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ON OBSERVATIONS OF THE TER-DIURNAL COMPONENT OF THE EARTH'S TIDAL STRAIN

By

Izuo OZAWA

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Abstract

The theoretical values of the earth tidal constants l_3 and h_3 have been calculated by use of the Gutenberg's model of the earth construction by H. Takeuchi et al. and I. M. Longman.

In this paper, the present author (I. Ozawa) observes the ter-diurnal components of the tidal linear strains of the earth by means of highly sensitive extensometers which have been devised by him at Osakayama Observatory and Kishu Mine.

From these observations, the M_3-components of linear strains in some directions are analyzed and the observed values of h_3-6l_3 are calculated as 0.638. According to this study, the observed magnitude of h_3-6l_3 is about twice as large as the theoretical values which have been calculated by Takeuchi et al. and Longman, but the observed phase lags of M_3-tide components of the strain elements have good agreements with the theoretical ones in the almost observations.

1. Introduction The tide generating potential is expressed by a series of spherical solid harmonics of 2nd, 3rd and so on degrees with coefficients which are periodic functions of the time. In general, the tide generating potential is estimated with the term (W_2) of the spherical harmonic function of the second order. The earth tidal numbres (h_2 and l_2) of the second degree were defined by A. E. H. Love [1909] and T. Shida [1912], respectively. Recently, following Love's and Shida's definitions, I. M. Longman [1963] and H. Takeuchi et al. [1962] have discussed theoretically on the higher terms of the earth tidal constants. And R. Lecolazet [1963] has shown the elements of the tidal strain depending on W_3, but he has only shown these elements as the functions of the zenith distance. His formulas only show the additional terms depending on W_3 for the terms depending on W_2. It is difficult, however, to observe directly the terms depending on W_3 as the additional terms to the ones depending on W_2. And it is necessary to obtain the formulas which show the observable quantities in practice.

The best method to obtain the numerical values of h_3 and l_3 is that the ter-diurnal components of the tidal strains are observed. The present author (I. Ozawa) has calculated the formulas of the ter-diurnal components of the tidal strain on the earth's surface. And also the present author has observed with highly sensitive extensometer the strain components at Osakayama Observatory and Kishu Mine, and then he has obtained the observed values of h_3 and l_3. According to his results, the magnitude of h_3-6l_3 is about two or three times as large as the ones which Longman and Takeuchi et al. have calculated theoretically by use of Gutenberg's model of the earth's construction. But these observed phase lags of ter-diurnal components of the tidal strains from the ones which are expected theoretically are very little.

2. Theory The radial component u_r, the colatitudinal component u_θ and meridional component u_ϕ of the tidal displacement at the earth's surface are expressed as

$$\left.\begin{aligned} u_r &= \frac{1}{g}(h_2 W_2 + h_3 W_3 + \cdots), \\ u_\theta &= \frac{1}{g}\frac{\partial}{\partial\theta}(l_2 W_2 + l_3 W_3 + \cdots), \\ u_\phi &= \frac{1}{g}\frac{\partial}{\sin\theta\,\partial\phi}(l_2 W_2 + l_3 W_3 + \cdots), \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots(1)$$

where g is the acceleration of the gravity, r is the radial vector, θ is the colatitude, ϕ is the longitude, W_2, W_3,$\cdots$ are the tide generating potentials of second, third,... degree. In the formulas (1), h_2 is Love's number and l_2 is Shida's number.

The total tide-generating potential U is expressed as follows:

$$\begin{aligned} U &= W_2 + W_3 + \cdots \\ &= k'M\left\{\frac{1}{2}\left(\frac{r^2}{c^3}\right)\cdot(3\cos^2 z - 1) + \frac{1}{2}\left(\frac{r^3}{c^4}\right)(5\cos^3 z - 3\cos z) + \cdots\right\}, \end{aligned} \qquad \cdots\cdots(2)$$

where k' is the gravitational constant, M is the mass of the depending celestial body as the moon or the sun, c is the distance between the center of the earth and the one of the the pending celestial body, and z is the geocentric zenith distance of the celestial body at the given observatory.

