MECHANISM OF LOCAL EARTHQUAKES IN KWANTO REGION, JAPAN, DERIVED FROM THE AMPLITUDE RELATIONS OF *P* AND *S* WAVES

BY

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1. Introduction

The study on earthquake mechanism has rapidly been developed in both theoretical and observational sides by many seismologists, since T. Shida (1917) found out a systematic pattern of quadrant type for dilatation and compression of first motions of longitudinal waves in a Japanese earthquake. Nowadays, there are two prevailing hypotheses about the earthquake mechanism, although a number of hypotheses have been given from various standpoints.

The one is analytically represented by two equal and opposite forces or a single couple with moment, indicating motion along a fault. The other is the force system of double couples with moment, being perpendicular to each other, which is equivalent to two sets of compressional and tensile stresses of equal magnitude working at right angle at a focus. From different standpoint some investigators presented focal models analogous to the double couples case. These two types of force systems each give an identical quadrant distribution of *P* waves, but the pattern of the first motion of *S* waves differs for the two cases. Therefore, we cannot judge which of them better represents the real focus without reliable observations of both waves. On the other hand, another mechanism called cone-type was proposed by some Japanese seismologists from different aspects. In this case, the distribution of condensation and rarefaction of *P* waves is not separated by two orthogonal planes as in the former two cases, but by a set of conical surfaces with its vertex at a focus.

Most of the observational studies in this line have been associated with great earthquakes and not with earthquakes of small magnitude. This may be by reason that it is difficult to apply the usual way to the latter case, owing to a lack of sufficient data over a wide area. It is, however, an interesting problem to investigate whether the mechanism of these earthquakes is identically solved or it depends upon the earthquake magnitude, its focal depth and some other factors. To approach the problem we shall study the focal mechanism of minor earthquakes taking place in southern Kwanto region, one of seismically active areas in Japan.

2. Theory

In the present study, we shall introduce a way to deduce a focal mechanism from the amplitude relations of body waves observed at a few stations, instead of the usual method using distribution of first motion directions of P and S waves. Assuming the type of mechanism or the applied force system to be the three models mentioned above, namely, a single couple, double couples and a cone-type, respectively, the expected field of displacement amplitudes of P, SV and SH waves will be calculated in convenient forms for practical use of observed data. The theoretical amplitude rations in comparison with the observed ones on which the effects of crustal structure are taken into account, lead to determine dynamic parameters of the focus. The respective mean errors in a statistical average of the estimated values will serve to identify what type of the mechanism is most reasonably fitted to the real source.

(1) Single couple[1), 2), 3)]

We shall first consider a single couple of forces $\pm K(t)$ acting on two points, each distance being infinitesimally small. Based on Nakano's theory, the displacement caused by the forces, at a large distance from the origin in an infinite elastic medium, can be expressed as follows in a spherical coordinate;

$$u_r = \frac{A_p}{r}(\lambda\sin\Theta\cos\Phi+\mu\sin\Theta\sin\Phi+\nu\cos\Theta)(l\sin\Theta\cos\Phi+m\sin\Theta\sin\Phi+n\cos\Theta)$$

$$u_\Theta = \frac{A_s}{r}(\lambda\sin\Theta\cos\Phi+\mu\sin\Theta\sin\Phi+\nu\cos\Theta)(l\cos\Theta\cos\Phi+m\cos\Theta\sin\Phi-n\sin\Theta)$$

$$u_\Phi = -\frac{A_s}{r}(\lambda\sin\Theta\cos\Phi+\mu\sin\Theta\sin\Phi+\nu\cos\Theta)(l\sin\Phi-m\cos\Phi) \qquad (1)$$

where $A_p=\dfrac{K'(t-r/a)}{4\pi\rho a^3}$, $A_s=\dfrac{K'(t-r/b)}{4\pi\rho b^3}$

For a coordinate system in which the x-axis is directed north, the y- east and the z- upward, Θ is the emergent angle of seismic ray at the focus, and Φ is the azimuth of a station relating to the epicentre, which is measured clockwise from northward. The coefficients l, m, n and λ, μ, ν are the direction cosines of the positive force and the line connecting the working two points, respectively. Let the angle of inclination of motion direction to horizontal surface be ψ, and the azimuth of horizontal trace of the direction be β, then we have $l=\cos\psi\cos\beta$, $m=\cos\psi\sin\beta$, and $n=\sin\psi$. We note that u_r takes a positive sense for motion away from epicentre, u_Θ increases for motion up and toward epicentre and u_Φ for clockwise motion looking from epicentre to station. The components u_r, u_Θ and u_Φ

correspond to the amplitudes of P, SV and SH waves in a homogeneous earth, respecsively.

