SOME CONSIDERATIONS ON THE WAVE FORMS OF *ScS* PHASES

BY

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1. Introduction

The seismic wave forms recorded by seismographs are determined by the form of shock at the origin, characteristics of seismic wave paths and those of the seismographs used. When we aim to get informations regarding the nature of seismic origin and/or seismic wave paths from seismic records, it is necessary to know how these factors take part in construction of the seismogram. Ricker (e.g. 1953), in his laborious series of paper, has worked out the problem of how impulsive shock at the origin suffers change in shape on the way of its propagation through the medium represented by Stokes' model. He showed that there are often found, in records of artificial explosions, wave forms predicted by his theory, in favour of his assumptions, and he showed further that some of the wave forms in seismograms are to be regarded as a consequence of superposition of such simple "unit" wave forms named wavelets by himself. It seems natural to deal the shock of the artificial origin as impulsive, but the assumption may be too simple for application to the origin of natural earthquakes. What are the forms of shock at the source of natural earthquakes, then? Can we not apply the idea of wavelet to elucidation of the source mechanism of natural earthquakes? Recently, source function of natural earthquakes are closely discussed from the observed surface waves by various authors (Sato 1955, 1956, Aki 1960 a, b, c, d), but the attack on the problem by use of body waves is not well executed. The reason may be that body waves incident on the Earth's crust give rise to various kinds of transformed and boundary waves, making it quite difficult to distinguish the contribution of the source function from seismic records.

Wave forms of natural earthquakes, however, are not always of complicated wave trains but very simple solitary wave forms occur from time to time especially in records of deep seated earthquakes. We deduce herein the characteristics of the wave paths on an assumption that these simple waves stand for the impulsive responses of the material constituting the Earth's mantle. Once an impulsive response is known, it would be possible to deduce the forms of shock at the origin of natural earthquakes. Preliminary examination of the forms of *S*- (including *sS*-, *S*-, *ScS*-, *SKKS*- etc.) wavelets of various earthquakes shows it is likely that the shock forms of *S* at the origins can be represented by a few types of fairly simple displacements.

2. Data and Analysis

We deal here chiefly with *ScS* phases of two deep seated earthquakes. The occurrence times and locations of each earthquakes are listed in Table 1.

Table 1

	times of occurrence	locations of epicenters	focal depths
a	1957 Jan. 3^d 12^h 48^m 35^s (GMT)	$43\frac{1}{2}$N $131\frac{1}{2}$E (near Vladivostok)	600 km.
b	1957 Apr. 16 04 04 04 (GMT)	$4\frac{1}{2}$S $107\frac{1}{2}$E (Java Sea)	600

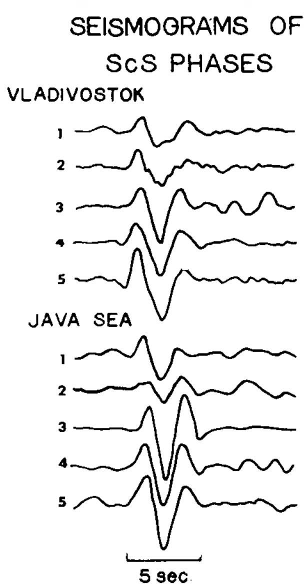

Fig. 1 *ScS* phases recorded at the Meteorological Observatories at 1: Morioka, 2: Fukushima, 3: Sumoto, 4: Kōchi, 5: Kumamoto, for Vladivostok earthquake and at 1: Morioka, 2: Fukushima, 3: Owashi, 4: Sumoto, 5: Hiroshima, for Java Sea earthquake, respectively.

They both occurred at the depth of 600 km. but their epicenters are geographically quite different. In Fig. 1 are shown a few of the examples of *ScS* phases of respective earthquakes recorded by Wiechert type seismographs equipped at the Meteorological Observatories in Japan. Seismograms of horizontal components are exclusively used throughout our present study, because the traces of vertical component seismograms are very small in amplitude due to their angles of incidence. It is noticable not only that they are quite similar in shape from record to record of the same earthquake, but that similarity also holds between the records of each earthquake. Taking differences of location and of the azimuth of epicenters into consideration, this similarity permits us a speculation that these wave forms are molded during their travel through the mantle of the Earth, and that they are not much degenerated by locality of observation stations nor by instrumental differences. Encouraged by such circumstances, we have made some considerations on the form of this phase in the following. As is easily verified, the simplest trace to be recorded by usual mechanical seismograph, like Wiechert type, must pass the equilibrium position at least twice under the condition that no dislocation and no particle velocity of the ground exist before and after the wave transit. Hence, we know that the traces shown in Fig. 1 belong to the simplest wave patterns to be found in seismograms. Fourier spectra of these waves are shown in Fig. 2 (a) and (b), and the same, corrected to those of displacement of the ground by use of instrumental characteristics are shown in Fig. 3 (a) and (b). In Fig. 3, ordinate is logarithmically scaled and each of them is arbitrally shifted one by one for convenience sake.

