

## DISPERSION AND ADSORPTION OF TRACER MATERIAL IN TRACING GROUND WATER FLOW

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### Abstract

Dispersion and adsorption of tracer material encountered in tracing ground water flow were investigated theoretically and experimentally. According to this result a method of determining the velocity of ground water flow was proposed for the case in which the tracer stays in the injection borehole for a time. The effect of partial adsorption of tracer in the soil on the ground water tracing is shown qualitatively.

### Introduction

Tracer method is frequently adopted in following ground water movement. This method gives the direction of ground water flow correctly, when the number of detecting points is sufficient, while it does not directly give the velocity of the flow because dispersion of the tracer makes the arrival time vague. The time of the first detection or that of the maximum concentration does not show correctly the mean arrival time, though they are often used.

It is preferable for the tracer material to be stable (chemically and physically), harmless, economical, easy to detect, and suitable for automatic recording; so, our laboratory has used NaCl or uranine (sodium fluorescein) according to the necessity of automatic recording or high sensibility. However the effect of dispersion still takes place, and adsorption of uranine in clayey soil causes some trouble. The ground water tracing made at Kamenose landslide area (Fig. 1) showed discrepancy between the results obtained by tracing NaCl and uranine (Kinki Bureau of Construction, 1963, 1965).

In this paper the effects of dispersion and adsorption of tracer on the time change of the concentration of the tracer at detecting points are examined experimentally and theoretically, then the method for the determination of the correct value of the velocity of ground water flow is proposed.

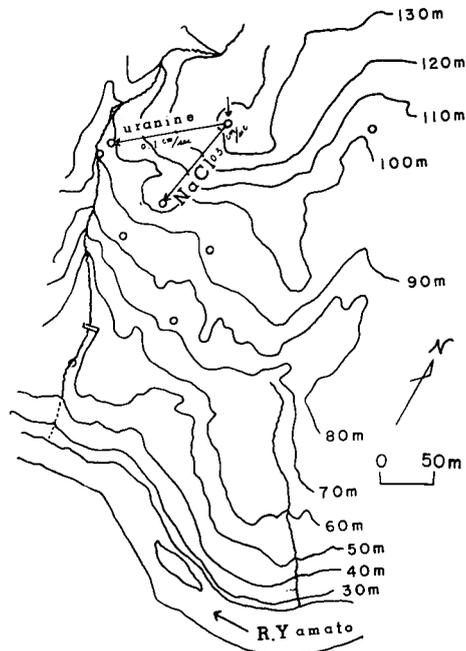


Fig. 1. The results of ground water tracing at Kamenose landslide area, showing the direction of the earliest detection and the velocity along it. The circles show the boreholes.

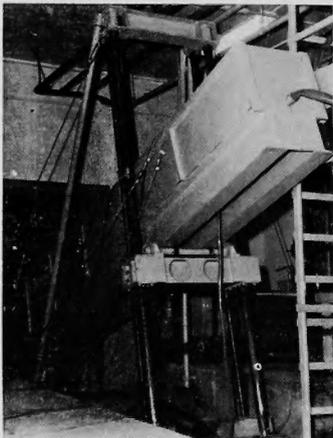


Fig. 2. A general view of the experimental apparatus.

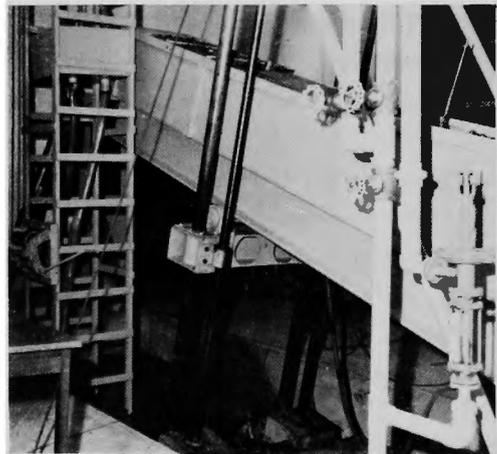


Fig. 3. Water supply equipments.