(2) Double couples[4), 5), 6)]

We shall next consider double couples of forces. In the same coordinate system as adopted above, the displacement components at a large distance from the origin are derived in the following form, by superposition of the single couple expressed in Eq. (1);

$$u_r = 2\frac{A_p}{r}(\lambda\sin\Theta\cos\Phi+\mu\sin\Theta\sin\Phi+\nu\cos\Theta)(l\sin\Theta\cos\Phi+m\sin\Theta\sin\Phi+n\cos\Theta)$$

$$u_\Theta = \frac{A_s}{r}[(\lambda\sin\Theta\cos\Phi+\mu\sin\Theta\sin\Phi+\nu\cos\Theta)(l\cos\Theta\cos\Phi+m\cos\Theta\sin\Phi-n\sin\Theta)$$
$$+(l\sin\Theta\cos\Phi+m\sin\Theta\sin\Phi+n\cos\Theta)(\lambda\cos\Theta\cos\Phi+\mu\cos\Theta\sin\Phi-\nu\sin\Theta)]$$

$$u_\Phi = -\frac{A_s}{r}[(\lambda\sin\Theta\cos\Phi+\mu\sin\Theta\sin\Phi+\nu\cos\Theta)(l\sin\Phi-m\cos\Phi)$$
$$+(l\sin\Theta\cos\Phi+m\sin\Theta\sin\Phi+n\cos\Theta)(\lambda\sin\Phi-\mu\cos\Phi)] \qquad (2)$$

If we write the angle of inclination of motion directions by ψ_1 and ψ_2, and the azimuth of the horizontal traces of them by β_1 and β_2, we find $l=\cos\psi_1\cos\beta_1$, $m=\cos\psi_1\sin\beta_1$, $n=\sin\psi_1$, $\lambda=\cos\psi_2\cos\beta_2$, $\mu=\cos\psi_2\sin\beta_2$ and $\nu=\sin\psi_2$. Other factors are quite analogous to the case (1).

(3) Cone-type[7), 8), 9)]

In most of the focal mechanism of a cone-type, the radial and tangential displacements at a large distance compared with the wavelength can exactly or approximately be expressed by the following equations involving spherical harmonics;

$$u_r = \frac{A}{r}[a_0P_0(\cos\theta)+a_2P_2(\cos\theta)]\cdot\exp[i\omega(t\text{-}r/a)]$$
$$u_\Theta = \frac{B}{r}\cdot\frac{d}{d\theta}[a_0P_0(\cos\theta)+a_2P_2(\cos\theta)]\cos\gamma\cdot\exp[i\omega(t\text{-}r/b)] \qquad (3)$$
$$u_\Phi = \frac{B}{r}\cdot\frac{d}{d\theta}[a_0P_0(\cos\theta)+a_2P_2(\cos\theta)]\sin\gamma\cdot\exp[i\omega(t\text{-}r/b)]$$

taking the before-mentioned coordinate with the z-axis as the polar axis. If we denote the angle of inclination of the polar axis by ψ, and the azimuth of it measured clockwise from northward by β, we obtain the relation

$$\cos\theta=\sin\Theta\cos\psi\cos(\Phi\text{-}\beta)+\cos\Theta\sin\psi$$

and

$$\frac{\sin(\Phi\text{-}\beta)}{\sin\theta}=\frac{\sin\gamma}{\sin(\pi/2-\psi)}$$

from spherical trigonometry.

In all cases Θ and Φ will be determined for every station when the hypocentre was located. The observed amplitude ratios concerning the three types of waves can be obtained from seismogram with some corrections.

In the case of a single couple, the two unknown factors, ψ and β, can theoretically be solved using Eq. (1) from the amplitude ratios observed at a single station. On the contrary, there are four unknowns ψ_1, ψ_2, β_1 and β_2 in the double couple case, and the same number of unknowns, ψ, β, α (angle of vertex of a nodal cone) and A/B in the cone-type. They are reduced to a quadratic equation, the coefficients of which can be determined from data at a single station. The solution may therefore be obtained by observation at more than two stations, by a graphical method or probably by the method of least squares.

