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Kyoto University
Towards greater building earthquake resilience using concept of critical excitation: a review

I. Takewaki

Department of Architecture and Architectural Engineering
Graduate School of Engineering, Kyoto University
Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8540, Japan
E-mail: takewaki@archi.kyoto-u.ac.jp

ABSTRACT

The words of ‘unexpected issue’ and ‘earthquake resilience’ are frequently used after the 2011 off the Pacific coast of Tohoku earthquake which occurred March 11, 2011. Although the unexpected issues are hard to include in the structural design stage of civil structures, those certainly decrease the earthquake resilience of those civil structures. Once these unexpected issues are taken into account in the structural design, those issues become expected issues. However these repetitions of cycles, i.e. experiences of unexpected issues during earthquakes and incorporation into design codes, never resolve the essential problems in structural earthquake engineering.

In this paper, a historical review is made on the development of critical excitation methods as worst-scenario analysis and some possibilities of application of this concept to upgrading of building earthquake resilience are discussed.

Keywords: Earthquake resilience, Critical excitation method, Uncertainty analysis, Robustness, Redundancy, Earthquake engineering
1. Introduction

The word of ‘earthquake resilience’ is frequently used especially after the 2011 off the Pacific coast of Tohoku earthquake which occurred March 11, 2011. Earthquake resilience is utilized in various fields including society, community and structural engineering etc. It implies the ability or capability to recover from certain damaged states or the toughness not to be damaged against various disturbances. As far as the earthquake structural engineering is concerned, when structural designers try to investigate the earthquake resilience, they have to evaluate the earthquake performances of building structures with various uncertainties under broader range of earthquake ground motions, preferably for critical excitation.

Since the Drenick’s pioneering work in 1970, the critical excitation methods have been tackled from various viewpoints. The critical input to a structure is a resonant wave to the structure. Most earthquake engineers believed that such phenomena never occur in a real world. However, some examples were actually observed during Mexico (1985), Northridge (1994), Kobe (1995), Tohoku (2011).

An efficient methodology is required to evaluate the robustness (degree of insensitiveness of response) of a building with uncertain structural properties under uncertain ground motions. It is well known (Fujita and Takewaki 2011a-c, 2012a, b, Takewaki et al. 2012b) that base-isolated buildings and structural controlled buildings have large structural uncertainties due to wide variability of base-isolation members and passive dampers for structural control caused by temperature and frequency dependencies, manufacturing errors and aging effect than earthquake resistant buildings. This procedure of taking large variability into account is well established in Japan in the actual structural design stage of high-rise and base-isolated buildings. Furthermore, after the devastating disaster of the 2011 off the Pacific coast of Tohoku earthquake in Japan, it is under discussion that base-isolated buildings are vulnerable against unexpected long-period ground motions. In fact, it is reported that some base-isolated buildings exhibited unfavorable behavior.

Under these circumstances, it is desired to evaluate the response variability caused by such structural variability and uncertain ground motions (Elishakoff and Ohsaki 2010, Takewaki et al. 2011). The method based on the convex model may be one possibility (Ben-Haim and Elishakoff 1990). This will be reviewed in the following section. Introduction of a bound on Fourier amplitude of input ground motions may be another approach (Takewaki and Fujita 2009). Independently, Kanno and Takewaki (2005, 2006) proposed an efficient and reliable method for evaluating the robustness of structures under uncertainties based on the concept of the robustness function (Ben-Haim 2001, Takewaki and Ben-Haim 2005). This will also be reviewed in the following section. However it does not appear that an efficient and reliable method for evaluating the robustness of structures has been proposed.

An interval analysis is believed to be one of the most efficient and reliable method to respond to this requirement. The interval analysis is aimed at finding the worst combination of uncertain parameters which attains the maximum or minimum objective function. While a basic assumption of "inclusion monotonic" is introduced in usual interval analysis, a possibility should be taken into account of occurrence of the extreme value of the objective function in an inner feasible domain of the interval parameters for more accurate and reliable evaluation of the objective function. This is very difficult because the number of combinations from finite to infinite. It is shown that the critical combination of the structural parameters can be derived.
explicitly by maximizing the objective function by the use of the second-order Taylor series expansion. This method is called the URP (Updated Reference-Point) method (Fujita and Takewaki 2011a, b, c). When nonlinear elastic-plastic responses are dealt with, it is useful to introduce an approximate objective function by the use of the method combining the URP method (Fujita and Takewaki 2011a, b, c) with a kind of response surface method (Fujita and Takewaki 2012a).

2. Robustness, redundancy and resilience

In the field of structural engineering, robustness, redundancy and resilience play an important role in order to guarantee the safety of infrastructures against severe disturbances, e.g. earthquakes, strong winds, impacts. Progressive collapse has to be avoided absolutely because progressive collapse often leads to a catastrophic damage. Progressive collapse is sometimes defined as follows:

*Spread of local damage, from an initiating event, from element to element resulting, eventually, in the collapse of an entire structure or a disproportionately large part of it; also known as disproportionate collapse* (ASCE 2005, Ellingwood 2006).

