VOLCANIC MICRO TREMOR OF THE SECOND KIND

—Nature of its wave generation and source condition—

BY

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Introduction

The characteristics of the volcanic micro tremors at Volcano Aso are closely related to the activity of the volcano.

According to K. Sassa[1], there are four kinds of volcanic micro tremors when classified from their periods, and these tremors each have different characteristics.

The volcanic micro tremor of the "2-nd kind" is a long period of 3.5 to 7.0 sec. The period of this tremor is maintained for the duration of unchanged activity of the volcano. The mode of this tremor corresponds to the compressional one and was observed in the form of several groups of regular wave trains during inactive periods, while during active periods it became a continuous train of somewhat irregular waves.

The wave forms of this tremor on the seismograms depend on, in general, the characteristics of the active origin of the volcano, the mode in which waves have propagated and observed instruments.

The spectrum of waves having a long period and compressional wave forms such as the 2-nd kind may be taken to indicate information concerning the non-distorted source conditions. For this reason, an attempt was made to determine the source conditions of the 2-nd kind of volcanic micro tremor from its frequency spectrum. The source condition of an explosive or seismic wave has been investigated as the stress change on the surface of a small spherical cavity by many authors[2]. They assumed the form of the stress change to be an impulsive or step function and discussed the relations between observed and predicted spectra.

From the investigation of K. Sassa[1], this tremor is understood to be guided waves because of the free oscillation of a magmatic chamber beneath the active crater and the instigation of chamber vibration due to explosions of gas involved.

Assuming the above mentioned origin which is proper in the volcanic region, a discussion will be made in this paper on the nature of the 2-nd kind of volcanic micro tremors from its observed frequency spectrum.

The Spectral Analysis of the Volcanic Micro Tremor of the 2-nd kind

Several records of the volcanic micro tremor of the 2-nd kind which had been observed at the Aso Volcanological Laboratory were selected, and spectra of the wave motion were made from these selected records. The records had been obtained during different durations with different instruments. The observed durations were October 1932, February 1933 and January 1962. The first two durations were the most active periods of this volcano. While, in the last duration, the inside of the volcano became active causing a rise in the amplitude of the volcanic micro tremor, but no eruptions have taken place. The observation instruments were Galitzin-type seismogram (its natural period of pendulum; T_0 and galvanometer; T_g being $T_0=8$ sec and $T_g=4$ sec and its damping constant of pendulum; h_0 and galvanometer; h_g being $h_0=h_g=1$) and Wiechert seismogram ($T_0=10$ sec its damping constant; $h=0.44$ in 1933 and $T_0=6$ sec, $h=0.35$ in 1962).

From the Fourier analysis, employing reduction of instruments, the spectra of the ground motion due to the volcanic micro tremor were obtained. Wave forms on seismograph did not indicate the ground motion itself but illustrated the response of the seismograph to it. The reduction method in this case was discussed by K. Kasahara[3] in detail and the reductions were made easily using the following equation.

$$S(\omega)=R(\omega)/Y(\omega) \tag{1}$$

where $R(\omega)$ and $Y(\omega)$ are the spectra of recorded wave forms and impulsive response of the seismograph, respectively. $S(\omega)$ is the true spectrum of the ground motion. In the equation (1), $Y(\omega)$ can be derived from the so-called characteristic curves of the the seismograph.

An example of an analysed spectrum of the volcanic micro tremor of the 2-nd kind is shown in Figure 1(a) and 1(b). In the Figure 1(a), Fourier components of wave on the seismogram vs. period are plotted. In the Figure 1(b), reduced amplitude vs. period are plotted on the logarithmic scale. Solid lines in the Figure 1(b) show the predicted spectra which will be explained in the following section. In Figures 2(a), 2(b) and 2(c), reduced spectra and predicted spectra are also shown.

Characteristics of the Spectrum of the 2-nd kind of Volcanic Micro Tremor

From the analysed spectra of the 2-nd kind of the volcanic micro tremor as shown in Figures 1 and 2, the observed spectrum seems to be consistent with the characteristic curve of a pendulum subjected to an exciting force, or more generally speaking, the curve of the impulsive response of the damped oscillation system.

