NOTES ON SEISMOGRAPH FEEDBACK SYSTEMS

BY

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1. Various methods to observe long-period earthquake waves have been greatly developed for the last several years.[1] Among these, some investigators[2,3] introduced a way applying a feedback to the seismographic observation. In this papar, theory and some notes will be given and compared with the experimental results, for the frequency characteristics of seismographic feedback systems with long-period galvanometers and a photoelectric amplifier[4].

2. We shall consider the system as illustrated in Fig. 1, which is composed of an electromagnetic seismograph with three transducers, two galvanometers of a long period and a highly sensitive photocell. The first circuit is a usual seismograph-galvanometer system. The deflection of the first galvanometer is projected as a rectangular light spot on the surface of the differential photocell, and then the induced current in it, which is proportional to the displacement of the light spot, is fed back into the second coil of the seismograph. Registration is made by the other galvanometer connected to the third coil of the seismograph. As a matter of course, the three coils, S_1, S_2 and S_3 indicated in Fig. 1 are mechanically coupled.

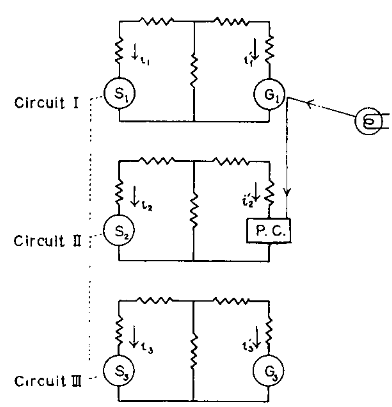

Fig. 1

Let the angular deflections of the seismograph pendulum and the two galvanometer coils be θ, φ_1 and φ_3, respectively, then we can find the equations of motion of the system in the following forms;

$$\left.\begin{aligned} &K\ddot{\theta}+D\dot{\theta}+U\theta=-MH\ddot{x}-G_1i_1-G_2i_2-G_3i_3 \\ &k_1\ddot{\varphi}_1+d_1\dot{\varphi}_1+u_1\varphi_1=g_1i'_1 \\ &k_3\ddot{\varphi}_3+d_3\dot{\varphi}_3+u_3\varphi_3=g_3i'_3 \end{aligned}\right\} \qquad (1)$$

The above notations are;

about the seismograph pendulum:

K ; the moment of inertia about the axis of rotation,

D ; the coefficient of damping in a open circuit,

U ; the moment of restitutive force per unit angle of rotation,

M ; the mass,

H ; the distance between the centre of gravity and the axis of rotation,

G_j ; the voltage sensitivity of the j-th transducer, given by the product of the circumference, the number of turns and the arm length, of its coil, and the strength of magnetic field,

about the j-th galvanometer:

k_j ; the moment of inertia,

d_j ; the coefficient of fluid resistance,

u_j ; the moment of restitutive force, and

g_j ; the electrodynamic constant, given by the product of the area, the number of turns and the strength of magnetic field.

i_j and i'_j are the currents going through the seismograph coil S_j and the galvanometer's coil G_j, and they are expressed by;

$$\left.\begin{aligned} i_1 &= \frac{G_1\dot{\theta}}{Z_1} - \mu'_1\frac{g_1\dot{\varphi}_1}{Z'_1}, \quad i'_1 = \mu_1\frac{G_1\dot{\theta}}{Z_1} - \frac{g_1\dot{\varphi}_1}{Z'_1} \\ i_2 &= \frac{G_2\dot{\theta}}{Z_2} - \mu'_2\frac{C\varphi_1}{Z'_2} \\ i_3 &= \frac{G_3\dot{\theta}}{Z_3} - \mu'_3\frac{g_3\dot{\varphi}_3}{Z'_3}, \quad i'_3 = \mu_3\frac{G_3\dot{\theta}}{Z_3} - \frac{g_3\dot{\varphi}_3}{Z'_3} \end{aligned}\right\} \qquad (2)$$

where

Z_j ; the total impedance of the j-th circuit seen from the seismograph side,

Z'_j ; the total impedance of the j-th circuit seen from the galvanometer's side, both including the coil resistance,

μ'_j ; the fraction of branch current from the galvanometer to the seismograph coil,

μ_j ; the fraction of branch current from the seismograph to the galvanometer's coil, and

C ; the constant relating to the sensitivity of photocell.

