*)\right].\end{aligned}\right\} \tag{33}$$

In the above, function $R(.)$ is rather complex, so that stability analysis of (33) is more cumbersome than analyzing the system of (K, K^*, q, q^*). However, since the solutions of (33) do not depend on $\bar{m}$, as shown in the next section, this alternative expression of dynamic system is useful for considering how $\bar{m}$ is determined.

4.3 Steady-State of the World Economy

We first characterize the stationary equilibrium of the world economy. In the steady state, all of K, K^*, p, p^*, B, B^*, q, q^* and λ stay constant over time. Inspecting the steady state conditions, we obtain the following:

Proposition 2 *Suppose that investment goods are not traded and international lending and borrowing are allowed. Then there is a feasible steady-state equilibrium where the steady-state levels of capital and relative price in each country satisfy $K = K^*$ and $p = p^*$ and they are uniquely given.*

Proof. See Appendix 1. ■

It is to be noted that while the steady-state levels of $K\,(= K^*)$ and $p\,(= p^*)$ are uniquely determined by the parameters involved in the model, the steady-state values of implicit prices of capital, q and q^*, cannot be determined by the parameter values alone. To see this, notice that from the optimization condition $(17b)$, in the steady state it holds that

$$p = \lambda q, \qquad p = \bar{m}^{\sigma} \lambda q^*, \tag{34}$$

From (28) in the steady state the implicit price of bond held in the home country, λ, is given by

$$\lambda = (1 + \bar{m})^{\sigma} \left[2y^2 (K, p)\right]^{-\sigma}.$$

Since λ depends on $\bar{m}$, we should know the value of $\bar{m}$ to determine the steady-state levels of λ, q and q^*. To find the value of $\bar{m}$, consider the current account of each country. Noting that the market equilibrium condition for the investment goods in (22) and the factor income distribution relation give $pY_1 + Y_2 = rK + w + \pi_1 + \pi_2$ and $p^* Y_1^* + Y_2^* = r^* K^* + w^* + \pi_1^* + \pi_2^*$, we see that the dynamic equation of bond holdings are expressed as

$$\dot{B} = RB + Y_2 - C, \qquad \dot{B}^* = RB^* + Y_2^* - C^*.$$

These equations represent the current accounts of both countries. In view of the no-Ponzi game and the transversality conditions, the intertemporal constraint for the current account of each country is respectively given by the following:

$$\int_0^{\infty} \exp\left(-\int_0^t R_s ds\right) C_t dt = \int_0^{\infty} \exp\left(-\int_0^t R_s ds\right) y^2 (K_t, p_t)\, dt + B_0,$$

$$\int_0^{\infty} \exp\left(-\int_0^t R_s ds\right) C_t^* dt = \int_0^{\infty} \exp\left(-\int_0^t R_s ds\right) y^2 (K_t^*, p_t^*)\, dt + B_0^*.$$

Since it holds that $C_t^* = \bar{m} C_t$ for all $t \geq 0$, the above equations yield

$$\bar{m} = \frac{\int_0^{\infty} \exp\left(-\int_0^t R_s ds\right) y^2 (K_t^*, p_t^*)\, dt + B_0^*}{\int_0^{\infty} \exp\left(-\int_0^t R_s ds\right) y^2 (K_t, p_t)\, dt + B_0}. \tag{35}$$

Equation (35) demonstrates that $\bar{m}$ depends on the initial holdings of bonds, B_0 and B_0^*, as well as on the entire sequences of $\{K_t, K_t^*, p_t, p_t^*\}_{t=0}^{\infty}$. Remember that the equilibrium paths of (K_t, K_t^*, p_t, p^*) determined by (33) and do not depend on $\bar{m}$. As a consequence, although the steady-state level of p depends only on the parameter values involved in the model, the steady state levels of $q\,(=\lambda p)$ and q^* $(=\bar{m}^{-\sigma}\lambda p^*)$ cannot be determined without specifying the initial holdings of bonds and the paths of the state variables. Therefore, if the dynamic system (33) exhibits indeterminacy, the value of $\bar{m}$ (so the steady-state values of q and q^*) are indeterminate as well.