### Experimental Apparatus

The apparatus used in the experiments is shown in Fig. 2, 3, and 4. This apparatus was made in order to investigate the cause of failure of mountain slope especially in respect to hydrologic circumstance (Okunishi,

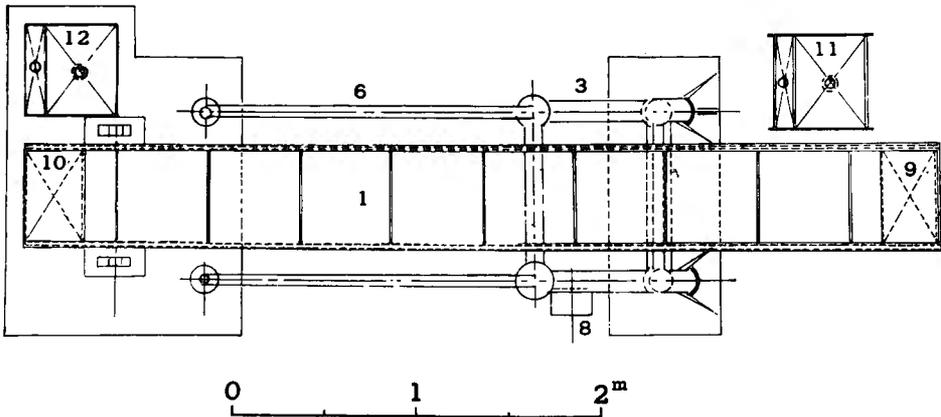


Fig. 4. Plan of the apparatus. 1-soil flume, 3-elevator, 9-upper end tank, 10-lower end tank, 11-upper head tank, 12-lower head tank.

1965). Simulated ground water flow can be applied from the variable head tank (upper head tank) and drained to a fixed head tank (lower head tank). Soil flume is 5m long, 50cm wide, and 50cm deep. Its inclination can be changed from  $0^\circ$  to  $35^\circ$  continuously by heaving one side of the soil flume. Auxilliary tanks (end tanks) are installed at both ends of the soil flume, and they are in contact with soil across the wall perforated all over, which maintains parallel flow of ground water near the ends of the flume.

The soil sample employed is weathered andesite of Kamenose landslide area (Kinki Bureau of Construction, 1963, 1965).

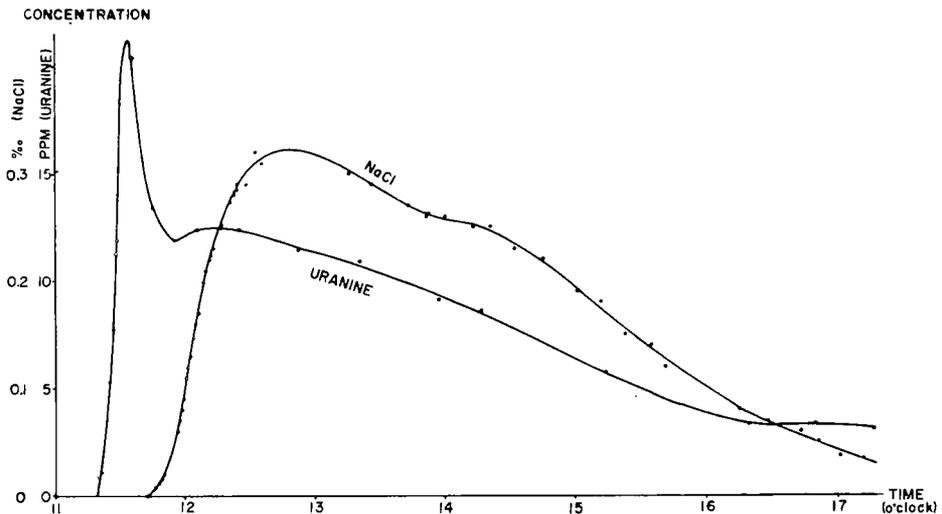


Fig. 5. Time change of the concentration of tracer at the lower end tank ( $x=500$  cm) at experiment 2.

