A Model Experiment for Saline Diffusion in Flow of Water through Porous Media [*]

by

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1. Introduction

The mean velocity of fluid flow through porous media is, in moderate ranges of the velocities, in proportion to the hydraulic gradient. That is, the Darcy law represents the relation between the hydraulic gradient and the mean velocity in those ranges. But, as the Darcy law is the macroscopic representation of fluid flow through porous media, if we consider the flow microscopically, the fluid motion is not simple and the fluid moves in complex voids which have various dimensions and sinuous shapes.

Consequently, if the velocities of each small fluid portion are taken in consideration, some of them would be faster or slower than the mean velocity. In other words, the velocities of each small fluid portion have a considerable fluctuation to the mean velocity. Therefore, we may treat the fluid motion through porous media as a sort of "eddy motion". The expression "eddy motion" has a meaning different from that used in hydraulics. Even whithin a range where the Reynold's number is smaller than 1, the flow is geohydrological laminer flow and then the Darcy law is established, we may call such flow of fluid through porous media "eddy motion", since the velocities of small fluid portions are considered to fluctuate fairly to the mean velocity.

If the flow of fluid through porous media could be regarded as the eddy motion, the diffusion phenomena which are analogous to those in true eddy motion, would be found there. The degree of such diffusion must be larger than that of molecular diffusion, and it has connection with the Thiem's method [1] for the investigation of ground water velocity. The phenomena arise in transversal and parallel direction to flow. The transversal diffusion is studied experimentally and theoretically by Dr. K. Kitagawa [2,3], but the diffusion in flow direction is not yet investigated. The latter offers not only a theoretical interest but also has a practical significance to the problems of

saline encroachment in the ground water near the sea shore, etc.

From such a point of view, we have made a model experiment to clarify the state of eddy diffusion and to determine the diffusion coefficient.

2. Experimental apparatus and methods

An upper-opened rectangle box (arranged by Dr. Y. Toyohara) is packed by sand. (see Fig. 1.) The water supply tank and the water drainage tank are equipped with both ends of the sand layer. They keep in contact with the layer through brass nets. By inclining the whole sand layer, and regulating the water levels of supply- and drainage-tanks, a steady water flow through the sand layer with constant flow sections is held. To see the water level gradient, five water manometers are connected with the sand layer at five points to measure the hydrostatic .pressure of the system.

The sand layer used consists of the Kamo-River-Sand with 35.9 percent porosity. The effective diameter of the sand is 0.315mm and the coefficient of uniformity is 4.9.

In order to measure the electric conductivity of the flowing water through the system, four pairs of Platinum cells are buried in the sand layer with equal interval.

Experimental apparatus and its size are shown in Fig.1.

Fig. 1.   Experimental apparatus.
   A_1 :   supply tank ; A_2 : drainage tank
   B :   sand layer ;   N : pedestal
   M_1, M_2 : brass nets : T_1, T_2 : water pipes
   O_1, O_2 : movable walls
   S_1, S_2, S_3 and S_4 : cells ;
   G : water manometers
   ab = 30.0cm ;   bc = 10.0cm ;   cd = 75.8cm
   ae = 4~5.5cm ;   l = 10cm ;   L = 17.3cm
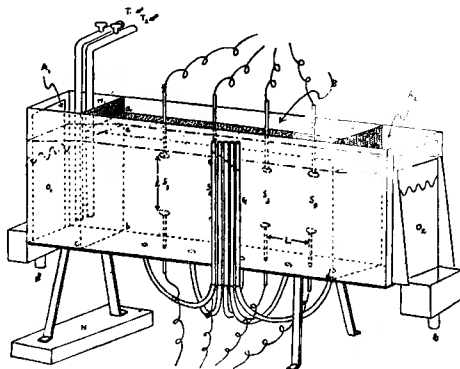
At the beginning of the experiment, the fresh water is allowed to flow through the system, after which it is replaced by the salt water in the supply tank. As the replacement is made within a minute, it may be regarded as a instantaneous replacement. Then, the electric resistances of each cell are measured at intervales of some minutes.

Apparent electric conductivities may be regarded as those of the solution only, since they are negligible small in case of flowing of fresh water.