A multiplex safety, often called fail-safe, may be of significance from the viewpoint of response to unexpected issues. The concepts of robustness, redundancy and resilience are closely interrelated. In general, robustness means insensitiveness of a system to parameter variation and implies toughness to disturbances (Ben-Haim 2001, Takewaki and Ben-Haim 2005, Kanno and Takewaki 2005, 2006a-c, 2007, Takewaki 2008a). On the other hand, redundancy indicates the degree of safety, frequently expressed by a safety factor (Doorn and Hansson 2011), of a system against disturbances or the connectivity of components. In the latter meaning, a parallel system is regarded as a preferable system able to avoid sudden overall system failure (the fail-safe system is a representative one). Resilience can be regarded as an ability of a system to recover from a damaged state or resist external disturbances and seems to be a more generic concept including robustness and redundancy (Takewaki et al. 2011a, 2012b). Recently the concept of resilience is getting much interest in broad fields of society (Ellingwood et al. 2006, Takewaki et al. 2011a, Committee on National Earthquake Resilience 2011, Poland 2012). In the report of Committee on National Earthquake Resilience (2011), there are some explanations. The followings are examples.

“The capability of an asset, system, or network to maintain its function or recover from a terrorist attack or any other incident” (DHS, 2006).

“The capacity of a system, community or society potentially exposed to hazards to adapt, by resisting or changing in order to reach and maintain an acceptable level of functioning and structure. This is determined by the degree to which the social system is capable of organizing itself to increase this capacity for learning from past disasters for better future protection and to improve risk reduction measures” (UN ISDR, 2006).

“The ability of social units (e.g., organizations, communities) to mitigate risk and contain the effects of disasters, and carry out recovery activities in ways that minimize social disruption while also minimizing the effects of future disasters. Disaster Resilience may be characterized by reduced likelihood of damage to and failure of critical infrastructure, systems, and components; reduced injuries, lives lost, damage, and negative economic and social impacts;
and reduced time required to restore a specific system or set of systems to normal or pre-disaster levels of functionality” (MCEER, 2008).

The term of resilience is often used loosely, vaguely and inconsistently (Committee on National Earthquake Resilience 2011). After some useful discussions, the following definition is summarized in the report of Committee on National Earthquake Resilience (2011).

*A disaster-resilient nation is one in which its communities, through mitigation and predisaster preparation, develop the adaptive capacity to maintain important community functions and recover quickly when major disasters occur.*

To investigate ‘earthquake resilience’ in more depth, eighteen tasks are treated in the report of Committee on National Earthquake Resilience (2011), e.g., physics of earthquake processes, earthquake early warning, earthquake scenarios.

3. Representation of uncertainty in selecting design earthquake ground motions

3.1 Origin and early stage of critical excitation method

Natural hazards are hard to understand clearly because there exist aleatory and epistemic uncertainties even now in the modern society. In particular, the properties of earthquake ground motions are highly uncertain both in epistemic and aleatory sense and it is believed to be a hard task to predict forthcoming events precisely (Geller et al. 1997, Stein 2003, Aster 2012). It has been made clear through numerous investigations that near-field ground motions (Northridge 1994, Kobe 1995, Turkey 1999 and Chi-Chi, Taiwan 1999) and the far-field motions (Mexico 1985, Tohoku 2011) have some peculiar, unpredictable characteristics.

In the history of earthquake resistant design of building structures, we learned a lot of lessons from actual earthquake disasters after Nobi earthquake 1891 (Japan) and San Francisco earthquake 1906 (USA). After we encountered a major earthquake disaster, we upgraded the earthquake resistant design codes many times. However the repetition of this revision does not resolve the essential problem. To overcome this problem, the concept of critical excitation was introduced. Although the concept of active structural control was developed in 1980-, the actual installation of those devices has further difficulties. Based on these observations, approaches based on the concept of "critical excitation" seem to be promising.

Through a conference discussion with a Japanese officer in 1960’s (Drenick 2002), Drenick (1970) formulated this problem in a mathematical framework and many researchers followed him. The terminology of ‘critical excitation’ may come from Penzien (Drenick 2002). The detailed history can be found in the reference (Takewaki 2007). It is natural to imagine that a ground motion input resonant to the natural frequency of the structure is a critical excitation. Drenick showed that the mirror image of the impulse response function of a single-degree-of-freedom (SDOF) system becomes the critical excitation of that SDOF system subject to a constraint on acceleration power of input acceleration (see Fig.1). Since Drenick’s problem and its solution were found to be conservative, Shinozuka (1970) discussed the same critical excitation problem in the frequency domain. He proved that, if an envelope function of Fourier amplitude spectra can be specified, a nearer upper bound of the maximum response can be obtained.
Various measures of criticality

Various quantities have been chosen and proposed as an objective function to be maximized in the critical excitation problems. Ahmadi (1979) posed another critical excitation problem including the response acceleration as the objective function to be maximized. He demonstrated that a rectangular wave in time domain is the critical one and recommended to introduce another constraint in order to make the solution more realistic. Westermo (1985) considered the input energy during $T$ divided by the mass $m$ as the objective function in a new critical excitation problem. He also imposed a constraint on the time integral of squared input acceleration. He introduced a variational approach and demonstrated that the critical input acceleration is proportional to the response velocity. His solution is not necessarily complete and explicit because the response velocity is actually a function of the excitation to be obtained. He pointed out that the critical input acceleration includes the solution by Drenick (1970). The damage of structures may be another measure of criticality. The corresponding problems have been tackled by some researchers.

