

ON THE M_1 -COMPONENT OBTAINED BY
GRAVIMETRIC TIDAL OBSERVATION
(SCREENING OF GRAVITATIONAL FORCES)*

By

Ichiro NAKAGAWA

(Received September 3, 1964)

Abstract

Absorption of gravitational forces has mainly been investigated during periods of solar eclipse by several investigators, and it has been established that the absorption effect of gravitation, if it exists, does not exceed the noise level.

The mass of the earth is larger than that of the moon and therefore the gravitational absorption due to the earth's mass itself should be larger than that due to the moon's mass.

In the present paper, a trial to detect the absorption effect of the lunar gravitational forces by the earth's mass is described by using data obtained with an Askania gravimeter No. 111, installed at Kyoto, during a period of one year. By the present investigation, it is ascertained that the gravitational absorption due to the earth's mass is about $4 \mu\text{gal}$ and that its value reaches a maximum two hours after the instant when the moon transited the meridian of the observation station. The value of amplitude obtained by observation is in good agreement with that predicted by theoretical calculation. But concerning the phase, there exists a difference of about 180° compared with that expected by theoretical calculation.

Introduction

A possible screening effect of gravitational forces by a third body interposed between two attracting bodies was suggested by Q. Majorana [1920]. His theory was thereafter thoroughly discussed and criticized by H. N. Russell [1921] and he discussed the effect on earth tides of such a gravitational absorption.

A solar eclipse affords the only opportunity for a simple investigation of the existence of this effect. But an actual observation to detect such an effect was not carried out for a long time due to insufficient instrumental accuracy. After advances in techniques for measuring earth tides, observations for such a purpose have recently been carried out by several investigators (Tomaschek, [1955]; Brein, [1957]; Nakagawa, [1962]; Dobrokhotov et al., [1961]; Caputo, [1961]; Sigl et al., [1961]). Except for the results of R. Brein [1957], their observations show that the screening

*) Read at the Fifth International Symposium on Earth Tides, June 1964, Brussels.

effect of gravitational forces observed during solar eclipse, even if it exists, does not exceed the mean noise level.

The author [1962] carried out a precise observation of gravity change during the annular eclipse of April 19, 1958, by means of an Askania gravimeter No. 111 at Naze, situated on one of the scattered islands in the south regions of the mainland of Japan, and established that any significant systematic effect which seemed to be related with the gravitational absorption was not observed during the solar eclipse, and also that the screening effect of gravitational forces, even if it existed, did not exceed $3 \mu\text{gal}$ which was the instrumental error of observation.

The screening effect of gravitational forces during a solar eclipse was theoretically estimated by R. Tomaschek [1955] and R. Brein [1957]. According to Brein's conclusion, the screening effect of gravitational forces during a total eclipse reaches a maximum of $2.1 \mu\text{gal}$, when the absorption constant of gravitation, 3×10^{-15} in c.g.s. units, determined by K. F. Bottlinger [1912] is adopted. According to Tomaschek's application of Majorana's theory, it is a maximum of $3 \mu\text{gal}$ for an absorption constant of gravitation less than 10^{-14} , which is about one thousandth of the value deduced by Q. Majorana from his experiments.

On the other hand, the gravimetric observational accuracy is of the order of $3 \mu\text{gal}$, at the present stage, although it has recently been increased. Under these circumstances, an attempt to detect the screening effect of the moon on the solar attraction is very difficult.

The mass of the earth is about 81.45 times as large as that of the moon and therefore the gravitational absorption by the earth's mass itself should be larger than that by the moon's mass. Consequently, it is advantageous to investigate the screening of gravitational forces by the earth's mass, regarding the earth itself as an absorbing body.

The amplitude of tidal constituents obtained by harmonic analysis of gravimetric tidal data is usually between 1.1 and 1.2 times as large as that calculated for the hypothetically perfect rigid earth. This value is usually called the "tidal factor of gravity" or, in short, "gravimetric factor." If the screening of the lunar gravitational forces by the earth's mass exists, it should be recognized in the M_1 -component. If the screening of the solar gravitational forces by the earth's mass exists, it should be recognized in the S_1 -component. The values of the tidal factor of gravity for diurnal tides obtained by actual observations show a divergence far in excess of the observational error, while those for semidiurnal tides usually show good agreement with the observational error. This fact suggests a possible screening of gravitational forces by the earth's mass.

