ON THE VERTICAL TEMPERATURE DISTRIBUTION IN RELATIVELY DEEP SNOW LAYER

BY

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1. Introduction

The vertical temperature distribution in snow layer is important for a study of the heat budget on the snow surface and for prediction of the beginning of snow melting. There are some reports on the relatively shallow snow layer (e.g. T. Seo[1]), and very deep glacier (e.g. H. Wexler[2]), but we can scarcely find the data in the layer of several meter depth. The present author joining the snow survey sponsored by the Kansai Electric Power Co., at the mountain district in the central Japan in winter or spring of 1961 to 1962, obtained the temperature distribution in the snow layer from 3 to 6 meter depth. The purposes of this paper are to present these data and to give some discussion on them.

2. Observations and results

Measurement of the vertical temperature distribution in the snow layer is practiced in the following way. A hole which has a rectangular horizontal section (about $1^m \times 3^m$) was bored in the snow layer, until the earth surface is exposed. A mercury thermometer is horizontally inserted into the snow layer through the surface of vertical section at a level. After the thermometer is read, it is pull out, and is again inserted at the new level apart 20 cm from the level where the temperature is previously measured, and the temperature is read. These procedures are successively carried out at levels of every 20 cm through the whole depth, and a vertical temperature distribution is obtained. These observations were carried out at the Happo-ridge in Nagano Prefecture in January 1961, at the catchment area of Kurobe-valley in Toyama Prefecture in April 1961, and at the same area as in April 1961, in March 1962. The results thus obtained are shown in Table 1 with some remarks.

In these observation, all the measurements of snow temperatures were intended to have been finished as soon as possible, but they took about 1 hour at latest after the vertical section revealed. The temperature in snow layer near the vertical section of the hole may be probably suffered from exposure to the air, and the

Table 1 The results of observations on the vertical temperature profile in snow layer.

Site	Kurobishi	Happo	Goshiki-hara	Taira	Kariyasu	Kariyasu	Taira
Date	10, Jan. '61	11, Jan. '61	9, Apr. '61	9, Apr. '61	10, Apr. '61	10, Apr. '61	19, Mar. '62
Time	14^h30^m -16^h15^m	10^h30^m -13^h00^m	10^h10^m -12^h25^m	14^h07^m -15^h10^m	10^h10^m -12^h25^m	11^h30^m -12^h04^m	13^h35^m -14^h00^m
M.S.L. (m)	1660	1930	2500	1380	1885	1750	1400
Orographic Condition	North facing slope	Ridge	Plain	Plain	Ridge	South facing slope	North east facing slope
Weather	Snow	Snow	Drifting snow	Snow	Clear	Clear	Cloudy
Air Temp. (°C)	−1.0	−9.5	−6.4	−2.3	3.0	1.3	1.0
Total Depth (cm) / Obs. Depth (cm)	285	345	565	205	325	305	365
20	−4.2 (°C)	− 3.5 (°C)	−0.2 (°C)	−0.3 (°C)	−0.3 (°C)	−0.3 (°C)	−2.7 (°C)
40	−6.3	− 6.9	−0.2	−0.3	−0.4	−0.5	−2.8
60	−7.9	− 9.5	−0.3	−0.4	−0.4	−0.5	−2.0
80	−7.9	−10.8	−1.1	−0.3	−0.4	−0.4	−2.0
100	−8.7	−10.5	−3.0	−0.2	−0.4	−0.3	−1.9
120	−7.8	−11.0	−3.7	−0.3	−0.3	−0.3	−2.2
140	−7.4	−11.0	−4.0	−0.3	−0.3	−0.3	−2.4
160	−6.3	− 9.7	−4.7	−0.3	−0.5	−0.3	−2.5
180	−5.4	− 8.8	−4.5	−0.2	−0.5	−0.3	−2.5
200	−4.3	− 9.7	−5.6	0.0	−0.5	−0.3	−2.0
220	−2.9	− 8.0	−7.0		−0.5	−0.3	−2.0
240	−2.6	− 5.9	−6.1		−0.5	−0.3	−2.0
260	−1.6	− 4.3	−5.5		−0.5	−0.3	−1.6
280	−0.4	− 3.3	−5.8		−0.4	−0.3	−0.7
300		− 2.5	−6.0		−0.4	−0.3	−0.5
320		− 1.8	−6.6		−0.4		−0.5
340		− 0.3	−6.4				−0.3
360			−6.5				0.0
380			−6.3				
400			−5.0				
420			−4.5				
440			−4.3				
460			−3.6				
480			−3.1				
500			−2.5				
520			−2.0				
540			−2.2				
560			−0.5				

observed temperature may have some deviation from the temperature in natural state.