Takewaki (2004, 2005) treated the earthquake input energy as the objective function in a new critical excitation problem. It has been shown that the formulation of the earthquake input energy in the frequency domain is essential for solving the critical excitation problem and deriving a bound on the earthquake input energy for a class of ground motions. The criticality has been expressed in terms of degree of concentration of input motion components on the maximum portion of the characteristic function defining the earthquake input energy.

Srinivasan et al. (1991) extended the basic approach due to Drenick (1970) to multi-degree-of-freedom (MDOF) models. They used a variational formulation and selected a quantity in terms of multiple responses as the objective function. They demonstrated that the relation among the critical displacement, velocity and acceleration responses is similar to the well-known relation among the displacement, velocity and acceleration response spectra.

Subcritical excitation

It was suggested that the critical excitation introduced by Drenick (1970) is conservative compared to the recorded ground motions. To resolve this problem, Drenick, Wang and their

Abdelrahman et al. (1979) extended the idea of subcritical excitation to the method in the frequency domain. They proposed a set of Fourier spectra of accelerograms has been expressed as a linear combination of Fourier spectra of recorded accelerograms. They pointed out clearly that the frequency-domain approach is more efficient than the time-domain approach.

An optimization technique was used by Pirasteh et al. (1988) in one of the subcritical excitation problems. They superimposed accelerograms recorded at similar sites to construct the candidate accelerograms, then used optimization and approximation techniques in order to find the most critical accelerogram. The most critical accelerogram was defined as the one which satisfies the constraints on peaks, Fourier spectra, intensities, growth rates and maximizes the damage index in the structure. The damage index has been defined as cumulative inelastic energy dissipation or sum of interstory drifts.

Abbas and Manohar (2002, 2005) employed a Fourier-series expression as a set of candidate ground motions. This is a new kind of subcritical excitation methods. Furthermore, in Japan, a procedure is used in the structural design of high-rise and base-isolated buildings to select several representative ground motions (recorded and simulated). This procedure is a kind of sub-critical excitation.

3.4 Stochastic excitation

Since an earthquake ground motion can be regarded as a realization of a random process, it seems rational to describe the earthquake ground motion using a stochastic model. The concept of critical excitation was extended to probabilistic problems by Iyengar and Manohar (1985, 1987), Iyengar (1989), Srinivasan et al. (1992), Manohar and Sarkar (1995), Sarkar and Manohar (1996, 1998), Takewaki (2000a-d, 2001a-c) and Abbas and Manohar (2002). The papers due to Iyengar and Manohar (1985, 1987) may be the first to discuss probabilistic critical excitation methods. They used a stationary model of input ground acceleration in the paper (Iyengar and Manohar 1985) and utilized a nonstationary model of ground accelerations expressed as $\ddot{u}_c(t) = c(t)w(t)$ in the paper (Iyengar and Manohar 1987). $c(t)$ is a deterministic envelope function and $w(t)$ is a stochastic function representing a stationary random Gaussian process with zero mean.

3.5 Convex models

A convex model is defined mathematically as a set of functions. Each function is a realization of an uncertain event. Several interesting convex models were proposed by Ben-Haim and Elishakoff (1990), Ben-Haim et al. (1996), Pantelides and Tzan (1996), Tzan and Pantelides (1996a) and Baratta et al. (1998) for ground motion modeling which can be constructed versatilely depending on the level of prior information available. Although the convex models lead to a simple treatment of complex problems, it is also true that a special class of problems, sometimes important and useful in realistic situations, cannot be dealt with by the convex models.
3.6 Nonlinear or elastic-plastic SDOF system

Takewaki (2001d, e) developed a new type of probabilistic critical excitation methods for SDOF elastic-plastic structures. He introduced the equivalent linearization method to evaluate elastic-plastic responses statistically. Although deterministic critical excitation methods are more direct than probabilistic methods, the latter methods have some advantages of (1) stability of response evaluation and (2) clear understanding of criticality of excitation.

Deterministic critical excitation methods are desired in order to make clear the time-history based critical characteristics of ground motions. Phase parameters are quite important in this aspect. The deterministic critical excitation methods are discussed in Section 3.8.

Au (2006a, b) presented an interesting method for finding a critical excitation for SDOF elastic-plastic structures. He proved the criticality in a smart manner by using an energy concept.