J. C. Harrison [1963] has recently described an abnormality in the amplitude of

the observed M_1 -component. Using data obtained at Bunia, situated near the equator, he has detected an anomalous tide with amplitude of $2 \mu\text{gal}$, and he has established that the second gravitational constant h , which is related to absorption of gravitational forces, must be less than 10^{-15} in c.g.s. units.

The author tried to detect the screening effect of the lunar gravitational forces by the earth's mass in 1962 and got an anomalous tide with amplitude of $4 \mu\text{gal}$ as a preliminary result. A further investigation has recently been carried out by the author using data obtained with an Askania gravimeter No. 111 during a period of one year.

In the following, an investigation concerning the observed M_1 -component is described. However, no analysis of the observed S_1 -component is given in the present paper because it is disturbed by a thermal effect.

Data and method of analysis

Data observed at Kyoto were used in the present research. They were obtained by means of an Askania gravimeter No. 111 with automatic recording apparatus. Observations were continuous during a period of about one year from August 1959 to August 1960. The observation station was situated in a special underground room of the Geophysical Institute of Kyoto University. Temperature and humidity in this room were maintained at $19.5^\circ\text{C} \pm 0.2^\circ\text{C}$ and $55\% \pm 1\%$, respectively, throughout a year by means of a thermostatically controlled apparatus. The gravimeter and recorder were installed on a concrete block isolated from the building. Therefore, the observation was carried out always under the best conditions. The position of the observation station is shown in Table 1.

Table 1. Description of the observation station

Observation station :	Geophysical Institute of Kyoto University, Kyoto, Japan
Geographic latitude :	$35^\circ 01.8' \text{ N}$
Longitude :	$135^\circ 47.2' \text{ E}$
Altitude :	59.9 metres above mean sea level
Depth :	2.4 metres below ground surface
Observation room :	International Reference Station of Gravity in Japan

Hourly values obtained during a period of about one year from August 1, 1959 to August 17, 1960, were used in the present analysis. First, the drift was eliminated from all the hourly observed values by Pertzev's method [1957]. The average value of the drift speed was about $14 \mu\text{gal}$ per day.

After elimination of the drift, hourly values were arranged on a special form for reading values in lunar hours. The values thus obtained were the amplitudes

of the first, second, third and higher harmonics of the principal lunar tides. Then the values were harmonically analyzed. Origin of the present analysis was August 9, 00h, 1959 (UT) and harmonic analysis itself was carried out using 8868 values commencing from the origin.

Results of analysis and discussion

Results of the harmonic analysis are shown in Table 2. In Table 2, phase of a component, ζ , denotes the time interval from the instant when an imaginary celestial body corresponding to that component tide has transited the meridian of the observation station until the instant when the observed tide actually reaches the maximum value, and phase, ζ' , denotes that from the origin time until the instant when the observed tide actually reaches the maximum value. Both time intervals are shown in Table 2 in unit of degrees.

Table 2. Results of harmonic analysis
 Origin of analysis: August 9, 00h, 1959 (UT)
 Period of analysis: August 9, 00h, 1959 —
 August 12, 11h, 1960 (UT)

Component	Amplitude (μgal)	Phase ζ (degree)	Phase ζ' (degree)
M_1	6.207	185.64	128.33
M_2	57.967	181.68	24.75
M_3	0.725	157.11	101.72
M_4	0.382	202.78	248.93
M_6	0.235	118.06	7.28
M_8	0.151	73.10	165.40

As described above, harmonic analysis was carried out for the period of August 9, 1959 to August 12, 1960. Therefore the values obtained by the harmonic analysis correspond to that analysis period. But the values in Table 2 correspond to an inclination of the moon's orbit to the equator equal to its average value of $23.^\circ 45'$.

Although six lunar component tides are obtained by the present harmonic analysis, only three component tides, M_1 , M_2 and M_3 , are discussed in the following. The theoretical amplitude and phase for these three component tides are calculated as shown in Table 3. The amplitudes of the other three are too small to be discussed in detail.

