# CHECKING COMPLEX MULTIPLE REFLECTIONS 

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## Introduction

Many writers have described the appearance of multiple reflections on seismic records. However, it has been thought that these phenomena are not common, and furthermore, that they must have good and unsteep primary reflection interfaces.

On the other hand, the present writers checked the results of their own reflection surveys, and concluded as follows. Multiple reflections (estimated) appear unexpectedly many times in areas of comparatively steep dip, so that areas of multiple reflections are not limited. These multiple reflections include not only simple, but also complex ones.

For checking multiple reflections the arrival times of these phases (i.e. normal times), and curvatures of their travel time curves (i.e. normal moveouts) are used in general. The relation between arrival times of phase suggested multiple and suggested primary, and relation between average reflection velocities estimated from curvatures of their line ups must be regular.

In this procedure comparatively large error amounting to several tens of millisecond will come into reading at normal times, because it is not possible quite to catch the exact first arrival of these phases. And in spite of the fact that the relative reading error of each phase time amounts to several milliseconds, the total observation value of normal moveout will be usually several tens of millisecond at most. Therefore, the percentage error of curvature will necessarily grow large. These two ambiguities are weak points of this procedure.

In the case mentioned above the dip of reflection interfaces is usually neglected. Then, when dip gets steep, not only will computation be troublesome, but its accuracy will not be reliable.

Meanwhile, use can be made of inclination of line ups of these phases (stepout times) to compute dip of primary reflection interface and apparent dip of multiple reflections between this interface and other auxiliary interface or interfaces (including topographic surface); then one can discuss the relation of these dips. After this it can be pointed out more easily that these phases are not multiple, or may
be multiple.
In this paper the writers presented methods for computation of complex multiple reflections, and their applications.

## Fundamental Relation Between Primaries and Multiples

The symbols used to distinguish the various diagrams of multiple reflections are composed of sequences of the letter designations of the interfaces, using capital letters for upward reflections and small letters for downward reflectons (after Bennet, 1962). But there is no letter for downward reflection from the topographic surface (or the base of low velocity layer near surface). For example, primary reflections are represented as $A, B, C$, simple multiple reflections as $A A, B B$ (1st order), $\mathrm{AAA}, \mathrm{BBB}$ (2nd order), AAAA (3rd order), complex multiple reflections as $\mathrm{AB}, \mathrm{AC}$ (1st order), $\mathrm{ABC}, \mathrm{BaB}, \mathrm{BaC}, \mathrm{AAC}$ (2nd order), $\mathrm{AAAB}, \mathrm{ABCD}, \mathrm{ABaC}$ (3rd order), ....

These multiples may be divided in another manner, in which the symbols including small letters are called conveniently type $B$, and others are called type $A$. Accordingly type A includes simple multiples.

Assuming that each interface has reflection coefficient of the same order, the topographic surface has reflection coefficient of nearly 1 , and effects of transmission coefficients, absorption, incidence angle, and so on are negligible, it can be roughly estimated that higher order multiple has smaller amplitude. But the prominence of the reflection signal on the usual AGC record depends not only on the absolute signal amplitude but also on the background noise level and other signal level during the same time interval. Therefore the writers concluded that the appearance possibility of a multiple is not known exactly, but the higher its order is, the lower the possibility is in general, and the more the number of reflections is as listed in Table 1 up to 3 rd order.

In the case of sloping interfaces every path length depends on the order of encountered reflection interfaces from shotpoint to receiving point. Meanwhile, assuming that every interface is horizontal, each path lengh will be independent of the order of encountered interfaces, and then the number of reflections will be reduced as shown in this table.

Rarely the writers got higher order multiple up to 3rd order in most favourable condition like sea bottom as shown in Fig. 12. However, no good multiple of higher order than this has been recognized on field records, so there may be no need to consider a 4th order and higher.

Table 1 Number of Reflections


Remark: Type A includes simple multiple reflections.

## Computation in the Case of Dip Indication

Assuming constant velocity as usual multiple computation one can construct geometrical relation, and get analytical solution of the depths and dips for the 1 st order multiples after Bortfeld (1956). This relation for the AB type (general case) is shown in Fig. 1, in which, for example, I $(O, A)$ indicates the image point of $O$ with respect to the interface $A$.