3.7 Elastic-plastic MDOF system

Several interesting approaches for MDOF systems were proposed as natural extensions of the method for SDOF systems. Philippacopoulos (1980) and Philippacopoulos and Wang (1984) took full advantage of a deterministic equivalent linearization technique in critical excitation problems of nonlinear MDOF hysteretic systems. Takewaki (2001f) extended the critical excitation method for elastic-plastic SDOF models to MDOF models on deformable ground by employing a statistical equivalent linearization method for MDOF models. The linearization method was used to simulate the response of the original elastic-plastic hysteretic model.

3.8 Damage index as target

Moustafa and Takewaki (Moustafa 2011, Moustafa and Takewaki 2010, Mustafa et al. 2010) developed some deterministic theories for critical excitation problems including elastic-plastic responses. Mustafa et al. 2010 modeled an earthquake ground motion as a superposition of a body wave in the former part and a surface wave in the latter part. They used a sequential quadratic programming method to solve those problems. This formulation enables the expression of a scenario that a structure damaged by an intensive shaking in the former part is shaken further by a wave with a longer-period component resonant to the natural period of the damaged structure. Various problems including these aspects are explained in the reference (Takewaki et al. 2012b).

3.9 Role of critical excitation method

A single-period, amplitude increasing ground motion similar to the critical excitation was observed in Niigata-ken Chuetsu-oki earthquake (2007) in Japan (Takewaki 2008b). Another similar phenomenon was observed in Osaka during Tohoku earthquake (2011). These phenomena strongly support that the critical excitation is truly a physically possible ground
There were some discussions on the existence of long-period ground motions in the field of earthquake structural engineering of large and high-rise building structures with long fundamental natural periods (Ariga et al. 2006, Takewaki 2011b, 2012a-c, Takewaki et al. 2013). However there was a little attention because actual records did not exist clearly. After the Tokachi-oki earthquake in 2003, long-period ground motions are getting much interest in the field of earthquake resistant design. The most difficult problem is that these long-period ground motions are highly uncertain in nature and the existence of these ground motions was not known during the construction of high-rise and super high-rise buildings. In order to take into account such highly uncertain long-period ground motions, a new paradigm is desired.

There are various buildings in a city as shown in Fig.2(a). Since building structures are not mass-produced ones in general, each building has its own natural period of amplitude-dependency and its original structural properties. When an earthquake occurs, a variety of ground motions are induced in the city, e.g. combination of body waves (including pulse wave) and surface waves (Moustafa et al. 2010), long-period ground motions. The relation of the building natural period with the predominant period of the induced ground motion may lead to disastrous phenomena in the city (see Fig.2(a)). In other words, the most critical issue in the seismic resistant design is the resonance. Many past earthquake observations demonstrated such phenomena repeatedly, e.g. Mexico 1985, Northridge 1994, Kobe 1995. One of the promising approaches to this is to shift the natural period of the building through structural control (Takewaki 2009) and to add damping in the building. However it is also true that the structural control is developing now and more sufficient time is necessary to respond properly to uncertain ground motions.

It is believed that earthquake has a bound on its magnitude and the earthquake energy radiated from the fault has a bound (Trifunac 2008). The problem is to find the most unfavorable ground motion for a building or a group of buildings under a certain constraint (see Fig.2(b)). There are two possibilities in the specification of such bounds. One is to define a velocity power ‘at the bottom of the basin’ based on the fault rupture mechanism and wave propagation characteristics. The other is to set the velocity power ‘at the ground surface level’ (Takewaki and Tsujioto 2011). In the case of definition at the bottom of the basin, the surface ground wave propagation has to be considered properly. However this procedure may include another uncertainty. In this sense, the specification of the velocity power at the ground surface level may be preferable and seems to be acceptable for more rational treatment of uncertainty.

Let us consider a case study on the specification of such bound. The Fourier spectrum of a ground motion acceleration has been proposed at the rock surface depending on the seismic moment $M_0$, distance $R$ from the fault, etc. (for example Boore 1983).

$$|A(\omega)| = C M_0 S(\omega, \omega_C) P(\omega, \omega_{\max}) \exp(-\omega R / (2\beta Q_\rho)) / R$$

$(1)$

$C$ is a constant and $S(\omega, \omega_C)$ indicates the source spectrum $S(\omega, \omega_C) = \omega^2 / \{1 + (\omega / \omega_C)^2\}$. Furthermore $P(\omega, \omega_{\max})$ is a high-cut filter and $\beta$ is a velocity of shear wave at rock. $Q$ is the Q-value. Such Fourier spectrum may contain uncertainties (Tokmechi 2011a-c). One possibility or approach is to specify the acceleration or velocity power (Takewaki 2007) as a global measure and allow the variability of the spectrum. The velocity power is related to the earthquake energy passing through a given area. As for the Great East Japan earthquake,
$|A(\omega)|$ is reported to be about 0.5(m/s) near the fault region (Yamane and Nagahashi 2012). However this treatment has a difficulty in confirming the reliability of the theory and of specification of the fault site. The change of ground motion by surface soil conditions is another difficulty. Based on this observation, a concept of critical excitation is introduced.