Combining Eqs. (1) and (2) and rearranging the equations, we obtain,

$$\left.\begin{aligned} \ddot{\theta}+2\varepsilon_0\dot{\theta}+\omega^2_0\theta &= -\alpha\ddot{x}+\gamma_1\dot{\varphi}_1+\gamma_3\dot{\varphi}_3+p\varphi_1 \\ \ddot{\varphi}_1+2\varepsilon_1\dot{\varphi}_1+\omega^2_1\varphi_1 &= \beta_1\dot{\theta} \\ \ddot{\varphi}_3+2\varepsilon_3\dot{\varphi}_3+\omega^2_3\varphi_3 &= \beta_3\dot{\theta} \end{aligned}\right\} \qquad (3)$$

where

$$2\varepsilon_0=\frac{1}{K}\left(D+\frac{G^2{}_1}{Z_1}+\frac{G^2{}_2}{Z_2}+\frac{G^2{}_3}{Z_3}\right),\quad \omega^2{}_0=\frac{U}{K},\quad \alpha=\frac{MH}{K}=\frac{1}{l}$$

$$2\varepsilon_1=\frac{1}{k_1}\left(d_1+\frac{g^2{}_1}{Z'_1}\right)\qquad ,\quad \omega^2{}_1=\frac{u_1}{k_1}$$

$$2\varepsilon_3=\frac{1}{k_3}\left(d_3+\frac{g^2{}_3}{Z'_3}\right)\qquad ,\quad \omega^2{}_3=\frac{u_3}{k_3}$$

$$\beta_1=\mu_1\frac{G_1g_1}{k_1Z_1},\quad \gamma_1=\mu'_1\frac{G_1g_1}{KZ'_1}\qquad P=\mu'_2\frac{G_2C}{KZ'_2}$$

$$\beta_3=\mu_3\frac{G_3g_3}{k_3Z_3},\quad \gamma_3=\mu'_3\frac{G_3g_3}{KZ'_3}$$

ε_0, ε_1 and ε_3 are the damping coefficients and ω_0, ω_1 and ω_3 are the natural angular frequencies, of the pendulum and the two galvanometers, respectively. The theoretical galvanometric motion on a recorder will be obtained by solving Eq. (3). We shall suppose the ground motion to be a simple harmonic one with amplitude x_m, angular frequency ω; $x=x_me^{i\omega t}$, and write the corresponding motions of the pendulum and the galvanometers by $\theta=\theta_me^{i\omega t}e^{i\delta_0}$, $\varphi_1=\varphi_{1m}e^{i\omega t}e^{i\delta_1}$ and $\varphi_3=\varphi_{3m}e^{i\omega t}e^{i\delta_3}$, with the phase angle δ_0, δ_1 and δ_3. φ_3/x can be deduced from Eq. (3) as follows;

$$\begin{aligned}
&\varphi_3/x_3=(\varphi_{3m}/x_m)e^{i\delta_3}=\Delta_3/\Delta\\
&\quad \Delta_3=-i\alpha\beta_3\omega^3(-\omega^2+2i\varepsilon_1\omega+\omega^2{}_1)\\
&\quad \Delta=(-\omega^2+2i\varepsilon_0\omega+\omega^2{}_0)(-\omega^2+2i\varepsilon_1\omega+\omega^2{}_1)(-\omega^2+2i\varepsilon_3\omega+\omega^2{}_3)\\
&\qquad -(i\beta_3\omega)(i\gamma_3\omega)(-\omega^2+2i\varepsilon_1\omega+\omega^2{}_1)-(i\beta_1\omega)(p+i\gamma_1\omega)(-\omega^2+2i\varepsilon_3\omega+\omega^2{}_3)
\end{aligned}$$