### Results of the Experiments

NaCl and uranine dissolved in water were used as tracer and put into the upper end tank, and then detected at the lower end tank in Fig. 4. Electrical conductivity meter and fluorometer were used to detect NaCl and uranine, respectively. Fig. 5 shows the time change of the concentration of the tracers at the second experiment. NaCl was put into the upper end tank 20 minutes later than uranine. Unit of concentration was adjusted so that both curves coincided with each other when two tracers took identical behavior. A steep peak in the curve of uranine is due to the surface flow which took place at the beginning of the experiment and vanished soon, so it must be excluded. Then the maximum relative concentration of NaCl in the ground water is higher than that of uranine. Later the concentration of NaCl receded to zero, while that of uranine appears to have receded to a certain constant.

Fig. 6 shows the integrated concentration of two tracers at the detecting point in fraction of total quantity used. In the first experiment the inclination of the soil flume was  $13^\circ$ , and filtering velocity  $0.36 \text{ cm/min}$ . NaCl and uranine were applied simultaneously in the quantity of 100 grams and 5 grams, respectively. Most of the NaCl passed the detecting point during 30 hours after injection, whereas most of the uranine remained undetected during the same time, though it continued to flow in a scanty but steady rate. In the second experiment the inclination of the soil tank was about  $20^\circ$  and filtering velocity  $1.03 \text{ cm/min}$ . NaCl and uranine were used in the quantity of 200 grams and 10 grams, respectively. Though NaCl was put into the upper end tank

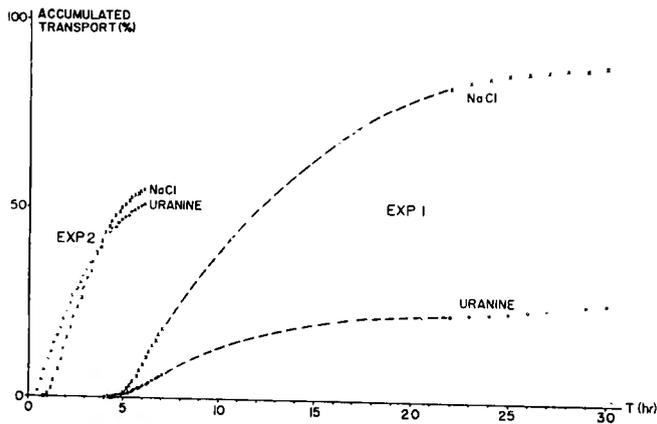


Fig. 6. Accumulated transport of tracer at the lower end tank in fraction of the total quantity of it put into the upper end tank.

20 minutes later than uranine, its integrated concentration exceeded that of uranine, and then the two curves became nearly parallel.

From the facts described above, it appears that there exists a sort of equilibrium between dissolved and absorbed state of uranine. It is absorbed or re-released whether or not the concentration exceeds the equilibrium point which depends on the total quantity of uranine absorbed before then. The difference of the behavior of uranine in the first and second experiments seen in Fig. 6 can be explained as follows.

In the first experiment the soil had not yet absorbed uranine and had enough capacity of adsorption (i. e. the equilibrium point was low). In the second experiment it had already absorbed so much uranine that it absorbed the latter only when the concentration of the latter was high (i. e. the equilibrium point was high), and so, the deficit of uranine was insignificant.