Fig. 1

Using a method similar to Bortfeld's the following equation (1) will be introduced for Fig. 1 :

$$
\begin{align*}
& \text { (total path length) } \\
= & \left\{x+2\left(d_{2}-x \cdot \sin \theta_{2}\right) \cdot \sin \theta_{2}-2 d_{1} \sin \theta_{1}\right\}^{2} \\
+ & \left\{2\left(d_{2}-x \cdot \sin \theta_{2}\right) \cdot \cos \theta_{2}+2 d_{1} \cos \theta_{1}\right\}^{2} \tag{1}
\end{align*}
$$

and by geometrical construction in Fig. 1,

$$
\begin{array}{r}
\tan \varphi=\left\{x+2\left(d_{2}-x \cdot \sin \theta_{2}\right) \cdot \sin \theta_{2}\right. \\
\left.-2 d_{1} \sin \theta_{1}\right\} /\left\{2\left(d_{2}-x \cdot \sin \theta_{2}\right) \cdot \cos \theta_{2}\right. \\
\left.+2 d_{1} \cos \theta_{1}\right\} \tag{2}
\end{array}
$$

The emergence angle at the receiving point $G$ is

$$
\begin{equation*}
E_{G}=2 \theta_{2}-\varphi \tag{3}
\end{equation*}
$$

similarly, the incidence angle at the shotpoint $O$ is

$$
\begin{equation*}
E_{0}=2 \theta_{1}+\varphi \tag{4}
\end{equation*}
$$

accordingly,

$$
\begin{equation*}
E_{G}+E_{0}=2\left(\theta_{1}+\theta_{2}\right) \tag{5}
\end{equation*}
$$

When the receiving point coincides with the shotpoint $(x=0)$,

$$
\begin{align*}
& \text { (normal path length to apparent dip on cross section) })^{2} \\
= & \left(d_{2} \sin \theta_{2}-d_{1} \sin \theta_{1}\right)^{2}+\left(d_{2} \cos \theta_{2}+d_{1} \cos \theta_{1}\right)^{2} \tag{6}
\end{align*}
$$

and

$$
\begin{equation*}
\tan \varphi=\left(d_{2} \sin \theta_{2}-d_{1} \sin \theta_{1}\right) /\left(d_{2} \cos \theta_{2}+d_{1} \cos \theta_{1}\right) \tag{7}
\end{equation*}
$$

Usually normal time and normal moveout time are used to obtain the position and dip of reflection interface, therefore the 1st order multiple appears on cross section as shown in Fig. 2.

Namely, substituting eq. (7) into eqs. (3) and (4), one gets two kinds of emergence angle. Then the length of (6) is taken in the direction of these emergences, and one can get apparent position of multiple; these emergence angles are equal to apparent dip angles. In the case of simple multiple (AA type) two dip angles coincide, and total amplitude becomes twice as large.

In the case of similar dipping primaries two apparent dips of multiples are nearly the same, while in the case of much different dip primaries they are very different as shown in Fig. 3.


Fig. 2


Fig. 3


For 2nd order multiples geometrical relation can be constructed as shown in Fig. 4 (type A), and Fig. 5 (type B).

In the case of AAC multiple, in which the interface A coincides with interface B of Fig. 4, it is possible to reduce this procedure to a 1st order one using the simple multiple AA as primary. In this case eqs. (6) and (7) will be changed to

$$
\begin{align*}
& \text { (normal path length to apparent dip on cross section) } \\
& =\left(d_{2} \sin \theta_{2}-2 d_{1} \cos \theta_{1} \sin 2 \theta_{1}\right)^{2}+\left(d_{2} \cos \theta_{2}+2 d_{1} \cos \theta_{1} \cos 2 \theta_{1}\right)^{2} \tag{8}
\end{align*}
$$

and

$$
\begin{equation*}
\tan \varphi=\left(d_{2} \sin \theta_{2}-2 d_{1} \cos \theta_{1} \sin 2 \theta_{1}\right) /\left(d_{2} \cos \theta_{2}+2 d_{1} \cos \theta_{1} \cos 2 \theta_{1}\right) \tag{9}
\end{equation*}
$$

In the case of BaB multiple, in which the interface B coincides with the interface C of Fig. 5, the following equations will be obtained:

$$
\begin{aligned}
& \text { (normal path length to apparent dip } \\
& \text { on cross section) } \\
& =\left\{2 d_{2}-d_{1} / \cos \left(\theta_{2}-\theta_{1}\right)\right\} \cdot \cos \left(\theta_{2}-\theta_{1}\right)(10)
\end{aligned}
$$

and

$$
\begin{equation*}
\text { (apparent dip angle) }=2 \theta_{2}-\theta_{1} \tag{11}
\end{equation*}
$$

In other cases analytical computations will be more complicated. An example of geometrical construction up to 2nd


Remarks , For example ( $A A B$ ) includes $A A B, A B A$, and BAA.
CAC, CBC, and CCC are Imaginary in this case.
Fig. 6
order multiples for 3 dipping primaries is illustrated in Fig. 6.

## Example of Checking Multiple on the Dip Indication

Fig. 7 (a) is a cross section transcribed from line ups on reflection records, using constant velocity assumption ( $1,900 \mathrm{~m} / \mathrm{s}$ ). On this cross section the dip indication plotted as about $1,100 \mathrm{~m}$ deep is continuous and predominant. Therefore, the appearance of multiple reflections caused by this horizon must be more probable than others. Then various 1st order multiples due to this horizon are shown in Fig. 7(b). Comparing these figures, one can omit estimated multiples, and get Fig. 7 (c), This figure fits other profiles near this section very well.


