STUDIES ON PEAK GUST

BY

Yasushi MITSUTA

1. Introduction

Turbulence of wind makes our studies on wind very complex one. To avoid this complexity, we use mean wind speed in weather analysis or in climatology. Averaging time interval of mean wind speed of 5 or 10 minutes (rarely 60 minutes) is chosen to be long enough to filter out the effect of turbulence but short enough not to lose sight of the change caused from synoptic or meso meteorological scale conditions. Although this time interval is often too long for the practical purpose, for example, very short duration wind forces, say a few seconds gusts, are said to be effective for destruction of constructions even in statical consideration. The peak gust speed has been commonly used for these purpose, which is read as the maximum value of the fluctuating trace of the anemometer. Although it is, strictly speaking, the maximum value of wind speed averaged over very short time interval which depends on the dynamical character of the anemometer. If averaging time interval of gust, or analysis time of gust, is different or if different kind of anemometer is used the peak gust speed is also different even under the same condition. Therefore the peak gust speed without provision of the analysis time has little significance. However, in Japan, the peak gust speed had been measured only by Dines anemometer and its value had some comparative meanings.

But recent development of structural theory has become to require more accurate knowledge of gust, and moreover several kinds of anemometers have become to be commonly used. From these points, new requirement on the studies on peak gust has arisen. And it is required to standardize the analysis time of the peak gust speed in common use.

2. Review of the Problem

In the studies of the peak gust, the ratio of the peak gust speed to the mean wind speed, called gust factor, is often used to simplify the consideration, because its value depends only on the turbulent structure of wind and its variation with mean wind speed can be ignored practically in high wind. And under such stormy

conditions, the air is well mixed so that the effect of the temperature lapse rate on turbulence can be disregarded. Therefore, the gust factor might be the function of analysis time of gust, height from the ground and the ground feature in high wind.

The gust factor, whether it is called by this name or not, has been studied or evaluated on many occasions. But most of the studies are description of the observational facts, and its dependency on the analysis time of gust had been remained untouched or unsolved for long time. This problem was studied by the present author [1], and a new experimental formula was established. Furthermore its variation with height was also studied and an experimental formula was proposed. While Durst [2] discussed this problem in a littele different way, but he did not show the functional relation. As will be shown later, his results can be also expressed by the formula of the present author.

The variation of the peak gust speed with height may be expected substantially smaller than the mean wind speed, and therefore the gust factor should show a slight decrease with height. Sherlock [3] and Deacon [4] have expressed the variation by power laws, but they did not discussed the problem in relation to the analysis time of gust. The result of the present author mentioned above will be shown in the next section.

It is natural to regard that the gust factor may vary with the ground feature. But this problem is still hard to discuss today, because we have little observational data over various ground features and they might show very complex nature. The studies mentioned above and in the followings are also restricted on the situation over flat level grounds.

In usual meteorological observations the peak gust speed is read from the trace of the ordinal speed run of the anemometer, therefore we can not resolve the duration of gust but read the maximum value. This maximum value is decided by the dynamical response of the anemometer to turbulence. The response of the commonly used anemometer to fluctuating wind speed may be expected to become to decrease rapidly at the period of a few seconds. Therefore the cut off in short period might approximately coincide with the minimum effective gust duration for destruction of the constructions. The response of the anemometer to periodically changing wind speed has been studied in many cases, but the response to the peak gust has not been discussed in detail.

Sherlock [3] has adopted 10 sec gust as the standard gust for the design problem, while Deacon [4] has used 2 sec gust for the design of the radio mast. This standard gust shall be determined as the minimum duration of gust which extends enough to affect the constructions with most common dimension, Sherlock [3]

considered that the longitudinal dimension of the minimum effective gust is eight times of that of the construction, while Ishizaki and the present author [5] considered that the minimum effective gust is the gust which has the same transversal dimension as the wind faced dimention of the construction, and together with the hypothesis of homogeneity discussed the duration of the minimum effective gust.