Knopoff (1956) investigated on the change of shape of elastic plane pulse in the medium possessing solid friction. With his model, amplitude decay of elastic sinusoidalwave is expressible in terms of exp $(-kD)$, where D denotes the distance along the wave paths, and k, coefficient of absorption depending on the first power of wave frequency:

$$k=\frac{S}{QTV}=\frac{S}{2\pi QV}\omega$$

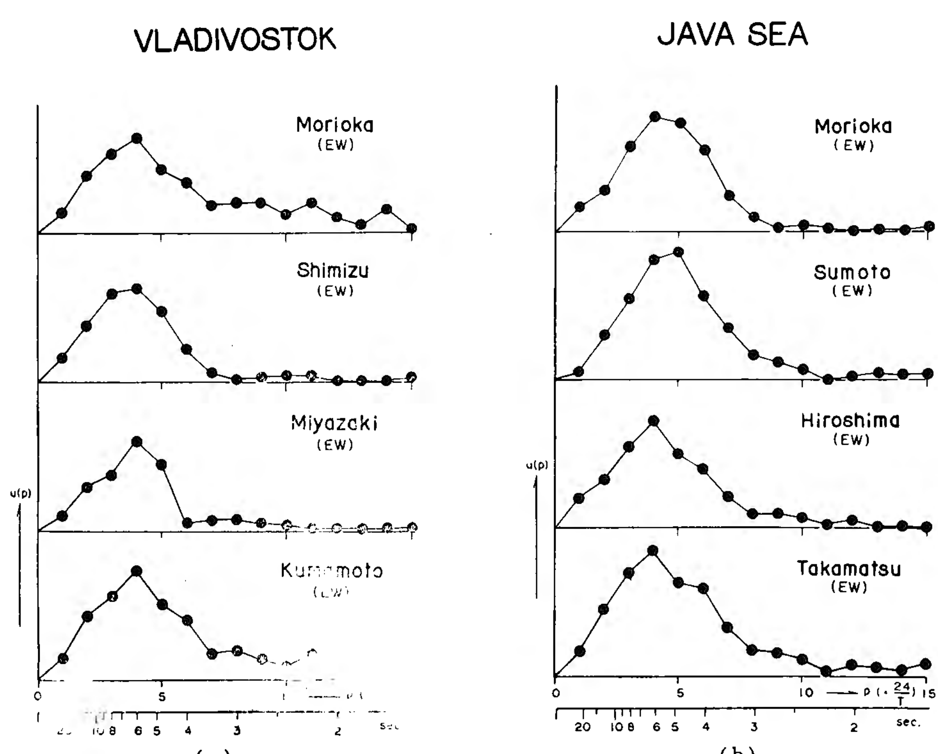

Fig. 2 Spectra of *ScS* phases of respective earthquakes.

