PROPAGATION VELOCITY OF THE MICROBAROMETRIC WAVES PRODUCED BY THE EXPLOSION OF HYDROGEN BOMB*

BY

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1. Introduction

Propagation velocity of the microbarometric wave due to a large explosion is nearly equal to that of sound [3, 9]. The present author [10] had discovered the microbarometric waves due to nuclear bomb explosion in 1954, and calculated their propagation velocities as values ranged in 284-310 m/sec, by a method of phase identification of barograms of comparatively small-scale network. The accurate time and site of the detonations by U.S. have recently announced from the National Academy of Science, U.S. [8], and we are able to calculate more accurately the propagation velocity. The results of the re-calculation will be presented in this paper.

In the previous paper [10], the present author noticed a tendency of increase of the wave velocity with the progress of season, cold to warm, remarking insufficient accuracy in calculating the velocity. T.N.B. Gaffney and K.N. Bullen [1] re-estimated the velocity of the same microbarometric waves by the aid of time and site of the explosion presumed from the study of seismic wave due to the same explosion, and stated that all the estimated velocities of the microbarometric waves were nearly equal to 320 m/sec, and could not find any systematic variation. The problem whether such seasonal variation exists or not will be also studied in this paper.

G.I. Taylor [7] showed that non-circular shape of isochronous line of travel of the waves due to Krakatoa eruption could be explained as the effect of distribution of the prevailing winds rather than that of the air temperature, without establishment of rigorous theory. It is natural to presume that the propagation velocity may be influenced by the atmospheric structure. A simple dyanmical theory on the wave travel will be presented, associating with the effect of the prevailing winds.

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2. Estimation of propagation velocity of the microbarometric waves and consideration of its seasonal variation

Shida's microbarographs in Japan have, until now, detected the waves due to more than 100 nuclear explosions by U.S., the Soviet and the United Kingdom. Of all the detected waves, very marked ones which have the accurate data of the explosion itself are taken up in this paper (Fig. 1 and Table 1). These waves were caused by the U.S. explosions in the Marshall Islands, and the station recording them is Kyoto or Shionomisaki, the both of which are located roughly northwestwards the explosion site.

The propagation velocity can be calculated as a ratio of travel distance to

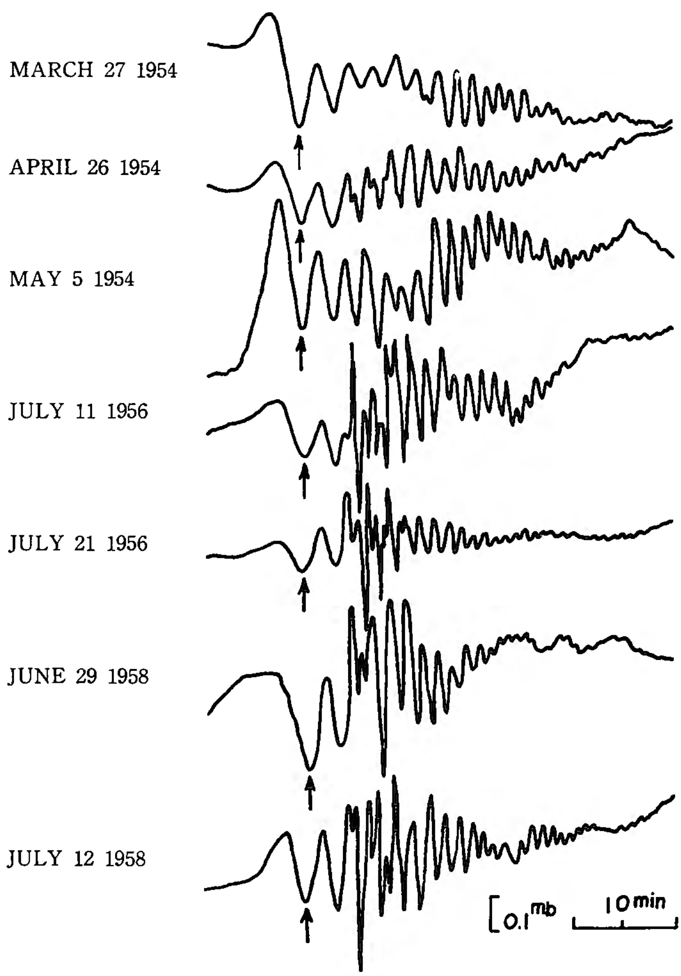

Fig. 1 Microbarograms due to the nuclear explosions.

