<table>
<thead>
<tr>
<th>Title</th>
<th>The macroeconomic effects of the wage gap between regular and non-regular employment and of minimum wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Sasaki, Hiroaki; Matsuyama, Jun; Sako, Kazumitsu</td>
</tr>
<tr>
<td>Citation</td>
<td>Structural Change and Economic Dynamics (2013), 26: 61-72</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2013-09</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/178154">http://hdl.handle.net/2433/178154</a></td>
</tr>
<tr>
<td>Rights</td>
<td>© 2013 Elsevier B.V.; This is not the published version. Please cite only the published version.</td>
</tr>
<tr>
<td>Type</td>
<td>Journal Article</td>
</tr>
<tr>
<td>Textversion</td>
<td>author</td>
</tr>
</tbody>
</table>

Kyoto University
The Macroeconomic Effects of the Wage Gap between Regular and Non-Regular Employment and of Minimum Wages

Hiroaki Sasaki*  Jun Matsuyama‡  Kazumitsu Sako‡

Abstract

We develop a disequilibrium macrodynamic model in which two types of labor (regular and non-regular employment) are incorporated. We analyze how the expansion of the wage gap between regular and non-regular employment affects the economy. If the steady state equilibrium exhibits the wage-led demand regime, an increase in the wage gap does not affect the stability of equilibrium. In this case, the size of the reserve army effect affects the stability of the equilibrium. If the reserve army effect is strong, the steady state equilibrium is unstable. On the other hand, if the steady state equilibrium exhibits the profit-led demand regime, an increase in the wage gap destabilizes the equilibrium. It is possible that depending on conditions, an increase in the wage gap produces endogenous and perpetual business cycles. The introduction of the minimum wage is desirable in that it mitigates business cycle fluctuations. However, the introduction of an inappropriate minimum wage policy leads to a real wage and an employment rate that are lower than the steady state values.

Keywords: wage gap; regular and non-regular employment; minimum wage; demand-led growth model

JEL Classification: E12; E24; E25; E32; J31; J83

*Corresponding author. Graduate School of Economics, Kyoto University. Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501, JAPAN. E-mail: sasaki@econ.kyoto-u.ac.jp. Phone: +81-75-753-3446
†Graduate School of Economics, Hitotsubashi University, JSPS Research Fellowships for Young Scientists (PD)
‡Department of Economics, North Asia University
1 Introduction

While a long time has passed since the debate over the so-called regular and non-regular employment began in Japan, this issue has become an increasingly urgent problem.\(^1\) We can find as reasons for the aggravation of this situation not only the increase of non-regular employment due to a prolonged economic depression but also the prevalence of discriminatory treatment based on the employment pattern, that is, regular employment and non-regular employment.\(^2\) The progress of such an undesirable situation motivates us to address the following questions: how the expansion of the wage gap between regular and non-regular employees influences a macroeconomy, and what is a consequence of a greater increase in non-regular employees. To investigate these questions, we build a model in which regular and non-regular workers coexist. Further, we consider the minimum wage policy one of the policies adopted by a government to alleviate the situation. Through analyzing how the introduction of the minimum wage policy affects the economy, we clarify the relevance of minimum wages as stabilizing mechanisms.

Flaschel and Greiner (2009) have justified a minimum wage policy based on human rights.\(^3\) Flaschel and Greiner’s model is an application of Goodwin’s (1967) growth cycle model, which assumes a two-class economy that consists of working and capitalist classes. They show the existence of an endogenous and perpetual business cycle with respect to the employment rate and wage share. They also show that the introduction of the minimum wage within certain bounds below the steady state wage share can increase the stability of the economy. In their model, the minimum wage corresponds to the minimum wage share exogenously determined by the government because labor productivity is constant. Based on Flaschel and Greiner (2009), a line of research has been developed. Flaschel and Greiner (2011) and Flaschel et al. (2012) introduce into the Goodwin model heterogeneous labor, that is, skilled and unskilled workers. The labor market is segmented. On the one hand, there is a market for skilled workers only. On the other hand, there is a market for unskilled workers and skilled workers who do not find a job in the first market. It is assumed that the skilled workers do not lose their jobs because they can always find a job in the second labor market even if they do not find a job in the first one. Thus, the excess labor

\(^1\)For instance, according to the labor force survey of February 2012 by the Ministry of Internal Affairs and Communications (http://www.stat.go.jp/data/roudou/longtime/03roudou.htm), the average ratio of non-regular employees to all employees (excluding executives of companies or corporations) from October 2011 to December 2011 is 35.7 \%, the highest such value ever registered. Moreover, that of the younger generation (ages 15 to 34) is 33.1\%.

\(^2\)Japan has not ratified the International Labor Organization (ILO) convention 111, which prescribes a prohibition of discrimination with respect to employment and occupation, in particular, discrimination between regular and non-regular employees.

\(^3\)As a theoretical contribution to research on human rights, see, for example, Sen (2004).
supply in the first labor market implies that some of the unskilled workers lose their jobs under the assumption that the labor supply is abundant. In addition, it is also assumed that only the skilled workers in the first labor market participate in the wage bargain. That is, the wages of the workers in the second labor market depend on the bargain by the skilled workers. Thus, the skilled workers have a large influence on the unskilled workers in the second labor market with respect to their wage determination and employment. In this framework, endogenous business cycles can occur like in Flaschel and Greiner (2009), and the introduction of the minimum wage into the second labor market can alleviate fluctuations of the economy.

The model developed in the present paper can be seen as in line with the above contributions. We consider a two-class economy, as did Flaschel and Greiner (2009). In addition, two types of heterogeneous labor (i.e., regular and non-regular employment) are introduced within the working class. Thus, our model is a two-class economy comprising three units, a model similar to that of Flaschel and Greiner (2011). Our model that incorporates regular and non-regular employment is also motivated in order to focus on dual labor markets in Japan. Ishikawa and Dejima (1994) statistically examined the existence of the dual labor market in Japan by applying a switching regression analysis to micro data for Japanese workers from Basic Surveys on Wage Structure in 1980 and 1990. One of their main findings is that the labor market is characterized by two different wage equations rather than a single one. They concluded that their finding supported the existence of the dual labor market in Japan in this period.

We analyze the effect of the wage gap between regular and non-regular employment on the stability of the steady state equilibrium by using a Kaleckian model. We also analyze how business cycles are affected by the introduction of the minimum wage. Generally, the minimum wage system has three major roles: improving the living conditions of low-paid workers, ensuring fair competition, and promoting a stable management-labor relationship. In particular in Japan, as the number of non-regular workers increases, it is often said that the minimum wage system plays an important role as a social safety net for non-regular workers. However, few studies analyze whether the introduction of the minimum wage has stabilizing mechanisms in Japan.