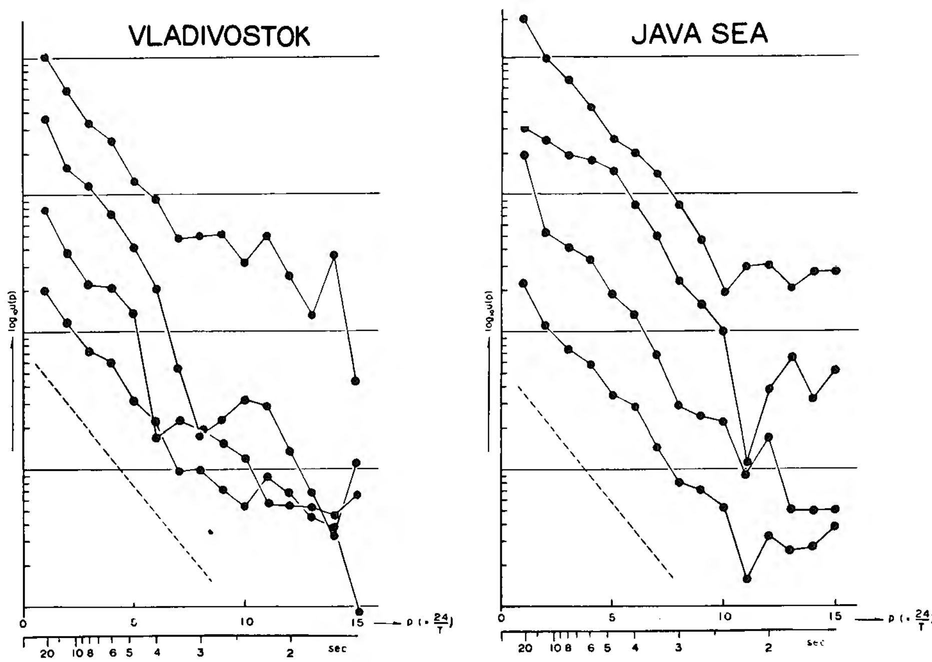

Fig. 3 Spectra of *ScS* phases of respective earthquakes corrected to those of ground displacements.

where

S: Stokes' constant of material
Q: dimensionless constant proper to material
T: wave period
ω: angular frequency ($2\pi/T$)
V: wave velocity

If displacement at the source is impulsive, including all frequency components equally in amplitude, spectrum of the displacement pulse at a distant point in Knopoff's medium may be of the form

$$\exp\left[-\frac{SD}{2\pi QV}\right].$$

This, plotted logarithmically against ω, will make a straight line of tangent of $-(SD/2\pi QV)\log_{10}e$. Again referring to Fig. 3, it may be noted that each of the spectrum is roughly on a line, if we ignore the small fluctuations. (Abscissae in Figs. 2 and 3 are not ω but $24/T$.) Representative slopes thus determined are shown in each figure by dotted lines. If we assume appropriate values for D, V, and S, the over-all values of Q of the wave paths can be obtained by use of said tangents of spectra. Q values thus calculated are 340 and 370 for Vladivostok and Java Sea earthquakes, respectively. S is assumed to be 4/3 after Stokes' criterion $S \geqq 4/3$ (Knopoff, 1954). Hence, Q values estimated in this way are the most conservative ones. D is the distance measured along the wave path of *ScS*. The effect of reflection at the mantle-core boundary on frequency is not considered here. Q values of material of the Earth's mantle are investigated by many authors, though their methods and standpoints are various. The determined values range from 400 to 2500. The values obtained here may be said to be in good accord with these results.

To obtain the informations on the forms of source pulses from seismograms, it is desireable to integrate the seismograms and get the ground displacement by use of instrumental constants, but for the purpose of scanning many seismograms, it seems to be more efficient to compare seismograms themselves with the calculated responses of seismographs against the incidences of theoretical wavelets. Knopoff's wavelet is mathematically expressed by the formula

$$u = \frac{\beta c D}{\pi} \cdot \frac{(1+1/Q^2)^{1/2}}{\tau^2 c^2 (1+1/Q^2)^{1/2} + D^2 \beta^2}$$

where $\beta = \left(\frac{-1+\sqrt{1+1/Q^2}}{2} \right)^{1/2}$ $\quad \gamma = \left(\frac{1+\sqrt{1+1/Q^2}}{2} \right)^{1/2}$

c: wave velocity, $\tau = t - \frac{\gamma D}{c(1+1/Q^2)^{1/2}}$ (local time)