3. Effects of Crustal Structure

Seismic waves emitted from a focus suffer considerable change in their amplitudes and direction of the ray paths by refraction and reflection, during their traveling through discontinuity surfaces within the earth's crust to an observation station on the ground surface. It is therefore necessary to eliminate these effects in determination of a focal mechanism, making use of the amplitudes recorded at the stations.

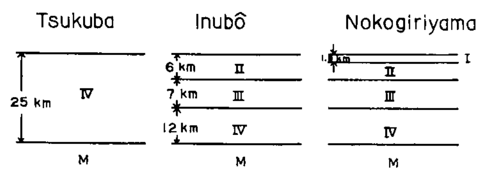
Under an assumption of a horizontally layered structure, the relation between horizontal and vertical components of the surface amplitude, which will be observed at the ground surface, and the original amplitude in a homogeneous medium can be written in the following form for the P, SV and SH waves, respectively,

$$\bar{A}_{P(H)}/A_P = f_1F_1F_2\cdots F_n = F_H(\Theta_P), \quad \bar{A}_{P(V)}/A_P = f_2F_1F_2\cdots F_n = F_V(\Theta_P)$$
$$\bar{A}_{SV(H)}/A_{SV} = g_1G_1G_2\cdots G_n = G_H(\Theta_S), \quad \bar{A}_{SV(V)}/A_{SV} = g_2G_1G_2\cdots G_n = G_V(\Theta_S)$$
$$\bar{A}_{SH}/A_{SH} = 2H_1H_2\cdots H_n = \Theta(\Theta_S)$$

neglecting the decrease in their amplitudes due to travel distance, if both P and S waves travel along the same path. F_j, G_j and H_j are the transmission rates of the respective waves in the case of refraction at each boundary, and f_j, g_j are the ratios of recorded amplitude to that in the uppermost layer.[10] All of these factors are related to the emergent angle at the focus, Θ_P or Θ_S.

In the present study, the data obtained at three stations of the Earthquake Research Institute in southern Kwanto region; Tsukuba, Inubo and Nokogiriyama, were used for our purpose. Crustal structure near the regions are presumed as illustrated in Fig. 1, from the results of explosion observations[11,12,13] and seismic prospectings.[14] Densities were determined from an empirical density *versus* seismic velocity curve compiled by Nafe and Drake.

Fig. 2 shows the ratios of the surface amplitudes at each station to the original one of the three kinds of waves, computed for the presented structure.



	V_p	V_s	ρ
I	2.0 km/sec	1.1 km/sec	2.00
II	4.9	2.9	2.55
III	5.6	3.3	2.70
IV	6.1	3.6	2.80
M	7.7	4.5	3.25

Fig. 1

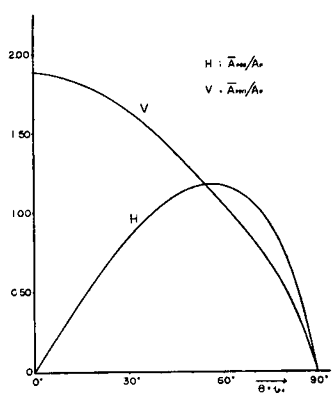


Fig. 2(a)

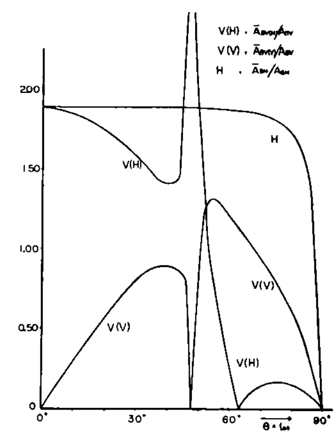


Fig. 2(b)

Tsukuba

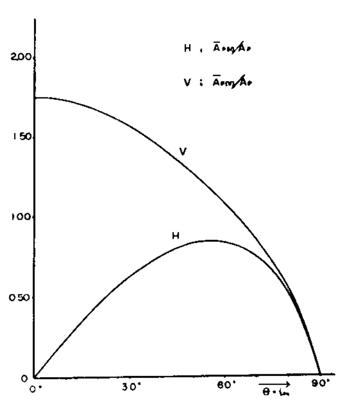


Fig. 2(c)