The actual formula of the second and third degrees of the tidal potential is shown numerically as follows:

$$\begin{aligned} &53.3g\left(\cos^2 z_m - \frac{1}{3}\right) \\ &+ 1.5g\left(\cos^3 z_m - \frac{3}{5}\cos z_m\right) \\ &+ 24.6g\left(\cos^2 z_s - \frac{1}{3}\right) \\ &+ 0.0018g\left(\cos^3 z_s - \frac{3}{5}\cos z_s\right), \end{aligned} \qquad \cdots\cdots\cdots\cdots\cdots(3)$$

where z_m is the geocentric zenith distance of the moon, and z_s is the one of the sun at any observatory. In this formula, the first term is the main term of the lunar tide, the second is the one depending on the fourth power of the parallax of the moon, the third is the main term of the solar tide, and the fourth is the one depending on the fourth power of the parallax of the sun. According to formula (3), it is evident that the second term is not so small, but the fourth term is negligible. Let δ be the declination of the moon and t be the Greenwich standard time of the moon. Then $\cos z$ is expressed as

$$\cos z = \cos\theta \sin\delta + \sin\theta \cos\delta \cos(t+\phi)\,. \quad \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

Using the equation (4), we can write as follows:

$$\left.\begin{aligned}
\cos^2 z - \frac{1}{3} &= \frac{3}{2}\left(\sin^2\delta - \frac{1}{3}\right)\left(\cos^2\theta - \frac{1}{3}\right) \\
&\quad + \frac{1}{2}\sin 2\delta \sin 2\theta \cos(t+\phi) + \frac{1}{2}\cos^2\delta \sin^2\theta \cos 2(t+\phi)\,, \\
\cos^3 z - \frac{3}{5}z &= \frac{1}{10}\cos\theta \sin\delta (3-5\cos^2\theta)(3-5\sin^2\delta) \\
&\quad + \frac{3}{20}\sin\theta\cos\delta\{(1-5\cos^2\theta)(1-5\sin^2\delta)\}\cos(t+\phi) \\
&\quad + \frac{3}{2}\cos\theta \sin^2\theta \sin\delta \cos^2\delta \cos 2(t+\phi) \\
&\quad + \frac{1}{4}\sin^3\theta \cos^3\delta \cos 3(t+\phi)\,.
\end{aligned}\right\} \quad \cdots\cdots (5)$$

According to formulas (5), it appears that the diurnal and semidiurnal components are dependent on W_2 and W_3. But the ter-diurnal component is independent of W_2, and it is chiefly dependent on W_3. So, in order to obtain the earth-tidal number l_3 and h_3, it is the best method to observe the ter-diurnal component. M_3-component is the principal constituent in the ter-diurnal tide and it is expressed as

$$M_3 = \frac{5}{8} f^{3/2} \frac{M}{E} g\left(\frac{a^2}{c^4}\right) r^3 \cos^6\frac{I}{2} \cdot \sin^3\theta \cdot \cos(3T+2h-3s+3\xi-\nu)\,, \quad \cdots\cdots (6)$$

where f is the reduction factor of M_3-component, E is the mass of the earth, a is the mean radius of the earth, I is the inclination of the lunar orbit for the equator, T is the hour angle of the mean sun, h is the longitude of the sun, s is the longitude of the moon, and ξ and ν are small angles relating to the moon's ascending node. Writing the east longitude ϕ_E, the formula of M_3-component of the potential is expressed as following form

$$M_3 = A_3 g r^3 \sin^3\theta \cos 3(t+\phi_E)\,,$$

where
$$A_3 = \frac{5}{8} f^{3/2} \frac{M}{E}\left(\frac{a^2}{c^4}\right)\cos^6\frac{I}{2} \approx \frac{1}{a}(0.0060) f^{3/2}\,. \quad \cdots\cdots\cdots\cdots\cdots (6')$$