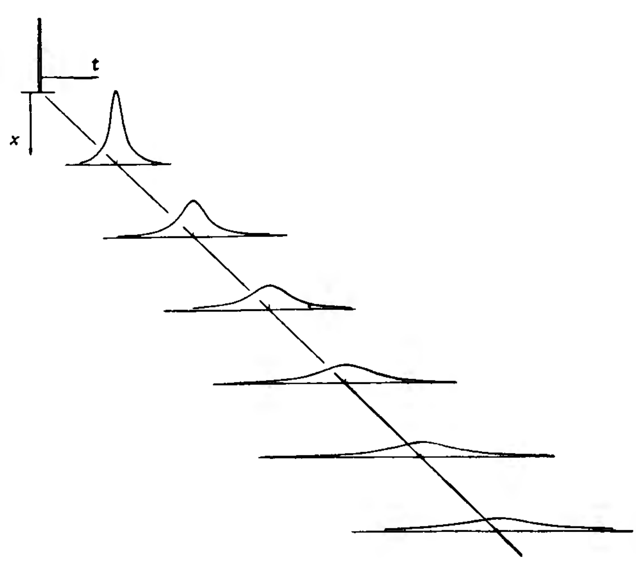

Fig. 4 Schematic displacement broadening for displacement pulse input, after Knopoff.

The pulse width increases linearly with distance. This broadening process is schematically depicted in Fig. 4 taken from Knopoff's paper. We calculated out, by use of a convolutional summing up of impulsive response of seismograph, the theoretical seismograms to be recorded by a Wiechert type seismograph when above Knopoff's wavelets of various breadth are incident. It was assumed that the period and damping constant of seismograph are 5 seconds and 0.6, respectively, to be compared with the actual seismograms in use for our analysis. In Fig. 5, some examples are shown. Widths of each incident pulse Δt in the figure are defined as the time interval between two zeros of the second derivative with time of the waves. It is interesting that the wave forms in Fig. 5, especially that of $\Delta t=1.60$ sec. is quite similar to the shape of *ScS* phase shown in Fig. 1. The similarity holds not only in shape but also in the absolute widths of the pulses.

However, contrary to our assumption, there is a widely recognized fact that periods of seismic waves have certain relation with the magnitudes of earthquakes, which simple wavelet theory assuming δ-source cannot explain. It seems necessary

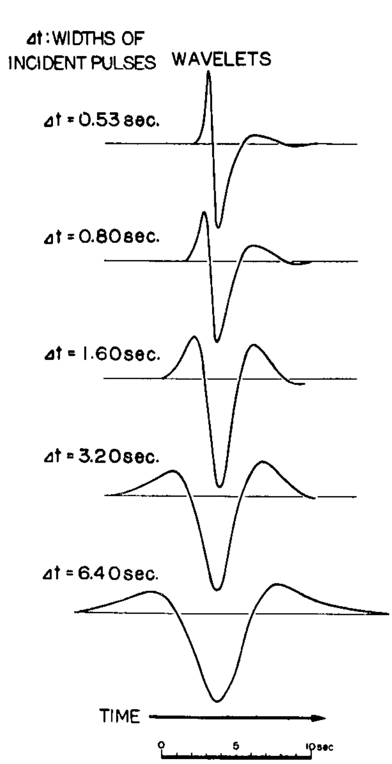

Fig. 5 Theoretical seismograms against Knopoff's displacement pulses.

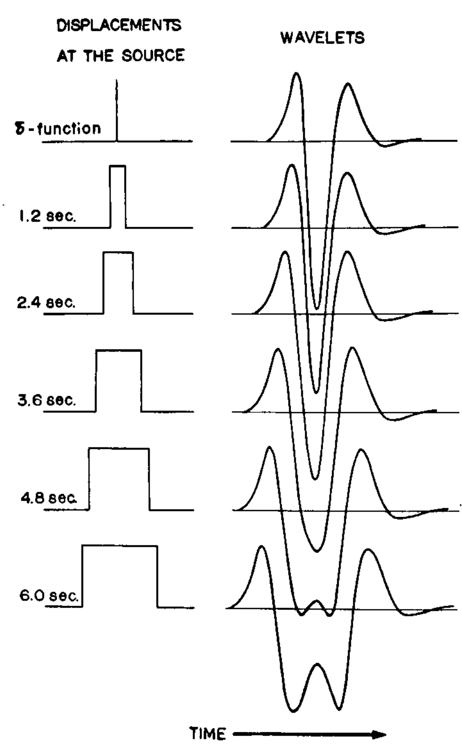

Fig. 6 Theoretical seismograms against Knopoff's displacement pulses when source pulses are rectangular.