A significance of critical excitation methods can be explained by investigating the role of buildings in a city. In general there are two classes of buildings in a city as shown in Fig.2(b). One is the important building which plays an important role during and after disastrous earthquakes. The other is the ordinary building. The former one should not be damaged during an earthquake and the latter one may be allowed to be damaged to some extent especially for critical excitation larger than code-specified design earthquakes. Of course this design philosophy depends on the return period of ground motions considered and on the society or economy of the country. Just as the investigation on limit states of structures plays an important role in the specification of response limits, safety factors and performance levels of structures during disturbances, the clarification of critical excitations for a given structure or a group of structures appears to provide structural designers with useful information in determining excitation parameters in a risk-based reasonable way. It is expected that the concept of critical excitation enables structural designers to make ordinary buildings more seismic-resistant and seismic-resilient (Takewaki et al. 2012b).

![Fig.2(a) Scenario to tackle various uncertainties in modeling design earthquake ground motions](image-url)
4. Uncertainty expression in terms of info-gap model

4.1 Info-gap model

In this section, let us introduce and explain a new concept of structural design which combines load and structural uncertainties (Ben-Haim 2001, Takewaki and Ben-Haim, 2005). For this purpose, it is absolutely necessary to identify the critical excitation and the corresponding critical set of structural model parameters. It is well recognized that the critical excitation inducing the worst response depends on the structural model parameters and it is quite difficult to deal with load uncertainties and structural model parameter uncertainties simultaneously. In order to tackle these difficult problems, info-gap models of uncertainty (non-probabilistic uncertainty models) introduced by Dr. Ben-Haim (2001) are used. This concept enables one to represent uncertainties which exist in the load (input ground acceleration) and in parameters of the vibration model of the structure (Ben-Haim 2001, Takewaki and Ben-Haim, 2005). This concept has been applied to various fields (Duncan et al. 2008, Hot et al. 2012).

As a simple example, let us consider a shear building model as a vibration model, as shown in Fig.3, with viscous dampers in addition to masses and story stiffnesses. It is well recognized in the field of structural control and health monitoring that viscous damping coefficients $c_i$ of dampers in a vibration model are quite uncertain compared to masses and stiffnesses. This is because most of such dampers possess high temperature and frequency dependencies and it is often difficult to restrain those properties into an acceptable range. By using a specific method for describing such uncertainty, the uncertain viscous damping coefficient of a damper can be expressed in terms of the nominal value $\tilde{c}_i$ and the unknown uncertainty level (band) $\alpha$ as shown in Fig.4(a) (Takewaki and Ben-Haim, 2005).
\[ \mathcal{C}(\alpha, \tilde{c}) = \left\{ c : \left| \frac{c_i - \tilde{c}_i}{\tilde{c}_i} \right| \leq \alpha, \quad i = 1, \cdots, N \right\}, \alpha \geq 0 \]  

(2a)

The inequality in Eq.(2a) can be rewritten as

\[ (1-\alpha)\tilde{c}_i \leq c_i \leq (1+\alpha)\tilde{c}_i. \]  

(2b)

This description is the same one used in the interval analysis (Moore, 1966; Mullen et al., 1999; Koyluoglu and Elishakoff, 1998).

Another definition of uncertainty may be expressed by

\[ \mathcal{C}(\alpha, \tilde{c}) = \left\{ c : \sum_{i=1}^{N} \left| \frac{c_i - \tilde{c}_i}{\tilde{c}_i} \right|^2 \leq \alpha \right\}, \alpha \geq 0 \]  

(2c)

The implication of Eq.(2c) can be found in Fig.4(b). It seems that equation (2c) is too mathematical-oriented and does not express the vagueness of a real problem. However, unless we express a matter in a mathematical form, we cannot formulate an uncertainty problem. Furthermore, a simple model shown in Fig.3 has been used to enable the draw a three-dimensional figure in Fig.4.

Fig.3 Shear building model with uncertain viscous dampers
4.2 Info-gap robustness function

It is necessary to relate the info-gap model explained in the previous section with the degree of robustness. An uncertainty analysis called ‘the info-gap uncertainty analysis’ was introduced by Dr. Ben-Haim (Ben-Haim 2001) for measuring the robustness (the degree of response insensitiveness to uncertain parameters) of a structure subjected to external loads. Simply speaking, the info-gap robustness is the greatest horizon of uncertainty, $\alpha$, up to which the performance function $f(\mathbf{c}, \mathbf{k})$ does not exceed a critical value, $f_c$ (Ben-Haim 2001). The performance function may be a peak displacement, peak stress, ductility factor, accumulated plastic deformation, damage index or earthquake input energy, etc.