Although it is possible to determine the absolute values of cell constants for the cells buried in the sand, it is sufficient to get their ratio. If we suppose the electric condudtivities of the solutions in the cells to be equal after flowing of salt water for a sufficiently long time,

$$\frac{C_1}{\mathit{\Omega}_1} = \frac{C_2}{\mathit{\Omega}_2} = \frac{C_3}{\mathit{\Omega}_3} = \frac{C_4}{\mathit{\Omega}_4}$$

where C_1, C_2, C_3 and C_4 are cell constants of each cell and $\mathit{\Omega}_1$, $\mathit{\Omega}_2$, $\mathit{\Omega}_3$ and $\mathit{\Omega}_4$ are the electric resistances measured in each cell. Therefore, if the $\mathit{\Omega}_1$, $\mathit{\Omega}_2$, $\mathit{\Omega}_3$ and $\mathit{\Omega}_4$ are known, the ratio $C_1 : C_2 : C_3 : C_4$ is determined. Then we can observe the time variation of electric conductivity for the flowing liquid at any cell.

At the water exit from the system, the flow rates and Cl-ion contents of the water are determined at intervals of scores of minutes.

After the conductivities at each cell keep the maximum values for a sufficiently long time, the supplied salt water is replaced by fresh water. Also, in this case, the same measurements as above mentioned are carried out.

These experiments are repeated three times and all constants for the experiments are shown in Table 1.

Table 1

	Gradient of sand layer (sin θ)	Flow section (cm²)	Cl-ion content in salt water (g/L)	Ratio of cell constants $C_1 : C_2 : C_3 : C_4$	Mean flow rate (cc/min.)
First experiment	0. 359	260	1. 58	1 : 2. 66 : 1. 1 : 5. 05	298. 7
Second experiment	0. 045	250	1. 61	1 : 1. 50 : 0. 69 : 3. 04	27. 8
Third experiment	0. 509	245	1. 65	1 : 2. 08 : 1. 47 : 3. 98	245. 0

3. Consideration to experimental results

In order to study the variations of the electric conductivity ρ, at each

cell, the simultaneous time for the supplied waters being allowed to change from fresh water to salt water is taken conveniently as $t = 0$ and such expression as $\rho = 1$, $\rho = 0$ is used. Figs. 2 and 3 are some examples of experimental results.

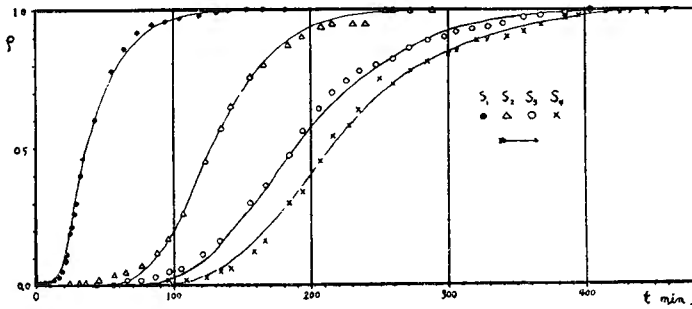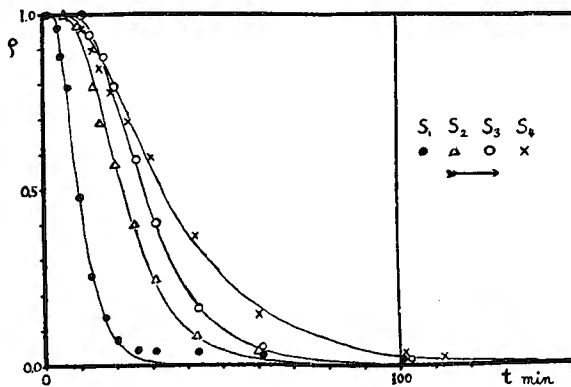Fig. 2 Experimental result (Second experiment) .

Fig. 3 Experimental result (Third experiment)

From Fig. 2, it is clear that in spite of the nearly instantaneous exchange of the supplied waters, a fairly long time is required for ρ for each cell to change from 0 to 1, and the these time intervals of the cells of the downstream are longer than those of the upper stream.

The variations of every cell are, to be brief, rapid in the earlier stage and slow in the later stage. In the close examination of the variations, the ρ of every cell increases slowly at the begining and then rapidly, and at $\rho = \dfrac{1}{2}$ its increase becomes slow again, and finally it approaches asymptotically to $\rho = 1$.

When the supplied waters change from salt water to fresh water, these relations among the conductivity variations are similar to those mentioned above.

On the basis of the foregoing facts, supposing the exsistence of the eddy diffusion phenomena in flow direction, a differential equation for the diffusion is

$$\frac{\partial \rho}{\partial t} + \bar{v}\, \frac{\partial \rho}{\partial \chi} = k\, \frac{\partial^2 \rho}{\partial \chi^2} \tag{1}$$

where t is the time, χ is the length measured in flow direction from the end of the sand layer, $\bar{v}$ is the mean velocity of the flow and k is the diffusion coefficient.