4.4 Indeterminacy Conditions

We now examine the local dynamics of the world economy around the steady state. A set of sufficient conditions for equilibrium indeterminacy for the model with nontraded investment goods is as follows:

Proposition 3 *Suppose that investment goods are not traded and international lending and borrowing are allowed. If the investment good sector is more capital intensive than the consumption good sector from the social perspective but it is less capital intensive from the private perspective, that is,* $\frac{a_2}{b_2} - \frac{a_1}{b_1} > 0$ *and* $\alpha_2 - \alpha_1 < 0$, *then the steady state of the world economy where investment goods are nontradable exhibits local indeterminacy.*

Proof. See Appendix 2 . ■

Proposition 3 claims that in our model equilibrium indeterminacy may emerge regardless of the magnitude of σ. This is in contrast to Proposition 1 for the indeterminacy conditions for the case of free trade of both consumption and investment goods without international lending and borrowing. When both investment and consumption goods are freely traded, in addition to the factor-intensity ranking conditions, the intertemporal elasticity in consumption $(1/\sigma)$ should be sufficiently high to hold indeterminacy. Since the closed economy version of our model is the same as the integrated world economy model discussed by Nishimura and Shimomura (2002), we need the same condition for holding indeterminacy if our model economy is closed. Hence, our result shows that the financially integrated world with non-tradable capital goods may produce indeterminacy under a wider range of parameter spaces than in the closed economy counterpart. In this sense, our model indicates that opening up international trade may enhance the possibility of belief-driven economic fluctuations, if investment goods are nontradables. In Section 5.1 we present an intuitive implication of the difference in the indeterminacy conditions in Propositions 1 and 3.

4.5 Long-Run Wealth Distribution

In the steady state it holds that $\dot{B} = \dot{B}^* = 0$ and $R = \rho$. Considering that $C + C^* = 2y^2(K, p)$ and $C^* = \bar{m}C$, we find that the steady-state level of bond holdings in the home and foreign countries are respectively given by

$$B = \frac{C - y^2(K,p)}{\rho} = \frac{1-\bar{m}}{\rho(1+\bar{m})} y^2(K,p), \tag{36a}$$

$$B^* = \frac{C^* - y^2(K,p)}{\rho} = \frac{\bar{m}-1}{\rho(1+\bar{m})} y^2(K,p). \tag{36b}$$

The above expressions show that once $\bar{m}$ is selected, the long-run asset position of each country is also determined. The asset holdings in the steady state are:

$$\Omega = B + pK, \quad \Omega^* = B^* + pK.$$

Thus the long-run wealth distribution between the two countries depends on the long-run levels of B and B^*. It is obvious that whether the home country becomes a debtor or a creditor in the long run depends solely on whether or not $\bar{m}$ exceeds one. Consequently, determinant of $\bar{m}$ plays a pivotal role for the long-run distribution of wealth.

Now consider (3335). If the steady state of (33) is locally determinate (i.e. the linearized dynamic system has two stable roots), then the equilibrium path of p_t and p_t^* are uniquely expressed as functions of K_t and K_t^* on the two-dimensional stable manifold. When we denote the relation between the relative prices and capital stocks on the stable saddle path as $p = \phi(K, K^*)$ and $p^* = \phi^*(K, K^*)$, the behaviors of capital stocks on the stable manifold are expressed as

$$\begin{aligned} \dot{K} &= y^1(K, \phi(K, K^*)) - \delta K, \\ \dot{K}^* &= y^1(K^*, \phi^*(K, K^*)) - \delta K^*. \end{aligned}$$

These differential equations show that once the initial capital stocks, K_0 and K_0^*, are specified, the paths of $\{K_t, K_t^*\}_{t=0}^{\infty}$ are uniquely determined. As a result, the paths of $\{p_t, p_t^*, R_t\}_{t=0}^{\infty}$ are also uniquely given under the specified levels of K_0 and K_0^*. This means that when equilibrium determinacy holds, the left hand side of (35) that depends on the entire sequences of $\{p_t, p_t^*, K_t, K^*\}_{t=0}^{\infty}$ is also determinate, so that $\bar{m}$ has a unique value under the given initial levels of K_0, K_0^*, B_0 and B_0^*. In this case, the long-term distribution of wealth between the two countries is uniquely determined. For example, if the initial stocks of capital and bonds held by the home households are relatively large, then the home country tends to be a creditor in the long-run equilibrium.

By contrast, if the converging path of (33) is indeterminate (that is, the linearly approximated dynamic system of (33) has three or four stable roots), then the given initial levels of K_0 and K_0^* alone cannot pin down the equilibrium paths of p_t and p_t^*. Therefore, the level of $\bar{m}$ given by (35) becomes indeterminate as well. In this situation, an extrinsic shock that affects expectations of agents in the world market may alter the equilibrium path and, therefore, it changes the level of $\bar{m}$.