The Kaleckian model is a type of Keynesian dynamic model based on the principle of effective demand, whose basic framework was developed by Rowthorn (1981). In his model, two kinds of labor, that is, direct labor and indirect labor, which are emphasized by Kalecki himself, are considered. Direct labor refers to labor that varies with changes in output, while

---

4 Their study is based on the method developed by Dickens and Lang (1985), who showed the existence of the dual labor market in the US by using a switching regression analysis.
indirect labor refers to fixed labor that is independent of changes in output. In this sense, we can interpret direct (indirect) labor as regular (non-regular) labor. Although Rowthorn (1981) emphasized two types of labor, surprisingly, almost all Kaleckian models have neglected the consideration of different types of labor. In particular, only a few contributions analyze the effect of direct and indirect labor on the stability of the steady state equilibrium. Raghavendra (2006), one of the exceptions, extends Rowthorn’s model and presents a model in which an endogenous business cycle with respect to the two variables (the capacity utilization rate and profit share) occurs. Specifically, although the income distribution (the wage and profit shares) is exogenously given in Rowthorn’s model, it is endogenously determined in Raghavendra’s. In place of Rowthorn’s investment function, which is an increasing function of the capacity utilization and the profit rate, Raghavendra introduces an investment function presented by Marglin and Bhaduri (1990), which is an increasing function of the capacity utilization and the profit share. Moreover, he adopts a non-linear investment function.

In Raghavendra’s model, a limit cycle occurs under certain conditions due to the interaction between the nonlinearity of the investment function and the increasing returns to scale caused by the existence of fixed labor. The occurrence of an endogenous and perpetual business cycle is proved by the Poincaré-Bendixson theorem. This suggests an alternation of the profit-led demand and wage-led demand regimes, where the profit-led demand (wage-led demand) regime means that an increase in the profit share increases (decreases) the capacity utilization rate. In this paper, fixed and variable labor in the Raghavendra model are interpreted as regular and non-regular employment, respectively. Furthermore, we extend the model in order to analyze the stability of the steady state equilibrium and the effect of introducing minimum wages on a business cycle.

There are two differences between our model and that of Raghavendra. First, our assumptions concerning wage differ. While Raghavendra (2006) assumes equal pay for fixed labor and variable labor, we assume that regular workers earn a higher wage than do non-regular workers. Second, our formulations regarding the income distribution dynamics differ. In his model, the income distribution dynamics (the profit share) are derived from the dynamic markup pricing rule of firms and the wage curve. In contrast, we adopt the conflicting-claims theory of inflation to obtain the income distribution dynamics. In par-

5For the literature considering the two types of labor in a Kaleckian model, see Lavoie (1992). Lavoie (2009) is a recent contribution to the field.
6See Blecker (2002) for further discussion concerning various regimes in the Kaleckian model.
7The wage curve in Raghavendra (2006) means that wage level is an increasing function of the capacity utilization rate.
8Flaschel and Greiner (2009) present an extended version of the Goodwin model in which the income distribution dynamics are given by how the dynamic markup pricing rule and the wage rate of change depends
ticular, we introduce the reserve army effect that the target profit share of labor unions is a
decreasing function of the capacity utilization rate.

We analyze the dynamics of the capacity utilization rate and the profit share by using
the extended model. In the case where the steady state equilibrium is unstable in the profit-
led demand regime, under certain conditions, an endogenous and perpetual business cycle
occurs by the Hopf bifurcation theorem. Generally, the longer a downturn of a business
cycle is, the worse the quality of life of a non-regular worker becomes. One of the policies
that a government should adopt to alleviate such an undesirable situation is a minimum wage
policy, which sets a lower bound to the wage share, that is, an upper bound to the profit share.

Next, we explain the minimum wage policy. We introduce the minimum wage by setting
an upper bound to the profit share. Such formalization can be regarded as the minimum
wage in our model because the determination of the maximum profit share is equivalent to
that of the non-regular employment minimum wage. The rationale is as follows. First, by
definition, the maximum profit share is equal to the minimum wage share. Second, given
labor productivity, the determination of the minimum wage share is equivalent to that of the
minimum average wage of the whole economy. Third, in our model, the average wage of
the whole economy is proportional to the non-regular employment wage. Thus, from these
three facts, the equivalence between the determinations of the maximum profit share and the
non-regular employment minimum wage holds.

In our analysis, we focus on the case where the steady state equilibrium is locally un-
stable. The government does not accurately know the equilibrium profit share. Thus, the
minimum wage determined by the government does not necessarily coincide with it. In this
case, the following results are obtained by our analysis.

Under the profit-led demand regime, if the government sets a minimum wage that is
lower than the steady state real wage, the size of fluctuation decreases. If a minimum wage
is set at the same level as the steady state real wage, the economy converges to the steady
state almost without fluctuations. If the government sets a minimum wage that is higher
than the steady state real wage, the economy converges to a point that is different from the
steady state. In this case, although fluctuations can be diminished, both the profit share
and the capacity utilization rate that are obtained in the long run become smaller than those
obtained before the introduction of the minimum wage policy.

Under the wage-led demand regime, the steady state is a saddle point. If the government
simultaneously introduces a minimum wage and a maximum wage, depending on whether
the initial value is located above or below the stable arm, the economy reaches different
steady states. If the economy starts from an initial value above the stable arm, it converges to a point where the capacity utilization rate is smaller than its equilibrium value whereas the profit share is larger than its equilibrium value. On the other hand, if the economy starts from an initial value below the stable arm, it converges to a point where the capacity utilization rate is larger than its equilibrium value whereas the profit share is smaller than its equilibrium value.

The rest of the paper is organized as follows. Section 2 describes our model. Section 3 analyzes the properties of the steady state equilibrium and its stability. Section 4, given that the steady state equilibrium is locally unstable, analyzes how the introduction of the minimum wage policy affects the economy. Section 5 concludes.

2 The model

We consider an economy with two heterogeneous workers (regular and non-regular employment) and capitalists. Workers’ income consists of wages only, and they spend all their income on consumption. On the other hand, capitalists obtain their income from profits only, which is saved at a constant rate $s$. Then, the saving function is given by

$$g^s = sr, \quad 0 < s < 1,$$

where $g^s = S/K$ is the ratio of real savings $S$ to the capital stock $K$, and $r$ is the profit rate.

Following Marglin and Bhaduri (1990), we assume that the firms’ investment function is an increasing function of the capacity utilization rate $u$ and the profit share $m$.

$$g^d = g^d(u, m), \quad g^d_u > 0, \quad g^d_m > 0,$$

where $g^d = I/K$ denotes the ratio of the real investment $I$ to the capital stock, $g^d_u$ denotes the partial derivative of the investment function with respect to the capacity utilization, and $g^d_m$ denotes the partial derivative of the investment function with respect to the profit share. For simplicity, we do not consider capital depreciation.