to examine the physical meaning of δ-source assumed in the cource of our preceding study. In Fig. 6 are shown some of the examples of calculated responses of seismograph (constants of which being the same as above), against Knopoff's wavelets (Δt=1.6 sec.) when displacements at the source are of rectangular function of time. It is seen that the wavelet broadens as Δt increases, but wave forms, as a whole, do not differ much from that of δ-source until source width of 3.6 or 4.8 sec. is reached. This is understandable from the standpoint of spectrum. The spectrum $f(\omega)$ of rectangular pulse of width 2τ and amplitude $\frac{1}{\tau}$ is given by

$$f(\omega)=2\frac{\sin \tau\omega}{\omega}$$

The rough sketch of $|f(\omega)|$ is shown for a few values of τ in Fig. 7. Spectrum changes with τ as a parameter. From observational point of view, however, components which take part in construction of the wavelets do not extend from zero to infinity of frequency, but are limited on both sides. Seismograph is insensible for low frequency components and high frequency components are lost during their propagation. Hence, some changes in components outside the said range of frequency have no effects on the form of wavelets. This may be a reason why

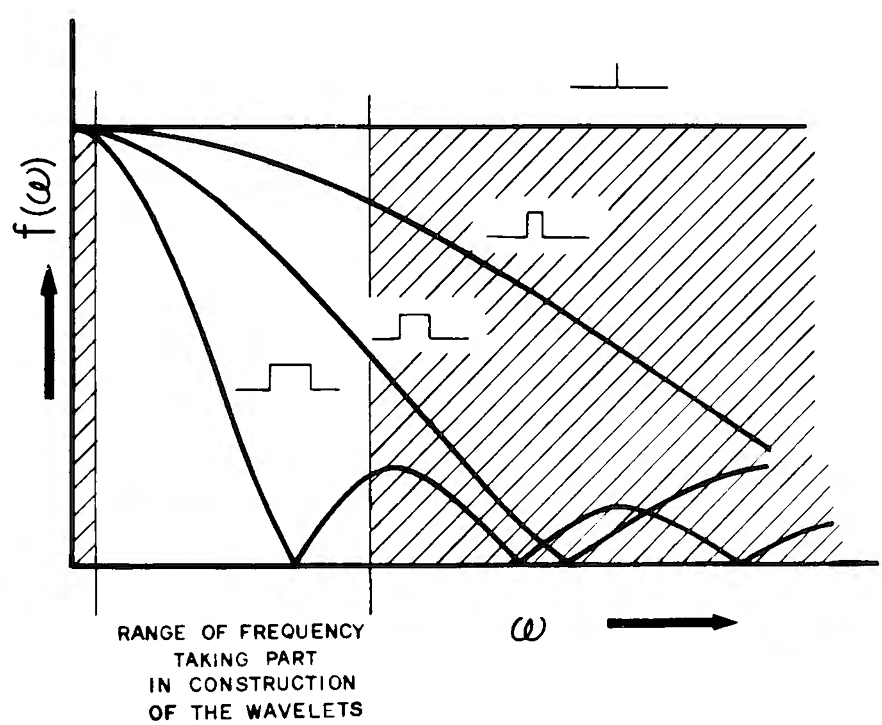

Fig. 7 Spectra of rectangular pulses.

some alteration of pulse width at the origin from δ-function does not effect on the forms of wavelets, and will give room for introducing the magnitude factor into our problems. If pulse at the source is narrower than a cirtain limit of width, minute structure of the form of the pulse takes little part in the form of wavelet, and whether the pulse at the origin is trapezoidal or of halfcircle, or others in form can hardly be detected from the form of wavelets. From physical point of view, these circumstances are related with the power of resolution of the system. With regard to our problem, power of resolution is limited by the damping characters and lengths of the wave paths. The shorter the lengths of wave paths are, the more will power of resolution be improved. The circumstances might illustrate the fact that the seismograms of near earthquakes are generally more complicated than those of teleseisms. That is to say, in case of teleseisms, minute features of the source pulses are lost on the way, and only rough configurations are identified. Fig. 8 shows some examples of wavelet forms when source pulses are not single but are doublets of rectangular pulses of various widths. It may be seen that wave forms in this case are entirely different from those in Fig. 6. Benioff (Richter, 1955) suggested the possible mechanism for azimuthal effect on wavelet emission by fault progression. It is schematically depicted in Fig. 9 taken from his paper. With his model, the lengths of wavelets emitted out of the source are different depending on the azimuth. The case shown in Fig. 6 are calculated after such model of origin in mind. The cases of couple of pulses as shown in Fig. 8 is also ones expected from the fault model.