For rough estimation of this temperature deviation, we treat one-dimensional conduction of heat in semi-infinite solid, under the reasonable assumption that the temperature deviation due to the horizontal conduction of heat through the vertical section of hole is much greater than that due to vertical heat conduction. The coordinate system is shown in Fig. 1. It is assumed that the temperature of snow is uniform and equals to θ_s before the vertical section is constructed ($t<0$), and the temperature at the surface ($x=0$) of the section becomes θ_a after the section is constructed ($t>0$). Thus, the temperature at x, and t after the section is constructed is obtained by solving the heat transfer equation:

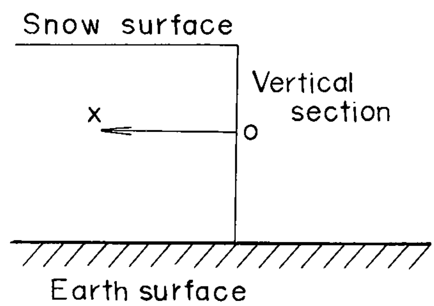

Fig. 1 Coordinate system.

$$\frac{\partial \theta}{\partial t}=K\frac{\partial^2 \theta}{\partial x^2}$$

under the conditions:

$$\theta=\theta_s \quad \text{when} \quad t=0$$
$$\theta=\theta_a \quad \text{when} \quad x=0, \quad t>0 .$$

The solution is easily obtained:

$$\theta=\theta_a+(\theta_s-\theta_a)\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{Kt}}} e^{-\xi^2}d\xi$$

Therefore

$$\frac{\theta-\theta_a}{\theta_s-\theta_a}=\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{Kt}}} e^{-\xi^2}d\xi \qquad (1)$$

Since the exposed time of the vertical section until the measurement is finished is equal to or less than 1 hour, and the inserted length of the thermometer is about 10 cm, the left-hand side of (1) equals to 0.83. Then, the error containing in the data in Table 1 is at most about 20 % of the difference between real value and surface temperature at the section.

3. Discussion

The data listed in Table 1 give vertical temperature profiles in snow layer.

We find that snow temperature in the bottom layer approach to 0°C in all cases. The profiles may be classified into two types. The first is such profile that the temperature is nearly equal to 0°C throughout the layer, which is found in the observation at relatively low altitude or in mid-spring. The second is such profile that has a marked minimum temperature region in the midway of the snow layer, which is found in the observation at relatively high altitude or in mid-winter. Since the difference between snow and the air temperature near the vertical section falls in the range of 0.1°C to 9.2°C, existence of such types of profile can be confirmed even when the error estimated in Section 2 is taken into account.

Establishment of these temperature profiles requires some explanations. The first type profile may be caused by warmness of snow itself at the time of falling or by heat conduction during long time interval. Our attentions are directed to the profile of the second type.

Although several theoretical treatments[3,4] were already published about temperature profile in snow layer, they are limited in heat conduction in semi-infinite body, because their purpose is a discussion on temperature profile in very deep snow layer or glacier (order of magnitude of 10 m or more). Such treatment seems to give little available informations about our second-type profile.

Assuming that horizontal conduction of heat in snow layer is negligible, and taking into account of growth of snow layer due to snow-fall through winter, we may affix the origin of our coordinate system to the rising surface, the following equation controls the temperature profile θ in snow layer,

$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial z}\left(k\frac{\partial \theta}{\partial z}\right)-v\frac{\partial \theta}{\partial z},$$

where v is the rate of rising of snow surface which is generally a function of time. Our aim may be attained if we can solve the above equation with suitable conditions, but it is difficult, and we must view from another angle.

