ON THE DETECTION OF EARTH MOVEMENTS BY MEANS OF THE GRAVITY METHOD

BY

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It is not easy to find out the existence of slowly occurring secular earth movements by measuring continuously or intermittently the change of gravity at fixed points. However, the possibility for the detection may be presumed when the gradient of gravity or the second derivative of gravity is measured instead of gravity itself in case that the change of underground mass distribution occurs near the surface. The problem will be dealt with as below.

We assume simply that a change of gravity field at a certain observation point $Q(0, 0, \zeta)$ is attributed to a displacement by δx, δy, δz of a material point m at $P(x, y, z)$, here the origin of the coordinates being taken at sea level just below the observation point and the z-axis downward as shown in Fig. 1. We do not take into account the effect due to the deficiency of mass at P after the shifting of the material point, as the pore must be filled up instantaneously with following mass.

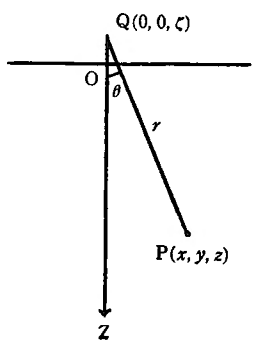

Fig. 1

The change of gravity due to a displacement δx, δy, or δz of the material point is expressed as

$$\delta g_x = -3k^2 m \frac{x(z-\zeta)}{r^5}\delta x\,,$$

$$\delta g_y = -3k^2 m \frac{y(z-\zeta)}{r^5}\delta y\,,$$

$$\delta g_z = k^2 m \frac{1}{r^3}\left\{1 - 3\frac{(z-\zeta)^2}{r^2}\right\}\delta z\,,$$

where δg_x, δg_y, or δg_z indicates the change of gravity under discussion due to δx, δy, or δz respectively, k^2 the universal gravitational constant, and r the distance between the observation point and the mass.

The change of gravity gradient $\frac{\partial g}{\partial \zeta}$ due to the same displacement of the mass is

$$\delta\left(\frac{\partial g}{\partial \zeta}\right)_x = 3k^2 m \frac{x}{r^5}\left\{1 - 5\frac{(z-\zeta)^2}{r^2}\right\}\delta x\,,$$

$$\delta\left(\frac{\partial g}{\partial \zeta}\right)_y = 3k^2 m \frac{y}{r^5}\left\{1 - 5\frac{(z-\zeta)^2}{r^2}\right\}\delta y\,,$$

$$\delta\left(\frac{\partial g}{\partial \zeta}\right)_z = 3k^2 m \frac{(z-\zeta)}{r^5}\left\{3-5\frac{(z-\zeta)^2}{r^2}\right\}\delta z\,,$$

where the left hand side indicates the change of gravity gradient due to the displacement δx, δy, or δz respectively.

The proportion of the change to the original gravity field determines the degree of difficulty of the measurement by which we want to know the existence of underground mass displacement.

For the measurement of gravity change at a fixed point, the ratio of the increment δg and the original gravity g is concerned. In this case the average gravity 980 gal at sea level can be approximately substituted for g.

For the case of dealing with the change of gravity gradient, the ratio of $\delta\left(\frac{\partial g}{\partial \zeta}\right)$ to $\frac{\partial g}{\partial \zeta}$ is taken into consideration, the latter being approximately estimated to be 3×10^{-6} gal/cm.

Now we compare the above two ratios. When the mass displaces horizontally by a distance $\delta\rho$, which is connected with δx and δy by the relation $\delta\rho^2=\delta x^2+\delta y^2$, the change of gravity and that of gravity gradient may be obtained from the above equations as

$$\delta g_\rho = -3k^2 m \frac{(z-\zeta)}{r^5}(x\delta x + y\delta y)\,,$$

$$\delta\left(\frac{\partial g}{\partial \zeta}\right)_\rho = 3k^2 m \frac{1}{r^5}\left\{1-5\frac{(z-\zeta)^2}{r^2}\right\}(x\delta x + y\delta y)\,.$$

Denoting the proportions δg_ρ : 980 and $\delta\left(\frac{\partial g}{\partial \zeta}\right)_\rho : 3\times 10^{-6}$ with $[g_\rho]$ and $\left[\left(\frac{\partial g}{\partial \zeta}\right)_\rho\right]$ respectively and the angle between the z-axis and the radius vector r with θ as in Fig. 1, we have

$$\left[\left(\frac{\partial g}{\partial \zeta}\right)_\rho\right] : [g_\rho] = 3.3(5\cos^2\theta - 1)\cdot\frac{1}{z-\zeta}\cdot 10^8\,,$$

whose absolute value is always larger than $\frac{1}{z-\zeta}\cdot 10^8$ except when the mass lies in the domain between about 59° and 69° of θ.

When the displacement of the mass occurs vertically by a distance δz, we have

$$\left[\left(\frac{\partial g}{\partial \zeta}\right)_z\right] : [g_z] = 30\cos\theta\frac{1-\frac{5}{3}\cos^2\theta}{1-3\cos^2\theta}\cdot\frac{1}{r}\cdot 10^8\,,$$

the notations being used similarly as above. The absolute value of this ratio is always larger than $\frac{1}{r}\cdot 10^8$ except when the mass lies between about 38° and 40° of θ. When $\theta=55°$, its denominator becomes approximately zero. That is, $\frac{\partial g}{\partial \zeta}$

has no change.

These results show that the change of mass distribution affects the gravity gradient much more than the gravity in most cases when $z-\zeta$ or r is small, or in other words, the change occurs in a shallow zone under the surface.

Around a volcano, the source of eruption is usually located at several kilometers below the surface. It is not rarely that the depth of the source is less than 1 km. or so.