It is purely the structual engineering problem to enact the the standard gust which is the most convenient to the practical use. And it is the duty of the meteorologist to make the anemometer which we can easily measure the standard peak gust with. In the present paper enactment of the standard gust is not discussed but the method of the conversion to a temporary standard of 2 sec gust.

3. Gust Factor in Relation to the Analysis Time of Gust and Height from Ground

The gust factor G_s is defined as the ratio of the s sec peak gust speed or the maximum value of wind speeds whose analysis time is s sec (V_s max) and the mean wind speed over the total sampling time interval of D sec (V mean) so

$$G_s = V_s \max / V \text{ mean}. \qquad (1)$$

If the analysis time s is short, the value of V_s max may be large, therefore G_s is larger for a smaller value of s. While if the total sampling time interval is large the difference of the peak gust speed and the mean wind speed may be large, therefore G_s is larger for a larger value of D. The relation between the gust factor G_s and the analysis time of gust s or the total sampling time interval D depends on the structure of wind turbulence, and if we have full knowledge on turbulence it shall be deduced. But it is very hard because of the complex feature of the problem.

Empirical approach of this problem was accomplished by the present author [1] by the use of the data of the very quick run records reported by Sherlock [3] and Deacon [4]. And the following experimental formula was derived within the range of s from 0.5 to 60 sec in the cases of D is 300 and 600 sec;

$$G_s = (s/D)^{-p} \qquad (2)$$

where p is a new constant which indicates the state of gustiness and whose value is about 0.097 over level flat ground at the height of 10 m and decrease with increasing height. The variation of the value of p with height is approximately represented by the experimental formula,

$$p = p_0 (z/z_0)^{-0.41} \qquad (3)$$

where p_0 is the value of p at the standard height z_0 and in this case $p_0 = 0.97$ at

$z_0=10$ m.

While Durst [2] derived the peak gust speeds for various analysis time of gust. His consideration is such that the distribution of wind speeds may be assumed to be Gaussian and the peak gust speed is determined by the standard deviation of the distribution and the mean wind speed. He calculated standard deviations for various analysis time from several data of the quick run records. His conclusion is only presentation of a table of peak gust speeds for various analysis time and mean wind speed, but functional relation was not shown as mentioned before. Though he did not use the idea of gust factor, he also considered that the peak gust speed is proportional to the mean wind speed. Gust factors derived from his conclusion are shown with dots in Fig. 1, axes of which are plotted in logarithmic scales. In this case the sampling interval is one hour or $D=$ 3600 sec. The distribution of these dots may be satisfactorily approximated by the straight line which passes the point $G=1.00$ and $s=3600$ sec $(=D)$. This means that the conclusion of Durst is also explained by the formula (2). In this case the value of p is 0.058. Furthermore the gust factors which correspond to the 10 min mean wind speed for the same case can be calculated from the data shown in his paper. The results are shown in Fig. 1 with circles. The distribution of the circles is also approximated by the straight line which passes the point $G=1.00$ and $s=600$ sec $(=D)$. In this case p is 0.059, which is almost equal to the value of p for $D=3600$ sec. This means that the formula (2) holds good for the value of D as large as one hour. The height of the observation which corresponds to the conclusion of Durst's is not clear, therefore the value of $p=0.058$ cannot be compared whith the formula (3).

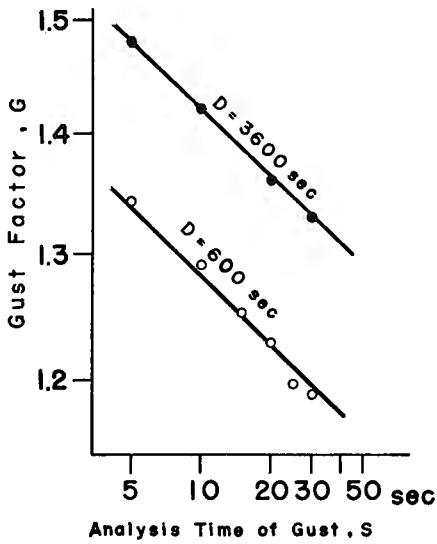

Fig. 1 Variation of gust factor with analysis time of gust calculated from the conclusion of Durst (2).