To sum up, we have shown:

Proposition 4 *If the steady-state equilibrium of the world economy is locally determinate (indeterminate), then the steady-state level of asset position of each country is determinate (indeterminate).*

4.6 Non-Tradable Consumption Goods

Now consider the opposite situation where the consumption goods are not internationally traded, but the investment goods are tradable and financial capital mobility is possible. In this case the commodity market equilibrium conditions are given by

$$I + I^* = Y_1 + Y_1^*, \quad C = Y_2, \quad C^* = Y_2^*. \tag{37}$$

We take the tradable investment good as a numeraire. Then the net wealth (in terms of investment good) held by the households in the home country is $\Omega = B + K$ and the flow budget constraint is written as

$$\dot{B} = R(B+K) + w + \pi_1 + \pi_2 - \widetilde{p}C - I,$$

where $\widetilde{p}$ $(= 1/p)$ denotes the domestic price of consumption good in terms of tradable investment good. The Hamiltonian function for the households in the home country is given by

$$H = \frac{C^{1-\sigma} - 1}{1-\sigma} + \lambda\left[RB + rK + w + \pi_1 + \pi_2 - \widetilde{p}C - I\right] + q(I - \delta K)$$

and the key first-order conditions for an optimum are:

$$C^{-\sigma} = \lambda \widetilde{p}, \tag{38a}$$

$$\lambda = q \tag{38b}$$

$$\dot{\lambda} = \lambda(\rho - R), \tag{38c}$$

$$\dot{q} = q(\rho + \delta - r). \tag{38d}$$

Conditions $(38b)$, $(38c)$ and $(38d)$ lead to $R = r - \delta$.

Since households in both country face the common interest rate, R in the international bond market, the rate of return to capital in both countries satisfy

$$r^* = R + \delta = r. \tag{39}$$

Thus $r(1/\widetilde{p}) = r(1/\widetilde{p}^*)$ holds in each moment, implying that $\widetilde{p}$ always equals $\widetilde{p}^*$. Consequently, it holds that $k_i(1/\widetilde{p}) = k_i(1/\widetilde{p}^*)$ $(i = 1, 2,)$, so that the world-market equilibrium condition of investment good yields the dynamic equation of the aggregate capital exactly the same as (20). In addition, from the equilibrium condition for consumption goods in each country in (37) we obtain

$$C = y^2(K, \widetilde{p}), \qquad C^* = y^2(K^*, \widetilde{p}).$$

In this case it holds that $\lambda^*/\lambda = q^*/q^*$ stay constant over time. Therefore, we obtain $C^* = \bar{s}C$, where $\bar{s} = (\lambda^*/\lambda)^{-1/\sigma}$. This leads to

$$(1+\bar{s})\,C = y^2\left(K,\widetilde{p}\right) + y^2\left(K^*,\widetilde{p}\right).$$

Consequently, the dynamic system of the world economy is the same as that of the Nishimura-Shimomura model.

Proposition 5 *If consumption goods are not traded and international lending and borrowing are possible, the indeterminacy conditions are the same as those for the case where both goods are traded without financial capital mobility.*

Therefore, in this case opening up international trade does not enhance the possibility of belief-driven business cycles. An intuitive implication of this result is as follows. If investment goods are tradable, a unit of bond is equivalent to a claim to the future capital good. Since bonds and capital are perfect substitutes, bonds yield the same rate of return as that of capital. Thus the interest rate of bond equals the net rate of return to capital. The interest rate in the integrated financial market is common for both countries, which means that the rate of return to capital in both country is the same as well. Since both countries have identical technologies, the relative price in each country is also the same, so that the integrated world economy behaves exactly the same manner as that of the economy in the Heckscher-Ohlin environment.

When only consumption goods are internationally traded, one unit of bond is a claim to the future consumption good. Hence, the non-arbitrage condition between holding of bond and capital shows that the rate of return to capital diverges from the world interest rate when the relative price between consumption and investment changes. Hence, the factor prices (so that the relative price) in each country are not identical during the transition. The failure of factor-price equalization makes the system with non-traded investment goods diverge from the Heckscher-Ohlin setting.