We can express the radial, co-latitudinal and longitudinal components of the tidal displacement being as, respectively, u'_r, u'_θ and u'_ϕ depending on W_3 in the following form

$$\left.\begin{aligned} u'_r &= \frac{H_3(r)}{g} W_3, \\ u'_\theta &= \frac{L_3(r)}{g}\frac{\partial W_3}{\partial \theta}, \\ u'_\phi &= \frac{L_3(r)}{g\sin\theta}\frac{\partial W_3}{\partial \phi}, \end{aligned}\right\} \qquad \cdots\cdots(7)$$

where $H_3(a) = h_3, \quad L_3(a) = l_3.$

Supposing the earth is a homogeneous elastic sphere, the h_3 and l_3 are given by use of the result of Kelvin and Tait [1923] in the following relations

for $$W_2;\ \frac{h_2}{ag} = \frac{5k+\frac{2}{3}\mu}{\mu\left(19k+\frac{4}{3}\mu\right)}, \qquad \frac{l_2}{ag} = \frac{3k}{2\mu\left(19k+\frac{4}{3}\mu\right)}, \qquad \cdots\cdots(8)$$

where k and μ are incompressibility and rigidity, respectively,

and for $$W_3;\ \frac{h_3}{ag} = \frac{3\left(\frac{7}{2}k+\frac{2}{3}\mu\right)}{2\mu(33k+4\mu)}, \qquad \frac{l_3}{ag} = \frac{3k}{4\mu(33k+4\mu)}. \qquad \cdots\cdots(8')$$

Supposing also that the earth is incompressible, these relations are more simplified as follows:

for $$W_2;\ \frac{h_2}{ag} = \frac{5}{19\mu}, \quad \frac{l_2}{ag} = \frac{3}{38\mu}, \quad \frac{h_2}{l_2} = \frac{10}{3}, \qquad \cdots\cdots(9)$$

and for $$W_3;\ \frac{h_3}{ag} = \frac{21}{13\mu}, \quad \frac{l_3}{ag} = \frac{1}{44\mu}, \quad \frac{h_3}{l_3} = 7, \quad h_3 - 6l_3 = l_3. \qquad \cdots\cdots(9')$$

Estimating $\mu \approx 2\times 10^{12}$ c.g.s., from above relations, it is given as follows:

$$h_3 \approx 0.605 h_2, \quad l_3 \approx 0.288 l_2.$$

Recently, H. Takeuchi et al. and I. M. Longman have calculated theoretically as $l_3 = 0.014$ or 0.010 and $h_3 = 0.290$ or 0.274 respectively, by use of the Gutenberg's earth's interior model.

From the formulas (6′), (7) and the relations between strain components and elastic displacements, the tidal strain components depend on W_3 have been calculated as

$$e_{\theta\theta} = \{l_3(6\sin\theta\cos^2\theta - 3\sin^3\theta) + h_3\sin^3\theta\} A_3 r^2 \cos 3(T+\phi) = \frac{D_1}{ag\sin^3\theta} W_3,$$

$$e_{\phi\phi} = \{-9l_3\sin\theta + 3l_3\sin\theta\cos^2\theta + h_3\sin^3\theta\} A_3 r^2 \cos 3(T+\phi) = \frac{D_2}{ag\sin^3\theta} W_3,$$

$$e_{\theta\phi} = -6l_3 \sin 2\theta r^2 A_3 \sin 3(T+\phi) = \frac{D_3}{ag \sin^3\theta} W_3 ,$$

$$e_{rr} = \left\{ a\left(\frac{\partial H_3}{\partial r}\right)_{r=a} + 3h_3 \right\} \frac{W_3}{ag} ,$$

$$e_{r\phi} = e_{r\theta} = 0 ,$$

$$e_{\theta\theta} - e_{\phi\phi} = 6l_3 \sin\theta(1+\cos^2\theta) A_3 r^2 \cos 3(T+\phi) = \frac{6l_3(1+\cos\theta)}{\sin^2\theta} \frac{W_3}{ag} ,$$

$$e_{\theta\theta} + e_{\phi\phi} = 2(h_3 - 6l_3) \frac{W_3}{ag} ,$$

$$\Delta = e_{\theta\theta} + e_{\phi\phi} + e_{rr} = \left\{ a\left(\frac{\partial H}{\partial r}\right)_{r=a} + 5h_3 - 12l_3 \right\} \frac{W_3}{ag} ,$$

where

$$D_1 = l_3(6 \sin\theta \cos^2\theta - 3 \sin^3\theta) + h_3 \sin^3\theta ,$$
$$D_2 = l_3(-9 \sin\theta + 3 \sin\theta \cos^2\theta) + h_3 \sin^5\theta ,$$
$$D_3 = -6l_3 \sin 2\theta .$$