Tidal factor of gravity G and phase lag κ can easily be calculated by combining Tables 2 and 3. The values obtained for G and κ are shown in Table 4. Here, the negative sign of κ shows that the observed tide leads the theoretical one, while the positive sign shows that the former lags behind the latter.

Table 3. Theoretical amplitude and phase at Kyoto

Component	Amplitude (μ gal)	Phase ζ (degree)
M_1	1.650	180.00
M_2	50.307	180.00
M_3	0.664	180.00

Table 4. Values of G and κ
Origin of analysis : August 9, 00h, 1959 (UT)

Component	G	κ (degree)
M_1	3.762	+ 5.64
M_2	1.152	+ 1.68
M_3	1.092	-22.89

The tidal factor of gravity and phase lag for the M_2 -constituent, based on the data used in the present analysis, have been discussed in detail by the author [1962] where values of 1.143 and 2.°24, respectively, were obtained for G and κ for the M_2 -constituent. In that case, Lecolazet's method was used, which differs from the present method. Therefore there was a difference in method of the analysis, but the results concerning the M_2 -constituent were in good agreement. Considering this, the values of the tidal factor G of the M_2 - and M_3 -component tides are considered to be valid. But the value of G for the M_1 -component tide, shown in Table 4, is too large compared to the expected value. This means that the amplitude obtained for the M_1 -component tide is considerably disturbed by certain effects. A screening effect of gravitational forces may be considered as a possible cause of this discrepancy. An attempt to explain this curious result will be made in the following.

The observed M_1 -component is considered to consist of two terms. One is the primary component which is a real tidal component originating from change in distance between the earth and the moon, and is called the "smaller lunar elliptic diurnal tide." The other is the secondary component which is an effect due to other causes.

The observed M_1 - and M_2 -components obtained by the present analysis are

$$\begin{aligned} M_1 \text{ (observed)} \dots & 6.207 \cos (t - 185.^\circ 64) \mu\text{gal} \quad \text{and} \\ M_2 \text{ (observed)} \dots & 57.967 \cos (2t - 181.^\circ 68) \mu\text{gal}, \end{aligned}$$

respectively. According to the theory, the ratio of amplitude of M_1 -component to that of M_2 -component is 0.0328. As described above, the values of G and κ for M_2 -component obtained by the present analysis are considered to be reasonable. Assuming then that the theory concerning the amplitude ratio is applicable in the

present case and that phase lag of M_1 -component is equal to half of that of M_2 -component, observed M_1 -component to be expected is calculated as follows:

$$M_1 \text{ (expected)} \dots 1.901 \cos (t - 180.^\circ 84) \mu\text{gal.}$$

Subtracting the expected M_1 -component from the observed M_1 -component, the residual M_1 -component can be derived as follows:

$$M_1 \text{ (residual)} \dots 4.315 \cos (t - 187.^\circ 73) \mu\text{gal.}$$

Moonrise, moonset and transit of the meridian at the point of latitude 35°N and longitude 135°E are 00h45m, 12h41m and 06h46m, respectively, for the origin date of the analysis. The relation between these three M_1 -components and position of the moon at the origin time are shown in Figure 1.

Figure 1 shows that the residual M_1 -component has an amplitude of about $4 \mu\text{gal}$ and that the phase of the residual M_1 -component is perfectly followed with movement of the moon. Specifically, the sign of the residual M_1 changes from negative to positive two hours after moonrise and changes from positive to negative two hours after moonset. Also, the amplitude of the residual M_1 has its maximum two hours after the instant when an imaginary celestial body corresponding to the residual M_1 -component had transited the meridian of the observation station. Consequently, the residual M_1 -component is positive while the moon is above the horizon, and negative while the moon is below the horizon.

On the other hand, assuming that the earth is a perfect sphere, the screening effect of the lunar attraction by the earth's mass can theoretically be calculated by Majorana's theory. According to its result, the tide-generating force at the point facing the moon is about $4 \mu\text{gal}$ as large as that at the opposite point, on the line connecting the centres of the earth and moon.

The amplitude for the residual M_1 -component obtained by the present analysis is a little larger than that obtained by J. C. Harrison [1963] in his spectrum of gravity record, but is of the same order in magnitude, and is also in good agreement with that calculated by Majorana's theory. But concerning the phase of the residual M_1 -component, there exists a difference of about 180° compared with the phase expected by the theoretical calculation.

