# ON THE VERTICAL DISTRIBUTION OF SNOW DENSITY IN RELATIVELY DEEP SNOW LAYER 

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#### Abstract

Measurements of vertical distribution of snow in deposited snow layer of depth from 2 meters to 6 meters are carried out. It is shown that the results are roughly in agreement with those expected from K. Kojima's theory.


## I. Introduction

An actual state of the vertical density distribution in deposited snow layer will give us important knowledges on the physics of snow. Although many informations have been already obtained for the distribution in relatively shallow snow layer, there are scanty observations in layer of several meter depth, because of many difficulties in accurate measurement. The most reliable method of measuring the snow density is, at the present time, sampling one, and it could give many valuable data to us for shallow layer. Application of this method to relatively deep layer is not so easy, because an appreciable compression of sampled snow may not be probably avoided if sampling will be made from the upper surface of snow layer.

The present author has joined the team of snow survey sponsored by Kansai Electric Power Co. at the mountainous district in the central Japan in mid-winter and early spring of 1961. In that survey, fairly reliable data of vertical distribution of density could be obtained for the snow layer of depth from 2 meters to 6 meters, by sampling the snow not from the upper surface of the layer but from the vertical wall surface of a hole bored in the deposited snow layer. The results of the temperature distribution acquired in the survey was already reported by the present author (K. Sahashi [1964]). The purposes of this paper are to present the data of density distribution in snow layer and to give some discussion on them.

## II. Observation and Results

Measurement of the vertical density distribution in the deposited snow
layer is carried out by employing a hole which is bored for temperature measurement. The hole has a rectangular horizontal section about 1 meter $\times 3$ meters, and extends to the soil surface. Sampling the snow is made from the vertical wall surface of the hole at levels of every 20 cm . The special snow sampler designed by Y. Mitsuta [1960] is used. The sampler is composed of three parts as shown in Fig. 1. The shape of the edge part is the same as that of the standard snow sampler of Kamuro-type (Tohoku Electric Power Co., et. al. [1959]). The diameter of the circular edge is 5.04 cm . The second part is measuring part, and has length of 15.0 cm , and the obtained volume of sampled snow is $30.0 \mathrm{~cm}^{3}$. The last part is the tail. This part is added in order to reduce compression of snow sample in the measuring part.


Fig. 1. Snow sampler.

In the measurement of snow density, the edge part puts perpendicular on the pre-determined position on the vertical snow wall, and beats into the wall by wood hammer until the tail goes in the snow, then, the whole body pull out, and after we confirm through the slit of the measuring part that the snow sample fill the sampler, the measuring part disconnect from the edge and tail parts. The snow core sticked out from measuring part is cut out by a knife. The weight of the snow sample in the measuring part of the sampler weigh by a spring balance. The density of the sample is obtained by divided the weight by the volume of the sample.

These procedures are successively carried out at the levels of every 20 cm through the whole depth, and the vertical distribution of deposited snow density is obtained.

The results are listed in Table 1 with some remarks.

## III. Discussion

The data given in Table 1 show that the vertical density distribution in deposited snow layer may be divided into two different types. The first

Table 1. The results of observations on the vertical density profile in snow layer.