### Theoretical Analysis

In order to determine the velocity of ground water flow by tracer method, the distribution of the concentration of the tracer must be described as a function of time, location, and the mean velocity of ground water flow. Such a function has been given by Yuhara (1954) in the study of diffusion in ground water. He treated the case in which the flow is one dimensional and constant flux of tracer is emitted continuously from the source point and deduced a solution of error function type. This result was verified by the model experiment described in the same paper. However, in the actual exercise of ground water tracing, constant flux of tracer is used rarely, and usually a certain quantity of tracer is injected into a borehole. Moreover, it does not immediately mingle with the flow of ground water, but gradually mixes with the ground water passing through the borehole. Especially in these model experiments the upper end tank is large, and it takes a considerably long time for the most of the tracer to go out of the tank into the flow of ground water. Therefore, the solution of Yuhara must be developed so as to be applicable to these conditions.

Assumptions : The flow of the ground water is assumed to be uniform, steady, and one dimensional. Coefficient of dispersion ( $D$ ) is assumed to depend only on the mean velocity of flow ( $u$ ) and to be constant because  $u$  is constant. Dispersion is one dimensional in the model experiment because the flow is confined both vertically and laterally, and two dimensional for the field conditions because the flow is confined vertically.

In this section one dimensional dispersion is accounted for. This result can be applied to the case of two dimensional dispersion with minor modifica-

tions (see **Appendix**). Tracer injected into the borehole is diluted at constant rate by the flowing water and its concentration decreases exponentially. Basic equation is the same as that of Yuhara, and written as follows :

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} - D \frac{\partial^2 C}{\partial x^2} = 0 \quad \dots\dots\dots(1)$$

Boundary conditions are :

$$C=0 \quad \text{at } t \leq 0 \quad \dots\dots\dots(2)$$

$$C=C_0 \exp(-\alpha t) \quad \text{at } t > 0, \quad x=0 \quad \dots\dots\dots(3)$$

where  $C$  is the concentration of tracer,  $t$  is the time, and  $\alpha$  and  $C_0$  are constants.

These equations can be solved exactly by Fourier series, but an approximate solution using source solution of heat conduction is adopted for the sake of simplicity (Carslaw and Jaeger, 1959). The source solution applies to the case in which porous medium extends from  $-\infty$  to  $+\infty$ . Therefore, the error is significant at the neighborhood of the boundary when  $\partial C/\partial x$  is large there. However, when  $t$  becomes large  $\partial C/\partial x$  becomes small because the effect of dispersion represented by the third term of eq. (1) makes the distribution of  $C$  gradually even, then the error is insignificant except for the region where both  $x$  and  $t$  is small.

The source solution for a source of  $uC_0dt$  is written as follows :

$$C = \frac{uC_0dt}{2\sqrt{\pi Dt}} \exp [-(x-ut)^2/4Dt] \quad \dots\dots\dots(4)$$

Therefore, the contribution from the source emitted from  $t=\tau$  to  $t=\tau+d\tau$  becomes,

$$C = \frac{uC_0e^{-\alpha\tau}d\tau}{2\sqrt{\pi D}} \phi(x, t-\tau) \quad \dots\dots\dots(5)$$

where,

$$\phi(x, t) = t^{-1/2} \exp [-(x-ut)^2/4Dt]$$

The final solution is obtained by integrating eq. (5) :

$$\begin{aligned} C &= \frac{uC_0}{2\sqrt{\pi D}} \int_0^t e^{-\alpha\tau} \phi(x, t-\tau) d\tau \\ &= \frac{uC_0}{2\sqrt{\pi D}} e^{-\alpha t} \int_0^t e^{\alpha\tau} \phi(x, t-\tau) d\tau \quad \dots\dots\dots(6) \end{aligned}$$

Integration of eq. (6) is not easy. In order to determine only the coeffi-

icients  $u$  and  $D$  from the concentration of tracer observed at detecting point, it will be convenient to use a differential form of eq. (6). From eq. (6),

$$\frac{\partial C}{\partial t} + \alpha C = \frac{uC_0}{2\sqrt{\pi Dt}} \exp [-(x-ut)^2/4Dt] \quad \dots\dots\dots(7)$$