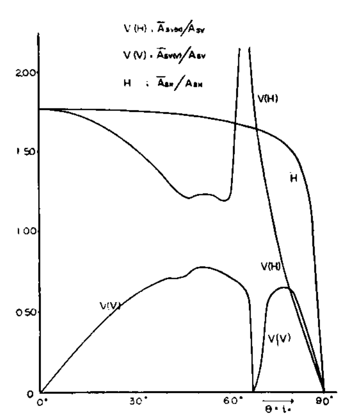


Fig. 2(d)

Inubo

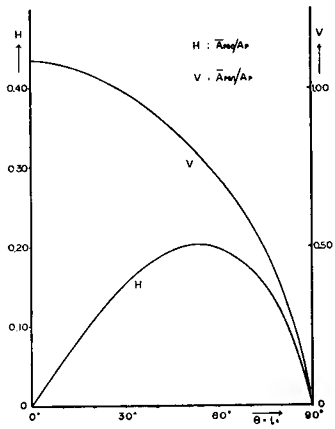


Fig. 2(e)

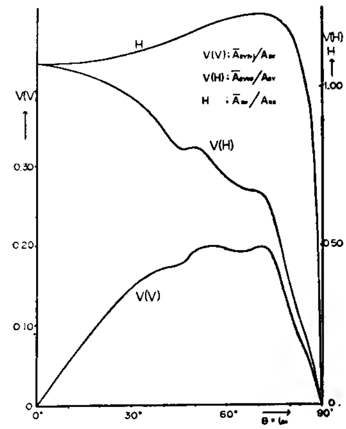


Fig. 2(f)

Nokogiriyama

4. Observed Data

In this study, seismograms were analysed which were recorded with electromagnetic seismographs of HES type at the above-mentioned stations. The natural periods of both pendulum and galvanometer are 1 sec and both are critically damped.

Among a great many earthquakes observed during the period from July, 1961 to March, 1962, 10 local earthquakes were selected for the present purpose, for which all components of both P and S wave first motions were clearly recorded at the said three stations. Their foci could be located graphically, as shown in Fig. 3, using a diagram of isochronic lines of $P \sim S$ times, which was constructed by I. Kayano based on the stated structure. Table 1 summarizes the elements of foci referring to each station. The emergent angle was slightly corrected, taking the increasing rate of the seismic velocity in the earth's upper mantle.

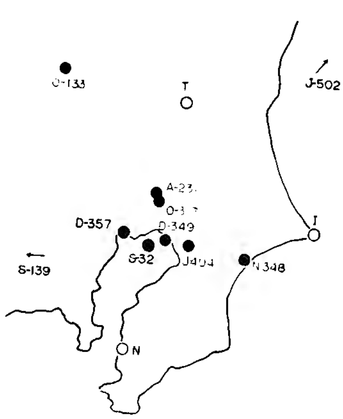


Fig. 3

5. Results and Discussion

The amplitude ratios of the three kinds of waves in the state of a homogeneous earth are computed from the observed displacements

Table 1

Shock No.	Δ (km)			Θ			Φ			h (km)
	T	I	N	T	I	N	T	I	N	
J-494	65.0	62.8	60.3	47.5°	47.5°	45.5°	355.0°	81.5°	210.5°	68
J-502	225.0	227.5	327.5	83.5	84.0	86.0	229.0	207.0	217.0	47
A-231	41.5	81.8	78.0	36.5	58.0	56.0	17.0	101.5	189.0	63
S- 32	67.0	82.5	52.0	90.0	90.0	90.0	12.5	82.0	190.5	25
S-139	184.5	238.0	150.5	73.0	78.0	69.0	65.5	85.0	105.5	73
O-133	63.8	148.5	139.5	41.5	68.0	65.5	106.5	120.0	164.5	79
O-317	45.5	79.3	74.0	39.0	58.0	53.5	13.0	98.0	190.5	63
N-348	77.8	37.2	74.5	90.0	90.0	90.0	333.5	61.0	232.0	25
D-349	63.0	75.0	56.8	51.0	57.5	47.5	6.0	84.5	198.5	60
D-358	70.5	95.8	51.5	54.0	64.0	45.0	22.5	84.0	176.5	60

by means of the graphs in Figs. 2. According to the foregoing theory, solution for a single couple can easily be obtained from data at a single station, and those for double couples and a cone-type can be deduced graphically by use of the data at two stations. In every assumption, data from the said three stations give three sets of solutions. The mean errors in the three assumptions may serve to find out what type of the force systems is best fitted to the observed results. The final results estimated for the 10 earthquakes are tabulated in Table 2.