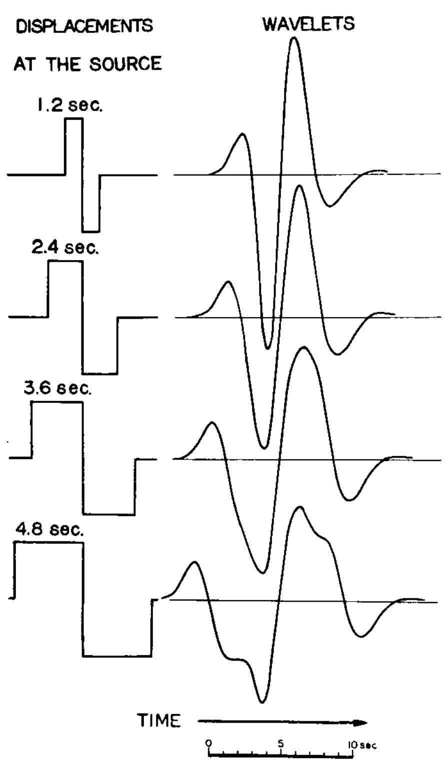

Fig. 8 Theoretical seismograms against Knopoff's displacement pulses when source pulses are doublets of rectangular impulses.

In Fig. 10 are shown some examples of wavelets recorded by Wiechert type seismograph at Kamigamo Observatory in Kyōto. Natural period and damping constant of the instrument are around 10 sec. and 0.5, respectively. The difference in natural period of the instrument needs no alteration of our preceding discussion so long as we are concerned only with wave forms, since the form of magnification curve remains the same. It is ascertained that the difference in damping constant gives a slight but not serious effects to the form of wavelets. In Fig. 10 are included not only *ScS*, but *S*-, *sS*-, *SKKS*-wavelet etc., and it is to be noted that these wavelets are similar in form to any of those in Figs. 5, 6 and 8. It is likely that the forms of shocks of *S* at the origin can be represented by a few types of fairly simple displacements. Seismogram of earthquake at Afghanistan in 1921 is an example which includes various phases distinctly and each of the phases has its peculiar form of wavelets. It might be related with the source mechanism of this earthquake. It is also of interest, in this seismogram that core phases are clearly recorded, and especially *PKKP*- and *SKKS*-wavelet are quite similar in form. Wave forms of the phases of P type are more or less complicated than those of *S* type in general, as far as we have examined, but above fact seems to suggest that the same kind of discussion may be applicable to P type wavelets. However, more detailed study will be needed to draw any decisive conclusions regarding the nature of seismic sources from the form of wavelets. Kasahara (1960) found out the azimuthal difference in the spectrum of *S* phase of Alaskan earthquake of 1958, and Ben-Menahem (1962) suggested the moving-source