Table 1. Data of the nuclear explosions and of the microbarometric waves

Date*	Explosion site		Explosion time*	Microbarographic station	Arrival time of the first trough*	Travel time	Travel distance	Velocity
1954 March 27	11°41′N	165°16′E	03 h 30.0 m	Shionomisaki**	07 h 01.4 m	3 h 31.4 m	3930 km	310 m/sec
1954 April 26	11 40	165 23	03 10.0	Shionomisaki	06 39.7	3 29.7	3941	313
1954 May 5	11 40	165 23	03 10.0	Shionomisaki	06 40.9	3 30.9	3941	311
1956 July 11	11 40	165 23	02 56.0	Kyoto***	06 23.8	3 27.8	3953	317
1956 July 21	11 40	165 20	02 46.0	Kyoto	06 13.3	3 27.3	3949	318
1958 June 29	11 36	162 06	04 30.0	Kyoto	07 44.0	3 14.0	3715	319
1958 July 12	11 41	165 16	12 30.0	Kyoto	15 54.0	3 24.0	3943	322

* Japan Standard Time at 135°E

** Shionomisaki: 33°27′N, 135°46′E

*** Kyoto: 35°03′N, 135°46′E

travel time of the wave. Travel distance falls in range of 3715-3966 km with error less than 2 km. Travel time of the wave may be considered as the time interval between the arrival time of the wave at the microbarographic station and the explosion time. The arrival time of the first marked trough of the wave indicated by arrow in Fig. 1 is adopted here. The true first arrival of the wave may be probably earlier than that of the first trough by several minutes, but we cannot specify the first arrival accurately beacuse of ambiguity of the phase. Travel time of the first trough falls in the range of 3 h 14.0 m–3 h 32.2 m with error less than 0.2 minutes. Thus, the propagation velocity is in range of 310–322 m/sec, with error less than 0.5 m/sec (Table 1).

The greatest difference among the values of velocity listed in Table 1 is 12 m/sec and is very much greater than the expected error. This fact does not agree with Gaffney and Bullen's result of equal velocity in all the cases. The velocity in March is equal to 310 m/sec, those in June and July equal to or higher than 317 m/sec, and the values in April and May 311 or 313 m/sec. Such velocity variation supports the existence of seasonal variation suggested by the present author previously, although the number of cases is not sufficient. All the waves taken up here travel through approximately same route, if we disregard the slight difference of locations of the explosion site and the microbarographic station. Presumption that such seasonal variation may be caused by variation of the atmospheric structure in the region where the waves travel is reasonable.

3. A simple theoretical treatment of the wave propagation

The atmospheric external gravity waves have been treated by the present author [12] and others [4, 5, 6] as a cause of the pressure waves considered here. In order to confirm the relationship between the variation of the wave velocity and that of the atmospheric structure, some theoretical consideration is required. Although the predominant influence of the prevailing winds may be supposed from the analysis by Taylor [7] and the present author [11], we can find no appropriate theory of the external gravity wave taking into account of prevailing winds. Actually, the hydrodynamical equations necessary to discuss the dynamic characters of the wave are of confluent hypergeometric type even in the case of no prevailing winds, and the solution cannot be easily obtained without use of high speed computer. A reasonable treatment of a case with the winds will be undertaken in an approximate manner.

External gravity wave in the atmospheric layer on the flat and non-rotating earth will be considered. In the undisturbed state, constant lapse rate of temperature and hydrostatic equilibrium are assumed. After the results of Pekeris' [4] and Penny's [5] treatment of the waves with vertical acceleration in an atmospheric model without prevailing winds are compared with that of Taylor's [7] one of the waves without vertical acceleration in a similar model, it is seen that neglection of the vertical acceleration gives no serious change to the results for the waves of period of several minutes or longer. Since the first part of the wave or the part with highest velocity is mainly composed of period of several minutes or longer, and the wave velocity is very much higher than the wind velocity, the neglection of the vertical acceleration is permitted for our purpose. Then, the hydrodynamical equations for two-dimensional wave motion (x: horizontal, z: downwards vertical) propagating in x-direction are as follows:

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}=-\frac{1}{Q}\frac{\partial p}{\partial x}\\
&\qquad 0=-\frac{1}{Q}\frac{\partial p}{\partial z}+\frac{q}{Q}g\\
&\frac{\partial p}{\partial t}+U\frac{\partial p}{\partial x}+w\frac{dP}{dz}=c^2\left(\frac{\partial q}{\partial t}+U\frac{\partial q}{\partial x}+w\frac{dQ}{dz}\right)\\
&\frac{\partial q}{\partial t}+U\frac{\partial q}{\partial x}+w\frac{dQ}{dx}=-Q\chi
\end{aligned}\right\} \qquad (1)$$

where the wave motion is assumed to be frictionless, adiabatic and of infinitesimal amplitude. And U or the undisturbed x-component of wind is assumed uniform throughout the layer considered, Q and c are air density and Laplacian sound velocity in the undisturbed state, respectively, and g gravitational acceleration.

u, w, p and q are small perturbations of χ-component of wind, vertical velocity, pressure and air density, respectively and χ is velocity divergence.