Therefore,

$$\frac{uC_0}{2\sqrt{\pi D}} \exp [-(x-ut)^2/4Dt] = \sqrt{t} \left( \frac{\partial C}{\partial t} + \alpha C \right) \quad \dots\dots\dots(8)$$

Denoting the right side of eq. (8) as  $\psi$ ,

$$\ln \psi = \ln \left( \frac{uC_0}{2\sqrt{\pi D}} \right) - \frac{x^2}{4Dt} - \frac{u^2t}{4D} + \frac{ux}{2D} \quad \dots\dots\dots(9)$$

$$\frac{\partial(\ln \psi)}{\partial t} = \frac{x^2}{4Dt^2} - \frac{u^2}{4D} \quad \dots\dots\dots(10)$$

From eq. (9) and (10) it is seen that  $\ln \psi$  becomes maximum at  $t=x/u$ , and when  $t$  becomes large the right side of eq. (10) approaches  $-u^2/4D$ . Therefore,  $u$  and  $D$  are determined from the time of maximum of  $\ln \psi$  and the slope of the asymptotic line of  $\ln \psi$ . The value of  $\ln \psi$  can be obtained by the continuous observation of the concentration of tracer at the injection point and

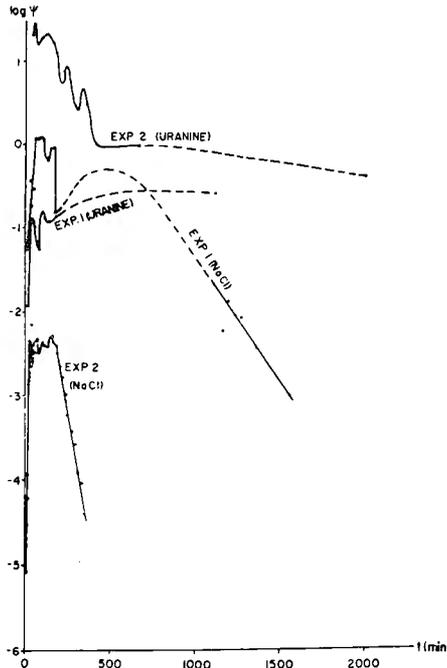


Fig. 7. Time change of  $\log \psi$ . Different constants are added to each curve.

Table 1. Determination of constants.

	Experiment 1	Experiment 2
Inclination of soil flume, $\theta$	13°19'	19°33'
Discharge, $Q$	0.87 l/min	1.80 l/min
Cross sectional area of ground water flow, $A$	1900 cm <sup>2</sup>	1775 cm <sup>2</sup>
Filtering velocity, $u_q$	0.358 cm/min	1.025 cm/min
Time of maximum of $\ln \psi$ , $t_m$ (at $x=500$ cm)		160 min
Mean velocity, $u$	1.09 cm/min	3.12 cm/min
Specific yield of the soil, $\beta$		0.328
Dispersion coefficient, $D$	108 cm <sup>2</sup> /min	213 cm <sup>2</sup> /min

the detecting point.

The time change of  $\ln \psi$  obtained in the first and second experiments is shown in Fig. 7. The peaks are seen only on the curve of NaCl in the second experiment, among which the last one is adopted, because it makes the mean velocity consistent with filtering velocity calculated from discharge. The others make the former velocity too large compared with the latter one and make specific yield of the soil too small. The results of the calculation of  $u$  and  $D$  are given in Table 1. In this table the value of the specific yield in the second experiment was used to calculate the mean velocity in the first experiment. The broken line of NaCl for the first experiment in Fig. 7 is drawn in accordance with the time of maximum value of  $\ln \psi$  calculated from the value of mean velocity in Table 1.

The value of dispersion coefficient ( $D$ ) in Table 1 is about ten times as large as that expected from the experimental curve given by Yuhara (1954) for sand. The curves of uranine in Fig. 7 do not clearly show their characteristic property, but it appears that the asymptotic slope is smaller than that of NaCl, which leads to a larger value of apparent dispersion coefficient.