The differential equation of motion for the oscillation system with damping

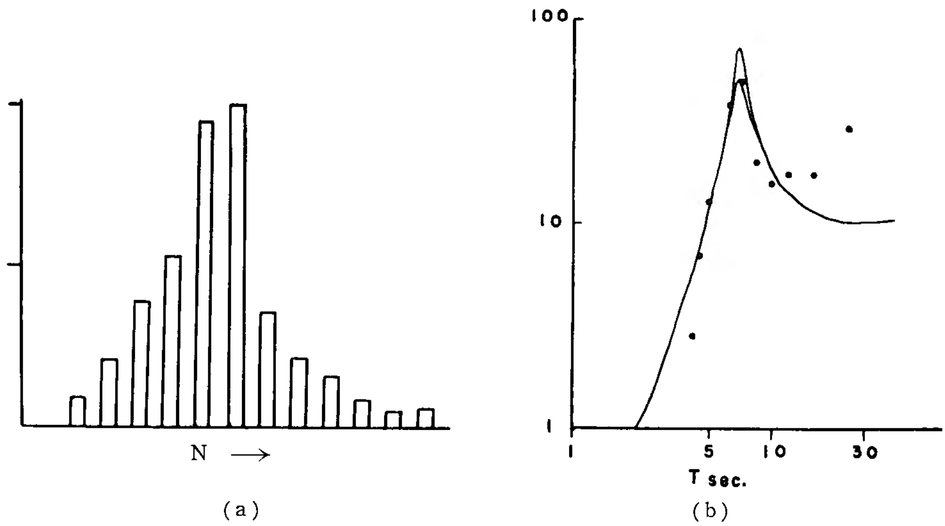

Fig. 1 Spectra of the volcanic micro tremor of the 2-nd kind observed during Oct. 1932.
(a) before reduction
(b) reduced spectrum
Fourier components vs. period are plotted and solid lines show the predicted spectra.

subjected an exciting force may be expressed as the following equation

$$\ddot{\theta}+2\varepsilon\dot{\theta}+n^2\theta=F(t) \qquad (2)$$

where

$$\varepsilon/n=h \qquad n=2\pi/T$$

ε, n, h and T are all usual notations.

Assuming $h=0.1$ and 0.07, solid lines in Figures 1 and 2 have been predicted. This predicted spectrum is compared with the analysed one. As shown in the Figures 1(b) and 2, the plotted points are distributed almost equally between two predicted curves. The agreement is reasonable except at the low frequency end. The increased low frequency component of the analysed signal is probably due to the fact that the base line of the seismogram is not taken accurately in making the Fourier analysis. This fact suggests that this oscillation system has a very small damping factor, since the predicted damped oscillation system can be regarded as a free oscillation system in approximate.

From auto-correlograms (Figure 3) it can be also presumed that the predicted damped oscillation system has a very small damping factor. The proper period of the oscillation system is consistent with the period of the auto-correlogram. These periods differ from the observed durations and are related to the dimension of the