And the conditions of the free surface are

$$a\left(\frac{dL_3}{dr}\right)_{r=a} + h_3 + 2l_3 = 0 , \qquad \text{etc.} \qquad \cdots\cdots\cdots\cdots\cdots(13)$$

These formulas look like the ones depending on W_2 (Ozawa, [1959]). Assuming the earth is a homogeneous and incompressible, the areal strain is equal to $2l_3 W_3/ag$.

3. Observations It is rarely observed the quasi-terdiurnal variations of the crust at observatory near the coast. But in this case, we treat the direct effect of the tide generating force. And so, the observations of the ter-diurnal component should be performed at a distant place from the sea.

The present author has carried out the observations of the tidal strains by use of the highly sensitive extensometers (Ozawa, [1960]) which he had devised, at Osakayama Observatory and Kishu Mine. Osakayama Observatory is situated at 135° 51.5′ of east longitude and 34° 59.6′ of north latitude, and is 65 km distant from the Japan Sea where is nearest from the observatory. Kishu Mine is situated at 135° 53.4′ of east longitude and 33° 51.7′ of north latitude, and is 14.8 km distant from the Kumano-nada (the Pacific Ocean). The both observatories are situated about 100 meters under the ground. The daily variations of the room temperature are less than 0.01°C in the both observatories.

The results of the observations are shown in Table 1. The results of S52°E and S38°W in this table have been analysed by means of the every 30 minute's method

Table 1. Observed values of M_3 and M_2 components of the tidal strains

Observatory	Component	Period of analyses	Sensitivity of instrument	Span of observation	M_3		M_2	
					Amplitude	Phase lag	Amplitude	Phase lag
			$\times 10^{-8}$/mm	m	$\times 10^{-8}$		$\times 10^{-8}$	
Osakayama	$e_{\phi\phi}$ (E-W)	1959, 3, 25. ---8, 7.	0.368	5.3	0.0041	314.0°	0.878	357.8°
"	$e_{\theta\theta}$ (N-S)	1959, 4, 23. ---7, 23.	0.566	6.6	0.153	2.3°	1.358	10.9
"	S52°E	1960, 2, 19. ---3, 19.	0.064	10.0	0.032	12.3°	1.63	0.9°
"	S38°W	1960, 8, 25. ---9, 23.	0.209	26.0	0.0033	23.2°	0.464	16.9°
Kishu	$e_{\theta\theta}$ (N-S)	1961, 3, 11. ---6, 6.	0.498	5.0	0.0095	346.7°	0.437	354.0°