An investigation, similar to the one described here, made at various stations in the world would contribute much to the study of the effect of gravitational shielding on the deformation of the earth.

Conclusions

The problem of a screening of gravitational forces is an exceedingly interesting

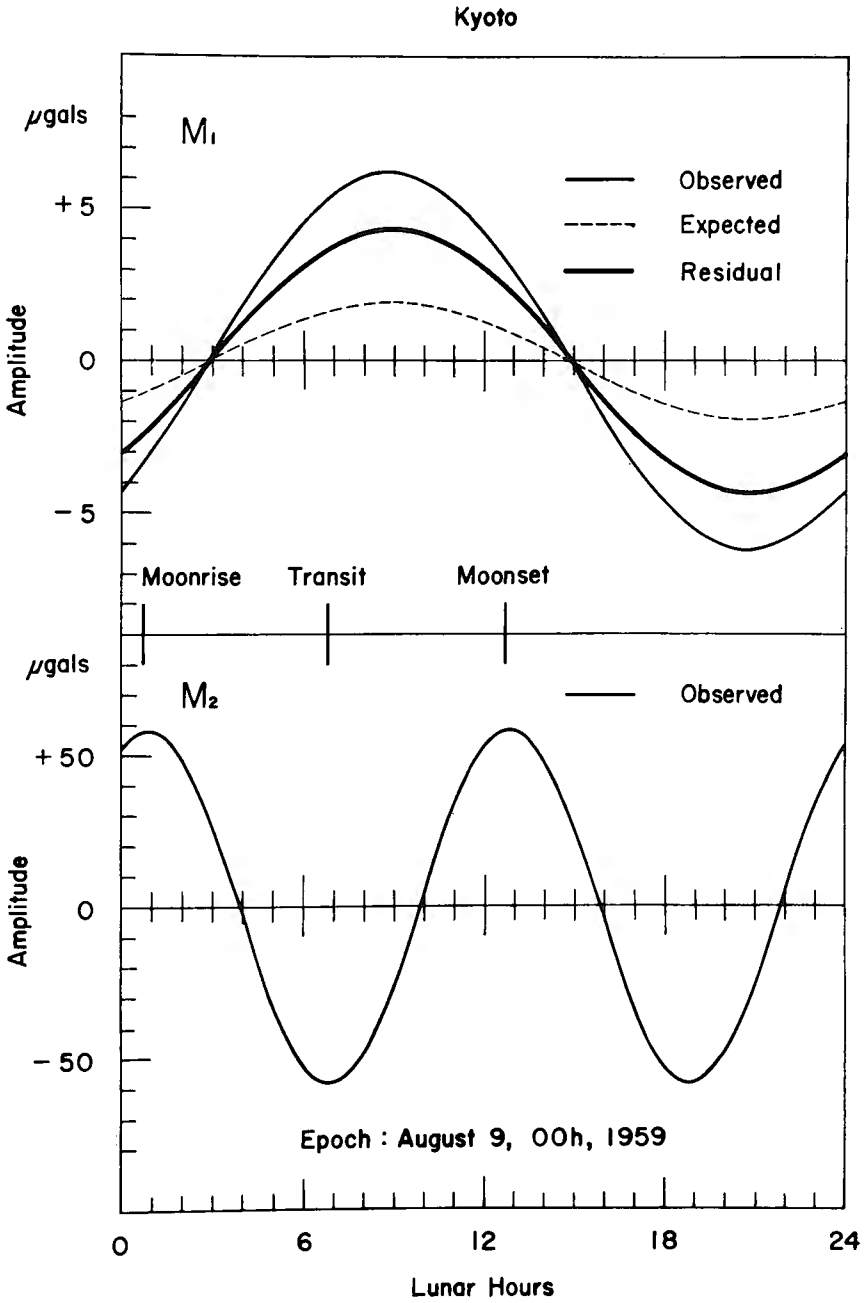


Fig. 1. M_1 -component, M_2 -component and position of the moon.

one. Solar eclipses have hitherto provided the only opportunity to investigate experimentally the existence of this effect.