Let us assume that the ratio of the potential output $Y^p$ to the capital stock is technically fixed, that is, always constant. Then, the capacity utilization rate can be represented as $u = Y/K$, where $Y$ denotes the actual output. Note that the relation $r = mu$ holds among the profit rate, the profit share, and the capacity utilization rate. In the following analysis,

---

10In the typical Kaleckian model, the investment function is formalized as an increasing function of the profit rate and the capacity utilization rate. On the contrary, Marglin and Bhaduri (1990) assert that one of the investment function variables should be the profit share rather than the profit rate.
without loss of generality, we assume that the ratio of the potential output to the capital stock is equal to unity.

We assume that the regular employment \( L_r \) is related to the potential output, while the non-regular employment \( L_{nr} \) is related to the actual output.

\[
L_r = \alpha Y^F, \quad \alpha > 0, \quad (3)
\]

\[
L_{nr} = \beta Y, \quad \beta > 0, \quad (4)
\]

where \( \alpha \) and \( \beta \) are positive constants. Regular employment is considered to be a fixed factor and it is not frequently fired or hired even if output fluctuates. Regular employment changes when scale of plants changes, and scale of plants changes when the potential output changes. Hence, we can consider that regular employment changes when the potential output changes. In contrast, non-regular employment is considered to be a variable factor and it is fired or hired according as output fluctuates. Thus, we can consider that non-regular employment changes when the actual output changes. In this paper, for simplicity, we assume that regular employment and non-regular employment are proportional to the potential output and the actual output, respectively.\(^{11}\)

The ratio of the regular employment to the non-regular employment leads to \( \frac{L_r}{L_{nr}} = \frac{\alpha}{(\beta u)} \). This means that an increase in the capacity utilization rate leads to a decrease in this ratio. In other words, a relatively large number of non-regular workers are employed with a rise in the capacity utilization rate. Once the capacity utilization rate is determined at the steady state equilibrium, the ratio of the regular employment to the non-regular employment is also determined.

We assume a quantity adjustment that the capacity utilization rate increases (decreases) in accordance with an excess demand (supply) in the goods market.

\[
\dot{u} = \phi(g^d - g^s), \quad \phi > 0, \quad (5)
\]

where the parameter \( \phi \) denotes the speed of adjustment of the goods market.

We define the level of the average labor productivity of the economy as \( a = Y/L \), where \( L \) denotes the aggregate employment, that is, \( L = L_r + L_{nr} \). From this, we obtain the following:

\[
a = \frac{Y}{\alpha Y^F + \beta Y} = \frac{u}{\alpha + \beta u}. \quad (6)
\]

\(^{11}\)An empirical study of Uni (2009) shows that in Japan, the employment elasticity of output, that is, how much employment changes when the actual output changes, is small for regular employment and large for non-regular employment. This evidence supports our specification.
This implies that the average labor productivity is an increasing function of the capacity utilization rate, that is, increasing returns to scale prevail. Since \( u \) is constant at the steady state equilibrium, the corresponding average labor productivity is also constant. Thus, there is no perpetual technical progress in our model.\(^{12}\)

The nominal wage of the regular employment \( w_r \) is supposed to be higher than that of the non-regular employment \( w_{nr} \) at a certain rate \( \gamma \).\(^ {13}\)

\[
w_r = \gamma w_{nr}, \quad \gamma > 1.
\] (7)

From these, the average wage of the economy yields

\[
w = \frac{L_r}{L} w_r + \frac{L_{nr}}{L} w_{nr} = \frac{\alpha}{\alpha + \beta u} w_r + \frac{\beta u}{\alpha + \beta u} w_{nr} = \left[ \frac{\gamma \alpha + \beta u}{\gamma (\alpha + \beta u)} \right] w_r.
\] (8)

The average wage is given by the weighted average of the regular and non-regular employment wages. Each weight corresponds to the corresponding employment share. While the weight of regular employment is a decreasing function of the capacity utilization rate, the weight of non-regular employment is an increasing function of the capacity utilization. The component in the square bracket is a decreasing function of the capacity utilization rate.

Next, we formalize the equation of the price of goods \( p \) and the equation of the regular employment wage by using the theory of conflicting-claims inflation.\(^ {14}\) First, firms set their price so as to narrow the gap between firms’ target profit share \( m_f \) and the actual profit share, and accordingly, the price changes. Second, labor unions negotiate so as to narrow the gap between the labor unions’ target profit share \( m_w \) and the actual profit share, and accordingly, the nominal regular employment wage changes. The two assumptions can be written as follows:

\[
\frac{\dot{p}}{p} = \theta (m_f - m), \quad 0 < \theta < 1, \ 0 < m_f < 1,
\] (9)

\[
\frac{\dot{w}_r}{w_r} = (1 - \theta) (m - m_w), \quad 0 < m_w < 1,
\] (10)

where \( \theta \) is a positive parameter. We interpret \( \theta \) and \( 1 - \theta \) as the bargaining power of firms and that of labor unions, respectively. Further, by taking the reserve army effect into consid-

\(^{12}\) We can introduce perpetual technical progress by using the theory of induced technical change. See the Appendix.

\(^{13}\) A similar formalization is adopted by Lavoie (2009).

\(^{14}\) The theory of conflicting-claims inflation is presented by Rowthorn (1977). For previous studies on Kaleckian models with this theory, see, for example, Dutt (1987) and Cassetti (2003).
eration, we assume that \( m_w \) is a decreasing function of the capacity utilization rate.

\[ m_w = m_w(u), \quad m_w' < 0. \] (11)

As the capacity utilization rate (a proxy variable of the employment rate) increases, workers’ demands in the bargaining are likely to increase, which leads workers to set a higher target wage share, and accordingly, set a lower target profit share.

Since the profit share is given by \( m = 1 - (wL/pY) \) by definition, by taking the derivative of \( m \) with respect to time, we obtain

\[ \frac{\dot{m}}{1-m} = \frac{\dot{p}}{p} - \frac{\dot{w}}{w} + \frac{\dot{\alpha}}{\alpha}. \] (12)

From equations (8) and (10), the rate of change of the average wage in the whole economy is given by

\[ \frac{\dot{w}}{w} = -\frac{(\gamma - 1)\alpha\beta}{(\gamma\alpha + \beta u)(\alpha + \beta u)} \cdot \dot{u} + (1 - \theta)[m - m_w(u)]. \] (13)

From equation (6), the rate of change of the labor productivity is given by

\[ \frac{\dot{a}}{a} = \frac{\alpha}{(\alpha + \beta u)u} \cdot \dot{u}. \] (14)

By substituting equations (1) and (2) into equation (5), and also equations (9), (12), and (13) into equation (11), we can obtain the following dynamic equations with respect to the capacity utilization rate and the profit share.