5 Discussion

5.1 Implication of the Indeterminacy Conditions

Intuition behind the difference in indeterminacy condition between Propositions 1 and 3 is as follows:

(i) Free Trade of Commodities

First consider the case where both consumption and investment goods are traded in the absence of international lending and borrowing. Suppose that a positive sunspot shock hits the world economy and all the households in the world expect that the rate of return to their capital will rise in the future. Such an impact makes the households reduce their current consumption and invest more. If the intertemporal elasticity of substitution in consumption, $1/\sigma$, is sufficiently high, there is a large increase in the future consumption. Meanwhile, the households expand their current investment and the world-wide capital stock will rise. Since we have assumed that the private technology of consumption good sectors is more capital intensive than that of the investment good sector, a higher capital stock will expand the consumption production in both countries through the Rybczynski effect. However, the strong intertemporal substitution effect yields a large increase in future consumption demand and, hence, the relative price p must increase to equilibrate the world-wide consumption good market. (Remember that from (10b) under our assumptions of $\frac{a_1}{b_1} - \frac{q_2}{b_2} < 0$ and $\alpha_1 - \alpha_2 > 0$, a higher p increases Y_2 and Y_2^*.) Noting that a rise in p increases $\widetilde{r}\,(p)$ under $\alpha_1 - \alpha_2 > 0$, a higher p actually raises the rate of return to capital, so that the initial expectations can be self-fulfilled.

By contrast, if $1/\sigma$ is not high enough, the above mechanism of adjustment will not work. If $1/\sigma$ is small, the intertemporal substitution effect is small and thus the expected rise in the future rate of return produce a relatively small amount of increase in the future consumption. If this is the case, an increase in consumption good production generated by a rise in K through the Rybczynski effect may exceed the increase in consumption demand. As a result, the relative price will decline to curtail the production level of consumption goods to meet the relatively small increase in demand. Hence, in contrast to the case with a high $1/\sigma$, a lower p reduces $\widetilde{r}\,(p)$ .This means that the initial expectations are not self fulfilled, and thus the equilibrium path of the world economy is determinate.

(ii) Nontradable Investment Goods

Next, consider the case where only consumption goods are traded and international lending and borrowing are allowed. In this case, the relative price in each country is not the same during the transition. Suppose that households in the home country expect that the rate of return to their capital will rise. As before, the households intend to raise their saving to invest more. In the Heckscher-Ohlin environment, this requires that households reduce their current consumption, and thus the magnitude of σ plays an important role. However, in the presence of international financial market, the households in the home country may increase their investment by borrowing from foreign households rather than by lowering their current consumption. Hence, investment demand will increase even if σ is not small. Then the households in the home country pay their debt by exporting consumption goods to the foreign country. Hence, the consumption good production in the home country will expand. This means that the relative price p may increase to complement the positive effect of a higher K on consumption good production. If this is the case, the rate of return to capital in the home country actually rise to fulfill the initial expectations of the households.

5.2 Empirical Plausibility of the Basic Assumptions

(i) Distinction between Traded and Nontraded Goods

In this paper we have considered three types of trade structures: (i) both investment and consumption goods are internationally traded; (ii) only consumption goods are traded, and; (iii) only investment goods are traded. In reality, considerable portions of both consumption and investment goods are traded in the domestic markets alone. For example, Coeurdacier (2009) claims that more than 50% of US consumption goods are not traded in the international markets, because the value added of services most of which are nontradables shares 55% of the aggregate value of consumption goods. Similarly, Baxter et al. (1998), Jin (2011), and Stockman and Tesar (1995) point out that more than 50% of consumption goods are not internationally traded in the US.

As for investment goods, Bems (2008) finds that the share of investment expenditure on nontraded goods is about 60% and that this figure has been considerably stable over the last

50 years both in developed and developing countries. Since construction and structures share a large part of investment goods, Bems' finding seems to be a plausible one.

Judging from those empirical facts, the traditional assumption of free trade of all commodities (trade structure (i)) is far from the reality. At the same time, it is rather hard to determine which of trade structures (ii) or (iii) is more realistic. Probably, it is safe to conclude that both (ii) and (iii) have roughly the same distance from the reality. However, from the theoretical viewpoint, the key condition for the relation between openness of an economy and belief-driven fluctuations is whether or not investment goods are freely traded. As we have seen in Section 4.6, if investment goods are tradables, the indeterminacy conditions do not diverge from those for the case of free intratemporal trade of both commodities.[14]

(ii) Externalities and Factor Intensity Ranking

The indeterminacy conditions in Proposition 1 and 3 require that constant returns prevail in each production sector at the social level and that there is a factor-intensity reversal between the private and social technologies. The production technologies assumed in this paper demonstrate that equilibrium indeterminacy may emerge even in the absence of strong increasing returns associated with large degree of external effects.[15] Several investigations on scale economies and factor-intensity ranking have suggested that our indeterminacy conditions are empirically plausible ones. For example, the well-cited study by Basu and Fernald (1997) finds that most industries in the US approximately exhibit constant returns to scale. Using the US data, Based on a detailed investigation of disaggregated data of the US industries, Harrison (2003) claims that consumption good industries exhibits decreasing internal returns to scale. Their aggregate returns including external effects are close to constant. Investment goods industries, on the other hand, show weak returns to scale. Those findings suggest that our assumption of social constant returns in both consumption and investment good sectors is not far from the reality.