Let us define the following info-gap robustness function corresponding to the info-gap uncertainty model represented by Eq.(1a).

$$\mathcal{C}(\alpha, \bar{\alpha}) = \left\{ \mathbf{c} \mid \sum_{i=1}^{N} \frac{(c_i - \bar{c}_i)^2}{\bar{c}_i} \leq \alpha, \alpha \geq 0 \right\}$$
\[
\hat{\alpha}(k, f_c) = \max \left\{ \alpha : \left\{ \max_{c \in \mathcal{C}(\alpha, \hat{\alpha})} f(c, k) \right\} \leq f_c \right\} 
\] (3)

An illustrative explanation of the info-gap robustness function can be seen in Fig.5. Examples of computation of this equation can be found in References (Takewaki and Ben-Haim 2005, 2008; Kanno and Takewaki 2006a-c). For complex structures, the semi-definite programming approach is powerful as shown in references (Kanno and Takewaki 2006a-c).

Let us put \( f_{C_0} = f(\bar{c}, k) \) for the nominal damping coefficients. Then one can show that \( \hat{\alpha}(k, f_{C_0}) = 0 \) for the specific value \( f_{C_0} \), as shown in Fig.6. Furthermore let us define \( \hat{\alpha}(k, f_C) = 0 \) if \( f_C \leq f_{C_0} \) (see Fig.6). This means that, when the performance requirement is too small, we cannot satisfy the performance requirement for any admissible damping coefficients. The definition in Eq.(3) also imply that the robustness is the maximum level of the structural model parameter uncertainty, \( \alpha \), satisfying the performance requirement \( f(c, k) \leq f_C \) for all admissible variation of the structural model parameter represented by Eq.(1a).

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Fig.5 Illustrative representation of concept of info-gap robustness function

Fig.6 Info-gap robustness function for tight and loose design requirements
Consider a six-story shear building model as shown in Fig. 7(a). A viscous damper is installed only in the first story as shown in Fig. 7(b). A rectangular Fourier amplitude is specified for the input base acceleration and an uncertainty model is defined for the amplitude of the Fourier amplitude under a constant area. Fig. 8 shows a plot of the info-gap robustness function $\hat{\alpha}_m$ with respect to the level of the load spectral uncertainty $\alpha_s$ for the model with a supplemental damper in the first story. From this figure, the designer can understand the effect of the load spectral uncertainty $\alpha_s$ on the info-gap robustness function. It is also interesting to note that the info-gap robustness function $\hat{\alpha}_m$ and the level of the load spectral uncertainty $\alpha_s$ introduce a new trade-off relationship.

![Fig. 7 Six-story shear building model: (a) bare frame; (b) frame with a supplemental damper in the first story](image)

![Fig. 8 Info-gap robustness function $\hat{\alpha}_m$ with respect to the level of the load spectral uncertainty $\alpha_s$ for various requirements of earthquake input energies $E_i$ = $4 \times 10^6$, $6 \times 10^6$, $8 \times 10^6$ Nm](image)
5. Worst combination of structural parameters and input parameters

Consider a general problem, as shown in Fig.9, of finding the worst case under complicated hybrid uncertainties of structural parameters and input ground motion parameters. The problem without uncertainty in input ground motion parameters was considered by Fujita and Takewaki (2012b) and the problem without uncertainty in structural parameters was investigated by Fujita et al. (2010). While the domain satisfying the constraints is referred to as the feasible domain, the domain defined by the info-gap model due to Dr. Ben-Haim is called the info-gap domain. The case is meaningful and defines a key concept where the info-gap domain is just included in the feasible domain, i.e. at which both domains have a common tangent. The edge point (point with common tangent) corresponds to the worst case and the problem to find this leads to a principal problem.

![Fig.9 Info-gap domain and worst case](image)

The most challenging part is how to find such worst case in which both uncertainties of structural parameters and input ground motion parameters are taken into account. The worst case of input ground motion parameters is a function of structural parameters and their uncertainty levels. This relationship is extremely complicated and this problem can be a principal subject in the future in the field of critical excitation.

As a promising method for investigating this subject, interval analysis and related methods have been developed (see, for example, Moore, 1966; Alefeld and Herzberger, 1983; Qiu et al, 1996; Mullen et al., 1999; Koyluoglu and Elishakoff, 1998; Qiu, 2003; Chen and Wu, 2004; Chen et al, 2009, Fujita and Takewaki 2011a, b, c). The interval analysis is aimed at finding a critical combination of uncertain parameters which attains the maximum or minimum objective function. The classical interval analysis is limited to the solution at the interval bounds only. On the other hand, the modern interval analysis includes the solution at the inner point of interval regions.

Fig.10(a) and (b) show the objective functions in the cases of monotonic inclusion and non-monotonic inclusion, respectively. In order to solve this problem of interval analysis, Fujita and Takewaki (2011a-c) developed two new methods. One is the fixed reference-point method (FRP method) shown in Fig.11 and the other is the updated reference-point method (URP method) shown in Fig.12. In the FRP method, the evaluation point is not changed at which the second-order Taylor series approximation is made for the objective function. On the
other hand, in the URP method, the corresponding evaluation point is changed in the iteration of evaluation. Any optimization algorithm can be used if desired. However, this problem is a highly nonlinear problem and many local minima may exit. An attempt has been made to explore a new computational tool even though it is an approximate one.