4. The Response of the Anemometer to the Peak Gust

The response of the anemometer to the sinusoidally changing wind speed has been studied in many occasions as mentioned before (for example [6], [7], [8] and [9]). The response of the anemometer to the peak gust is also able to be

studied from their results by means of Fourier analysis of the peak gust. But it is very complex in procedure and almost impossible. When we restrict our discussion on the response to the peak gust, the problem may be reduced to the response to the single pulsative wind fluctuation. That is, the peak gust can be roughly approximated by a single pulse, whose width is s and height is Vmax overlapping on the constant value of Vmean. With this simplification we can avoid the complexity. The response of commonly used Dines, Cup and Propeller type anemometers to the peak gust are discussed in the followings.

i) Dines Anemometer: The equation of motion of the float (of the pen) of Dines anemometer in unsteady state of motion may be written as following after Sanuki [6],

$$dZ/dt=C_1\sqrt{Z_e^2-Z^2} \qquad (4)$$

where Z is the position of the float (or the height of the pen), Z_e is the terminal position of the float, and C_1 is the constant which is characteristic for each anemometer and is determined by the radius of the float, length and inner radius of connecting pipes and so on. The mean value of C_1 of Dines anemometers of thirteen observatories of Japan Meteorological Agency in Hokkaido in the normal state of atomospheric pressure and temperature are reported to be 0.182 ($\sec^{-1}$) by Yajima [7].

If we assume that wind speed changes suddenly from V_0 to V_e at $t=0$. The motion of the float which corresponds to this wind change is described by the solution of (4) with boundary conditions $Z=Z_0$ at at $t=0$ and $Z=Z_e$ at $t=\infty$, where Z_0 and Z_e is the position of the float for the wind speeds of V_0 and V_e in steady state ($Z=1/360\ V$ for Dines anemometers used in Japan). The solution is as following,

$$\begin{cases} Z=Z_e\sin(C_1t+A) & \text{for } 0\leqq t\leqq 1/C_1\ (\pi/2-A) \\ Z=Z_e & \text{for } \quad t>1/C_1\ (\pi/2-A) \end{cases} \qquad (5)$$

where $A=\sin^{-1}(Z_0/Z_e)$.

The maximum displacement of the float (or the pen) during the single pulse of s sec in width and V_e in height overlapping on the constant wind speed of V_0 would be the value of Z for $t=s$ in equation (5). Therefore the gust factor for s sec gust measured on the trace of Dines anemometer is $Z(s)/Z_0$, while the true gust factor G is Z_e/Z_0. The dynamical magnification, M (observed gust factor/true gust factor, or observed peak gust speed/true peak gust speed) of Dines anemometer for s sec gust is $Z(s)/Z_e$, and is written,

$$\begin{cases} M=\sin(C_1s+\sin^{-1}1/G) & \text{for } 0\leqq s\leqq 1/C_1\ (\pi/2-\sin^{-1}1/G) \\ M=1 & \text{for } \quad s>1/C_1\ (\pi/2-\sin^{-1}1/G) \end{cases} \qquad (6)$$

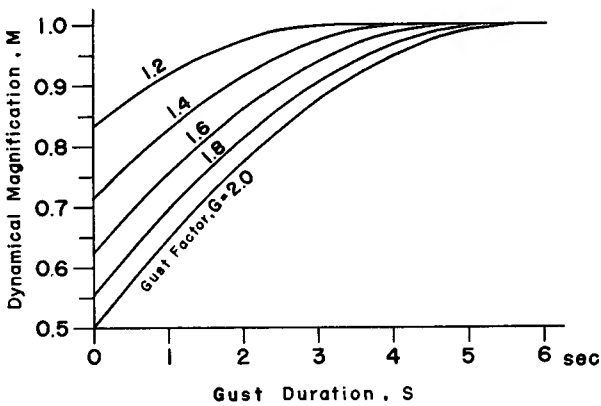

Fig. 2 Response of Dines anemometer to peak gust.