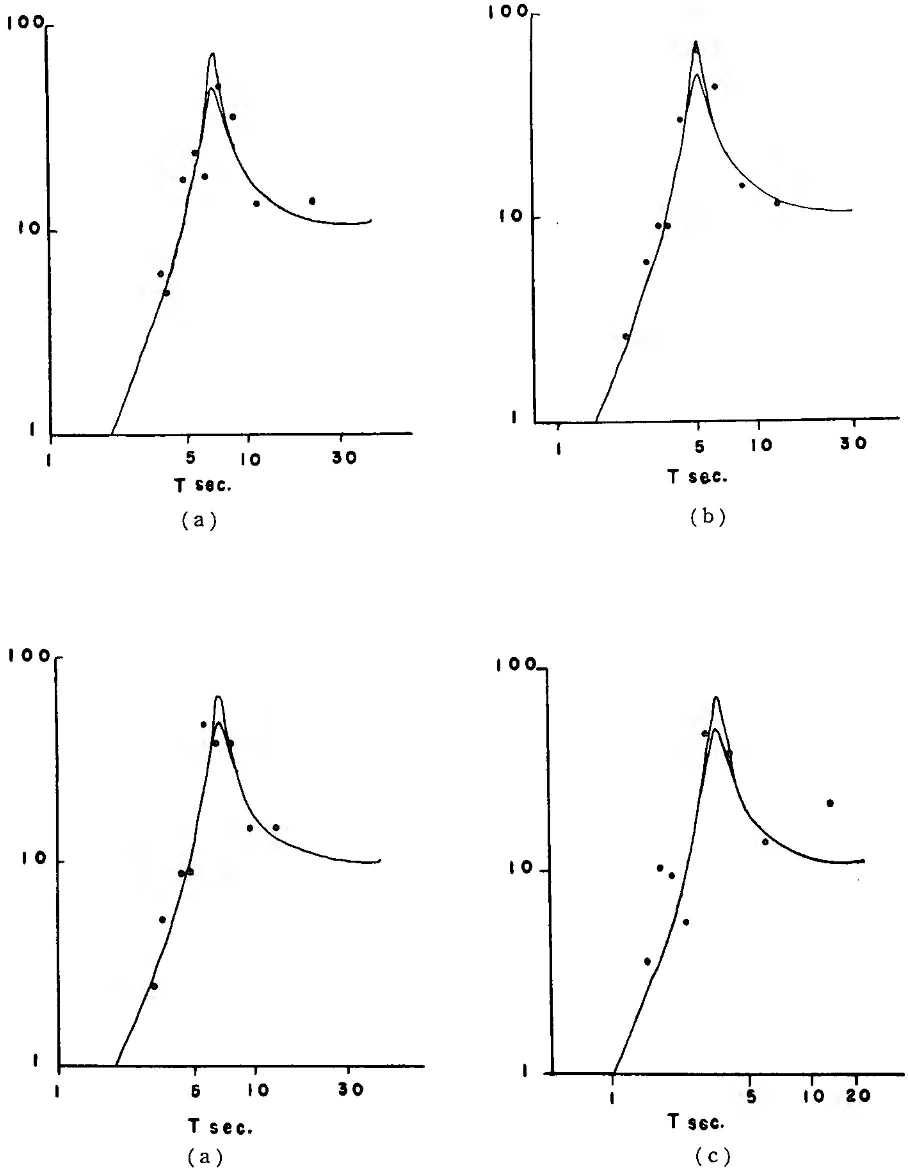

Fig. 2 Spectra of the volcanic micro tremor of the 2-nd kind observed during various durations.
(a) Oct. 1932
(b) Feb. 1933
(c) Jan. 1962

oscillation system.

The spectrum of the ground motion derived from the wave forms of the volcanic micro tremors depend on, in general, the characteristic of the active origin of the volcano and the mode in which waves have propagated. The 2-nd kind has a long period and characteristics of a compressional wave, therefore, the main feature of the spectrum would not be greatly influenced by the conditions of propagation. Subsequently, we might regard it, to a certain degree of approximation, as non-distorted information of the source condition.

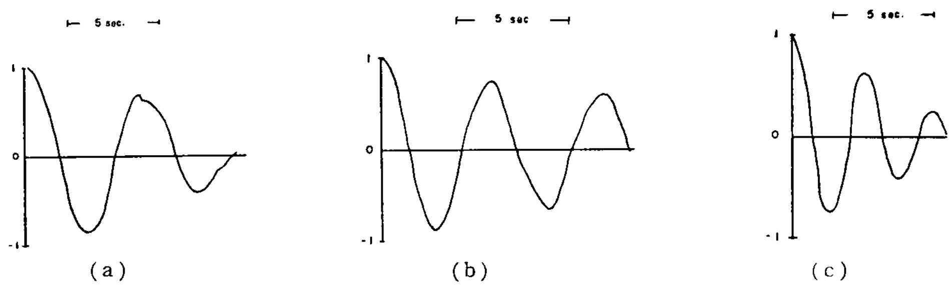

Fig. 3 Auto-correlograms of the 2-nd kind volcanic micro tremor.
(a) observed in 1932
(b) obsefved in 1933
(c) observed in 1962

Models of the Origin of the Volcanic Micro Tremor of the 2-nd kind

The source condition of an explosive or seismic origin has been investigated as the stress change on the surface of a small spherical cavity. According K. Kasahara[3], the relation between the energy of such a source and the characteristic period of a spectrum has been obtained as is shown in the following formula

$$\log T_0=\alpha+\beta M \qquad (3)$$

where T_0 is the characteristic period of the spectrum, and M is the magnitude of the earthquake refering to its energy. When the characteristic period is $T_0=$ 3.5 to 7.0 sec, $M=4.7$ to 5.8 may be estimated from the equation (3). This value, however, is greater than the observed results.

