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ON MOVEMENT OF THE TROPICAL CYCLONE*

By

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Abstract

The tendency equation is applied to a numerical model of tropical cyclone embedded in a general current, assuming that the fields of wind and temperature proper to the cyclone can be superimposed upon those of the general current.

It is shown that the speed and direction of movement of tropical cyclone in barotropic general current agree with those of the general current, which is an illustration of steering concept.

In the case of baroclinic current, tropical cyclone has a more or less component of movement velocity across the streamline of general current, directing to the region of low temperature. This may illustrate a factor of recurvature.

1. Introduction

Numerous investigations on movement of the tropical cyclone have been made from the various point of view. Almost all of the scientific works were restricted in treatment of two-dimensional horizontal motion (e.g., C. G. Rossby, [1948]; T. C. Yeh, [1950] and S. Syono, [1951]). Although such treatments could roughly elucidate the general processes of cyclone movement, no sufficient results are yet obtained.

Tropical cyclone has, in addition to the cyclonic circulation, a vertical circulation, that is, inflow in the lower troposphere, upward current near the center and outflow in the upper troposphere. It is a well-known fact that the vertical circulation does play very important role in the mechanism of tropical cyclone (E. Palmen and H. Riehl, [1957]). In the present paper, the tendency equation will be applied to the problem of cyclone movement, taking account of the vertical circulation.

2. Tendency equation and model of tropical cyclone

Local time change of the geopotential ϕ_0 at a pressure level p_0 in the lower troposphere can be expressed by the following equation (C. L. Godske et al., [1957]):

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$$\frac{\partial \phi_0}{\partial t} = -R\int_{p_1}^{p_0} \frac{\partial T}{\partial t} \frac{dp}{p} \, , \tag{1}$$

where R designates the gas constant of dry air, and p_1 is some pressure level in the upper troposphere or the lower stratosphere, the tendency at which is negligibly smaller than $\partial \phi_0 / \partial t$. It is assumed that the local time change of virtual temperature can be approximated by that of temperature $\partial T / \partial t$.

$\partial T / \partial t$ is determined by horizontal advection of temperature, adiabatic heating due to vertical motion and non-adiabatic heating. R. C. Gentry [1963] suggests that the advection may be the most effective factor for the tropical cyclone. Now, we assume the following form:

$$\frac{\partial T}{\partial t} = -\boldsymbol{V} \cdot \nabla_H T - W_s \, , \tag{2}$$

where $\boldsymbol{V}$ is the wind velocity and $-\nabla_H T$ horizontal temperature gradient. Total effect of all the factors except the advection is conventionally expressed by W_s. Then, the equation (1) becomes

$$\frac{\partial \phi_0}{\partial t} = R\int_{p_1}^{p_0} (\boldsymbol{V} \cdot \nabla_H T + W_s) \frac{dp}{p} \, . \tag{3}$$

This gives the distribution of $\partial \phi_0 / \partial t$, if we have three-dimensional distribution of temperature, wind velocity and W_s throughout the layer effective to the tropical cyclone.

Use of the tendency equation makes us possible to carry out numerical experiments on movement of tropical cyclone. As a model of the cyclone to be used in the experiment, E. Palmen and H. Riehl's [1957] wind field and E. S. Jordan and C. L. Jordan's [1954] temperature field are adopted (Table 1). Assuming that this

Table 1. Tropical cyclone model at radial distance of 5° lat. $-\partial T_c / \partial_r$ is the radial gradient of temperature. V_r and V_θ are the radial and tangential velocities and the positive values show the outward and cyclonic motion, respectively.

P (mb)	800	600	400	250	150
V_r (m/sec)	−0.8	0.0	0.4	1.8	1.4
V_θ (m/sec)	8.1	7.8	4.0	−2.0	−7.0
$-\partial T_c / \partial_r$ (°C/°lat)	0.0	0.1	0.2	0.3	0.0
W_s (m.°C/sec. °lat)	0.0	0.0	0.08	0.54	0.0

cyclone model might be in a steady state when this might not be influenced by any other large-scale current, we have the following relation between W_s, velocity $\boldsymbol{V}_c$ and temperature T_c in cyclone itself:

$$W_s = -\boldsymbol{V}_c \cdot \nabla_H T_c\,. \tag{4}$$

This makes possible to determine the value of W_s by the aid of distribution of $\boldsymbol{V}_c$ and T_c (Table 1).