The values of M for various gust factor, G and gust duration, s are shown in Fig. 2, where C_1 is taken to be 0.182.

This figure shows the response of Dines anemometer to the peak gust in normal state. But it is very perplexed fact that the constant, C_1 varies widely from station to station for example the maximum value in the study of Yajima's mentioned before is 0.33 and the minimum is 0.042. Therefore we should remember that the result of Fig. 2 is only a averaged charactor and that of each anemometer varies widely.

ii) Cup and Propeller Type Anemometer: The equation of motion of the rotating type (cup or propeller type) anemometer in unsteady state of rotation can be written as following after Shrenk [8] or Sanuki [9], disregarding the torque of the generator coupled to the sensor which might be small in comparing to the torque exerted by high wind,

$$Id\omega/dt=T \qquad (7)$$

where I is the moment of inertia of the anemometer, ω is the rotating speed and T is the torque exerted by wind. And this torque is written as following,

$$T=1/2\rho V^2FRC_{d0}(1-\lambda^2(\omega R/V)^2) \qquad (8)$$

where ρ is the density of air, V is wind speed, F is a constant determined by the shape of the cup or the propeller, R is the center radious of the cup or the radious of the propeller, C_{d0} is the static torque parmeter and λ is the proportion constant of rotating speed and wind speed.

If wind speed changes suddenly from V_0 to V_e at instant $t=0$, the response of the anemometer can be described by (7) and (8) under the conditions of $\omega=\omega_0$ at $t=0$ and $\omega=\omega_e$ at $t=\infty$ ($\lambda=V_0/\omega_0R=V_e/\omega_eR$). The solution of them is

$$\tanh^{-1}\omega/\omega_e=K\omega_et+\tanh^{-1}\omega_0/\omega_e \qquad (9)$$

where $K=1/2I\cdot\rho C_{d0}F\lambda^2R^3$.

The maximum rotating speed of the anemometer during a single pulse of s sec in width and V_e in height overlapping on the constant speed of V_0 is the value

of ω at $t=s$ in equation (9). Ordinarily the rotating speed of the sensor (cup or propeller) of the anemometer is measured by electric voltage of the generator coupled to the sensor. Therefore we should consider the response of the recording system of the anemometer. But we might assume that the response of the recording system is much better than that of the sensor within the range of time scale under discussion. So in the present paper the response of the recording system is neglected. But to study in detail this problem we shall indeed consider the joint response of the sensor and the recorder. With this simplification the gust factor for s sec gust measured on the trace of a rotating anemometer becomes $\omega(s)/\omega_0$, while the true gust factor is ω_e/ω_0. The dynamical magnification M of a rotating anemometer for s sec gust is $\omega(s)/\omega_e$ and is written,

$$M=\tanh(K\omega_e s+\tanh^{-1}\omega_0/\omega_e)\,. \qquad (10)$$

Because $\omega_e/\omega_0=G$ and $\lambda=V_0/\omega_0 R$,

$$M=\tanh(C_2 G V_0 s+\tanh^{-1}1/G)$$

where $C_2=1/2I\cdot\rho C_{d0}F\lambda R^2$. In this case the response varies with wind speed. This is different from the case of Dines anemometer.

The values of C_2 for commonly used cup and propeller type anemometer in normal state of the atmospheric pressure and temperature were determined by wind tunnel experiment. The tested anemometers were a standard type 3 cup anemometer with generator and a standard type propeller anemometer (Kôshinvane), both of them are made by Kôshin Electric Engineering Co.. The wind tunnel of the Disaster Prevention Research Institute of Kyoto Univ. was used. The maximum test wind speed was about 30 m/sec. And the generating voltage was recorded by a pen-oscillograph with the amplifier which can fully record oscillations up to 200 cycle and the input impedance is the same as that of the recorder of each anemometer. Averaged values of C_2 in this experiment are $C_2=0.065$ (m^{-1}) for the cup anemometer and 0.087 (m^{-1}) for the propeller anemometer.