\[ \dot{u} = \phi[g^d(u, m) - smu], \quad \phi > 0, \] (15)

\[ \dot{m} = -(1 - m)[m - \theta m_f - (1 - \theta)m_w(u) - f(u)\dot{u}], \quad f(u) = \frac{\alpha\gamma}{(\gamma\alpha + \beta u)u}, \quad f'(u) < 0. \] (16)

Firms set \( m_f \) as large as possible. Conversely, labor unions set \( m_w \) as small as possible. Hence, we can assume that \( m_f > m_w(u) \).
3 Characteristics of the steady state equilibrium and its stability

3.1 Characteristics of the steady state equilibrium

The steady state equilibrium is an equilibrium such that $\dot{u} = \dot{m} = 0$. From this, we have the simultaneous equations with respect to $u_{ss}$ and $m_{ss}$.

\begin{align*}
g^d(u_{ss}, m_{ss}) &= sm_{ss}u_{ss}, \quad (17) \\
m_{ss} &= \theta m_f + (1 - \theta)m_w(u_{ss}). \quad (18)
\end{align*}

There is also a steady state for $m_{ss} = 1$, where the wage share is zero. We exclude the steady state in the following analysis because every worker is always employed without payment. In the following analysis, we assume that there exists a unique pair of $u_{ss} \in (0, 1)$ and $m_{ss} \in (0, 1)$ that simultaneously satisfies equations (17) and (18).

The capacity utilization rate and the profit share at the steady state equilibrium depend on the bargaining power, the target profit share of firms, and the target profit share of labor unions. However, the steady state equilibrium does not depend on the four parameters $\phi$, $\gamma$, $\alpha$, and $\beta$. This property is used in the analysis of the next section.

3.2 The stability of the steady state equilibrium

To investigate the stability of the steady state equilibrium, we analyze the Jacobian matrix of the system of the differential equations. Let the Jacobian matrix be $J$. The matrix $J$ is given as follows:

\begin{equation}
J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial u} & \frac{\partial u}{\partial m} \\ \frac{\partial m}{\partial u} & \frac{\partial m}{\partial m} \end{pmatrix},
\end{equation}

where

\begin{align*}
J_{11} &= \phi[g^d_u(u_{ss}, m_{ss}) - sm_{ss}], \quad (20) \\
J_{12} &= \phi[g^d_m(u_{ss}, m_{ss}) - su_{ss}], \quad (21) \\
J_{21} &= (1 - m_{ss})[(1 - \theta)m_w'(u_{ss}) + f(u_{ss})J_{11}], \quad (22) \\
J_{22} &= -(1 - m_{ss})[1 - f(u_{ss})J_{12}]. \quad (23)
\end{align*}

Each component of $J$ is evaluated at the steady state equilibrium.
Let us assume the following condition:

**Assumption 1.** \( sm_{ss} > g^{d}_{m}(u_{ss}, m_{ss}) \).

This means that the response of savings to the capacity utilization rate is larger than that of investments. This assumption makes the quantity adjustment of the goods market stable. Assumption 1 is sometimes called the Keynesian stability condition (Marglin and Bhaduri, 1990), which is often imposed in Kaleckian models. With Assumption 1, we can obtain \( J_{11} < 0 \).

Let us classify the regime according to the effect of a profit share increase on the capacity utilization.

**Definition 1.** If the relation \( g^{d}_{m}(u_{ss}, m_{ss}) < su_{ss} \) holds, the steady state equilibrium is called the wage-led demand regime. If, on the other hand, the relation \( g^{d}_{m}(u_{ss}, m_{ss}) > su_{ss} \) holds, the steady state equilibrium is called the profit-led demand regime.\(^{15}\)

If the investment response to the profit share is less than the saving response, then the steady state equilibrium exhibits the wage-led demand regime. On the other hand, if the investment response to the profit share is more than the saving response, then the steady state equilibrium exhibits the profit-led demand regime. Depending on which regime is realized in the steady state equilibrium, the wage-led demand regime or the profit-led demand regime, we have \( J_{12} < 0 \) or \( J_{12} > 0 \). From Assumption 1 and Definition 1, the sign structure of the Jacobian matrix \( J \) is given as follows:

\[
J = \begin{pmatrix}
- & \pm \\
- & ?
\end{pmatrix}
\]

The steady state equilibrium is locally stable if and only if the determinant of the Jacobian matrix \( J \) is positive and the trace is negative. Let us confirm this fact in our model.

First, calculating the determinant, we obtain

\[
\det J = -(1 - m_{ss})[J_{11} + (1 - \theta)m'_{w}(u_{ss})J_{12}].
\]

If the steady state equilibrium exhibits the profit-led demand regime, that is, \( J_{12} > 0 \), then \( \det J > 0 \) always holds. On the contrary, if the steady state equilibrium exhibits the wage-led demand regime, that is, \( J_{12} < 0 \), then \( \det J < 0 \) holds if the absolute value of \( m'_{w}(u) \) is

\(^{15}\)The profit-led demand regime is generally defined as a regime such that an increase in the exogenously given profit share leads to a rise in the capacity utilization rate at the steady state equilibrium. However, we cannot apply this definition to our model because the profit share at the steady state equilibrium is an endogenous variable. Here, we follow the definition of Raghavendra (2006).
sufficiently large whereas \( \det J > 0 \) holds if the absolute value of \( m'_w(u) \) is sufficiently small. Further, when \( \det J < 0 \), the discriminant of the characteristic equation is always positive. Hence, we obtain the two distinct real roots.

Second, calculating the trace, we obtain

\[
\text{tr} J = J_{11} - (1 - m_{ss}) + (1 - m_{ss})f(u_{ss})J_{12}. \tag{26}
\]

By assumption, \( J_{11} < 0 \). However, since \( J_{12} \) can be positive or negative, \( \text{tr} J \) is not always negative. If the steady state equilibrium exhibits the wage-led demand regime, that is, \( J_{12} < 0 \), then \( \text{tr} J < 0 \) always holds. On the contrary, if the steady state equilibrium exhibits the profit-led demand regime, that is, \( J_{12} > 0 \), then \( \text{tr} J > 0 \) can hold.

From the above analysis, we obtain the following propositions.

**Proposition 1.** Suppose that the steady state equilibrium exhibits the wage-led demand regime. If the reserve army effect is small, then the steady state equilibrium is locally stable. On the other hand, if the reserve army effect is large, then the steady state equilibrium is locally unstable.

Remember that the reserve army effect is given by \( m'_w(u) < 0 \).