Fig. 10 Objective function in info-gap domain, (a) monotonic inclusion, (b) non-monotonic inclusion

Fig. 11 Fixed reference-point method
6. Reality of resonance and its investigation

The resonance of buildings with input ground motions is a long-time issue of great interest in the community of earthquake structural engineering. We can find some actual examples in Mexico 1985, Northridge 1994, Kobe 1995, Tokachi-oki 2003 and Tohoku 2011. Ariga et al. (2006) discussed that issue for base-isolated buildings subjected to long-period ground motions after the experience of the Tomakomai ground motion during the earthquake of Tokachi-oki in 2003. Fig.13 shows the velocity waves of Tomakomai EW and NS (Tokachioki Earthquake 2003) as a representative long-period ground motion and the corresponding sinusoidal velocity wave. The long-period ground motion of 6.5-7.0s was observed and caused a large sloshing response in oil tanks. The long-period ground motions drew an attention to large deformation of base-isolated high-rise buildings with rather long fundamental natural period. Takewaki et al. (2011b) investigated the response of super high-rise buildings in Tokyo and Osaka during Tohoku 2011. Some clear observations will be explained in the following.
The figure illustrates velocity waves of Tomakomai EW and NS (Tokachioki Earthquake 2003) as a representative long-period ground motion and the corresponding sinusoidal velocity wave (Takewaki and Tsujimoto 2011).

(a) time interval of 20 - 60s

(b) time interval of 120 - 180s

Fig. 13 Velocity waves of Tomakomai EW and NS (Tokachioki Earthquake 2003) as a representative long-period ground motion and the corresponding sinusoidal velocity wave (Takewaki and Tsujimoto 2011)
(a) Elevation of super high-rise building

Long-period ground motion of several intensive cycles is resonant with building fundamental natural period.

(b) Ground acceleration, velocity and top-story displacement (short span direction)

Fig.14 Reality of resonance in a 55-story building in Osaka

The response of a 55-story super high-rise steel building in Osaka (height=256m; $T_1=5.8s$ (long-span direction), 5.3s (short-span direction)) is very symbolic because this building is owned by a public community and the data are opened for many people. The building was shaken intensively for long time (more than 10min) regardless of the fact that Osaka is located about 800km far from the epicenter (about 600km from the boundary of the fault region) and the JMA instrumental intensity was 3 in Osaka (Takewaki et al. 2011b, Takewaki et al. 2012a, b). Through the post-earthquake investigation, the natural periods of the building were found to be...
longer than the design values mentioned above reflecting the flexibility of pile-ground systems, the increase of a mass at the top and the damage to non-structural partition walls etc. It should be pointed out that the level of velocity response spectra of ground motions observed here (first floor) is almost the same as that at the Shinjuku station (K-NET) in Tokyo and the top-story displacement are about 1.4m (short-span direction) and 0.9m (long-span direction). Most of the data in buildings at Shinjuku, Tokyo are not opened because of the data release problem. The maximum top-floor horizontal displacement in one super high-rise building with passive oil dampers in Shinjuku attained at 0.6m and exhibited a smaller response compared to that building in Osaka. However the response strongly depends on the resonance and careful examination of other building records will be inevitable for future directions of structural design of super high-rise buildings in earthquake-prone countries. Once these data are opened, the vibration equivalent to this Osaka’s building may be reported.

Fig.14 shows the ground acceleration (the first-floor acceleration more exactly), ground velocity (the first-floor velocity) and top-story displacement numerically integrated from the recorded acceleration in this building. The highlighted zoom plot indicates that the monotonically increasing resonant response at the top of the building just corresponds to the intensive ground velocity. It can be observed that a clear resonant phenomenon between the building vibration and ground vibration occurs during about eight cycles (ground fundamental natural period can be evaluated by $4H/V_s=4\times1.6/1.0=6.4$s). It seems that such clear observation of high intensity has never been reported in super high-rise buildings all over the world. This implies the need of consideration and code-specification of long-period ground motions in the seismic resistant design of super high-rise buildings in mega cities even though the site is far from the epicenter. It is also being discussed that the expected three consecutive occurrence of Tokai, Tonankai and Nankai events is closer to this building (about 160km from the boundary of the fault region) and several times of the ground motion may be induced during that consecutive events based on the assumption that body waves are predominant outside of the Osaka basin. However the nonlinearity of surface ground and other uncertain factors may influence the magnitude of amplification. Further investigation will be necessary. The seismic retrofitting using hysteretic steel dampers, oil dampers and friction dampers is being planned.