When the depth or the distance between the source and the observation point is 1 km, the change of gravity gradient is over 1,000 times as large as that of gravity except in the special cases that the change of the nearest mass occurs in the sectorial zone of $\theta=59°$ and $69°$ for the horizonal displacement and of $\theta=38°$ and $40°$ for the vertical displacement.

In general, the measurement of gravity gradient is far more effective for the detection of shallow earth movements than the measurement of gravity. For the prediction of volcanic eruptions, measuring the gravity gradient may be recommended.

Next we compare the change of $\frac{\partial^2 g}{\partial\zeta^2}$ and $\frac{\partial g}{\partial\zeta}$, when the material point displaces.

Differentiating $\frac{\partial g}{\partial\zeta}$ with respect to ζ, we have

$$\delta\left(\frac{\partial^2 g}{\partial\zeta^2}\right)_x=15\,k^2m\frac{x}{r^6}\cos\theta(1+7\cos^2\theta)\delta x\,,$$

$$\delta\left(\frac{\partial^2 g}{\partial\zeta^2}\right)_y=15\,k^2m\frac{y}{r^6}\cos\theta(1+7\cos^2\theta)\delta y\,,$$

$$\delta\left(\frac{\partial^2 g}{\partial\zeta^2}\right)_z=-3\,k^2m\frac{1}{r^5}(1+10\cos^2\theta-35\cos^4\theta)\delta z\,.$$

Since the average value of $\frac{\partial^2 g}{\partial\zeta^2}$ due to the normal earth is about 1.4×10^{-14} gal/cm^2, the proportion of $\delta\left(\frac{\partial^2 g}{\partial\zeta^2}\right) : 1.4\times10^{-14}$ to $\delta\left(\frac{\partial g}{\partial\zeta}\right) : 3\times10^{-6}$ is, for the horizontal displacement $\delta\rho$ of the mass

$$\left[\left(\frac{\partial^2 g}{\partial\zeta^2}\right)_\rho\right] : \left[\left(\frac{\partial g}{\partial\zeta}\right)_\rho\right]=\cos\theta\frac{1+7\cos^2\theta}{1-5\cos^2\theta}\cdot\frac{1}{r}\cdot10^9\,,$$

and for a vertical displacement δz of the mass

$$\left[\left(\frac{\partial^2 g}{\partial\zeta^2}\right)_z\right] : \left[\left(\frac{\partial g}{\partial\zeta}\right)_z\right]=2.1\frac{1+10\cos^2\theta-35\cos^4\theta}{5\cos^2\theta-3}\cdot\frac{1}{z-\zeta}\cdot10^8$$

The absolute value of the former of the above two ratios is always larger than $\frac{1}{r}\cdot10^9$, when θ has a value less than 72°, and that of the latter is also always larger than $\frac{1}{z-\zeta}\cdot10^8$, when θ is less than 79° and is other than around 53° which makes $\delta\left(\frac{\partial^2 g}{\delta \zeta^2}\right)_z$ null. It is naturally beyond question that $\delta\left(\frac{\partial g}{\partial \zeta}\right)_\rho$ or $\delta\left(\frac{\partial g}{\partial \zeta}\right)_z$ undergoes no change, when $1-5\cos^2\theta=0$, i.e., $\theta \doteqdot 63°$, or $5\cos^2\theta-3=0$, i.e., $\theta \doteqdot 40°$ respectively.

So, if we take 1 km. for $z-\zeta$ or r, for instance, of the source of a valcanic eruption as in the above example, the change of the second vertical derivative of gravity is over 10,000 times for a horizontal displacement of the mass and over 1,000 times for its vertical displacement as large as that of the vertical gradient of gravity except when the mass exists in a nearly horizontal direction from the observation point and in the narrow zone around $\theta=53°$ in case of vertical movements.

Although the theoretical value of $\frac{\partial^2 g}{\partial \zeta^2}$ is about 1.4×10^{-14} c.g.s., it is liable to be influenced by near disturbing mass more than $\frac{\partial g}{\partial \zeta}$. According to our observations around Kyoto, the actual values showed numbers of the order of 10^{-11} c.g.s. or so. For instance, the value has been obtained 2×10^{-11} c.g.s. at Tsutenkaku Tower in Osaka, that is situated in the plain covered by deltaic deposits and has almost no surface disturbance upon the observation.

If we take 1×10^{-11} c.g.s. as the steady value of $\frac{\partial^2 g}{\partial \zeta^2}$ in an unmovable state of underground mass, the above two equations are replaced by ones which have the numerical constants 1.5×10^6 and 3×10^5 instead of 10^9 and 2.1×10^8 respectively for the horizontal and vertical displacement. Then the value of $\left[\frac{\partial^2 g}{\partial \zeta^2}\right]$ does not differ so much from $\left[\frac{\partial g}{\partial \zeta}\right]$, when $z-\zeta$ or r is several kms. Of course, the nearer the mass approaches the surface or the observation point, the larger the value of $\left[\frac{\partial^2 g}{\partial \zeta^2}\right]$ becomes compared with $\left[\frac{\partial g}{\partial \zeta}\right]$.

The general conclusions are summarized as follows: in order to detect the earth movement in the earth's crust, for instance, for the prediction of earthquakes or volcanic eruptions, the measurement of $\frac{\partial g}{\partial \zeta}$ is generally more suitable than of g, and for the detection of shifting of mass located near the surface, or roughly speaking, within about 1 km. from the surface, for instance, in case of volcanic eruptions from shallow sources or the change of underground water the measurement of $\frac{\partial^2 g}{\partial \zeta^2}$, if possible, may be generally suitable more than of $\frac{\partial g}{\partial \zeta}$.