Let us explain Proposition 1 intuitively. This proposition means that in the wage-led demand regime, the higher reserve army effect has unstable effects on the economy. Consider the case where the capacity utilization rate deviates from the steady state capacity utilization rate due to the occurrence of an exogenous shock. Suppose that the capacity utilization rate is less than the steady state equilibrium, for instance. In this case, there exist two opposing effects. First, the capacity utilization rate increases due to \( J_{11} < 0 \), which is called the direct stable effect. Second, the fall in the capacity utilization rate leads to an increase in the profit share due to \( J_{21} < 0 \). As a consequence, due to \( J_{12} < 0 \), the capacity utilization rate decreases, which is called the indirect unstable effect. Thus, in the wage-led demand regime, whereas the direct effect is stable, the indirect effect is unstable. If the reserve army effect is large, the profit share fairly increases due to a strong effect of \( J_{21} < 0 \). As a result, the capacity utilization rate largely decreases via an effect of \( J_{12} < 0 \). That is, the indirect unstable effect is intense. In the unstable case, the steady state equilibrium is a saddle point.\(^{16}\)

**Proposition 2.** Suppose that the steady state equilibrium exhibits the profit-led demand regime. Then, the steady state equilibrium can be locally unstable.

\(^{16}\)We consider both the capacity utilization rate and the profit share as state variables rather than jump variables. Unlike models with dynamic optimization, our model has no mechanism to set initial values on the saddle path.
This proposition means that, in the profit-led demand regime, the higher response of the capacity utilization rate to a change in the profit share can destabilize the economy. First, in the profit-led demand regime, if the capacity utilization rate is less than the steady state value, then it increases due to $J_{11} < 0$. This is the direct effect. Second, if the capacity utilization rate is less than the steady state value, then the profit share increases due to $J_{21} < 0$, which increases the capacity utilization rate due to $J_{12} > 0$. This is the indirect effect. In the profit-led demand regime, both the direct and indirect effects seem stable. However, if the indirect effect is too large, that is, if the response of the capacity utilization rate to the profit share is sufficiently large, the capacity utilization rate fairly increases due to the strong effect of $J_{12} > 0$. In this case, the capacity utilization rate increases too much and deviates from the steady state, and consequently, the steady state equilibrium is unstable.

Now, we show that endogenous and perpetual business cycles occur. By rearranging $\text{tr } J$, we obtain

$$\text{tr } J = \phi A - (1 - m_{ss}), \quad A \equiv [g_1^d(u_{ss}, m_{ss}) - sm_{ss}] + (1 - m_{ss})f(u_{ss})(g_m^d(u_{ss}, m_{ss}) - su_{ss}).$$

(27)

The term $A$ does not depend on the parameter $\phi$, which represents the adjustment speed of the goods market. Both the capacity utilization rate and the profit share at the steady state equilibrium also do not depend on $\phi$. Thus, we can choose $\phi$ as a bifurcation parameter.

**Proposition 3.** Suppose that the steady state exhibits the profit-led demand regime and that the term $A$ is positive. Then, a limit cycle occurs when the speed of adjustment of the goods market lies within some range.

**Proof.** Set $\phi_0 = (1 - m_{ss})/A > 0$. Taking a positive value $\phi$ arbitrarily, we have $\text{tr } J = 0$ for $\phi = \phi_0$, $\text{tr } J < 0$ for $\phi < \phi_0$, and $\text{tr } J > 0$ for $\phi > \phi_0$. Thus, $\phi = \phi_0$ is a Hopf bifurcation point. That is, there exists a continuous family of non-stationary, periodic solutions of the system around $\phi = \phi_0$.

This proposition means that, as the adjustment speed of the goods market increases, there appears a point at which the stable steady state equilibrium switches to the unstable one. As Raghavendra (2006) has shown, we also obtain a similar result: given the phase such that both the profit share and the capacity utilization rate simultaneously either increase or decrease, and the phase such that one increases while the other decreases, the former phase alternates with the latter. This implies an apparent alternation of the profit-led demand and wage-led demand regimes.

Next, we consider the parameter $\gamma$, which represents the wage gap between regular and non-regular employment. Note that $\gamma$ does not affect the equilibrium. Therefore, $\gamma$ appears
only in the part \( f(u_{ss}) \) of \( \text{tr}\ J \). Furthermore, we find that

\[
\frac{\partial f(u; \gamma)}{\partial \gamma} = \frac{\alpha \beta}{u(\gamma \alpha + \beta u)^2} > 0.
\]  

(28)

This means that in the case in which the steady state equilibrium exhibits the profit-led demand regime, a rise in \( \gamma \) amplifies the instability of the steady state equilibrium.

**Proposition 4.** Suppose that the steady state equilibrium exhibits the profit-led demand regime. Then, an expansion of the gap between regular and non-regular employment makes the steady state equilibrium unstable.

Furthermore, we obtain the following proposition.

**Proposition 5.** Suppose that the steady state equilibrium is unstable in the profit-led demand regime. Then, a limit cycle occurs within a certain range of the wage gap between regular and non-regular employment.

**Proof.** Set

\[
\gamma_0 = \frac{\beta}{\alpha} \cdot \frac{(u_{ss})^2[(1 - m_{ss}) - J_{11}]}{[(J_{11} - (1 - m_{ss})u_{ss} + (1 - m_{ss})J_{12})]} > 0.
\]

Taking a positive value \( \gamma \) arbitrarily, we have \( \text{tr}\ J = 0 \) for \( \gamma = \gamma_0 \), \( \text{tr}\ J < 0 \) for \( \gamma < \gamma_0 \), and \( \text{tr}\ J > 0 \) for \( \gamma > \gamma_0 \). Thus, \( \gamma = \gamma_0 \) is a Hopf bifurcation point. That is, there exists a continuous family of non-stationary, periodic solutions of the system around \( \gamma = \gamma_0 \).

As the wage gap expands, there appears a point at which the stable steady state equilibrium switches to the unstable one. Thus, an increase in the wage gap makes the economy more unstable. To stabilize the economy, it is desirable to shrink the wage gap between regular and non-regular employment.

Note that the steady state capacity utilization rate does not depend on the parameters \( \alpha \) and \( \beta \). Given the steady state capacity utilization rate, we consider the effect of a rise in \( \beta/\alpha \) on the stability of the steady state equilibrium. Then, a rise in \( \beta/\alpha \) means that a relatively large number of non-regular employees are employed compared to the number of regular employees. The ratio \( \beta/\alpha \) appears only in the part \( f(u_{ss}) \) of \( \text{tr}\ J \). From equation (16), we find that

\[
\frac{\partial f(u; \beta/\alpha)}{\partial (\beta/\alpha)} = \frac{u^2}{u[\gamma + (\beta/\alpha)u]^2} < 0.
\]

(29)
This means that in the case in which the steady state equilibrium exhibits the profit-led demand regime, the higher the ratio $\beta/\alpha$ is, the smaller $\text{tr} \mathbf{J}$ is. From this, we obtain the following proposition.