Subsequently, we must assume a different type of source which is proper in the volcanic region.

Concerning the source of the volcanic micro tremor of the 2-nd kind, K. Sassa[1] has estimated the free oscillation of a magmatic chamber beneath the active crater.

A model of the magmatic chamber has been presented by M. Shima[4]. According to his model, the vibration of the magmatic chamber is substituted for the vibration of a liquid sphere. In this paper, similar models have been used. The chamber was assumed to be a spherical form having a radius "a" in an infinite perfect elastic medium and filled with material which deforms like a compressible fluid. The vibrational mode of the chamber is expressed as the formulas of spherical wave motions in a polar coordinate system which has been solved by K. Sezawa[5]. Using his solutions of the fundamental mode of vibration and introducing the boundary conditions consisting of the continuity of stress and displacement at the surface of the spherical magmatic chamber, the author obtained the following reduced equation

$$\left\{\left(\frac{pa}{V_{pi}}\right)^2-4\left(\frac{V_{so}}{V_{pi}}\right)^2\left(1+i\frac{pa}{V_{pi}}\cdot\frac{V_{pi}}{V_{po}}\right)\right\}\left\{\sin\left(\frac{pa}{V_{pi}}\right)-\frac{pa}{V_{pi}}\cos\left(\frac{pa}{V_{pi}}\right)\right\}\frac{1}{\rho_i}$$
$$=\left\{\left(\frac{pa}{V_{pi}}\right)^2\sin\left(\frac{pa}{V_{pi}}\right)\right\}\left\{\left(1+i\frac{pa}{V_{pi}}\frac{V_{pi}}{V_{po}}\right)\right\}\frac{1}{\rho_o} \qquad (4)$$

where

a is the radius of the spherical chamber;

V_{pi} and V_{po} are the longitudinal wave velocities of inner and outer parts of the chamber, respectively;

V_{so} is the transverse wave velocity of the solid medium of the outer part of the chamber; and

ρ_i and ρ_o are the densities of inner and outer parts of the chamber, respectively.

It is presumed that the damped vibration systems take place in the magmatic chamber, hence "p" becomes the complex number "$p=p_1+ip_2$" in which the real and imaginary parts express the circular frequency and damping factor of waves, respectively.

Putting

$$Z=\xi+i\eta=\frac{a}{V_{pi}}(p_1+p_2), \quad \left(\frac{V_{so}}{V_{pi}}\right)^2=A \quad \frac{V_{pi}}{V_{po}}=B \qquad (5)$$

$$\left.\begin{aligned}
L_1&=\sin\xi\cosh\eta-\xi\cos\xi\cosh\eta-\eta\sin\xi\sinh\eta\\
L_2&=\cos\xi\sinh\eta+\xi\sin\xi\sinh\eta-\eta\cos\xi\cosh\eta\\
K_1&=\cos\xi\cosh\eta-B\cos\xi\sinh\eta\\
K_2&=B\sin\xi\cosh\eta-\sin\xi\sinh\eta
\end{aligned}\right\} \qquad (6)$$

Introducing (5) and (6) into equation (4), the following equation has been obtained:

$$\begin{aligned}
&(L_1+iL_2)[\{(\rho_o-\rho_i)(\xi^2-\eta^2)+4\rho_oAB\eta-4\rho_oA\}\\
&-i\{2\xi\eta(\rho_o-\rho_i)-4\rho_oAB\xi\}]\\
&=(K_1+iK_2)[\rho_i\xi(\xi^2-3\eta^2)+i\rho_i\eta(3\xi^2-\eta^2)]
\end{aligned} \qquad (7)$$

Separating real and imaginary parts in equation (7), the following final two equations were obtained:

$$\left.\begin{aligned} &L_1Q_1-L_2Q_2-(K_1R_1-K_2R_2)\gamma=0\\ &L_2Q_1+L_1Q_2-(K_2R_1+K_1R_2)\gamma=0 \end{aligned}\right\} \quad (8)$$

where

$$Q_1=(1-\gamma)(\xi^2-\eta^2)+4AB\eta-4A$$
$$Q_2=\{2(1-\gamma)\eta-4AB\}\xi$$
$$R_1=\xi(\xi^2-3\eta^2)$$
$$R_2=\eta(3\xi^2-\eta^2)$$
$$\gamma=\rho_i/\rho_o$$

Numerical computions of the vibration of magmatic chamber

In order to compute vibration formulas of the magmatic chamber from the equations (5), the parameteric values of V_{po}, V_{so}, V_{pi} and $\rho_i/\rho_o=\gamma$ in the equation (8) have to be assumed.