For simple harmonic wave proportional to exp $(ikx\text{-}i\sigma t)$, the equations (1) are reduced to

$$\left.\begin{aligned} &\frac{\partial^2\chi}{\partial z^2}+\frac{n+2}{z}\frac{\partial\chi}{\partial z}+\frac{g}{W^2}\frac{\chi}{z}\left(n-\frac{n+1}{K}\right)=0 \\ &w=\frac{K}{n+1}\left[\frac{W^2}{g}z\frac{\partial\chi}{\partial z}+\chi\left\{\frac{W^2(n+1)}{g}-z\right\}\right] \\ &iku=K\left(\chi+\frac{z}{n+1}\frac{\partial\chi}{\partial z}\right) \\ &p=QWu \end{aligned}\right\} \qquad (2)$$

where $W\equiv V-U$, and V, k and σ are wave velocity, wave number and frequency, respectively, and $n\equiv\frac{\beta_h}{\beta}-1$, $\beta\equiv\frac{d\Theta}{dz}$, $\beta_h\equiv g/R$, and $\Theta(\equiv\beta z)$, K and R are undisturbed temperature, ratio of specific heats and gas constant of the air respectively. If we set $m=\frac{g\left(n-\frac{n+1}{K}\right)}{W^2}$, $\chi=z^{-\frac{n+1}{2}}\phi$, $4mz=y^2$, the first equation of the system (2) becomes

$$\frac{\partial^2\phi}{\partial y^2}+\frac{1}{y}\frac{\partial\phi}{\partial y}+\left\{1-\frac{(1+n)^2}{y^2}\right\}\phi=0 \qquad (3)$$

This equation is of Bessel type, and we have a solution from (2) and (3):

$$\left.\begin{aligned} &\phi=AJ_{n+1}(y)+BY_{n+1}(y) \\ &w=\frac{K(4m)^{\frac{n+1}{2}}y^{-n-1}}{2(n+1)}\left[\frac{W^2}{g}y\left\{AJ_n(y)+BY_n(y)-\frac{y^2}{2m}\left\{AJ_{n+1}(y)+BY_{n+1}(y)\right\}\right]\right. \\ &iku=\frac{K(4m)^{\frac{n+1}{2}}y^{-n-1}}{2(n+1)}y\left\{AJ_n(y)+BY_n(y)\right\} \\ &p=QWu \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots (4)$$

where A and B are arbitrary constants and J_n and Y_n n-th order Bessel function of the first kind and second kind, respectively.

First, a one-layer model with constant positive lapse rate ($\beta>0$) is taken up, which corresponds to the troposphere. Boundary condition of free surface is applied at the level $z=0$ or $y=0$, where temperature reduces to zero, and solid wall condition $w=0$ at the earth's surface $z=z_0$ or $y=y_0$. Then, we have a frequency equation:

$$\frac{2}{y_0}\left(n-\frac{n+1}{K}\right)=\frac{J_{n+1}(y_0)}{J_n(y_0)} \qquad (5)$$

This equation of frequency can be numerically solved and the results are shown

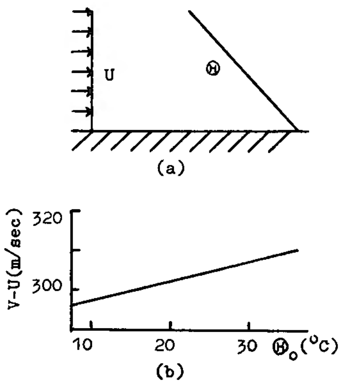

Fig. 2 One-layer model (a) and the wave solution for $\beta=6.8$°C/km (b). Θ_0 is the temperature at the earth surface.

in Fig. 2. That the wave velocity is algebraic sum of the undisturbed wind velocity U and the wave velocity V for the case without prevailing wind is easily seen in the equation (5) as well as in Fig. 2.