For the sake of simplicity, we shall here consider a special case when $\varepsilon_3=\varepsilon_1$ and $\omega_3=\omega_1$, that is, the case when the two galvanometers have the identical characteristics. If we denote the distance between the recording galvanometer and a recorder by A_3, the magnification of this system, V, can be expressed in the following;

$$\left.\begin{aligned}
&V=2A_3\varphi_{3m}/x_m\equiv Q\cdot f(\omega)\\
&Q=2A_3\alpha\beta_3=\frac{4\pi^2G_3/l}{S_3T_3{}^2Z_3/\mu_3}\\
&f(\omega)=\frac{\omega^3}{\sqrt{[A(\omega)]^2+[B(\omega)]^2}}\\
&\tan\delta_3=A(\omega)/B(\omega)\\
&A(\omega)=(\omega_0{}^2-\omega^2)(\omega_1{}^2-\omega^2)-4\varepsilon_0\varepsilon_1(1-\sigma_1{}^2-\sigma^2{}_3)\omega^2\\
&B(\omega)=\omega[2\varepsilon_1(\omega_0{}^2-\omega^2)+2\varepsilon_0(\omega_1{}^2-\omega^2)-\beta_1p]
\end{aligned}\right\}\qquad (4)$$

where $\sigma_1{}^2$ and $\sigma_3{}^2$ are the coupling factors in the circuits I and III and written by

$$\sigma_1{}^2=\frac{\beta_1\gamma_1}{4\varepsilon_0\varepsilon_1}\leqq\mu_1\mu'_1\frac{h_{01}}{h_0},\quad \sigma_3{}^2=\frac{\beta_3\gamma_3}{4\varepsilon_0\varepsilon_1}\leqq\mu_3\mu'_3\frac{h_{03}}{h_0}$$

h_{01}, h_{02} and h_{03} are the factors of electromagnetic damping due to the three circuits, respectively, S_3 the current sensitivity of the recording galvanometer and T_3 the natural period of it. The frequency response of the present system depends on the natural periods or angular frequencies and the damping constants, of the seismograph and galvanometers, the coupling factor and the feedback constant. If we know these quantities and the value of Q by measuring the related factors, the magnification can be determined from Eq. (4).

In order to determine the feedback constant, we shall adopt the following way. When the light spot of the first galvanometer on the surface of photocell is displaced statically by y_1, opening the circuits I and III, the seismograph pendulum is supposed to be deflected by s owing to the feedback current into the second coil of it. In this experiment, the following relation holds in Eq. (1), neglecting the sign of deflection. That is, $U\theta = G_2 i_2$. And we have, $s = L\theta$, $y_1 = 2A_1\varphi_1$ and $T_0 = 2\pi\sqrt{K/U}$. So we obtain,

$$p = \mu'_2 \frac{G_2 C}{KZ'_2} = \left(\frac{2\pi}{T_0}\right)^2 \frac{S}{L} \cdot \frac{2A_1}{y_1} \quad \text{and} \quad \beta_1 = \mu_1 \frac{G_1 g_1}{k_1 Z_1} = \frac{2\pi^2 G_1}{A_1 S_1 T^2_1 Z_1/\mu_1} \tag{5}$$

The feedback constant $\beta_1 p$ can therefore be estimated by the above formulae.

We can control the feedback current by regulating the T-type attenuator in the circuit I or III, without any change in the dampings of the three instruments. For this case, the frequency response depends only upon $\beta_1 p$ in Eq. (4). As the current increases, the system gradually becomes unstable and comes into oscillation at a finite period. The condition for oscillation can be inspected from Eq. (4). That is, $A(\omega_c) = B(\omega_c) = 0$ at $\omega = \omega_c$. The angular frequency for this state, ω_c, is obtained from the first equation,

$$2\omega_c^2 = \omega_0^2 + \omega_1^2 + 4\varepsilon_0\varepsilon_1(1-\sigma_1^2-\sigma_3^2) \pm \sqrt{[\omega_0^2+\omega_1^2+4\varepsilon_0\varepsilon_1(1-\sigma_1^2-\sigma_3^2)]^2 - 4\omega_0^2\omega_1^2}$$

The critical feedback constant is obtainable, combining this result and the second equation.

