Special Contributions, Geophysical Institute, Kyoto University, No. 6, 1966, 193–200

A METHOD FOR THE DYNAMICAL STUDY OF THE UNDERGROUND ELECTRICAL STATE BY A NETWORK OBSERVATION OF GEOMAGNETIC VARIATIONS

By

Akira SUZUKI and Hiroshi MAEDA

(Received November 14, 1966)

Abstract

A consideration is made for the practical application of Weaver's formulae and for recording separately the external and internal parts of short-period geomagnetic variations in a region of local anomaly, and the possibility for detecting the magnetic effect which might be expected from a time-change in the underground electrical state is discussed. It is shown that the effect would be very small, but it might be possible to find out the efeect on the record if we employ magnetometers of very high sensitivity and the location of anomaly is not so deep.

1. Introduction

It has long been an aim of geomagnetic researchers to separate the geomagnetic field observed on the ground into parts of external and internal origin. There are two methods for the separation; analytical and numerical. The analytical method usually called "Spherical harmonic analysis" was initiated by Gauss [1838] and followed by a number of workers for the study of the geomagnetic field and its variations on a worldwide scale. The numerical method often called "Surface integral method" was first discussed by Vestine [1941] and developed by several authors.

In recent years, the surface integral method has been employed and further developed for the study of local anomalies of the geomagnetic field. Siebelt and Kertz [1957], for example, made it possible to separate the field along a certain straight line in a confined region. Their method has been extended into the three dimensional case by Weaver [1963], and his result seems to be most convenient for the practical use.

This paper proposes a method for recording separately the external and internal parts (and also the ratio of them) of short-period geomagnetic variations by a network observation in a region of local anomaly, by employing

Weaver's formulae, and then discusses the possibility for detecting the magnetic effect which might be expected from a time-change in the electrical state of the earth's interior.

2. Separation of Short-Period Geomagnetic Variations by a Network Observation

The short-period anomaly in the geomagnetic field is a fact that has often been reported its existence in places, and the fact is thought to be caused by some conductivity anomaly in the earth's interior. This fact may suggest that we should separate, first of all, the short-period geomagnetic variations into parts of external and internal origin, in order to discuss not only the underground electrical state but also the cause of the variations of external origin.

On this account we first discuss the method of separation in a region where a local anomaly is expected to exist, by utilizing Weaver's formulae. Weaver [1963] supposed the earth as half space. Right-handed Cartesian coordinates (x, y, z) are taken on the earth's surface to be northward (x), eastward (y) and vertically downward (z), the magnetic-field components are designated respectively by corresponding capital letters (X, Y, Z). Each component has its external part (X_e, Y_e, Z_e) and internal part (X_i, Y_i, Z_i), and these two parts are separated at a point (x_0, y_0) on the earth's surface. He introduced two integral operators (M_1 and M_2) :

$$M_1[U(x, y)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(x, y)\frac{x_0 - x}{[(x-x_0)^2+(y-y_0)^2]^{3/2}}dxdy \quad \cdots\cdots\cdots\cdots(1)$$

$$M_2[U(x, y)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(x, y)\frac{y_0 - y}{[(x-x_0)^2+(y-y_0)^2]^{3/2}}dxdy \quad \cdots\cdots\cdots\cdots(2)$$

where U indicates one of the three components (X, Y, Z). Using these operators, the X, Y and Z components at the point (x_0, y_0) are separated as follows:

$$X_e = \frac{1}{2}\{X(x_0, y_0) + M_1 Z(x, y)\}, \qquad X_i = \frac{1}{2}\{X(x_0, y_0) - M_1 Z(x, y)\}$$

$$Y_e = \frac{1}{2}\{Y(x_0, y_0) + M_2 Z(x, y)\}, \qquad Y_i = \frac{1}{2}\{Y(x_0, y_0) - M_2 Z(x, y)\}$$

$$Z_e = \frac{1}{2}\{Z(x_0, y_0) - M_1 X(x, y) - M_2 Y(x, y)\} \quad \cdots\cdots\cdots(3)$$