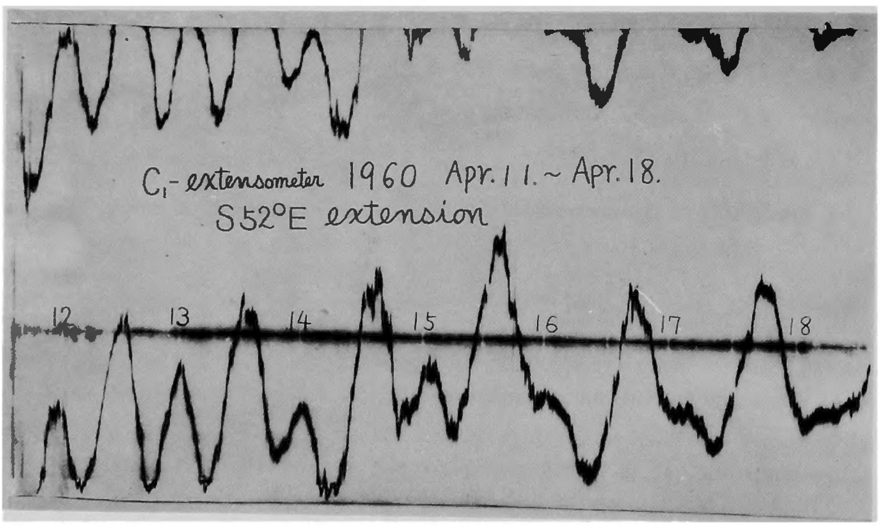

Photo. Photographic record of the tidal strain observed with the extensometer on the direction of S 52°E at Osakayama.

which had been devised by Ozawa [1963], and they are observed with the extensometers of longer spans than the others. So, it seems that their accuracies are higher than the results of the others ($e_{\theta\theta}$ and $e_{\phi\phi}$ components of Osakayama and one of Kishu).

4. Results From these results of Table 1, the areal strains are obtained as follows:

$$\text{Osakayama}; \quad e_{\theta\theta}+e_{\phi\phi} = \begin{cases} M_3: & 0.156\times 10^{-8}\cos(3t-\ \ 1.1°)\,, \\ M_2: & 2.15\ \ \times 10^{-8}\cos(2t-\ \ 6.6°)\,, \end{cases}$$

$$\text{Osakayama}; \quad e_{S52°E}+e_{S38°W} = \begin{cases} M_3: & 0.035\times 10^{-8}\cos(3t-14.4°)\,, \\ M_2: & 2.080\times 10^{-8}\cos(2t-\ \ 4.4°)\,. \end{cases}$$

From the values of areal strains and formula (12) and formula of one of the second order's tide generating potential, we have obtained as follows,

$$\text{from} \quad e_{\theta\theta}+e_{\phi\phi} = \left\{ \begin{array}{l} h_2-3l_2=0.448\,, \\ h_3-6l_3=2.82\,, \end{array} \right\} \quad \cdots\cdots\cdots\cdots\cdots(\mathrm{I})$$

$$\text{from} \quad e_{\mathrm{S52^\circ E}}+e_{\mathrm{S38^\circ W}} = \left\{ \begin{array}{l} h_2-3l_2=0.436\,, \\ h_3-6l_3=0.638\,. \end{array} \right\} \quad \cdots\cdots\cdots\cdots\cdots(\mathrm{II})$$

From the results of Kishu, assuming $e_{\theta\theta}+e_{\phi\phi}\approx 2e_{\theta\theta}$, we have obtained as

$$h_3-6l_3 \approx 0.343\,. \qquad \cdots\cdots\cdots\cdots(\mathrm{III})$$

As mentioned above, the accuracy is exceedingly good in the case of $e_{\mathrm{S52^\circ E}}+e_{\mathrm{S38^\circ W}}$, therefore we decide as

$$\left. \begin{array}{l} h_2-3l_2=0.436\,, \\ h_3-6l_3=0.638\,. \end{array} \right\} \qquad \cdots\cdots\cdots\cdots(\mathrm{II'})$$

and

If we assume that the earth is an incompressible and homogeneous eleastic sphere of which rigidity is 2×10^{12}, we get as $h_3=7l_3=0.0496$, $l_3=0.0071$, and $h_3-6l_3=0.0071$; these values are very less than our observed values. But, using theoretical values of H. Takeuchi, M. Saito and N. Kobayashi, and one of I. M. Longman, we get $h_3-6l_3=0.206$ and 0.214, respectively. Namely, our observed values are nearer with the results which are obtained by use of Gutenberg's earth's interior model than one which is calculated by use of the assumption of the incompressible and homogeneous elastic sphere; the observed value of h_3-6l_3 is about three times as large as the one of the theoretical values. However, the observed phase-lags of M_3-component for the ones of the theoretical values are almost small; these phase lags are less than about 20° except that one of them is $-46°$. We may regard that the phase lag of observed value is little. The present author is having many problems whether these analysed results are actual ter-diurnal tide without fail or not, and he is going to study precisely this problems.

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