Table 2

Shock No.		Double Couples		Cone-type		
		Strike	Dip	Strike	Dip	Vertical Angle
J-494	I	N 61.9°± 2.4 E	10.6°± 2.3° NE	N 28.4°±0.7° E	8.0°±1.3° SW	
	II	N 35.9± 4.3 W	35.6±21.2 NW			
J-502	I	N 46.2± 8.9 W	44.9± 2.5 NW			
	II	N 46.3± 8.9 W	45.1± 1.7 SE			
A-231	I	N 21.4± 8.1 W	21.0± 5.5 NW			
	II	N 56.1±22.8 E	29.6±20.7 SW			
S- 32	I	N 25.0± 7.9 W	32.3±11.1 SE			
	II	N 35.2±17.5 E	38.2±14.0 NE			
S-139	I	N 2.2± 3.7 W	52.6± 2.8 SE	N 59.1±1.0 E	32.1±1.5 SW	68.6°± 1.5°
	II	N 26.4±20.8 W	33.8± 5.3 NW			
O-133	I	N 39.2± 6.6 E	38.9± 3.8 NE	N 50.2±8.0 W	57.1±6.7 NW	84.2±21.4
	II	N 55.3± 7.3 E	50.0± 3.0 SW			
O-317	I	N 60.3± 0.5 E	5.9± 0.5 SW			
	II	N 35.5± 6.8 W	44.3±14.9 SE			
N-348	I	N 19.3± 2.5 E	12.0± 8.6 NE			
	II	N 69.4± 2.2 W	5.8± 5.0 NW	N 48.5±2.9 E	25.7±5.0 SW	
D-349	I	N 43.4± 5.0 E	6.6± 5.4 SW	N 44.5±6.0 E	50.1±4.6 NE	
	II	N 43.7± 0.9 W	23.5±12.5 NW			
D-358	I	N 32.7± 6.1 W	13.5± 5.2 NW	N 32.4±8.6 E	12.1±7.8 NE	78.8±22.2
	II	N 13.8± 7.2 E	71.2± 5.6 SW			

Satisfactory solutions could not be obtained under the assumption of a single couple, for all of the earthquakes treated here. The double couples, on the contrary, gave fairy good agreement with the observed data, allowing a certain extent of errors. To the only one earthquake (No. S-139) the mechanism of a cone-type, rather than of double couples, may be considered to slightly better fit. Although we cannot come to a definite conclusion on account of insufficient data, we may be allowed to consider that the double couples are most promissing than the other two systems, for minor earthquakes in Kwanto region, as far as the present data are concerned. This contrasts sharply with the mechanism of local earthquakes taking place in the upper crust in Wakayama area.

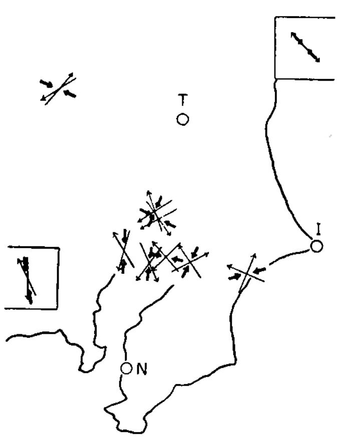


Fig. 4

Fig. 4 indicates the distribution of force directions and of horizontal component of maximum pressures, assuming the mechanism to be of double couples. No systematic pattern is recognized in this figure. This trend shows a little difference with the results for greater earthquakes in the same area, which were derived from distribution of P wave motions by an usual way.[15]

6. Concluding Remarks

A method to determine the focal mechanism of small earthquakes was studied by the use of amplitude ratios of P, SV and SH waves observed at a few stations, taking into consideration the effects of crustal structure. Applying the proposed method to local earthquakes in Kwanto region, Japan, the mechanism of them was estimated to be of double couples or type II mechanism rather than of the other two types.

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