$$Z_i = \frac{1}{2}\{Z(x_0, y_0) + M_1 X(x, y) + M_2 Y(x, y)\}$$

It is easily seen from (1) and (2) that the denominator of the integrand is equal to the cube of the distance from a point (x, y), at which an area element $dxdy$ is taken, to the separating point (x_0, y_0) at which we want to separate the field-variations. The numerator in equation (1) indicates the distance of a point (x, y) from the y axis, and that in equation (2) is the distance from the x axis. Thus if, for simplicity, the point (x_0, y_0) is taken to be an origin of the coordinates, equations (1) and (2) are easily translated into the form in two-dimensional polar coordinates (r, θ) (see Fig. 1). The results are

$$M_1 U = -\frac{1}{2\pi}\int_0^\infty \int_0^{2\pi} U(r, \theta)\frac{\cos\theta}{r} dr d\theta \qquad \cdots\cdots\cdots\cdots(4)$$

$$M_2 U = -\frac{1}{2\pi}\int_0^\infty \int_0^{2\pi} U(r, \theta)\frac{\sin\theta}{r} dr d\theta \qquad \cdots\cdots\cdots\cdots(5)$$

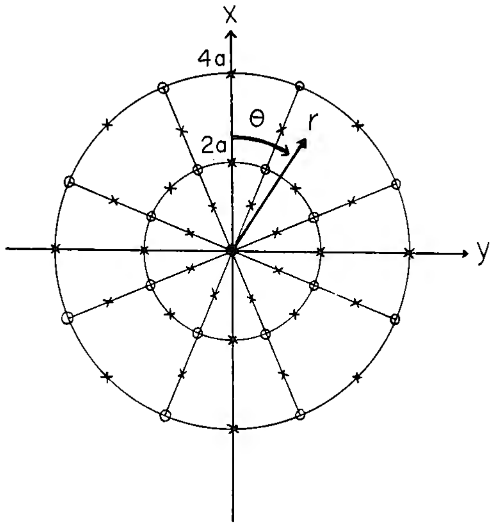

Fig. 1. Showing an observation network, where open circles indicate observing points, and crosses show points at which the fields are estimated from two neighbouring points.

We now intend to calculate this integral from the actual network of observations. An example of such a network is shown in Fig. 1, where open circles indicate observing points. The procedure of actual calculations is as follows: Taking the Z component as U, for example, and the separating point (x_0, y_0) to be the origin $(0, 0)$, we have approximately

$$M_1 Z = -\frac{1}{2\pi}\int_0^\infty \frac{1}{r}\Big[Z\Big(r, \frac{\pi}{8}\Big)\cos\frac{\pi}{8} + Z\Big(r, \frac{3\pi}{8}\Big)\cos\frac{3\pi}{8} + \cdots\cdots + Z\Big(r, \frac{15\pi}{8}\Big)\cos\frac{15\pi}{8}\Big] dr$$

Assuming that at points indicated by crosses in Fig. 1 the fields are taken to be an average of them at two neighbouring observation-points, that is, for example

$$Z\Big(2a, \frac{\pi}{4}\Big) = \frac{1}{2}\Big\{Z\Big(2a, \frac{\pi}{8}\Big) + Z\Big(2a, \frac{3\pi}{8}\Big)\Big\}$$

we have the final result as follows:

$$\begin{aligned} -M_1 Z = {} & 0.058\Big\{Z\Big(2a, \frac{\pi}{8}\Big) - Z\Big(2a, \frac{7\pi}{8}\Big) - Z\Big(2a, \frac{9\pi}{8}\Big) + Z\Big(2a, \frac{15\pi}{8}\Big)\Big\} \\ & + 0.020\Big\{Z\Big(4a, \frac{\pi}{8}\Big) - Z\Big(4a, \frac{7\pi}{8}\Big) - Z\Big(4a, \frac{9\pi}{8}\Big) + Z\Big(4a, \frac{15\pi}{8}\Big)\Big\} \\ & + 0.024\Big\{Z\Big(2a, \frac{3\pi}{8}\Big) - Z\Big(2a, \frac{5\pi}{8}\Big) - Z\Big(2a, \frac{11\pi}{8}\Big) + Z\Big(2a, \frac{13\pi}{8}\Big)\Big\} \\ & + 0.010\Big\{Z\Big(4a, \frac{3\pi}{8}\Big) - Z\Big(4a, \frac{5\pi}{8}\Big) - Z\Big(4a, \frac{11\pi}{8}\Big) + Z\Big(4a, \frac{13\pi}{8}\Big)\Big\} \quad \cdots\cdots(6) \end{aligned}$$

A similar expression for M_2Z is also easily obtained, and we can calculate X_e, X_i, Y_e and Y_i from (3), by using observed values $X(0, 0)$ and $Y(0, 0)$ at the origin. It is also possible to obtain Z_e and Z_i from M_1X and M_2Y in the same way, if necessary. These calculations would be automatically carried out by an appropriate electronic circuit, and we will be able to record separately the external and internal parts of the short-period geomagnetic variations.