Integration of the right-hand side of the equation (3) can be approximately carried out as follows:

$$\left.\begin{aligned}\frac{\partial\phi_0}{\partial t} = R\Bigg[\sum_{i=8,6,4}(\boldsymbol{V}\cdot\nabla_H T+W_s)_i \times \frac{200}{i\times 100} \\ + \sum_{j=2.5,1.5}(\boldsymbol{V}\cdot\nabla_H T+W_s)_j \times \frac{100}{j\times 100}\Bigg],\end{aligned}\right\} \tag{5}$$

where $(\boldsymbol{V}\cdot\nabla_H T+W_s)_k$ designates the value at $k \times 100$ mb level. The layers from the surface to 900 mb and from 100 mb to the top of atmosphere are obliged to be omitted, because the temperature data in such layers are not presented in Jordan's results. The fact that the temperature gradient is usually small in these layers of tropical cyclone makes us to presume that this omission may give no essential bias to the result.

3. Cyclone embedded in barotropic current

It is assumed that the wind field of cyclone itself can be vectorially superimposed upon that of the general current embedding the cyclone. Asymmetric features of the tropical cyclone structure found in moving cyclone are commonly considered as a result of the cyclone movement or superposition with the general current. T. N. Krishnamurti [1962] has used generalized Rankine vortex where the symmetric velocity field of the ordinary Rankine vortex is superimposed upon the field of general current. This may be supported by such result of B. I. Miller's [1958] statistics that the wind field around hurricane subtracted vectorially by that of the general current is almost axi-symmetric, except just near the center. Although these facts give only unsufficient reasoning, the superposition may be permitted for our preliminary purpose. Furthermore, it is assumed that the value of W_s is not affected by such superposition.

For the cyclone model embedded in a straight barotropic current, U, the equation (5) is reduced to the following form:

$$\left.\begin{aligned}\frac{\partial\phi_0}{\partial t} = R\Bigg[\sum_{i=8,6,4}\left\{\frac{\partial T_c}{\partial r}(v_r+U\cos\alpha)+W_s\right\}\times\frac{200}{i\times 100} \\ + \sum_{j=2.5,1.5}\left\{\frac{\partial T_c}{\partial r}(v_r+U\cos\alpha)+W_s\right\}\times\frac{100}{j\times 100}\Bigg],\end{aligned}\right\} \tag{6}$$

where v_r and $-\partial T_r/\partial r$ are radial velocity and radial gradient of temperature of cyclone itself, respectively. β is an angle between the direction of general current and the line connecting the cyclone center and the grid point for numerical computation.

Numerical experiments are made for $U = 5$, 10, 15 and 20 m/sec, at radial distance of 5° lat. The results are shown in Fig. 1. In all the cases, distribution of the tendency is sinusoidal for the angle α and the minimum and the maximum tendencies appear at the points of $\alpha = 0°$ and $\alpha = 180°$, respectively.

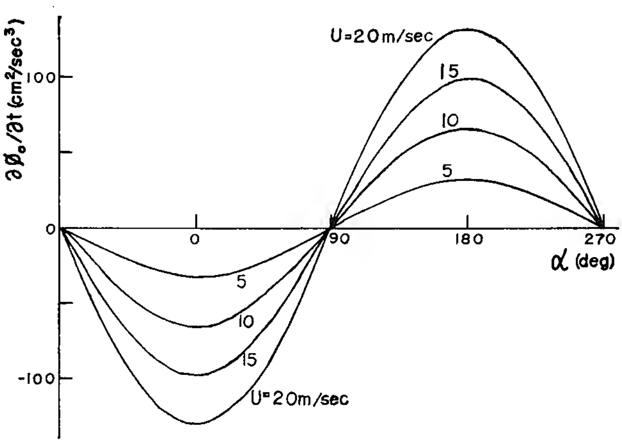

Fig. 1. Distribution of the computed value of $\partial\phi_0/\partial t$ for the angle α, for the tropical cyclone model embedded in barotropic current U.

It is a reasonable assumption that the cyclone associated without change of intensity and deformation of pressure pattern moves in the direction from the point of maximum tendency to that of the minimum one. Thus, it is concluded that the movement direction of cyclone coincides with that of the general current.