[14] It is worth pointing out that Meng and Velasco (2004) show that indeterminacy still holds even if there are both traded and nontraded investment goods. Reconsidering their finding in the context of our world economy model would be a useful extension.

[15] In our notation, external effects associated with capital and labor in i-th sector are respectively given by $\varepsilon_i = \alpha_i - a_i$ and $\eta_i = 1 - \alpha_i - b_i \quad (i = 1, 2)$. The factor-intensity ranking conditions in Proposition 3 mean that

$$\frac{a_1}{b_1} < \frac{a_2}{b_2} \quad \text{and} \quad \frac{a_1 + \varepsilon_1}{b_1 + \eta_1} > \frac{a_2 + \varepsilon_2}{b_2 + \eta_2}.$$

Notice that the above inequalities can hold, even though the magnitudes of external effects, ε_i and η_i, are sufficiently small.

As to the factor-intensity ranking between the two sectors, a recent study by Takahashi et al. (2012) find that in most of the OECD countries, the consumption good sector uses a more capital intensive technology than the investment good sector.[16] They also show that the gap in capital intensities between the two production sectors is generally small. If there is a large difference in factor intensities from the private perspective, then we need large degree of production externalities to establish the factor-intensity reversal between the social and private technologies. The relatively small difference in the capital-labor ratios between the consumption and investment good sectors means that the factor-intensity reversal may hold even in the presence of small-scale external effects. Although the empirical studies cited above do not directly support our assumptions, they indicate that the indeterminacy conditions given in Proposition 3 can hold under a set of empirically plausible magnitudes of parameter values involved in our model.

6 Final Remarks

The world economy as a whole is a closed economy in which there are multiple countries. Therefore, its model structure is similar to that of a closed, single economy model with heterogeneous agents. In particular, if consumption and saving decisions are made by the representative household in each country, the world economy model is closely connected to the closed economy model with heterogeneous households. There is, however, an important difference between the world economy models and the single country setting: when dealing with the world economy model, we should specify the trade structure between the countries. This paper has revealed that the specification of trade structure plays an important role as to the presence of equilibrium indeterminacy, even if there is no international heterogeneity in technologies and preferences.

Our research topic can be explored further in several directions. First, we may reconsider indeterminacy of equilibrium without assuming symmetric technologies and preferences between the two countries. Recently, several authors have explored how the presence of heterogeneous preferences and technologies alter the determinacy/indeterminacy conditions in

[16] In their estimation, Takahashi et al. (2012) do not assume the presence of production externalities. This means that their finding would support our assumption in Proposition 3, that is, the consumption good sector employes more capital intensive technology than the investment good sector from the private perspective.

the equilibrium business cycle models with market distortions. These studies have shown that the heterogeneity in preferences and technologies often affects stability condition in a critical manner.[17] In a similar vein, Sim and Ho (2007) find that introducing technological heterogeneity into the Nishimura-Shimomura model may produce a substantial change in equilibrium indeterminacy results. In addition, even if taste and technologies are identical in both countries, introducing financial frictions, policy distortions and adjustment costs of investment also breaks the symmetry between the home and foreign countries at least during the transition process. Further investigation of our problem in a more general modelling would be promising.[18]

Second, it would be useful to examine our topic in a general model in which both consumption and investment goods are partially traded. In such a framework, we may investigate how changes in the shares of tradables affect indeterminacy conditions. This research would provide us with useful results as to the relation between the degree of 'globalization' and belief-driven, international business cycles.

Finally, our discussion may apply to the regional economy setting as well. Judging from our findings, we may conjecture that specific patterns of commodity trade and factor mobility between multiple regions would produce equilibrium indeterminacy of the entire economy. It is also an interesting topic for our future research to study the relation between intranational trade patterns and sunspot-driven business fluctuations. [19]

Appendices

Appendix 1: Proof of Proposition 2

When $\dot{q} = \dot{q}^* = 0$ in $(26a)$ and $(26b)$, it holds that

$$a_1 A_1 k_1 (p)^{\alpha_1 - 1} = a_1 A_1 k_1 (p^*)^{\alpha_1 - 1} = \rho + \delta.$$

[17] See, for example, Ghiglino and Olszak-Duquenne (2005).