The observational results concerned with V_{po} have been obtained by two different experimental methods, as is described in the following. From the seismic prospecting tests which have been carried out near the active crater of Volcano Aso, Nakadake, by S. Yoshikawa and others[6], P-wave velocity was found to be 2.5 to 3.5 km/sec beneath the surface layer, having a P-wave velocity of 1.4 to 1.6 km/sec, while laboratory measurements of P-wave velocities made by the writer using ultrasonic wave, show 2.82 to 5.56 km/sec for Aso lava specimen[7].

On the basis of these observations it is assumed that P-wave velocity of the outer part of the chamber is 3.0 km/sec.

When the assumption $\lambda=\mu$ in this medium is added, V_{so} can be determined directly from V_{po}.

The sound velocity of magma (V_{pi}) has been taken as $V_{pi}=0.79$ km/sec by M. Shima[4]; however, this value is the sound velocity of the molten lava in the volcanic vent which was calculated by K. Sassa[1], while G.S. Gorshkov[8] has obtained $V_{pi}=1.6$ to 1.8 km/sec with respect to the sound velocity of magmatic reservoirs under the earth's crust this being calculated by the difference in the time of arrival of direct and composite seismic waves in the volcanic region, these two different values of V_{pi} may be considered as the lower and upper limits of the sound velocity of magma which has been made an object of this compution. For the reason above mentioned, three different values of V_{pi} were used in this compution, i.e. $V_{pi}=0.79$, 1.0 and 1.6 km/sec.

On the other hand, the density in the magmatic chamber is unknown, depending perhaps on the internal condition of its contents. Here various values of ρ_i/ρ_o, i.e. $\rho_i/\rho_o=\gamma=0.1$, 0.2 0.5 and 1.0, have been selected.

The numerical computions have been made by using the above mentioned assumptions consisting of:

$V_{po}=3.0$ km/sec

$V_{pi}=0.79$, 1.0 and 1.6 km/sec

$\rho_i/\rho_o=0.1$, 0.2, 0.5 and 1.0

ξ and η in the equation (5) have been computed by the "trial and error" method.

As the result of this compution it was found that there are two different solutions having a large damping factor and a small one. The former is not in agreement with the observed results because the observed wave form shows a small damped oscillation as has been discussed in the previous section. The later solutions of ξ and η are shown in Fig. 4 and Table 1.

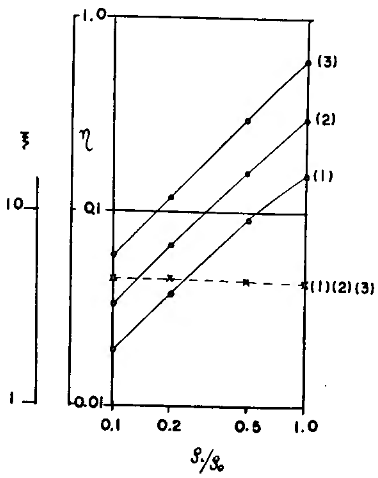

Fig. 4 Computed results of ξ and η

(1) $V_{pi}=0.79$ km/sec

(2) $V_{pi}=1.0$ km/sec

(3) $V_{pi}=1.6$ km/sec

Table 1

V_{pi} km/s.	$\gamma=\rho_i/\rho_o$	ξ	η	η/ξ
0.79	0.1	4.47	0.0199	0.0045
	0.2	4.45	0.038	0.0085
	0.5	4.38	0.089	0.020
	1.0	4.26	0.151	0.036
1.00	0.1	4.47	0.034	0.0076
	0.2	4.45	0.068	0.015
	0.5	4.37	0.162	0.037
	1.0	4.20	0.290	0.069
1.60	0.1	4.48	0.059	0.013
	0.2	4.46	0.117	0.026
	0.5	4.39	0.301	0.069
	1.0	4.16	0.614	0.148