The movement speed of cyclone, C, is given as follows:

$$C = -\frac{(\partial\phi_0/\partial t)_{min}}{\partial\phi_0/\partial s}, \tag{7}$$

where s-axis is taken along the movement direction, and $(\partial\phi_0/\partial t)_{min}$ is the minimum value of tendency. The numerator in the equation (7) can easily be obtained from the equation (6). The denominator $\partial\phi_0/\partial s$ can be determined by the following relationship:

$$\frac{\partial\phi_0}{\partial s} = -R\int_{p_1}^{p_0}\frac{\partial T_c}{\partial r}\frac{dp}{p}, \tag{8}$$

where the geopotential gradient at p_1 is assumed to vanish and $\partial T_c/\partial s$ is equal to

$\partial T_c/\partial r$. The computed values of C are listed in Table 2. Equality of the values of C and U can clearly be seen in high accuracy.

Table 2. The movement speed C obtained in the numerical experiment for tropical cyclone embedded in barotropic current of the speed U

U (m/sec)	5.0	10.0	15.0	20.0
C (m/sec)	4.8	9.7	14.5	19.3

This result can be obtained without any numerical computation. Taking account of the relation (4), the equation (3) is now reduced to

$$\left(\frac{\partial \phi_0}{\partial t}\right)_{min} = R\int_{p_1}^{p_0}\left(U\frac{\partial T_c}{\partial r}\right)\frac{dp}{p}.$$

This gives the following relation:

$$C = -\frac{(\partial\phi_0/\partial t)_{min}}{\partial\phi_0/\partial s} = -\frac{+R\int_{p_1}^{p_0}\left(U\frac{\partial T_c}{\partial r}\right)\frac{dp}{p}}{-R\int_{p_1}^{p_0}\left(\frac{\partial T_c}{\partial r}\right)\frac{dp}{p}} = U.$$

The above mentioned results that the velocity of movement of tropical cyclone agrees with that of the barotropic general current embedding it gives us an illustration of steering concept.

4. Cyclone embedded in baroclinic current

Another numerical experiments are carried out for cyclone model embedded in a baroclinic general current U along x-axis with a horizontal temperature gradient $-\partial T_f/\partial y$, which is assumed to be perpendicular to the general current. Here, x- and y-axis are in right-hand system. Since, there exists, of course, thermal wind corresponding to $-\partial T_f/\partial y$, the general current is determined by the wind speed at 1,000 mb level U_{1000}, the temperature gradient $-\partial T_f/\partial y$ and the latitude. Assuming that the superposition of both temperature fields of cyclone itself and the general current is permitted, the equation (3) is reduced to

$$\frac{\partial \phi_0}{\partial t} = R\int_{p_1}^{p_0}\Big[(v_r+U\cos\alpha)\left(\frac{\partial T_c}{\partial r}+\frac{\partial T_f}{\partial y}\sin\alpha\right) + \frac{\partial T_f}{\partial y}\cos\alpha\,(v_\theta - U\sin\alpha)+W_s\Big]\frac{dp}{p}. \tag{9}$$

Some results of experiment for the case of $-\partial T_f/\partial y = 0.05°\mathrm{C}/°$ lat are illustrated in Fig. 2. The tendency is not always minimum, at $\alpha=0$, differing from the barotropic case. The minimum tendency appears in the region between $\alpha = 0°$ and

$\alpha = 180°$, and the maximum one between $\alpha = 180°$ and $\alpha = 360°$, respectively. This result implies that the movement direction of cyclone deflects from that of the general current to colder region.