3. As an examle, numerical results are given for a seismograph feedback system of the above described type designed by K. Aki.[4)]

$T_0 = 12$ sec, $K = 3.74 \times 10^6$ c.g.s.
$G_1 = G_2 = 1.24 \times 10^9$, $G_3 = 4.00 \times 10^9$ c.g.s. e.m.u.
$Z_1 = 500\,\Omega$, $Z_3' = 5{,}980\,\Omega$
$h_{01} = 0.787$, $h_{03} = 0.683$, $h_{02} \doteqdot 0.000$, $h_0 = 1.470$
$T_1 = T_3 = 90$ sec,
$S_1 = S_3 = 5.0 \times 10^{-11}$ amp/mm
$Z_1' = 2{,}280\,\Omega$, $Z_3' = 2{,}160\,\Omega$,

$h_1 = h_3 \doteqdot 1.00$

$\mu'_1 = 0.040,\ \mu_1 = 0.00138,$

$\mu'_3 = 0.082,\ \mu_3 = 0.227,$

$\sigma^2_1 = 3.00 \times 10^{-5},$

$\sigma^2_3 = 8.65 \times 10^{-3}.$

$Q = 3.59 \times 10^2,$

$\omega_c = 5.236 \times 10^{-2}$

$(T_c = 120 \text{ sec})$

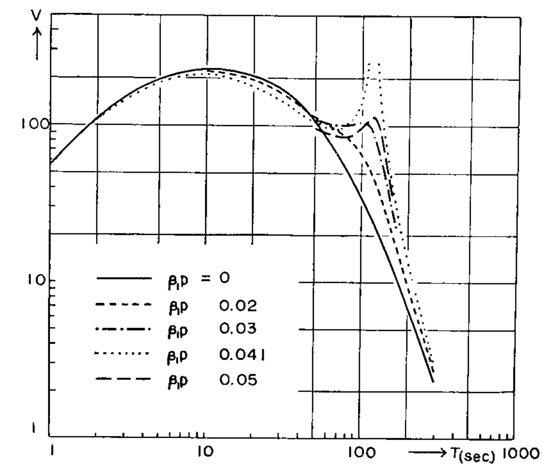

Fig. 2

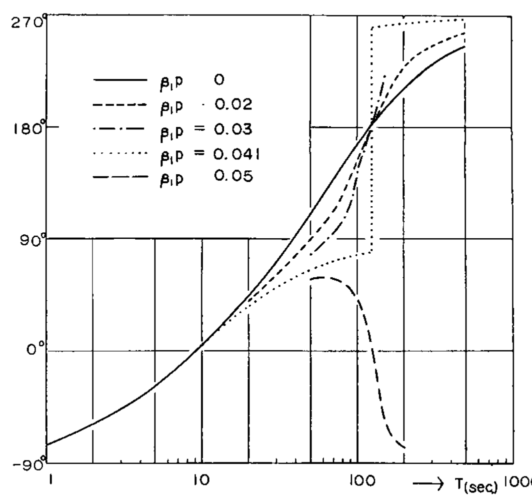

Fig. 3

Frequency response curves are shown in Figs. 2 and 3. The critical period for oscillation will be increased, as the damping of the system becomes larger, as understood from the before-mentioned condition. The response characteristics with a higher magnification at longer periods will therefore be attained, if the factors in $f(\omega)$ are selected for the purpose.

4. Alternative feedback systems are briefly described in the following.

(1) We make the first galvanometer serve also a recording purpose, omitting the circuit III. In this case, the frequency response has quite the same form as in the before-stated system, replacing the suffix 3 for 1 in Eq. (4), because the quantities concerning the galvanometer in the circuit III vanish. That is,

$$Q = \frac{4\pi^2 G_1/l}{S_1 T^2_1 Z_1/\mu_1}, \qquad \sigma^2_3 = 0$$

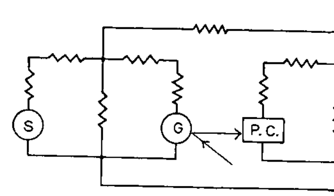

Fig. 4

(2) We omit the circuit III and the second coil of the seismograph. The current induced in the photocell is fed back into the original circuit I, as indicated in Fig. 4. Registration

is made by the first galvanometer. Also in this system, the response characteristics are quite analogous to the former two cases, although only a natural period of the galvanometer suffers a change on account of the feedback.

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