**Proposition 6.** Suppose that the steady state equilibrium exhibits the profit-led demand regime. Then, a rise in the relatively large number of non-regular employees compared with the number of regular employees makes the steady state equilibrium stable.

This proposition shows that an increase in the relatively large number of non-regular employees will have the effect of making the economy stable. Thus, if the government tries to decrease the relatively large number of non-regular employees, the economy becomes unstable. However, we cannot conclude that a government should adopt an employment policy to increase non-regular employment in order to stabilize the economy. Rather, from propositions 4 and 6, we emphasize a policy mix between wage and employment policies; even though the economy becomes unstable due to a decrease in non-regular employees, a government makes an effort to stabilize the economy by conducting a policy to shrink the wage gap between regular and non-regular employment.

### 4 Introduction of minimum wages

As one of the policies adopted by the government, a minimum wage policy is considered next. The minimum wage is introduced by setting an upper bound to the profit share. We consider such formalization justifiable because the determination of the maximum profit share is equivalent to that of the minimum wage of the non-regular employment.\(^{17}\) The equivalence is obtained from the following three facts. First, by definition, the introduction of the maximum profit share ($m_{\text{max}}$) is equal to that of the minimum wage share ($1 - m_{\text{max}}$). Second, given the labor productivity ($a = Y/L$), the determination of the minimum wage share is equivalent to that of the minimum average wage of the whole economy, that is, $1 - m_{\text{max}} = w/(pa)$. Third, by substituting equation (7) into equation (8), the average wage of the whole economy is proportional to the wage of the non-regular employment, that is, $w = \left(\frac{\gamma \alpha + \beta u}{\alpha + \beta u}\right) w_{nr}$, where ($\cdot$) > 1 for all $u \in (0, 1)$. Note that the minimum average wage of the economy $w_{\text{min}}$ corresponding to $m_{\text{max}}$ lies between $w_{nr}$ and $w_r$ from equation (8) and the third fact, that is, $w_r > w > w_{\text{min}} > w_{nr}$.\(^{18}\)

\(^{17}\)However, if policymakers set the upper bound to the profit share, firms might choose to relocate in another country to seek higher profits.

\(^{18}\)Concretely, the value in the square brackets in equation (8) is smaller than unity for any $u \in (0, 1)$. This means that the relation $w < w_r$ holds. Again, the value $\left(\frac{\gamma \alpha + \beta u}{\alpha + \beta u}\right)$ of the third fact is greater than unity for any $u \in (0, 1)$, which means that the relation $w > w_{nr}$ holds.

14
Here, as did Flaschel and Greiner (2009, 2011) and Flaschel et al. (2012), we analyze how the introduction of the minimum wage affects the dynamics of the profit share and the capacity utilization rate.

In this section, to facilitate our analysis, numerical simulations are used. For this purpose, we need to specify the investment function and the target profit share of labor unions. Here, we assume the following functional forms:19

\[ g^d = \psi u^\delta m^\varepsilon, \quad \psi > 0, \quad 0 < \delta < 1, \quad \varepsilon > 0, \]  
\[ m_w = \sigma_0 - \sigma_1 u, \quad 0 < \sigma_0 < 1, \quad \sigma_1 > 0. \]  

If Okun’s law, that is, a positive correlation between the employment rate and the capacity utilization rate, holds, we can regard the capacity utilization rate as a proxy variable of the employment rate. Such a method is adopted by Tavani et al. (2011).20

4.1 The case of the profit-led demand regime

In this subsection, we assume that the steady state equilibrium exhibits the profit-led demand regime and that a limit cycle occurs. Moreover, we conduct a numerical simulation that is roughly consistent with business cycles in the Japanese economy.

Figure 1 plots data for the capacity utilization rate and the profit share between the period 1980–2007. These data are smoothed by means of the Hodrick-Prescott filter. From Figure 1, we see that the capacity utilization and the profit share show a clockwise movement along an upward-sloping orbit, which implies that the profit share is pro-cyclical to the capacity utilization, that is, the wage share is counter-cyclical to the capacity utilization. Moreover, the capacity utilization fluctuates between 75% and 100% and the profit share between 15% and 30%.

[Figure 1 around here]

From Figure 1, we find that the Japanese economy exhibits the profit-led demand regime. In fact, by using VAR models, Azetsu et al. (2010) and Nishi (2010) empirically show that the Japanese economy exhibits the profit-led demand regime. Using a method similar to Flaschel et al. (2007) and Barbosa-Filho and Taylor (2006), Sonoda (2013) estimates the demand and distribution regimes in Japan between the period 1977–2007. He concludes

---

19 In the case where the investment function is specified like in equation (30), there is also a steady state for \( m_{ss} = 0 \). We will also exclude it because in this case, the profit rate leads to zero and hence, capital stock will not be accumulated.

20 For Kaleckian models that strictly consider the endogenous determination of the employment rate, see Sasaki (2010, 2011, 2012, 2013)
that labor productivity is pro-cyclical to capacity utilization, real wage is counter-cyclical, and in total, the wage share is counter-cyclical.\textsuperscript{21} Consequently, the profit share becomes pro-cyclical to the capacity utilization rate.

Next, we set the parameters to produce a cyclical pattern like in Figure 1.

\textit{Wage gap:} According to the Basic Survey on Wage Structure by the Ministry of Health, Labour and Welfare, the ratio of wages for ordinary workers to wages for part-time workers is calculated as $100/60 = 1.61$. Therefore, we set $\gamma = 1.6$.

\textit{Labor input coefficients of regular and non-regular workers:} According to data by the Ministry of Internal Affairs and Communications, the ratio of regular workers to non-regular workers is calculated as $L_r/L_{nr} = 7/3 = 2.33$. Note that in our theoretical model, the ratio is given by $L_r/L_{nr} = \alpha/\beta u$. If the equilibrium capacity utilization rate is 0.7, we have $L_r/L_{nr} = 2.77$ if we set $\alpha = 2$ and $\beta = 1$. Since the percentage of non-regular workers has recently increased, it was less than 30% between the period 1980–2007. Therefore, the value 2.77 seems reasonable.

\textit{Saving rate:} According the estimation of Naastepad and Storm (2007), the propensity to save from profits in Japan between the period 1960–2000 is calculated as 0.5, and hence, we set $s = 0.5$.

\textit{Parameters of investment function:} Naastepad and Storm (2007) also estimate the investment function of the Japanese economy. However, their specification of the investment function differs from ours, and accordingly, we cannot employ their results as they are. Nevertheless, their results indicate that the elasticity of investment with respect to the profit share is much larger than that with respect to the output (a proxy variable of the capacity utilization rate). Hence, we use a much larger value for $\varepsilon$ than for $\delta$. With our specification of the investment function, $\varepsilon > 1$ corresponds to the profit-led demand regime, and hence, we set $\varepsilon = 2$ to satisfy this restriction.