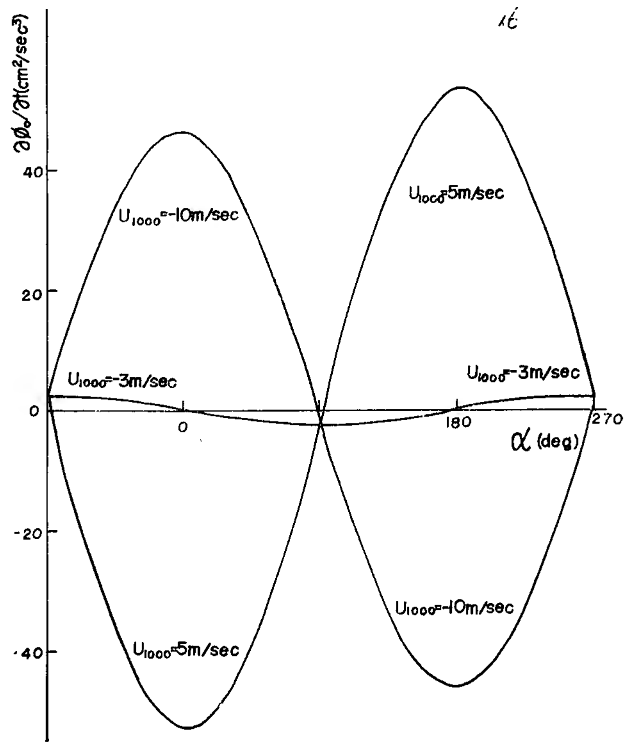

Fig. 2. Distribition of the computed value of $\partial\phi_0/\partial t$ for the angle α, for the tropical cyclone model embedded in baroclinic current of $-\partial T_f/\partial y$=0.05°C/°lat.

The movement direction is estimated from the location of the minimum and maximum tendencies. The speed is computed by a similar method in the previous section, excpet $\partial\phi_0/\partial s$ is determined by the following relation:

$$\frac{\partial\phi_0}{\partial s} = -fU_1 \sin\beta - R\int_{p_1}^{p_0}\left(\frac{\partial T_c}{\partial r}+\frac{\partial T_f}{\partial y}\sin\beta\right)\frac{dp}{p}\,, \tag{10}$$

where U_1 is the speed of general current at p_1, and f the Coriolis parameter. The results for the cases of $-\partial T_f/\partial y = 0.05$, 0.1 and 0.2°C/°lat are shown in Fig. 3. Deflection of the movement direction from that of the general current is only several degrees for positive value of U_{1000}. The direction does rapidly vary with U_{1000} near some negative value which is dependent of $-\partial T_f/\partial y$. Movement in the direction $\beta \fallingdotseq 180°$ occurs for negative U_{1000} smaller than some value. The movement speed C is very

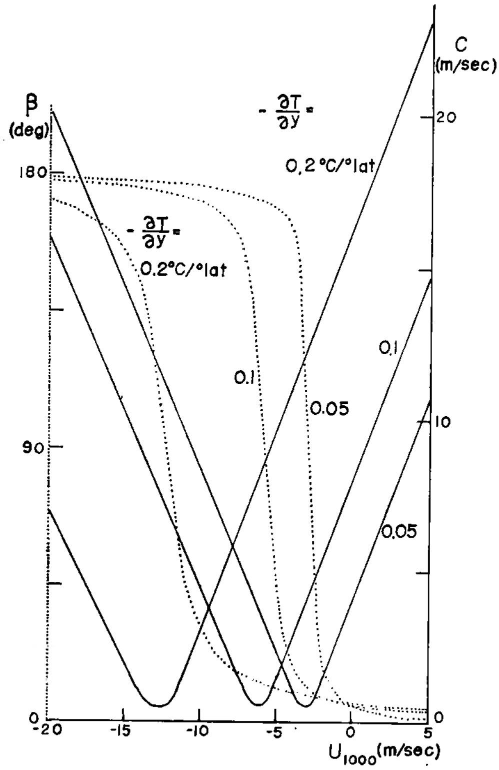

Fig. 3. The movement speed C (full curve) and the direction angle β (dot curve) obtained for the tropical cyclone model embedded in baroclinic current.

small for U_{1000} near which rapid change of the movement direction occurs. In the region of U_{1000} where the movement direction is nearly parallel to that of the general current, the speed is in a linear relation with U_{1000}.

If we take x-axis directing eastwards and y-axis northwards, the above results may be interpreted as follows: Tropical cyclone moves westwards in the region where the prevailing easterly wind is not so weak at the surface. When the cyclone enters into the area of weak easterly wind, it may have an appreciable northward component of velocity. In the zone of surface westerlies, the cyclone displaces eastwards with fairly large speed. These results outline roughly the average feature of actual movement of typhoon and hurricane, including the recurvature.

5. Summary

Tendency equation is applied to the tropical cyclone model with vertical circulation as well as cyclonic circulation, embedded in the general current. Under assumption of superposing temperature and wind fields of cyclone itself upon those of the general current, several numerical experiments are carried out. For barotropic general current, the movement direction and speed of cyclone coincide with those of the general current. This gives us an illustration of steering concept of the tropical cyclone. For cyclone embedded in baroclinic current, the movement direction does generally deflect from that of the general current to colder region. This may probably disclose a factor causing the recurvature.

These results must be verified by comparison with the actual movement of typhoon and hurricane.

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