| Survey | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Site | Kurobishi | Kurobishi | Goshiki-hara | Taira | Kariyasu | Kariyasu | Kariyasu |
| Data | 9, Jan. '61 | 10, Jan. '61 | 9, Apr. '61 | 9, Apr. '61 | 10, Apr. '61 | 10, Apr. '61 | 10, Apr. '61 |
| Time | $13^{\mathrm{h}} 30^{\mathrm{m}}-14^{\mathrm{h}} 30^{\mathrm{m}}$ | $14^{\mathrm{h}} 30^{\mathrm{m}}-16^{\mathrm{h}} 15^{\mathrm{m}}$ | $10^{\mathrm{h}} 10^{\mathrm{m}}-12^{\mathrm{h}} 25^{\mathrm{m}}$ | $14^{\mathrm{h}} 07^{\mathrm{m}}-15^{\mathrm{h}} 10^{\mathrm{m}}$ | $10^{\mathrm{h}} 10^{\mathrm{m}}-12^{\mathrm{h}} 25^{\mathrm{m}}$ | $11^{\mathrm{h}} 30^{\mathrm{m}}-12^{\mathrm{h}} 04^{\mathrm{m}}$ | $13^{\mathrm{h}} 45^{\mathrm{m}}-14^{\mathrm{h}} 30^{\mathrm{m}}$ |
| Above M.S.L. (m) | 1660 | 1660 | 2500 | 1380 | 1885 | 1750 | 1830 |
| Orographic Condition | North facing slope | North facing slope | Plain | Plain | Ridge | South facing slope | North facing slope |
| Weather | Cloudy | Snow | Drifting snow | Snow | Clear | Clear | Clear |
| Air Temp. $\left({ }^{\circ} \mathrm{C}\right)$ | 0.0 | $-1.0$ | -6.4 | $-2.3$ | 3.0 | 1.3 | 1.0 |
| $\qquad$ | 260 | 285 | 565 | 205 | 325 | 305 | 370 |
| $-20$ | 0.24 | 0.15 | 0.25 | 0.52 | 0.47 | 0.47 | 0.29 |
| 40 | 0.24 | 0.15 | 0.42 | 0.52 | 0.42 | 0.44 | 0.42 |
| 60 | 0.34 | 0.22 | 0.41 | 0.49 | 0.47 | 0.47 | 0.51 |
| 80 | 0.31 | 0.15 | 0.35 | 0.52 | 0.42 | 0.47 | 0.45 |
| 100 | 0.31 | 0.22 | 0.35 | 0.54 | 0.47 | 0.42 | 0.51 |
| 120 | 0.36 | 0.25 | 0.44 | 0.55 | 0.47 | 0.44 | 0.49 |
| 140 | 0.31 | 0.25 | 0.44 | 0.55 | 0.48 | 0.49 | 0.42 |
| 160 | 0.37 | 0.25 | 0.44 | 0.54 | 0.49 | 0.49 | 0.38 |
| 180 | 0.37 | 0.25 | 0.42 | 0.51 | 0.49 | 0.54 | 0. 48 |
| 200 | 0.44 | 0.36 | 0.44 | 0.48 | 0.52 | 0.54 | 0.49 |
| 220 | 0.44 | 0.36 | 0.44 |  | 0.52 0.49 | 0.49 | 0.47 0.48 |
| 240 | 0.44 | 0.39 0.39 | 0.45 0.46 |  | 0.49 0.48 | 0.54 0.54 | 0.48 0.48 |
| 260 280 |  | 0.39 0.39 | 0.46 0.49 |  | 0.48 0.48 | 0.54 0.44 | 0.48 0.48 |
| 300 |  | 0.3 | 0.49 |  | 0.51 | 0.54 | 0.49 |
| 320 |  |  | 0.48 |  | 0.45 |  | 0.49 |
| 340 |  |  | 0.48 |  |  |  | 0.49 0.49 |
| 360 380 |  |  | 0.49 0.51 |  |  |  | 0.49 |
| 400 |  |  | 0.51 |  |  |  |  |
| 420 |  |  | 0.52 |  |  |  |  |
| 4.10 |  |  | 0.52 |  |  |  |  |
| 160 |  |  | 0.52 |  |  |  |  |
| 480 |  |  | 0.49 |  |  |  |  |
| 500 |  |  | 0.51 |  |  |  |  |
| 520 |  |  | 0.51 |  |  |  |  |
| 540 |  |  | 0.49 |  |  |  |  |
| 560 |  |  | 0.51 |  |  |  | - |



Fig. 2. (a) Density profile in snow layer. Fig. 2. (b) Density profile in snow layer.
type is such that snow density gradually increases as it goes deep, as shown in Fig. 2 (a). The second is that the density is almost constant throughout the whole layer, as shown in Fig. 2 (b).

On the vertical distribution of deposited snow density, there is excellent work by K. Kojima 〔1957〕. The essential point of his theory is as follows.

Contraction strain $d s$ of a snow layer of which the thickness is $h$ at $t_{1}$ day after deposited, is defined by

$$
d s=-\left(\frac{d h}{h}\right)_{t_{1}}
$$

if we assume that $\left(\frac{d s}{d t}\right)_{t_{1}}$ is proportional to the weight $w(t)$ acts upon the snow layer

$$
\left(\frac{d s}{d t}\right)_{t}=\frac{\omega(t)}{\eta(t)}
$$

where, $\eta(t)$ is called the constant of compressive viscosity of snow. If water equivalent of individual snow layer is conserved throughout the period concerned, that is, there is no sublimation and no melting away, the relation between $\gamma$ and the snow density $\rho$ must be written as

$$
\begin{equation*}
\frac{\omega(t)}{\eta(t)}=-\left(\frac{d h}{h d t}\right)_{t_{1}}=\left(\frac{d \rho}{\rho d t}\right)_{t_{1}} \tag{1}
\end{equation*}
$$

and the experiments show

$$
\begin{equation*}
\eta(\rho)=\eta_{0}^{\prime} e^{k \rho} \tag{2}
\end{equation*}
$$

where $\gamma_{0}^{\prime}$ and $k$ are the constants.