The author tried to elucidate this problem independently of a solar eclipse, regarding the earth itself as an absorption body. Data obtained with an Askania gravimeter No. 111 at Kyoto during a period of about one year, from August 9, 1959 to August 12, 1960, were used in this analysis. Analysis of the M_1 -component tide permits the following conclusions:

- (1) The screening effect of the lunar gravitational forces by the earth's mass, if it exists, is about 4 μ gal.
- (2) The residual M_1 -component, which is considered to be related to the screening effect, is in phase with the movement of the moon.

By the gravimetric observations, only the vertical component of the deformation of the earth can be detected. For further investigation of the screening effect of gravitational forces, a precise, continuous and simultaneous observation with various instruments, for example, gravimeter, tiltmeter, extensometer and others, must necessarily be carried out at the same place for a long time. Needless to say, it is necessary to increase the accuracy of the instrument and to select an observation station which is as quiet and stable as possible and which is insulated from various disturbances. To carry out such observations at more than three stations is urgently required.

Acknowledgements

The author is indebted to the late Professor E. Nishimura for his generous advice and encouragement throughout the present research. The author wishes to express his thanks to Professors M. Ewing and J. Oliver of the Lamont Geological Observatory, where this research was made, for their encouragement, and to Dr. J. Dorman who read the manuscript and offered helpful suggestions. The author is also indebted to the Goddard Institute for Space Studies of the National Aeronautics and Space Administration for generous permission to use an IBM 7090.

This research was partially supported by Grant No. AF-AFOSR-62-303 of the Air Force Office of Scientific Research as part of the Advanced Research Projects Agency's Project VELA-UNIFORM.

References

- Bottlinger, K. F., 1912; Die Gravitationstheorie und die Bewegung des Mondes, Inaugural-dissertation, Universität der München, Freiburg/Brigau.
- Brein, R., 1957; Die Schwerkraftregistrierungen. Beitrag zur Frage einer Absorption der Schwere,

- Deutsche Geodätische Kommission bei der Bayerischen Akademie der Wissenschaften, Reihe B, Heft Nr. 34, 30-45.
- Caputo, M., 1961; Observations taken at Trieste, with the great horizontal pendulums at the time of the solar eclipse of February 1961, Communications de l'Observatoire Royal de Belgique N° 188, Série Géophysique N° 58, 64-65.
- Dobrokhotov, Y. S., N. N. Pariisky and V. I. Lysenko, 1961; Observations of tidal variations of gravity in Kiev during the solar eclipse of February 1961, *ibid.*, N° 188, Série Géophysique N° 58, 66-69.
- Harrison, J. C., 1963; A note on the paper 'Earth-tide observations made during the International Geophysical Year', Journal of Geophysical Research, 68, 1517-1518.
- Majorana, Q., 1920; On gravitation. Theoretical and experimental researches, Philosophical Magazine and Journal of Science, 39, 488-504.
- Nakagawa, I., 1962; Some problems on time change of gravity. Part 3. On precise observation of the tidal variation of gravity at the Gravity Reference Station, Bulletin, Disaster Prevention Research Institute, Kyoto University, 57, 2-65.
- Nakagawa, I., 1962; Some problems on time change of gravity. Part 4. On continuous and precise gravity observation during the period of annular eclipse on April 19, 1958, *ibid.*, 57, 67-84.
- Nakagawa, I., 1962; On M_1 -component, Meeting of Geodetic Society of Japan.
- Pertzev, B. P., 1957; On the calculation of the drift curve in observations of bodily tides, Marées Terrestres, Bulletin d'Informations, N° 5, 71-72.
- Russell, H. N., 1921; On Majorana's theory of gravitation, The Astrophysical Journal, 54, 334-346.
- Sigl, R. und O. Eberhard, 1961; Horizontalpendelbeobachtungen in Berchtesgaden während der Sonnenfinsternis vom 15. 2. 1961, Communications de l'Observatoire Royal de Belgique N° 188, Série Géophysique N° 58, 70-75.
- Tomaschek, R., 1955; Tidal gravity measurements in the Shetlands. Effect of the total eclipse of June 30, 1954, Nature, 175, 937-939.

Lamont Geological Observatory
Columbia University
Palisades, New York
Contribution No. 795