[18] Antras and Caballero (2009) introduce financial frictions into the two-county Heckscher-Ohlin model. Ono and Shibata (2010) and Jin (2011) introduce adjustment costs of investment into 2×2 × 2 models.

[19] We thank one of the referees for this suggestion.

Thus by use of $(3a)$ we find:

$$p = p^* = \left(\frac{A_2}{A_1}\right)\left(\frac{a_2}{a_1}\right)^{\alpha_2}\left(\frac{b_2}{b_1}\right)^{1-\alpha_2}\left(\frac{\rho+\delta}{a_1 A_1}\right)^{\frac{\alpha_2-\alpha_1}{\alpha_1-1}}.$$

These conditions show that the steady-state levels of p and p^* are uniquely given and it holds that $p = p^*$ in the steady state. The steady-state levels of capital stocks satisfying $\dot{K} = \dot{K}^* = 0$ in $(25a)$ and $(25b)$ are determined by the following conditions:

$$\frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1} = \delta K,$$

$$\frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_1 k_1(p^*)^{\alpha_1} = \delta K^*.$$

Using the conditions for $\dot{p} = \dot{p}^* = 0$ and the fact that $p = p^*$ holds in the steady state, we confirm that the steady-state level of capital stock in each county has the same value, which is given by

$$K = K^* = \frac{(aA_1)^{\frac{1}{1-\alpha_1}}(\rho+\delta)^{\frac{\alpha_1}{\alpha_1-1}}}{\rho + \delta\left(1 - \delta + \frac{a_2 b_1}{b_2}\right)}\left(\frac{a_2 b_1}{a_1 b_2}\right),$$

which has a positive value. We also find that the steady-state values of labor allocation to the investment good sector are:

$$L_1 = L_1^* = \frac{a_1\delta\left(\frac{a_2 b_1}{a_1 b_2}\right)}{\rho + (1-a_1)\delta + a_1\delta\left(\frac{a_2 b_1}{a_1 b_2}\right)} \in (0,1).$$

Hence, (8) is fulfilled so that both countries imperfectly specialize.

Appendix 2: Proof of Proposition 3

Since the functional form of $R(K, K^*, p, p^*)$ in (33) is complicated, it is simpler to treat a dynamic system with respect to K, K^*, q and q^* displayed in Section 4.1. We thus focus on the dynamics system consisting of $(25a)$, $(25b\ )$, $(26a)$ and $(26b)$ with $p = \pi(K, K^*, q, , q^*; \bar{m})$ and $p^* = \pi^*(K, K^*, q, q^*; \bar{m})$, where $\bar{m}$ is fixed.[20]

[20] When the dynamic system of (K, K^*, q, q^*) satisfies equilibrium determinacy under a given level of $\bar{m}$, then the equilibrium paths of K and K^* are uniquely determined under given levels of K_0 and K_0^*. Therefore, equilibrium path of (33) is also uniquely determined. Conversely, if the dynamic system of (K, K^*, q, q^*) exhibits local indeterminacy, the equilibrium paths of K and K^* cannot be uniquely determined by selecting

To prove Proposition 3, the following facts are useful:

Lemma 1 *In the symmetric steady state where $K = K^*$ and $q = q^*$, the following relations are satisfied:*

$$y^i_K(K,p) = y^i_{K^*}(K^*,p^*), \quad i = 1,2,$$

$$y^i_p(K,p) = y^i_{p^*}(K^*,p^*), \quad i = 1,2,$$

$$\pi_K(K,K^*,q,q^*) = \pi^*_K(K,K^*,q,q^*) = \pi_{K^*}(K,K^*,q,q^*) = \pi^*_{K^*}(K,K^*,q,q^*),$$

$$\pi_q(K,K^*,q,q^*) = \pi^*_{q^*}(K,K^*,q,q^*),$$

$$\pi_{q^*}(K,K^*,q,q^*) = \pi^*_q(K,K^*,q,q^*).$$

Proof. By the functional forms of $y^i_j(\cdot)$ ($i = 1,2,\ j = K,K^*,p,p^*$), it is easy to see that $y^i_K(K,p) = y^i_{K^*}(K^*,p^*)$ and $y^i_p(K,p) = y^i_{p^*}(K^*,p^*)$ are established when $p = p^*$ and $K = K^*$. As for the rest of the results, we use $p\lambda(\cdot) = q$ and $p^*\lambda(\cdot)\bar{m}^{-\sigma} = q^*$. total differentiation of $p\lambda(\cdot) = q$ and $p^*\lambda(\cdot)\bar{m}^{-\sigma} = q^*$ yields the following:

$$\frac{\partial p}{\partial K} = \pi_K = -\frac{p\lambda_K}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \quad \frac{\partial p}{\partial K^*} = \pi_{K^*} = -\frac{p\lambda_{K^*}}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \tag{A1}$$

$$\frac{\partial p^*}{\partial K} = \pi^*_K = -\frac{p^*\lambda_K}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \quad \frac{\partial p^*}{\partial K^*} = \pi^*_{K^*} = -\frac{p^*\lambda_{K^*}}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \tag{A2}$$

$$\frac{\partial p}{\partial q} = \pi_q = \frac{\lambda + p^*\lambda_{p^*}}{\lambda(\lambda + p\lambda_P + p^*\lambda_{p^*})}, \quad \frac{\partial p}{\partial q^*} = \pi_{q^*} = -\frac{p\lambda_p}{\lambda(\lambda + p\lambda_P + p^*\lambda_{p^*})}, \tag{A3}$$

$$\frac{\partial p^*}{\partial q} = \pi^*_q = -\frac{p^*\lambda_{p^*}}{\lambda + p\lambda_P + p^*\lambda_{p^*}}, \quad \frac{\partial p^*}{\partial q^*} = \pi^*_{q^*} = \frac{\lambda + p\lambda_p}{\lambda + p\lambda_P + p^*\lambda_{p^*}}. \tag{A4}$$

Since $\lambda_K(\cdot) = \lambda_{K^*}(\cdot)$ and $\lambda_p(\cdot) = \lambda_{p^*}(\cdot)$ in the steady state where $K = K^*$ and $p = p^*$,, we obtain $\pi_K = \pi^*_K = \pi_{K^*} = \pi^*,\ \pi_q = \pi^*_{q^*}$ and $\pi_{q^*} = \pi^*_q$. ■

Under a given level of $\bar{m}$, let us linearize the dynamic system of $(25a)$, $(25b)$, $(26a)$ and

K_0 and K^*_0. This means that (33) also holds equilibrium indeterminacy.

(26b) at the steady state. The coefficient matrix of the linearized system is given by

$$J = \begin{bmatrix} y_K^1 - \delta + y_p^1 \pi_K & y_p^1 \pi_{K^*} & y_p^1 \pi_q & y_p^1 \pi_{q^*} \\ y_{p^*}^1 \pi_K^* & y_{K^*}^1 - \delta + y_{p^*}^1 \pi_{K^*}^* & y_{p^*}^1 \pi_q^* & y_{p^*}^1 \pi_{q^*}^* \\ -q\widetilde{r}' \pi_K & -q\widetilde{r}' \pi_{K^*} & -q\widetilde{r}' \pi_q & -q\widetilde{r}' \pi_{q^*} \\ -q\widetilde{r}' \pi_K^* & -q\widetilde{r}' \pi_{K^*}^* & -q\widetilde{r}' \pi_q^* & -q\widetilde{r}' \pi_{q^*}^* \end{bmatrix}.$$

By use of Lemma 1, we see that the characteristic equation of J is written as

$$\begin{aligned} \Gamma(\eta) &= \det[\eta I - J] \\ &= \det \begin{bmatrix} \eta - (y_K^1 - \delta + y_p^1 \pi_K) & -y_p^1 \pi_K & -y_p^1 \pi_q & -y_p^1 \pi_{q^*} \\ -y_p^1 \pi_K & \eta - (y_K^1 - \delta + y_p^1 \pi_K) & -y_p^1 \pi_{q^*} & -y_p^1 \pi_q \\ q\widetilde{r}'' \pi_K & q\widetilde{r}' \pi_K & \eta + q\widetilde{r}' \pi_q & q\widetilde{r}' \pi_{q^*} \\ q\widetilde{r}' \pi_K & q\widetilde{r}' \pi_K & q\widetilde{r}' \pi_q & \eta + q\widetilde{r}' \pi_q \end{bmatrix} \\ &= \det \begin{bmatrix} \eta - \left(y_K^1 - \delta\right) & 0 & \eta & 0 \\ 0 & \eta - (y_K^1 - \delta) & 0 & \eta \\ q\widetilde{r}' \pi_K & q\widetilde{r}' \pi_K & \eta + q\widetilde{r}' \pi_q & q\widetilde{r}' \pi_{q^*} \\ q\widetilde{r}' \pi_K & q\widetilde{r}' \pi_K & q\widetilde{r}' \pi_q & \eta + q\widetilde{r}' \pi_q \end{bmatrix} \\ &= \left[\eta - \left(y_K^1 - \delta\right)\right] \left[\eta + q\hat{r}'(\pi_q - \pi_{q^*})\right] \xi(\eta). \end{aligned}$$