The values of M for various values of gust frctor, G and gust duration, s in the cases of mean wind speed being 20, 30 and 40 m/sec are computed with these values of C_2 and are shown in Fig. 3. Fig. 3(a) is the case when mean wind speed is 20 m/sec in which the response of the 3 cup anemometer is shown by dotted lines and that of the propeller by solid lines. Fig. 3(b) and (c) are the cases when mean wind speeds are 30 and 40 m/sec respectively.

These figures show the responses of commonly used anemometers to the peak gust in normal state. And they show that the response of the propeller anemometer is a little better than that of the cup anemometer, and they are both much better than that of Dines anemometer. The difference of the response from station

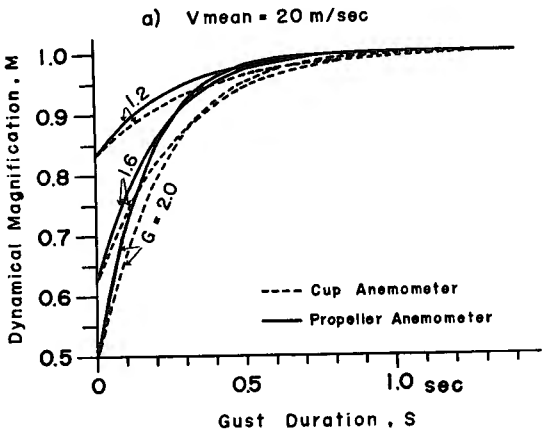

Fig. 3(a)

to station can be neglected in the case of the rotating type anemometer because the wiring from the sensor to the recorder does not affect the response of the system.

5. Duration of Peak Gust Speed Observed by the Commonly Used Anemometer

The gust speed observed by the commonly used anemometer is the value of true

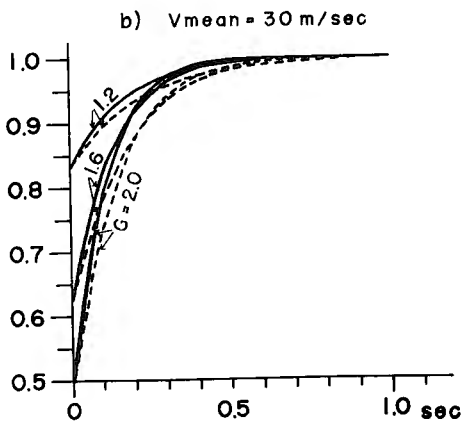

Fig. 3(b)

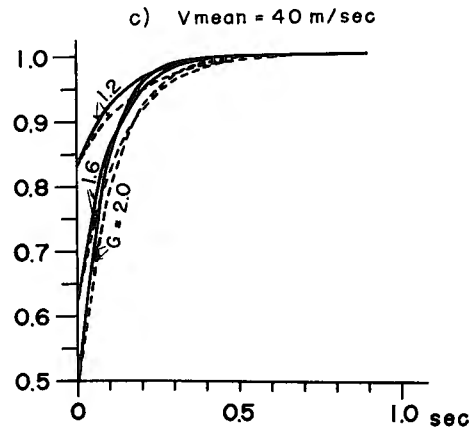

Fig. 3(c)

Fig. 3 Response of the cup and propeller anemometer to peak gust.

gust speed multiplied by the dynamical magnification. True gust speed in natural wind decreases with increasing analysis time of gust as is stated in the preceding section, while the dynamical magnification increases with increasing gust duration as is shown in the above section. Therefore the product of them has a maximum value at a certain value of analysis time. This maximum value corresponds to the peak gust speed on the trace of the anemometer.