For other parameters except the above-mentioned parameters, we cannot obtain estimation results. Accordingly, we set the parameters so that the equilibrium values of the capacity utilization rate and the profit share would roughly approximate the Japanese data.

Summarizing the above discussion, we set the parameters as follows:

\[ \psi = 1.3, \; \delta = 0.2, \; \varepsilon = 2, \; \phi = 5, \; s = 0.5, \; \theta = 0.4, \]
\[ m_f = 0.7, \; \sigma_0 = 0.1, \; \sigma_1 = 0.1, \; \alpha = 2, \; \beta = 1, \; \gamma = 1.6, \; m_{\text{max}} = 0.35. \]

Calculating the steady state with the above parameters, we obtain $u_{ss} = 0.72$ and $m_{ss} = 0.30$.

\textsuperscript{21}The fact that the wage share in Japan is counter-cyclical to the capacity utilization rate is also pointed out by Yoshikawa (1994).
Given the arbitrary initial value, a limit cycle occurs (see Figure 2). The limit cycle is stable. That is, if the initial value is set far from the steady state, it converges to the limit cycle. Even if the initial value is set in the neighborhood of the steady state, it converges to the limit cycle, which is shown in Figure 2.

Here, we introduce $m_{\text{max}}$. Depending on how the government sets $m_{\text{max}}$, we obtain different results.

First, we consider the case in which the exogenously given $m_{\text{max}}$ is more than the maximum value (i.e., the top) of the limit cycle without introducing minimum wages (for instance, $m_{\text{max}} = 0.5$). From the phase diagram, we can see that the initial value in this case converges to the limit cycle faster than does the initial value in the case without $m_{\text{max}}$ as long as the initial value is located outside of the limit cycle and its transition to the limit cycle reaches $m_{\text{max}}$ before it converges to the limit cycle.

Second, we consider the case in which $m_{\text{max}}$ is set between the maximum value of the limit cycle and the steady state equilibrium. Then, from the analysis of the phase diagram, we find that the size of the limit cycle becomes smaller. This means that the size of fluctuation decreases. In other words, by introducing the minimum wage, the size of business cycles can be reduced. Figure 3 shows the limit cycles in the case of $m_{\text{max}} = 0.35$. Figure 4 shows the limit cycles of these two cases before and after introducing the minimum wage. Clearly, we can see that the size of business cycles diminishes.

Third, we consider $m_{\text{max}}$ exactly the same as the steady state profit share. Then, from the phase diagram, the economy converges to the steady state almost without fluctuations. That is, if the government knows the steady state equilibrium in advance, by setting the minimum

---

As a robustness check, we examine the range of a parameter that produces limit cycles. First, we consider the adjustment speed of the goods market $\phi$. As long as the other parameters are constant, limit cycles can occur with the range of $\phi \in [4.2, 8.6]$. Second, we consider the elasticity of investment with respect to the profit share. As long as the other parameters are constant, limit cycles can occur with the range of $\varepsilon \in [1.92, 4.99]$. With a value of $\varepsilon > 5$, interior solutions are not obtained.

When the steady state equilibrium exhibits the profit-led demand regime and stable, the economy converges to the equilibrium with oscillations. In this case, if the minimum wage policy is effective in that the pair of $u$ and $m$ reaches the maximum profit share line along its transitional process, then the size of business cycles diminishes compared to the case where the minimum wage policy is ineffective. Therefore, even in the case where limit cycles do not occur, as long as the minimum wage policy is effective, the policy can alleviate business cycles.
wage at the same level of the steady state, it can lead the economy to converge to the steady state.

Fourth, we consider the case in which $m_{\text{max}}$ is set less than the steady state equilibrium. If the government does not actually know the equilibrium, it is significant to consider such a case. Then, from the phase diagram, the economy converges to a point on line $\dot{u} = 0$ almost without fluctuations. For example, as shown in Figure 5, the economy converges to point $P$. That is, although fluctuations can be diminished, both the profit share and the capacity utilization rate obtained in the long run become smaller than those obtained before the introduction of the minimum wage policy. Figure 5 shows the case of $m_{\text{max}} = 0.25$.

[Figure 5 around here]

Let us compare the real wage of point $E$ and that of point $P$. The real wage of regular employment and that of non-regular employment are given by using the capacity utilization rate and the profit share as follows:  

$$\frac{w_r}{p} = \frac{\gamma u}{\gamma \alpha + \beta u} (1 - m),  \quad (32)$$

$$\frac{w_{nr}}{p} = \frac{u}{\gamma \alpha + \beta u} (1 - m).  \quad (33)$$

From this, the real wage is an increasing function of the capacity utilization rate and a decreasing function of the profit share. By rearranging the formulae, we obtain the iso-real wage curves in the $(u, m)$-space.

$$m = 1 - \frac{1}{\gamma \frac{w_r}{p}} \left( \frac{\alpha \gamma}{u} + \beta \right),  \quad (34)$$

$$m = 1 - \frac{w_{nr}}{p} \left( \frac{\alpha \gamma}{u} + \beta \right).  \quad (35)$$

Each curve becomes a rectangular hyperbola. In the first quadrant of the $(u, m)$-space, the lower and the more to the right these curves are located, the higher are the associated real wages.

Whether point $E$ or $P$ gives a higher real wage depends on two things: the shape of the iso-real wage curve and the relative positions between points $E$ and $P$. Iso-real wage curves and points $E$ and $P$ are shown in Figure 6. The real wage associated with the dotted iso-real wage curve is higher than that associated with the solid one. In the left figure, point $E$ gives a higher real wage compared to point $P$. The employment rate (capacity utilization rate)  

\[\text{Taking the derivatives of equations (32) and (33) with respect to } \gamma, \text{ we obtain } \frac{\partial (w_r/p)}{\partial \gamma} > 0 \text{ and } \frac{\partial (w_{nr}/p)}{\partial \gamma} < 0, \text{ respectively. This means that an expansion of the wage gap between regular and non-regular employment leads to an increase in the real wage of regular employees and a decrease in the real wage of non-regular employees.}\]
in $E$ is also high compared to its counterpart because $u$ in $E$ is higher than $u$ in $P$. On the contrary, in the right figure, a higher real wage is given in point $P$ compared to point $E$. However, the employment rate in $P$ is low compared to that in $E$ because $u$ in $P$ is smaller than $u$ in $E$.

[Figure 6 around here]

### 4.2 The case of the wage-led demand regime

We consider the case in which the steady state equilibrium exhibits the wage-led demand regime.

First, we consider the case in which the reserve army effect is small and consequently the steady state equilibrium is stable. Figure 7 shows the phase diagram in the wage-led demand regime case.