As shown in Fig. 4 and Table 1, ξ is about 4.4 for each case, therefore, the dimension of the magmatic chamber hardly varies with the various densities and sound velocities of the magma; however, η, referring to the damping factor, rapidly increases accordingly as the density of the magmatic chamber increases. The ratios of η/ξ relating to the damping constant of the equivalent pendulum which has been considered as the origin of the 2-nd kind of volcanic micro tremor are also shown in Table 1. This value corresponds to the usual damping constant "h" referring to seismometers and has the following relation:

$$\eta/\xi=\frac{h}{\sqrt{1-h^2}} \qquad (9)$$

The Source Condition

From the comparison between computed results in the previous section and the predicted spectrum in the Figures 1(b) and 2, it is found that $\eta/\xi=0.1$ to 0.07 is equivalent to the predicted curves. The solutions having these η/ξ values correspond to the instances of comparatively large ρ_i/ρ_o values.

On the basis of these computed results only, the density of the inner part of the magmatic chamber may be of the same order or somewhat smaller than the outer part. In this compution, however, discussions of the viscous property of the magma which can be seen as its important characteristic, have not been made. Generally speaking, when the viscosity of the magma increases the damping factor of the oscillation system increases. But the computions containing the viscous factor have not been carried out due to their complexty.

As the result of this investigation, it is found that the oscillation of the magmatic chamber can be approximately substituted for the model of the oscillation of the liquid sphere.

The dimension of the magmatic chamber can be determined from the ξ and the characteristic period of the spectrum. The chamber's radius of "*a*" becomes about 2~4 km. These values are somewhat greater than the Shima's results[4].

The computed results using the Shima's assumptions[4], $V_{pi}=0.79$, $V_{po}=1.25$ and $V_{so}=0.98$ km/sec, are shown in the Table 2.

On the other hand, there are also the solutions having a large damping factor as has been mentioned. For instance, in the case of $V_{pi}=1.6$ km/sec, ξ and η change as shown in Table 3. From this table, η/ξ shows that the oscillation system is almost critically damped oscillation. Therefore, this solutions do not agree with the observed results.

Table 2

V_{pi} km/s.	$\gamma=\rho_i/\rho_o$	ξ	η
0.79	0.1	4.46	0.052
	0.2	4.43	0.100
	0.5	4.30	0.233
	1.0	4.06	0.378

Table 3

V_{pi} km/s.	$\gamma=\rho_i/\rho_o$	ξ	η	η/ξ
1.60	0.1	1.83	1.34	0.74
	0.2	1.88	1.41	0.75
	0.5	2.04	1.65	0.81
	1.0	2.48	2.29	0.93

Summary

1) Fourier spectral analyses of the ground motion due to the volcanic micro

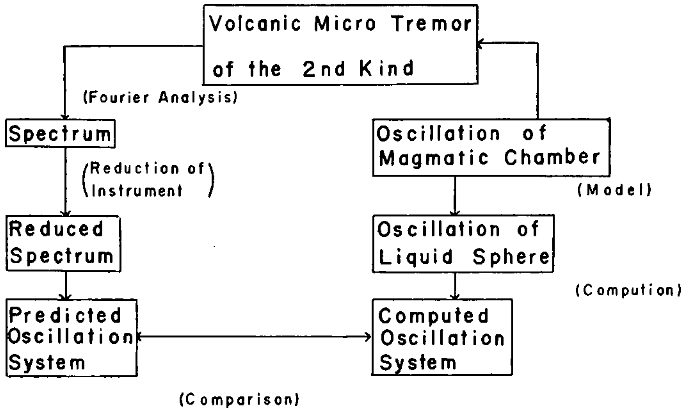

Fig. 5

tremor of the 2-nd kind have been made by using several selected seismograms.

This spectrum is consistent with the predicted spectrum of the damped oscillation system generated by the impulsive exciting force.

2) The predicted oscillation system with damping factor $h=0.1$ to 0.07 shows nearly free oscillation and may be regarded as nondistorted information of the source condition of this tremor.

3) The oscillation of a magmatic chamber generated by the involved gas explosions which had been considered by K. Sassa as the source of this tremor is substituted approximately for the model of the oscillation of a liquid sphere.

Numerical computions have been made by using this model and the relation between the damping factor and the density of the liquid sphere has been determined.

4) The predicted oscillation system from the computed results is in good agreement with the observed results.

Therefore, it is concluded that the volcanic micro tremor of the 2-nd kind originates from the free oscillation of a magmatic chamber as has been predicted by K. Sassa.

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