The apparent gust factor, G_{app} observed by the anemometer in natural wind is written by the use of equations (2) and (6) or (11) as followings;

Dines anemometer ;

$$G_{app}=M\cdot G$$
$$=\begin{cases}\sin(C_1 s+\sin^{-1}1/G)\cdot G & \text{for } s\leqq 1/C_1\cdot(\pi/2-\sin^{-1}1/G)\\ G & \text{for } s>1/C_1\cdot(\pi/2-\sin^{-1}1/G)\end{cases}$$
$$=\begin{cases}\sin(C_1 s+\sin^{-1}(s/D)^p)\cdot(s/D)^{-p} & \\ & \text{for } s+1/C_1\cdot\sin^{-1}(s/D)^p\leqq\pi/2C_1\\ (s/D)^{-p} & \text{for } s+1/C_1\cdot\sin^{-1}(s/D)^p>\pi/2C_1\end{cases} \qquad (12)$$

Cup or Propeller Anemometer;

$$G_{app}=M\cdot G$$
$$=\tanh(C_2 G V_0 s+\tanh^{-1}(1/G))\cdot G$$
$$=\tanh(C_2 s^{1-p}D^p V_0+\tanh^{-1}(s/D)s)\cdot(s/D)^{-p} \qquad (13)$$

The value of p at the height of 10 m over flat level ground in high wind is about 0.097 as is shown in eq. (3). Therefore the variation of G_{app} with s under these conditions can be calculated, which is shown in Fig. 4. Fig. 4(a) shows that the maximum value of the apparent gust factor measured on the trace of Dines anemometer is about 1.60 at $s=4.2$ sec, and this value corresponds to the true gust factor at $s=4.7$. Therefore the peak gust speed measured by Dines anemometer can be regarded to be the peak value of about 5 sec gust. But we shall remember that the value of C_1 varies widely from station to station and equivalent gust duration also varies from about 2 to 10 sec within the range of C_1 mentioned before. Fig. 4(b), which shows the response of 3 cup anemometer, shows that the maximum apparent gust factors appear at about 0.7, 0.5 and 0.3 sec when mean wind speeds are 20, 30 and 40 m/sec respectively. And their values are equivalent to true gust factors of about 0.9, 0.6 and 0.4 sec gust respectively. While the apparent maximum gust factors measured by the propeller anemometer appear at about 0.5, 0.3 and 0.2 sec when mean wind speeds are 20, 30 and 40 m/sec respectively, and their values are equivalent to the true gust factors of 0.7, 0.4 and 0.3 sec gust respectively.

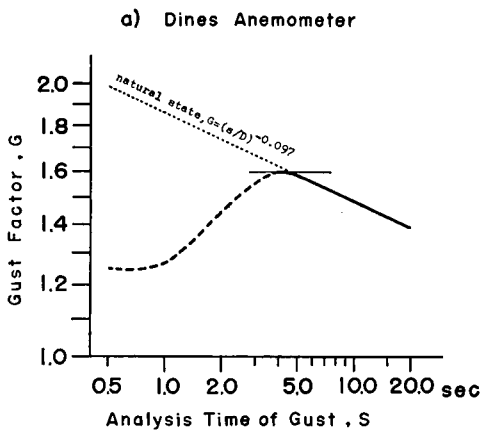

Fig. 4(a)

We can see from these results that gust factors observed by three kinds of

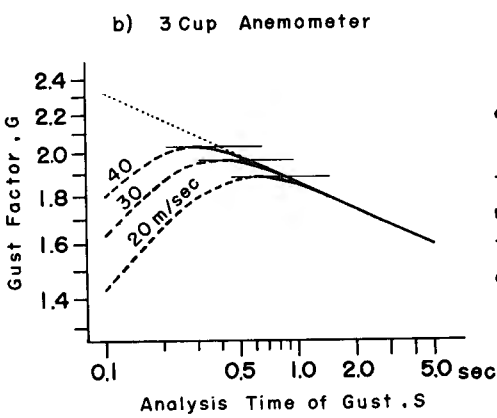

Fig. 4(b)

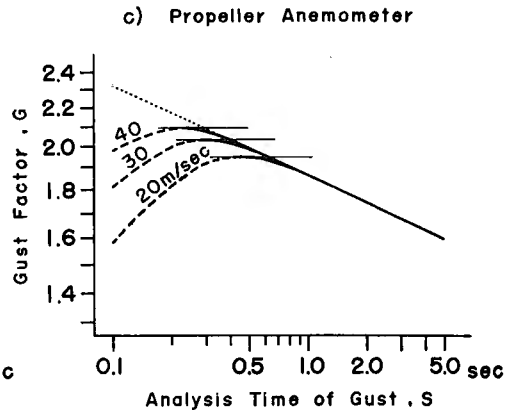

Fig. 4(c)