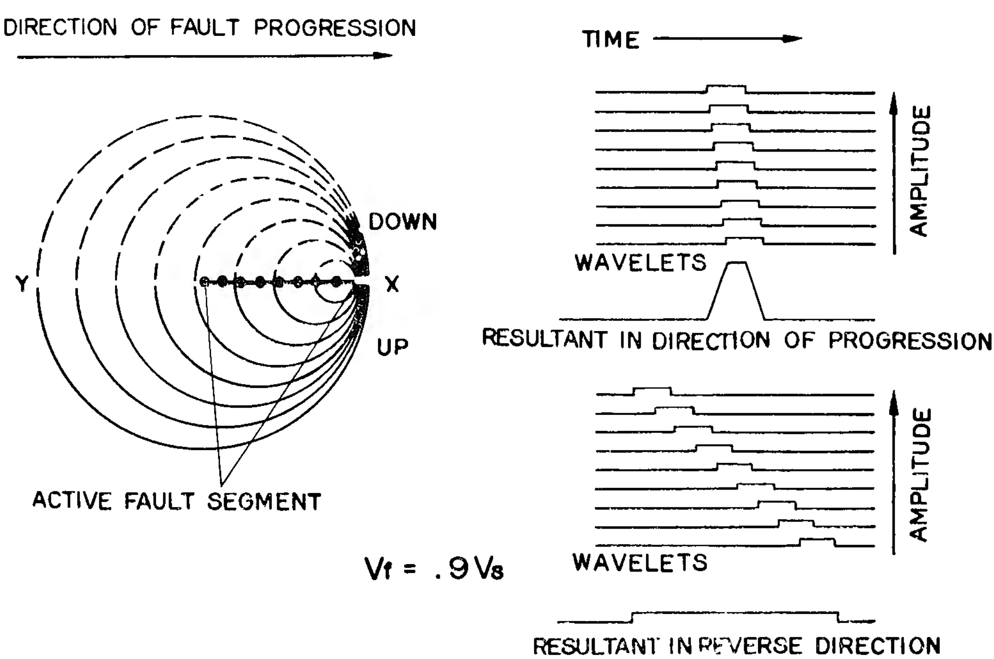

Fig. 9 Possible mechanism for azimuthal effect on wavelet emittion by fault progression, suggested by Benioff.

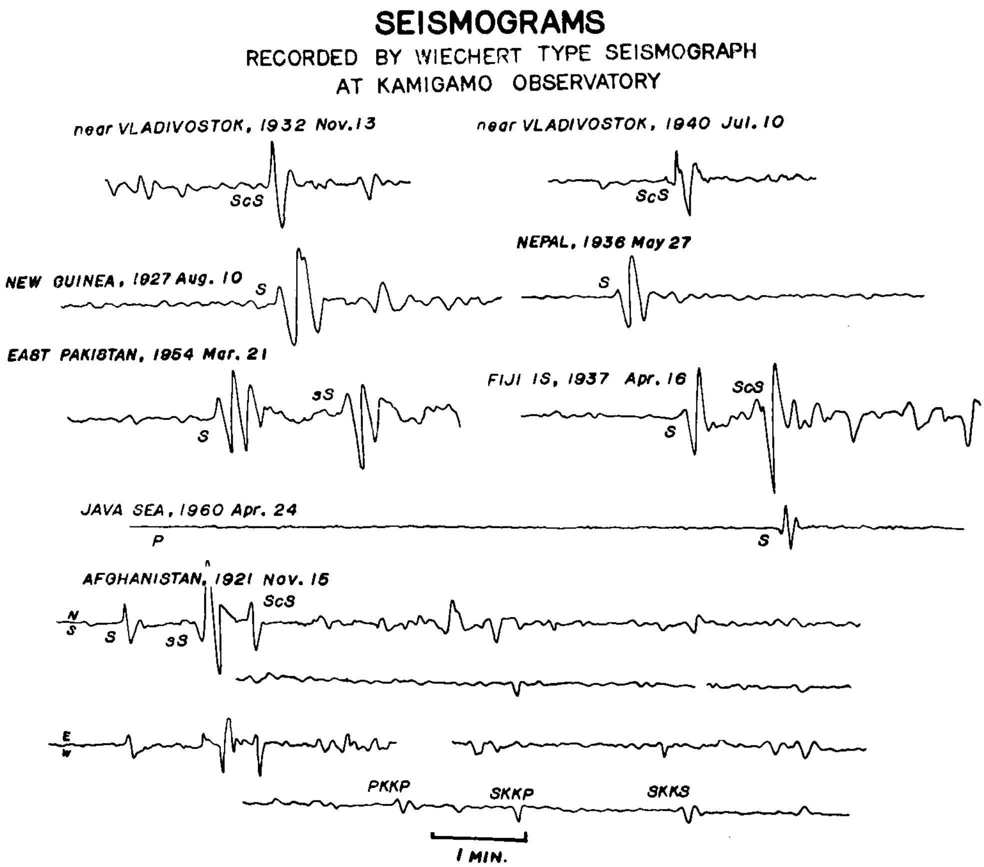

Fig. 10 Wavelets recorded by Wiechert type seismograph at Kamigamo Observatory in Kyoto.

model for the origin. That seismic sources of natural earthquakes have directivities in spectra has offered many interesting research items to the analysis of earthquake mechanisms.

Seismograms can not always be divided up into such simple wavelets even when we restrict our materials to the seismograms of teleseisms. However, it is also true that simple wavelets as shown in Fig. 10 are not rare instances but that they represent ones of the typical wave forms to be met in seismic records.

As a conclusion, author feels it is promissing to apply the idea of wavelets to elucidation of earthquake mechanisms.

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