Fig. 7(a)


Fig. 7(b)

## Computation in the Case of Continuous Section

In this case the reflection interface A is indicated by two normal path lengths at appropriate points of observation ( $\left.d_{1}, d^{\prime}\right)$; similarly the reflection interface B is indicated by $\left(d_{2}, d^{\prime}{ }_{2}\right)$. Then normal path lengths to a 1 st order multiple are given by eq. (6).
where

$$
\begin{equation*}
\sin \theta_{1}=\left(d_{1}-d_{1}^{\prime}\right) / x, \sin \theta_{2}=\left(d_{2}-d^{\prime}{ }_{2}\right) / x \tag{12}
\end{equation*}
$$

$x$ : distance between observation points.
The record section of data processing system or continuous seismic profiler is thought to be the cross section of appropriate scale, in which abscissa shows $x$, ordinate shows normal path length $d$. One can make the cross section like this,


Fig. 7 (c)


Fig. 8
write the results of computation on it, compare it with the original record section, and then discuss every phase on records directly. Fig. 8 shows example of the computation method mentioned above.
$A-A$, and $B-B$ show reflection interfaces $A$ and $B$, and another real line shows the false reflection interface computed for the 1 st order multiple AB. (There is no discrimination be-
tween $A B$ and $B A$, because the continuous expression does not involve an emergence angle explicitly.) The broken line shows the same multiple, but roughly computed In this case each normal path length is expressed as

$$
\begin{equation*}
d_{1}+d_{2}, \quad d_{1}^{\prime}+d_{2}^{\prime} \tag{13}
\end{equation*}
$$

where $\theta_{1}$, and $\theta_{2}$ are neglected.
There is not so much difference between the results of these two computation methods (exact and rough) in the case of moderate dip like this example. Numerical results of computation are shown on Table 2.

Table 2

|  |  | Observation point | Refle <br> A | on layer B | Multiple AB | Multiple $A B$ (rough estimation) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case <br> (a) | Normal path | right | 20.00 | 80.00 | 97.05 | 100.00 |
|  | length (one way) | left | 63.41 | 185.65 | 240.37 | 249.06 |
|  | Dip angle |  | $10^{\circ} 00^{\prime}$ | $25^{\circ} 00^{\prime}$ | $34^{\circ} 59^{\prime}$ | $36^{3} 3{ }^{\prime}$ |
| Case <br> (b) | Normal path | right | 20.00 | 185.65 | 205.01 | 205.65 |
|  | length |  |  |  |  |  |
|  | (one way) | left | 63.41 | 80.00 | 142.18 | 143.41 |
|  | Dip angle |  | $10^{\circ} 00^{\prime}$ | $-25 \cdot 00^{\prime}$ | -1433 | -14:25' |

## Complex Multiple Reflections on Sparker Records

Let the phases on sparker (a kind of continuous seismic profiler) records be checked next by the rough computation as shown in Fig. 8 and eq. (13).

Example 1
Line ups of predominant phases on sparker record in Fig. 9 are shown on the time section in Fig. 10. The scale of abscissa ( $m$ ) is estimated by the trigonometrical position location of vessel (observation point) from land, assuming its constant speed.

When constant velocity of medium along reflection travel path is assumed, the ordinate axis of time can be thought as normal path length of appropriate scale, and appearance of various kinds of multiple can be roughly estimated. In Fig. $10 a-a^{\prime}$ shows sea bed, and the broken line $b-b^{\prime}$ shows simple multiple of sea bed. In offshore survey the reflection of sea bed is especially predominant, and then 1st order multiple due to sea bed and another geological interface is computed as shown by broken line. Line ups $j-j^{\prime}, l-l^{\prime}, m-m^{\prime}$, and $o-o^{\prime}$ are very possibly multi-


Fig. 9


Fig. 10
ples, while no care need be taken for multiple possibility of $i-i^{\prime}$, and $n-n^{\prime}$.

Example 2
Line ups of predominant phases in Fig. 11 are shown in Fig. 12. Real line shows observed phase, and broken line shows computed multiple. After estimated multiples are omitted, line ups in Fig. 13 only are left, and this cross section is much more considerable.

## Example 3

Line ups of predominant phases in Fig. 14 are shown in Fig. 15.

These figures indicate comparatively steep dip of sea bed. The results are that there is no true geological information below the simple multiple of sea bed, and there are only some kinds of multiples.
Example 4
Line ups of predominant phases in Fig. 16 are shown in Fig. 17. Only arrow marked item may be true deep geological information, if this is not type B.


Fig. 11


Fig. 12


Fig. 13


Fig. 14


Fig. 15


Fig. 16


Fig. 17

## Conclusion

The appearance of complex multiple reflections on records is thought to be more probable than had been previously expected.

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