### Discussion

In these experiments the concentration of both uranine and NaCl is measured by reading. The ruggedness of the curves of Fig. 7 is mostly due to numerical differentiation of such a discrete data. Use of automatic recording of the concentration of tracers will remove this defect.

When there exist partial adsorption and re-release of tracer in the soil, the method proposed in this paper can not give the correct figure of the concentration of tracer in the ground water and leads to incorrect value of the

mean arrival time of tracer. Assuming a sort of balance between dissolved and absorbed state of tracer described above, eq. (1) may be improved in the form such as follows :

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial x^2} - A(C - C^*) \quad \dots\dots\dots (11)$$

$$C^* = \text{func.} \left\{ \int_0^t A(C - C^*) dt + C_0^* \right\} \quad \dots\dots\dots (12)$$

where  $A$  is probably some function of  $C$  and/or  $C^*$ . If the functional forms of  $C^*$  and  $A$  are known, eq. (11) and (12) can be solved numerically.

### Conclusion

A method of the analysis of the results of ground water tracing was found for the case in which the tracer injected into a borehole does not immediately go along with ground water flow, which is usually the case. The applicability of this method is verified by the model experiment.

When partial adsorption of tracer takes place in the soil, it appears that there exists a sort of balance between the dissolved and absorbed state, and when the concentration becomes very small, the tracer absorbed in the soil is re-released. These processes make the time curve of the concentration of tracer at the detecting point more even than in the case without adsorption.

### Appendix

The equation of two dimensional dispersion of the tracer for the field condition is written as follows :

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} - D_1 \frac{\partial^2 C}{\partial x^2} - D_2 \frac{\partial^2 C}{\partial y^2} = 0 \quad \dots\dots\dots (1)'$$

The boundary conditions are the same as that for one dimensional dispersion. Denoting the diameter of injection borehole as  $b$ , the source solution corresponding to eq. (4) is (Carslaw and Jaeger, 1959),

$$C = \frac{ubC_0}{4\pi t \sqrt{D_1 D_2}} \exp \left[ -\frac{(x-ut)^2}{4D_1 t} - \frac{y^2}{4D_2 t} \right]$$

assuming that the direction of the ground water flow coincides with that of the line from the injection borehole to a detecting borehole, we can put  $y=0$ , along the line and the above equation is rewritten as,

$$C = \frac{ubC_0}{4\pi t \sqrt{D_1 D_2}} \exp \left[ -\frac{(x-ut)^2}{4D_1 t} \right] \quad \dots\dots\dots (4)'$$

Then the equations corresponding to eq. (6)~(10) are :

$$C = \frac{ubC_0}{4\pi\sqrt{D_1D_2}} e^{-at} \int_0^t e^{at} \phi'(x, t) dt \quad \dots\dots\dots(6)'$$

where,

$$\phi'(x, t) = t^{-1} \exp\left[-\frac{(x-ut)^2}{4D_1t}\right]$$

$$\frac{\partial C}{\partial t} + \alpha C = \frac{ubC_0}{4\pi t\sqrt{D_1D_2}} \exp\left[-\frac{(x-ut)^2}{4D_1t}\right] \quad \dots\dots\dots(7)'$$

$$\frac{ubC_0}{4\pi\sqrt{D_1D_2}} \exp\left[-\frac{(x-ut)^2}{4D_1t}\right] = \left(\frac{\partial C}{\partial t} + \alpha C\right)t \equiv \psi' \quad \dots\dots\dots(8)'$$

$$\ln\psi' = \ln\left(\frac{ubC_0}{4\pi\sqrt{D_1D_2}}\right) - \frac{x^2}{4D_1t} - \frac{u^2t}{4D_1} + \frac{ux}{2D_1} \quad \dots\dots\dots(9)'$$

$$\frac{\partial(\ln\psi')}{\partial t} = \frac{x^2}{4D_1t^2} - \frac{u^2}{4D_1} \quad \dots\dots\dots(10)'$$

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