In the case of exchange where the fresh water changes into the salt water, the initial conditions are

$$\text{at } t = 0, \qquad \rho = \rho_0 = 1, \quad \chi < 0 \atop \rho = 0, \quad \chi > 0 \right\} \tag{2}$$

If we take a new independent variable X given by $X = \chi - \bar{v}t$, from the eq. (1) we have

$$\frac{\partial \rho}{\partial t} = k \frac{\partial^2 \rho}{\partial X^2} \tag{3}$$

and the initial conditions are replaced by

$$\text{at } t = 0, \qquad \rho = 1, \quad X < 0 \atop = 0, \quad X > 0 \right\} \tag{4}$$

The solutions of eq. (3) are

$$\rho = \frac{1}{2} - \frac{1}{2}\, \varPhi\!\left(\frac{X}{2k\sqrt{t}}\right), \qquad \text{at } X > 0$$
$$= \frac{1}{2} + \frac{1}{2}\, \varPhi\!\left(\frac{|X|}{2k\sqrt{t}}\right), \qquad \text{at } X < 0 \Bigg\},$$

where $\varPhi$ is the error function. Returning from X to χ,

$$\rho = \frac{1}{2} - \frac{1}{2}\, \varPhi\!\left(\frac{\chi - \bar{v}t}{2k\sqrt{t}}\right), \qquad \text{at } \chi > \bar{v}t$$
$$= \frac{1}{2} + \frac{1}{2}\, \varPhi\!\left(\frac{\bar{v}t - \chi}{2k\sqrt{t}}\right), \qquad \text{at } \chi < \bar{v}t \tag{5}$$

When the salt water changes into the fresh water, the solutions of eq. (3) are

$$\rho = \frac{1}{2} + \frac{1}{2} \, \emptyset \left(\frac{\chi - \bar{v}t}{2k\sqrt{t}} \right), \quad \text{at } \chi > \bar{v}t$$

$$= \frac{1}{2} - \frac{1}{2} \, \emptyset \left(\frac{\bar{v}t - \chi}{2k\sqrt{t}} \right), \quad \text{at } \chi > \bar{v}t \qquad (6)$$

On the positions distributed with the same interval, if $\bar{v}$ and k are constants, the $\rho - t$ curves of eqs. (5) and (6) have the same time interval at $\rho = \frac{1}{2}$.

The experimental results (Figs. 2 and 3) show that the intervals of measured values at $\rho = \frac{1}{2}$ are not the same, but the type of curves resembles to the theoretical. Therefore, supposing that the experimental results are satisfied by eqs. (5) and (6), theoretical curves may be described as solid curves in Figs. 2 and 3 according to the eqs. (5) and (6). In that case, the values of $\bar{v}$ may be determined from $\bar{v} = \frac{\chi}{t}$ at $\rho = \frac{1}{2}$ and the values of k may be chosen so that the theoretical curves may accord with the experimental results as perfectly as possible.

The theoretical curves drawn by such method accord fairly with the experimental results and the values of k are far larger than the molecular diffusion coefficient of Cl-ion in water. Hence, the exsistence of the eddy diffusion phenomena which occur in the flowing solution through the sand layer may be confirmed, and the flow of fluids through porous media may be regarded as a sort of eddy flow.

The theoretical curves in Figs. 2 and 3 have not the same interval at $\rho = \frac{1}{2}$, and the values of k for each cell are not necessarily the same even in one experiment. The reason may be due to the fact that since the sand layer used in the experiment is not ideally homogeneous and has a local difference of permeability, the flow has different mean velocities at the positions where the cells are buried. The inhomogeneity of the sand layer may be caused by using natural sand as it is. Therefore, if materials of the uniform size be packed homogeneously, the experimental results would approach to those of the ideal case.

The values of $\bar{v}$ and k for all theoretical curves are shown in Table 2. The relation between $\bar{v}$ and k is indicated in Fig. 4.

The values of k obtained from the experiments are 562 cm²/day $\sim$ 61056 cm²/day, and are roughly $6 \times 10^2 \sim 6 \times 10^4$ times as many as the molecular diffusion coefficient of Cl-ion in water.