A marked example of the second type of profile is shown in Fig. 2, which is obtained in the snow layer at "Goshiki-hara" Although meteorological data, and data of depth and surface temperature of snow layer for this profile are lacking, but some meteorological observations obtained at "Muro-do" in Mt. Tateyama in the last winter of our observation might give the general state of snow layer and its environment. "Muro-do" is situated at apart from "Goshiki-hara" to the north about 4 km and is nearly the same as "Goshiki-hara" on altitude and orographic condition. Of course, the states of snow layer and its environments at different place and in different year are not identical with each other. General tendency of the state, however, have not large variation from year to year. This

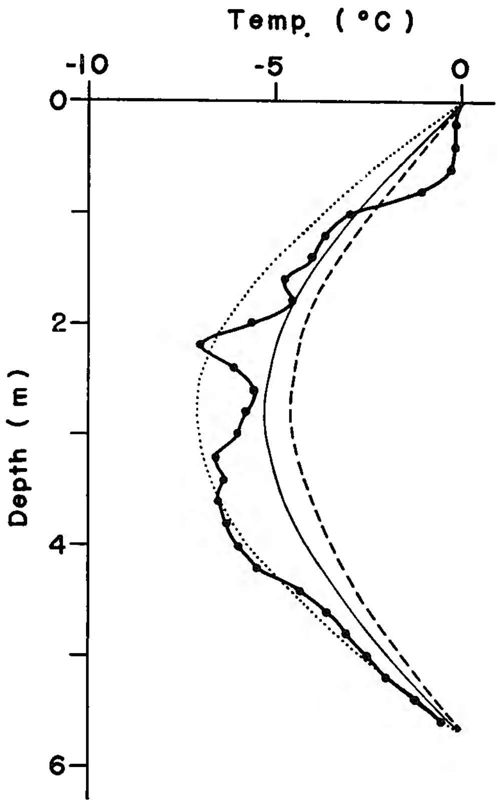

Fig. 2 Vertical temperature profile in snow layer.

(i) Thick line; observed profile at "Goshiki-hara" in Apr. 1961

(ii) Dotted line; calculated profile from eq. (8), $t=0$ is set at 20 th Feb.

(iii) Thin line; calculated profile from eq. (8), $t=0$ is set at 20 th Jan.

(iv) Broken line; calculated profile from eq. (8), $t=0$ is set at 20 th Dec.

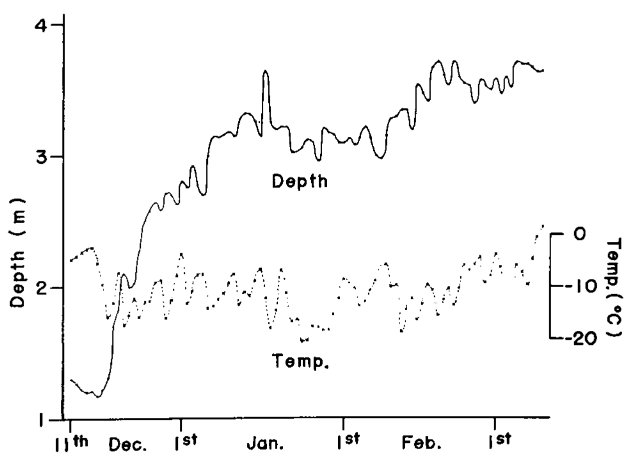

Fig. 3 Air temperature and snow depth data at "Muro-do" in Mt. Tateyama from Dec. 1959 to Mar. 1960.

permits us to combine the data of temperature profile at "Goshiki-hara" in 1961 and the meteorological data at "Muro-do" in 1960. The daily data of snow depth are shown in Fig. 3, which shows a sudden increase of the depth about 20 th December, and a gradual increase afterwards. The increase of the depth ceases about 20 th February. Daily data of mean air temperature at " Muro-do " are also shown in Fig. 3. Although there are somewhat large day-by-day fluctuations, general tendency of the air temperature has a sudden fall at about 20 th December, roughly constant temperature about $-12°$C from 20 th December to 20 th February, and gradual rise to 0°C afterwards. The fact that the temperature of snow-layer bottom are nearly equal to 0°C in all cases listed in Table 1 may permit an assumption that the bottom temperature might remain unchanged during snow season. Assuming that the difference between the temperature of falling snow and the air temperature at that time is not so large throughout the period of snow fall, these facts permit us to construct a simple model of establishment of the temperature profile in question here.