Secondly, a model of two layers which are separated by a surface of zero-order discontinuity of wind and of the first-order discontinuity of temperature is treated. The lower layer with positive lapse rate of temperature corresponds to the troposphere and the upper layer of negative lapse rate extending from tropopause to some height (e.g. 40 km which is adopted in the following numerical solution) the lower stratosphere. Boundary conditions are introduced; free surface condition at the upper boundary of the upper layer ($y'=y'_u$), kinematical and dynamical boundary conditions at the tropopause ($y=y_t$ for the lower layer, and $y'=y'_t$ for the upper layer) and solid wall condition at the earth's surface ($y=y_0$). We have a frequency equation:

$$\frac{2\Delta U}{g}=\frac{y_t}{mW}\,\frac{\alpha J_{n+1}(y_t)+Y_{n+1}(y_t)}{\alpha J_n(y_t)+Y_n(y_t)}+\frac{y_t'}{m'W'}\,\frac{\alpha' J_{1-n'}(y'_t)+Y_{1-n'}(y'_t)}{\alpha' J_{-n'}(y'_t)+Y_{-n'}(y'_t)} \tag{6}$$

where prime indicate the quantity for the upper layer, and β', n' and m' have the opposite sign to those in the lower layer. And $\Delta U\equiv U'-U$,

$$\alpha=-\frac{Y_{n+1}(y_0)-\dfrac{2}{y_0K}(2K-n-1)Y_n(y_0)}{J_{n+1}(y_0)-\dfrac{2}{y_0K}(nK-n-1)J_n(y_0)} \quad \text{and} \quad \alpha'=-\frac{Y_{1-n'}(y'_n)}{J_{1-n'}(y'_n)}$$

This equation can be also numerically solved for the given model (Fig. 3). The results for several models show that the existence of the warm stratosphere instead of extension of the troposphere to the level of $\Theta=0$ causes somewhat increase of wave veloity only by a few per cents. If the other conditions are remained unchanged in the both layers, the lapse rate of the upper layer affects the wave velocity by a few per cents; the model with the upper layer of lapse rate of

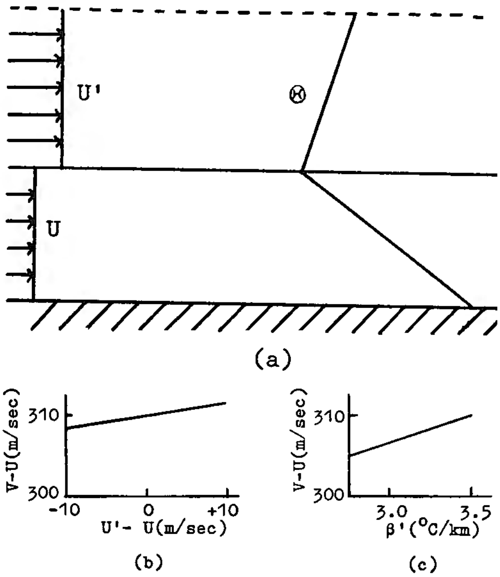

Fig. 3 Two-layer model (a). (b): the wave solution for $\beta=6.8°C/km$, $\Theta_0=26°C$ and $\beta'=3.4°C/km$. (c) the solution for $\beta=6.8°C/km$, $\Theta_0=26°C$ and $U=U'$.

$-4.0°C/km$ has the wave velocity higher than that of $-2.5°C/km$ by about 5 m/sec. The wind shear at the tropopause also results in variation of the wave velocity, the magnitude of which is only about 1/10 of the shear itself.

It is necessary that this theory will be verified by the actual data of the wave and the atmospheric structure. The latter data enough to make possible the verification are not unfortunately available for each case listed in Table 1. According to climatological data (e.g.H. Heastie [2]), warming in the troposphere and cooling in the stratosphere from spring to summer have values of several degrees in centigrade, respectively, and change the lapse rate in the both layers. The both variations of temperature affect the velocity only by a few m/sec respectively and the effects have the opposite sense each other, so we cannot expect appreciable change of velocity due to the temperature change. On the other hand, monthly mean westerly wind in April at 140°E averaged over the latitudinal belt of 10–30°N throughout the troposphere is higher than that in July by about 30 m/sec. According to the theory peesented above, this wind variation results in increase of the velocity of wave travelling northwestwards in question by about 20 m/sec. Such difference of wind can produce sufficiently the observed difference among the velocities listed in Table 1. It may be concluded that the observed variation of the wave velocity is mainly resulted from that of the prevailing westerly winds in the troposphere.

Acknowledgement

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