From Eqs. (1) and (2), we obtain

$$
\omega(t) d t=\eta_{0}^{\prime} e^{t_{\rho}} \frac{d \rho}{\rho}
$$

then, we hawe

$$
\begin{equation*}
\int_{0}^{t_{1}} \omega(t) d t=\eta_{0}^{\prime} \int_{\rho_{0}}^{\rho} \frac{e^{k \rho}}{\rho} d \rho=\gamma_{0^{\prime}}\left\{E_{\iota}(k \rho)-E_{i}\left(k \rho_{0}\right)\right\} \tag{3}
\end{equation*}
$$

where, $E_{i}(k \rho)$ is exponential integral, and $\rho_{0}, \rho$ is the snow density at $t=0$, and $t_{1}$, respectively.

Since the lelt hand side of Eq. (3) is determined by the falling snow during the period from the time of which the layer appeared ( $t=0$ ) to the time in question, $\eta_{0}{ }^{\prime}, k, \rho_{0}$ and the day to day snow fall are given, we can calculate the density of snow which was appeared at the time $t=0$.

Now, the present author attempt to compare the typical example of our data listed in Table 1 and above mentioned Kojima's theory. To apply Kojima's theory, we must set up appropriate model of snow fall. We take up the data at "Goshiki-hara" as an example. On this example, the present author discussed before about temperature distribution in the snow layer [1963], and showed that at "Goshiki-hara", it is reasonable that the time of first appearance of snow cover assumes 20th December. Then we set 20th December as $t=0$. The date of $t_{1}$ is clearly 9 th April, which is observation is carried out. A problem arises here, that is, how assume the snow fall during the period from $t=0$ to $t=t_{1}$. Two assumpions may be done as follows:

Assumption (i). The whole water equivalent observed at 9th April ( $t=t_{1}$ ) come down equally in every day throughout that period.

Assumption (ii). The water equivalent accumulate equally day by day until some days before observed time, and after that time, there is no snow fall.

The present author calculate the snow density distribution under both assumptions, and on the second assumption, the time of the last snow fall set at the end of February.

In our calculation, following values are employed for the constants containing in Eq. (3) ; $\eta_{0}{ }^{\prime}=1.0$ dyne day $\mathrm{cm}^{2}, k=21 \mathrm{~g}^{-1} \mathrm{~cm}^{3}, \rho_{0}=0.05 \mathrm{~g} \mathrm{~cm}^{-3}$.

The former two values are obtained experimentally by K. Kojima about many snow samples at Hokkaido, the northern part of The Japan Islands, and it is considered that the meteorological conditions are nearly same as that of our sampling area. The snow density of $0.05 \mathrm{~g} \mathrm{~cm}^{-3}$ is adopted as the typical value of dried fresh snow.

The results of calculation are plotted in Fig. 3. In this figure, white cir-


Fig. 3. Density profile in snow layer.
Circle; observed data at the 9 th April.
Solid line; calculated profile at the 9 th April under the Assumption (ii).
Broken line; calculated profile at the 9 th April under the Assumption (i).
Dotted line; calculated profile at the end of February under the Assumption. (ii).
cles show the observed density at "Goshiki-hara", broken line shows calculated density distribution at the 9th April, under the above mentioned first assumption, solid line shows that at the same day under the second assumption and dotted line shows that at the end of February under the second assumption.

This figure shows the calculated density distribution under the second assumtion is nearly coincide with that of observed. However, some systematic departures are exist, especially in the surface layer, calculated values are less than observed ones. These departures may be resulted from following reason. The Kojima's theory may be properly applied under the condition of that the increase of the snow density only caused by compression due to deposited snow above the snow layer in question. But in our case, in the surface layer, increase of the snow density may occur not only by compression due to deposited snow above it but also by that due to wind pressure or by modification of the structure of snow due to melting. The second type of density distribution as shown in Fig. 2 (b) may occur under these circumstances.

Conclusively, actual vertical density distribution of deposited snow can be approximately explained by Kojima's theory, but as time goes by, after the snow fall come to end, other factors must be taken into account.

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## References

Kojima, K. Viscous compression of natural snow layers III. Low Temp. Sci., Ser. A, Vol. 16, P. 167, (1957). (in Japanese)
Mitsuta, Y. A practical method of snow survey for deep snow layer. Tech. Rep., Tech. Res. Lab. Kansai Electric Power Co., No. 11, P. 260, (1960). (in Japanese)
Sahashi, K. On the vertical temperature distribution in relatively deep snow layer. Special Cont., Geophy. Geophy. Inst., Kyoto Univ., No. 2, P. 435 (1963).
Tohoku Electric Power Co. and Sendai Meteorological Office. Report on hydro-meteorological survey in Tadami area (1959). (in Japanese)