where η denotes the characteristic root of J and

$$\xi(\eta) \equiv \eta^2 + \left[q\widetilde{r}'(\pi_q + \pi_{q^*}) - \left(y_K^1 - \delta\right) - 2y_p^1 \pi_K\right] \eta - q\widetilde{r}' \left(y_K^1 - \delta\right)(\pi_q + \pi_{q^*}).$$

Our assumptions mean that $\frac{a_1}{b_1} - \frac{a_2}{b_2} < 0$ and $\alpha_1 - \alpha_2 > 0$. Thus from (10a) we see that $y_K^1 - \delta < 0$. In addition, the equations in (A3) mean that $\pi_q - \pi_{q^*} = 1/\lambda\,(> 0)$. Hence, using $\hat{r}(p) \equiv a_1 A_1 k_1(p)^{\alpha_1 - 1}$, we obtain:

$$\widetilde{r}'(\pi_q - \pi_{q^*}) = a_1(a_1 - 1) A_1 (k_1(p))^{a_1 - 2} \frac{k_1'(p)}{\lambda} > 0.$$

Thus at least two roots of $\Gamma(\eta)=0$ have negative real parts. In addition, (A3) shows

$$\pi_q+\pi_{q^*}=\frac{1}{\lambda+2p\lambda_p},$$

where

$$\begin{aligned}\lambda_p &= \frac{\partial}{\partial p}(1+\bar{m})^{\frac{1}{\sigma}}\left[y^2(K,p)+y^2(K^*,p^*)\right]^{-\frac{1}{\sigma}}\\ &= -\frac{y_p^2}{\sigma}(1+\bar{m})^{\frac{1}{\sigma}}\left[y^2(K,p)+y^2(K^*,p^*)\right]^{-\frac{1}{\sigma}-1}<0.\end{aligned}$$

Therefore, in the steady state equilibrium. the following holds:

$$\lambda+2p\lambda_p=\frac{1}{\sigma}\left[\sigma-\frac{py_p^2(K,p)}{y^2(K,p)}\right].$$

Notice that under our assumptions, it holds that $y_p^2(K,p)>0$. Suppose that σ is small enough to satisfy $\sigma<py_p^2/y^2$. Then $\lambda_p+2p\lambda_p>0$ so that $\pi_q+\pi_{q^*}<0$, which leads to

$$-q\widehat{r}'\left(y_K^1-\delta\right)(\pi_q+\pi_{q^*})<0.$$

This means that $\xi(\eta)=0$ has one positive and one negative roots. As a result, $\Gamma(\eta)=0$ has three stable roots. Hence, if σ is smaller than the price elasticity of supply function of consumption goods, then there locally exists a continuum of equilibrium paths converging to the steady state.

Now suppose that σ is larger than py_p^2/y^2. Then we obtain $\pi_q+\pi_{q^*}>0$. Furthermore, it holds that

$$\begin{aligned}-2y_p^1\pi_K &= -2y_p^1\left(-\frac{p\lambda_K}{\lambda+2p\lambda_p}\right)\\ &= -\frac{2py_p^1}{\lambda+2p\lambda_p}y_K^2\left[\frac{(1+\bar{m})^{\sigma^{-1}}}{\sigma}\right]\left(2y^2\right)^{-\sigma^{-1}-1}>0,\end{aligned}$$

because $y_p^1<0$ and $y_K^2>0$ under our assumptions. Consequently, the following inequalities are established:

$$-q\hat{r}'\left(y_K^1-\delta\right)(\pi_q+\pi_{q^*})>0,$$

$$q\widetilde{r}'\left(\pi_q+\pi_{q^*}\right)-\left(y_K^1-\delta\right)-2y_K^1\pi_K>0.$$

These conditions mean that $\xi(\eta)=0$ has two roots with negative real parts and, hence, all the roots of $\Gamma(\eta)=0$ are stable ones. In sum, if $\frac{a_1}{b_1}-\frac{a_2}{b_2}<0$ and $\alpha_1-\alpha_2>0$, then the characteristic equation of the linearized system involves at least three stable roots, regardless of the value of σ.

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