[Figure 7 around here]

Next, we consider the case in which the reserve army effect is large and consequently, the steady state equilibrium is unstable. Steady state $E$ is the saddle point in Figure 8. Thus, if the economy starts from the initial value above the stable arm, for instance, point $A_0$, it converges to a point where the capacity utilization rate is zero and the profit share is unity. If the economy starts from the initial value below the stable arm, for instance, point $B_0$, it converges to a point where the capacity utilization rate is unity and the profit share is zero.

Here, we introduce $m_{\text{max}}$ and $m_{\text{min}}$. If the government does not actually know the steady state equilibrium, the minimum wage given by the government does not necessarily coincide with the steady state profit share. The interval $[m_{\text{min}}, m_{\text{max}}]$ represents the feasible combinations of the profit share determined by the government as the minimum wage policy. Then, as shown in Figure 8, starting from point $A_0$, the economy converges to point $A$, while when starting from point $B_0$, it converges to point $B$. If we compare point $A$ with point $B$, point $B$ is superior to point $A$ because the real wage associated with $B$ is higher than that associated with $A$. Further, the employment rate in $B$ is also high compared to that in $A$ because $u$ in $B$ is higher than $u$ in $A$.

[Figure 8 around here]
5 Concluding remarks

In this paper, we have developed a Kaleckian model in which two types of labor (regular and non-regular employment) are incorporated. We have analyzed how the expansion of the wage gap between regular and non-regular employment affects the economy.

First, if the steady state equilibrium exhibits the wage-led demand regime, an increase in the wage gap does not affect the stability of equilibrium. In this case, the size of the reserve army effect affects the stability of the equilibrium. If the reserve army effect is strong, the steady state equilibrium is unstable. However, even if the steady state equilibrium is unstable, the introduction of the minimum and/or maximum wage prevents the capacity utilization and profit share from diverging. In this case, it is possible that depending on conditions, we obtain a real wage and an employment rate that are higher than the steady state equilibrium values.

Second, if the steady state equilibrium exhibits the profit-led demand regime, an increase in the wage gap destabilizes the equilibrium. It is possible that depending on conditions, an increase in the wage gap produces endogenous and perpetual business cycles. In addition, an increase in the non-regular employment relative to the regular employment stabilizes the steady state equilibrium. However, we must note that the government should not increase the non-regular employment literally. On the contrary, the government should adopt a policy mix that decreases both the non-regular employment and the wage gap. The introduction of the minimum wage is desirable in that it mitigates fluctuations of business cycles. However, the introduction of an inappropriate minimum wage policy that sets a minimum wage higher than the steady state equilibrium value consequently leads to a real wage and an employment rate lower than the steady state values.

Finally, we must note that our model is a short- to medium-run model, not a long-run model, in a strict sense. As stated in the text, if we assume Okun’s law, the capacity utilization rate has a one-to-one relationship with the employment rate. However, these two variables are strictly different. In this paper, for convenience, we identify the capacity utilization rate with the employment rate, but we cannot in a strict sense. To build a long-run model that distinguishes between the capacity utilization rate and the employment rate and investigate the employment in detail will be left for future research.

Acknowledgments

We would like to thank Peter Skott, Naoki Yoshihara, Takashi Ohno, Shinya Fujita, and seminar participants at Hitotsubashi University, Kyoto University, Nihon University, and
Ehime University for their useful comments and suggestions. We would also like to thank two anonymous referees for their constructive comments, which have helped us improve the paper.

Appendix: Introducing induced technical change

As the text shows, the average labor productivity changes with changes in the capacity utilization rate along the transitional dynamics toward the steady state equilibrium, but after the economy reaches the equilibrium, there is no perpetual technical progress in our model because the capacity utilization rate remains constant. However, we can introduce perpetual technical progress into the model by using the idea of induced technical change, which we turn to in this Appendix.25

Suppose, for simplicity, that the labor input coefficients $\alpha$ and $\beta$ are decreasing at a constant and the same rate $\lambda > 0$. Then, we can write $\alpha$ and $\beta$ as follows:

$$\alpha = \alpha_0 e^{-\lambda t},$$  \hspace{1cm} (36)

$$\beta = \beta_0 e^{-\lambda t},$$  \hspace{1cm} (37)

where $\alpha_0$ and $\beta_0$ are positive constants. In this case, the rate of change in the average labor productivity is given by

$$\frac{\dot{a}}{a} = \frac{\alpha}{(\alpha + \beta u)u} \cdot \dot{u} + \lambda.$$  \hspace{1cm} (38)

That is, $\lambda$ is added to equation (14).

Here, we introduce induced technical change and assume that $\lambda$ is an increasing function of the wage share, that is, a decreasing function of the profit share.

$$\lambda = \lambda(m), \quad \lambda' < 0.$$  \hspace{1cm} (39)

This idea is also adopted by Tavani et al. (2011) and similar to the ideas of Taylor (2004, ch. 7) and Foley and Michl (1999, ch. 14).

With equation (39), the element of the Jacobian matrix $J_{22}$ is modified to

$$J_{22} = -\left(1 - m_{ss}\right)[1 - f(u_{ss})J_{12} - \lambda'(m_{ss})].$$  \hspace{1cm} (40)

From this, we can see that $\lambda'(m_{ss}) < 0$ is likely to make $J_{22}$ negative, which makes the

25We are grateful to an anonymous referee for turning our attention to this issue.
dynamical system more stable. Indeed, the trace and the determinant of \( J \) are rewritten as follows:

\[
\begin{align*}
\text{tr} J &= J_{11} - (1 - m_{ss}) + (1 - m_{ss}) f(u_{ss}) J_{12} + (1 - m_{ss}) \lambda'(m_{ss}), \\
\text{det} J &= -(1 - m_{ss}) [J_{11} + (1 - \theta) m'(u_{ss}) J_{12} - \lambda'(m_{ss}) J_{11}].
\end{align*}
\]

(41)  (42)

Accordingly, the term \( \lambda'(m_{ss}) < 0 \) is likely to make the \( \text{tr} J \) negative and \( \text{det} J \) positive.

Therefore, introducing induced technical change makes the dynamical system more stable. However, qualitative results do not change considerably as long as the extent of induced technical change is small enough.

References


Figures

Figure 1: Capacity utilization and profit share in Japan (1980–2007). Sources: Indices of Industrial Production (Ministry of Economy, Trade and Industry) for the capacity utilization rate and Financial Statements Statistics of Corporations by Industry (Ministry of Finance) for the profit share.

Figure 2: The occurrence of the limit cycle.
Figure 3: Introduction of the minimum wage

Figure 4: Comparisons of the two cycles
Figure 5: Introduction of the minimum wage that is larger than the equilibrium

Figure 6: Iso-real wage curves and the two equilibria
Figure 7: Phase diagram in the case where the stable steady state equilibrium exhibits the wage-led demand regime

Figure 8: Phase diagram in the case where the unstable steady state equilibrium exhibits the wage-led demand regime