The bottom of snow layer keeps a constant temperature which equals to 0°C and the heating of the snow layer continuously proceeds from the first snowfall. While, the temperature of upper surface of snow layer always equals to the air temperature (θ_s), until the heating through the surface begins on 20 th February ($t=t_1$) when the air temperature abruptly rises. These model will be formulated in the following.

In the coordinate system shown in Fig. 4, the temperature in the snow layer is given by solving the equations:

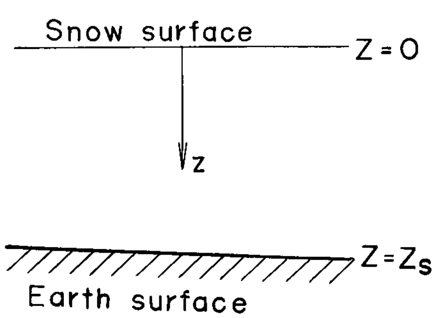

Fig. 4 Coordinate system.

For $t<t_1$

$$\frac{\partial\theta_1}{\partial t}=K\frac{\partial^2\theta_1}{\partial z^2}\quad(0<z<z_s) \tag{2}$$

under the conditions of

$$\left.\begin{array}{lll}\theta_1=\theta_s & \text{when} & t=0\\ \theta_1=\theta_s & \text{when} & z=0\\ \theta_1=0 & \text{when} & z=z_s\end{array}\right\} \tag{3}$$

and for $t\geqq t_1$

$$\frac{\partial\theta_2}{\partial t}=K\frac{\partial^2\theta_2}{\partial z^2}\quad(0<z<z_s) \tag{4}$$

under the conditions of

$$\left.\begin{array}{lll}\theta_2=\theta_1 & \text{when} & t=t_1\\ \theta_2=0 & \text{when} & z=0\\ \theta_2=0 & \text{when} & z=z_s\end{array}\right\} \tag{5}$$

The solution is well known as

$$\theta_1=\theta_s\frac{z}{z_s}+\frac{2}{\pi}\sum_1^{\infty}\frac{\theta_s\cos n\pi}{n}\sin\frac{n\pi z}{z_s}\exp\left(-K\frac{n^2\pi^2}{z_s^2}t_1\right)$$
$$+\frac{2}{z_s}\sum_1^{\infty}\sin\frac{n\pi z}{z_s}\exp\left(-K\frac{n^2\pi^2}{z_s^2}t_1\right)\int_0^h\theta_s\sin\frac{n\pi z'}{z_s}dz' \qquad (6)$$

$$\theta_2=\frac{2}{z_s}\sum_1^{\infty}\sin\frac{n\pi z}{z_s}\exp\left(-K\frac{n^2\pi^2}{z_s^2}\overline{t-t_1}\right)\int_0^h\theta_1(z')\sin\frac{n\pi z'}{z_s}dz' \qquad (7)$$

In the practical calculation, equation (7) is rewrited as

$$\theta_2=\frac{\theta_s}{\pi}\left[2\sin\frac{\pi z}{z_s}\exp\left(-K\frac{\pi^2\overline{t-t_1}}{z_s^2}\right)\left\{1+\exp\left(-K\frac{\pi^2 t_1}{z_s^2}\right)\right\}\right.$$
$$\left.-\sin\frac{2\pi z}{z_h}\exp\left(-K\frac{4\pi^2\overline{t-t_1}}{z_s^2}\right)\left\{1-\exp\left(-K\frac{4\pi^2 t_1}{z_s^2}\right)\right\}\right] \qquad (8)$$

this expression contains an error of 10 % at most. For calculation, t is set at 9 th April which is observed time, and the time $t=0$ is taken either at (i) 20 th December (beginning of growing period of snow layer), (ii) 20 th January (midway of the period) or (iii) 20 th February (end of the period). The value of thermal diffusivity K is taken as 6.0×10^{-3} ($cm^2\ sec^{-1}$) which is the middle of 7.3×10^{-3} for firn[3)] and 5.0×10^{-3} for fresh snow[5)], and we set $\theta_s=-12°C$.

Fig. 2 shows the comparison between temperature profile calculated by equation (8) and observational one at "Goshiki-hara". The main part of observed profile is almost exist between the profiles of (ii) and (iii) mentioned above, and it suggests that the introduced model may be reasonable.

Acknowledgement

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