In Fig. 4, the relation between $\bar{v}$ and k may be nearly shown by the equation,

$$k = \alpha \, \bar{v}^{\imath}$$

where, $n > 1$ and α is constant. Because the experiments are not so precise,

Table 2

	Cell	x (cm)	$\bar{v}$ (cm/min)	k (cm²/min.)
First experiment	S_1	12.03	3.01	20.7
	S_2	29.28	2.93	14.8
	S_3	46.53	2.66	42.4
	S_4	63.78	2.90	30.9
Second experiment (fresh to salt)	S_1	12.03	0.316	0.56
	S_2	29.28	0.230	0.39
	S_3	46.53	0.246	0.67
	S_4	63.78	0.298	1.00
(salt to fresh)	S_1	12.03	0.219	1.35
	S_2	29.28	0.262	0.51
	S_3	46.53	0.247	1.56
	S_4	63.78	0.238	1.56
Third experiment (fresh to salt)	S_1	12.03	1.93	17.30
	S_2	29.28	1.37	3.20
	S_3	46.53	2.09	6.45
	S_4	63.78	1.20	1.56
(salt to fresh)	S_1	12.03	1.93	22.80
	S_2	29.28	1.66	7.70
	S_3	46.53	1.33	4.35
	S_4	63.78	1.20	1.56

Fig. 4   Relation between $\bar{v}$ and k.
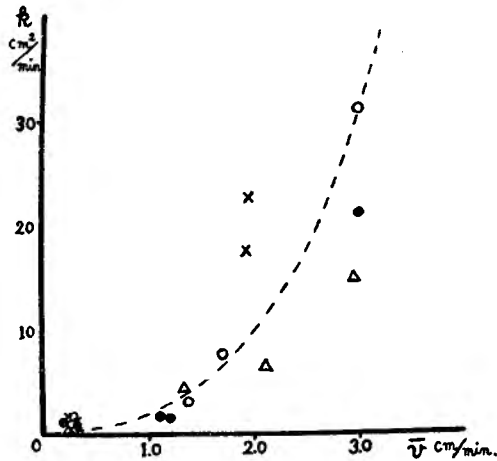
we may not mention it definitely, bnt it seems true that the values of k increase with $\bar{v}$. Also, the diffusion coefficient k must be a function for the coefficient of uniformity and the shape factor of sand.

Although we can not tell how the experimental result varies with the scale of model, the conclusions derived from these model experiments, if they are extended to a large scale, may be applicable to the problems in actual fields.

4. Conclusion

According to microscopical consideration, the flow of fluids through porous media even in the ranges where the Darcy law is established, should be regarded as a sort of eddy motion and the eddy diffusion phenomena be considered there.

The experimental methods which have been made to study the diffusion in flow direction are as follows : the fresh water flowing through the sand layer is allowed to change instantaneously into the salt water in the supply tank and the flow of salt water is investigated electrically. In these case, the exsistence of the eddy diffusion phenomena is confirmed.

The values of diffusion coefficient obtained from the experiment are roughly $6 \times 10^2 \sim 6 \times 10^4$ times as many as the molecnlar diffusion coefficient of Cl-ion in water. It is also found that the coefficients increase with the mean velocities of the flow.

References

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2) K. Kitagawa, Théorie de l'Écoulement des Eaux Souterraines. College of Scie., Kyoto Imperial University, A, vol. XV, 1932.
3) K. Kitagawa, Sur le Dispersment et l'Écart Moyen de l'Écoulement des Eaux Souterraines. College of Scie., Kyoto Imperial Univerity, A, vol. XVII, 1934.

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砂層流動水の塩分拡散に関する模型実験

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　砂層中流水の平均流速は圧力勾配に比例して，　Darcy 法則が成立する。　しかしこれは巨視的なものであつて，porous media 中の液体の運動を微視的に見れば，液体は個々の砂層空隙中を曲折して流動する。　したがつて流動液体の微少な実質部分の流速は平均流速に対して相当変動し，　乱流の場合と類似の拡散現象を呈すると思われる。この様な意味の乱流拡散の状況を明らかにし，　更にその係数の値を知るために一つの模型実験を行つた。矩形砂層に傾斜を与え，　定常流を作り，ある瞬間に供給水を淡水から塩水に切換えて，あらかじめ砂層中に埋設した4対の電極の電気抵抗を測定した。　その結果，各電極で測定した電気伝導度の時間的変化曲線は，　流動方向に塩分拡散が存在するとして得た理論曲線とかなり一致することが明らかになつた。　そしてその場合の拡散係数は分子拡散係数の約 6×10^2 乃至 6×10^4 倍の値であり，且つ平均流速が大になるに従つて拡散係数も大きくなる。　この実験では平均流速と拡散係数との間の函数関係を決定するには至らなかつたけれども，　より精密な実験装置によれば，この様な方法でその函数関係を決定することが出来ると思われる。