Fig. 4 Variation of apparent gust factor measured by the commonly used anemometer with analysis time of gust in natural wind.

anemometers are not the same, for example when mean wind speed is 20 m/sec they are 1.60, 1.88 and 1.94 for Dines, the cup and propeller anemometer respectively. Although gust factors measured by the cup and propeller anemometers seem a little larger than we expect in comparing with the observed data in past typhoon passages. This may be caused from the neglection of dynamical charactor of the generator or the recorder in this study, but it is not clear and the problem is left to the future studies. But the present results may be applicable to practical use.

6. Concluding Remarks

As the conclusion, some results, which may be used in practice, are summarized in the followings.

The values of the gust factor to 10 min mean wind speed over the flat level ground in high wind computed from eq. (2) and eq. (3) are as follows;

Height (m)	Gust Duration (sec)	0.5	1.0	2.0	5.0	10.0	20.0	p
10		1.99	1.86	1.74	1.59	1.49	1.39	0.097
20		1.68	1.60	1.52	1.42	1.35	1.28	0.073
50		1.43	1.38	1.33	1.27	1.23	1.19	0.050
100		1.32	1.28	1.25	1.21	1.17	1.14	0.039

The gust factor or peak gust speed measured by the commonly used anemometer should be corrected for dynamical response of the anemometer. If we take 2 sec gust for a temporal standard peak gust, the correction factor to the

gust factor or the peak gust speed measured on the trace of each anemometer at the height of 10 m over flat level ground is as follows,

Anemometer	Mean Wind Speed		
	20	30	40 m/sec
Dines	1.09	(regardless of wind speed)	
3 cup	0.92	0.89	0.86
Propeller	0.89	0.86	0.84

Acknowledgement

The author is indebted to Prof. K. Sassa, the Director of the Disaster Prevention Research Institute of Kyoto Univ. for his encouragement throughout the study. And the author also wishes to express his gratitude to Miss H. Kusakabe for her assistance in the course of preparing this manuscript.

References

1) Mitsuta, Y., 1962: Gust Factor and Analysis Time of Gust. J. Met. Soc. Japan, Ser. 2, Vol. 40, pp. 242–244.
2) Durst, C.S., 1960: Wind Speed Over Short Periods of Time. Met. Mag., Vol. 89, pp. 181–186.
3) Sherlock, R.H., 1952: Variation of Wind Velocity and Gusts with Height. Proc. Amer. Soc. Civ. Eng., Vol. 78, (Separate No. 126).
4) Deacon, E.L., 1955: Gust Variation with Height up to 150 m. Quart. J. Roy. Met. Soc., Vol. 81, pp. 562–573.
5) Ishizaki, H. & Y. Mitsuta, 1962: On the Scale of Peak Gust and Gust Factor. Disaster Prevention Research Institute (Kyoto Univ.) Annuals, No. 5, pp. 135–138.
6) Sanuki, M., 1952: A New Development of the Theoretical and Experimental Treatment of Lag of Dines Pressure Tube Exposed in Fluctuating Wind. Pap. in Met. & Geoph., Vol. 3, pp. 115–124.
7) Yajima, Y., 1960: On the Response of Anemometer in Fluctuating Wind. J. Meteo. Res., Vol. 12, pp. 55–57.
8) Schrenk, O., 1929: üeber die Trägheitsfhler des Schalenkreuz-Anemometers bei schwankender Windstärke. Zeit. Tech. Phys., Vol. 10, pp. 57–66.
9) Sanuki, M., 1952: Experiments on the Start and Stop of Windmill- and Cup- Anemometers with Particular Reference to Their Over-Estimation Factors. Pap. in Met. & Geoph., Vol. 3 pp. 41–53.