Fig.15 illustrates the schematic explanation of the influence of resonance between the building vibration and the input motion predominant frequency, duration of ground motion and damping capacity deterioration on input energy response. The mechanism is investigated in detail on increase of credible bound of input energy, based on the critical excitation method, for the velocity power constraint due to uncertainties in input excitation duration (lengthening) and in structural damping ratio (decrease). In other words, lengthening of input excitation duration and decrease of structural damping ratio lead to increase of credible bound of input energy and this is explained schematically in Fig.15. As for uncertainties in excitation predominant period and in natural period of a structure, the resonant case is critical and corresponds to the worst case. It can be understood that the lengthening of input excitation duration and decrease of structural damping ratio due to damping capacity deterioration may have caused large input in the super high-rise building in Osaka bay area mentioned above. Especially the decrease of structural damping ratio induces unsmoothing phenomenon of the energy spectrum and the period region to increase the energy spectrum happened to coincide with the fundamental natural period of the super high-rise building (see Takewaki et al. 2012a, c).
Fig. 15 Effect of input motion predominant period, input motion duration and structural damping ratio uncertainties on bound of input energy: Resonance between input and structure and increase of credible bound of input energy for velocity power constraint due to lengthening of input excitation duration and decrease of damping ratio of structure.

7. Input energy and energy spectrum

Fig. 16(a) presents the total earthquake input energy from the ground motion at K-NET Shinjuku station (TKY007, NS) during the Tohoku earthquake in 2011 for various damping ratios and Fig. 16(b) illustrates the corresponding energy spectrum. It can be seen that, as the damping becomes larger, the energy spectrum becomes smoother. This phenomenon is related to the so-called ‘smoothing of Fourier amplitude spectrum’ (approximately equivalent to zero-damping velocity response spectrum). This explains the lower part of Fig. 15.

(a) Total earthquake input energy
(b) Energy spectrum

Fig. 16 Total earthquake input energy from ground motion at K-NET Shinjuku (TKY007, NS) for various damping ratios and energy spectrum (Takewaki et al. 2013)
Fig. 17 illustrates the total input energy from the ground motion at Osaka bay area (NS) during the Tohoku earthquake in 2011 for various damping ratios together with the corresponding energy spectrum. The smoothing process with respect to damping level can be seen also in this ground motion and it is clear that a large energy input can be observed around 5.5-7.0s. This can be explained by the fact that the building treated in Fig. 14 has fundamental natural periods of 6.5-7.0s in both directions and the surface ground fundamental natural period is about 6.5s.

![Graph of total earthquake input energy and energy spectrum](image1)

(a) Total earthquake input energy  
(b) Energy spectrum

Fig. 17 Total earthquake input energy from ground motion at Osaka bay area (NS) for various damping ratios and energy spectrum (Takewaki et al. 2013)

In order to clarify the difference of earthquake input mechanism between the long-period ground motions and the near-field ground motions, the ground motion at JMA Kobe (NS) during Hyogoken-Nanbu earthquake 1995 has been treated. Fig. 18 presents the total input energy from the ground motion at JMA Kobe (NS) during Hyogoken-Nanbu earthquake 1995 for various damping ratios together with the corresponding energy spectrum. It can be seen that, different from the long-period ground motions, most energy is concentrated to the short period range around 1(s) and the variability of input energy due to change of damping is not remarkable in the long-period range.

![Graph of total earthquake input energy and energy spectrum](image2)

(a) Total earthquake input energy  
(b) Energy spectrum

Fig. 18 Total earthquake input energy from ground motion at JMA Kobe (NS) during Hyogoken-Nanbu earthquake 1995 for various damping ratios and energy spectrum (Takewaki et al. 2013)
8. Conclusions

The conclusions may be stated as follows.
(1) The critical excitation methods play an important role in improving the earthquake resilience of infrastructures under uncertain input environments.
(2) In the field of structural engineering, robustness, redundancy and resilience should be discussed deeply in order to guarantee the safety of infrastructures against severe disturbances. The concepts of robustness, redundancy and resilience are closely interrelated. In general, robustness means insensitiveness of a system to parameter variation. On the other hand, redundancy indicates the degree of safety of a system against disturbances or the parallel system avoiding overall system failure (the fail-safe system is a representative one). Resilience can be regarded as an ability of a system to recover from a damaged state or resist external disturbances and seems to be a more generic concept including robustness and redundancy.
(3) Several uncertainties in earthquake ground motions can be explained by a model of Boore which takes into account the seismic moment, attenuation model etc. in constructing a Fourier amplitude of ground motions. By introducing the input energy bound, structural designers can restrict such uncertainties to a limited level.
(4) The info-gap model introduced by Dr. Ben-Haim can be a vital approach to describe uncertainty in earthquake ground motions and structural parameters. The method to obtain the worst combination of structural parameters and input parameters is desired. The fixed reference-point method and updated reference-point method based on interval analysis can be promising methods.
(5) The influence of resonance, duration of ground motion and damping deterioration on input energy response can be investigated with the credible bound theory in the field of critical excitation.

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