From the result of continuous recording of X_i and X_e, for example, we can pick up an element of a particular period (T_p, say) of the geomagnetic variations by using a frequency analyser. Thus we will finally be able to record the ratio $X_i(T_p)/X_e(T_p)$ for the element of period T_p. It would be expected that if the electrical state of the earth's interior does not change with time, the ratio should be constant in time, within the limits of error. Thus the continuous recording of the ratio may enable us to detect any time-change in the underground electrical state on one hand, and to study quantitatively the external-origin part without any local anomaly of internal origin on the other.

3. Magnetic Effect of Induced Currents in the Earth

The problem detecting conductivity anomalies in the earth's interior by using induced currents has already been studied by Rikitake [1959, 1962]. He discussed the magnetic effect observed on the ground, when a conducting body is suddenly inserted in an earth of non-conducting, and showed that the effect is very small. We shall discuss another example of the magnetic effect of induced currents associated with a conductivity anomaly in a conducting earth. Some approximations are made for simplicity, one of which is the assumption that the background electric-field distribution does not change from that specified by skin effect, even when the conductivity in a limited region does change suddenly.

The earth is treated as a semi-infinite conductor the conductivity (σ_1) of which is 10^{-15} e.m.u. A left-handed Cartesian coordinates are taken on the earth's surface (see Fig. 2) ; x and y being northward and westward, respectively, and z vertically downward. Permeability (μ) is taken to be unity everywhere.

It is supposed that the external magnetic field which changes with time makes the electric field $E_g e^{i\omega t}$ in the x direction on the surface, where E_g is the field intensity at time $t=0$. Time factor $e^{i\omega t}$ is omitted hereafter for simplicity. The electric field $\vec{E}_0$ induced in the earth would be decreased exponentially downward for the sake of skin effect (see, e.g., Price [1965]), so that it is expressed in the form

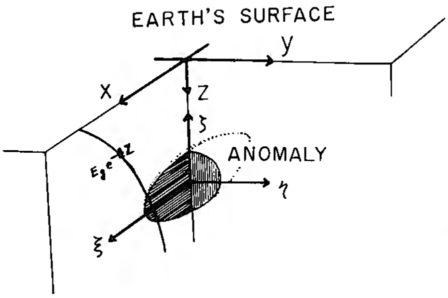

Fig. 2. Half-space earth and anomaly in it. Coordinate systems are also shown together with induced electric field.

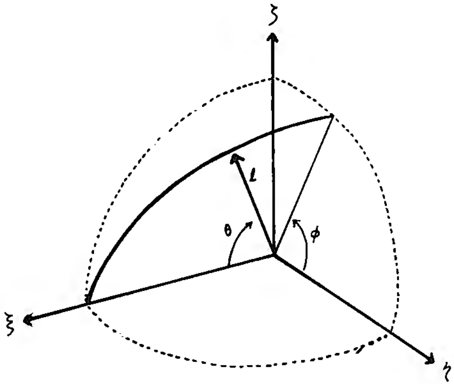

Fig. 3. Coordinate systems which have their origin at the center of the anomaly.

$$\vec{E}_0=(E_g e^{-kz},\ 0,\ 0) \qquad \cdots\cdots\cdots\cdots(7)$$

where

$$k=\sqrt{2\pi\sigma\omega\mu}$$

Now let us consider the case in which a spherical anomaly of conductivity σ_2 occurs at a depth z_0. The field $\vec{E}_p$ caused by this anomaly may have a potential Φ_p, i.e.,

$$\vec{E}_p=-\operatorname{grad}\Phi_p \qquad \cdots\cdots\cdots\cdots(8)$$

Here two coordinate systems, (ξ, η, ζ) and (l, θ, ϕ), having their origins at the center of the anomaly are introduced as in Fig. 2 and Fig. 3, where ξ is northward, η westward, and ζ upward. Then z and ζ has a relation $z=z_0-\zeta$, and $\vec{E}_0$ and $\vec{E}_p$ makes the total field $\vec{E}$. From (8) and $\operatorname{div}\vec{i}=0$, we have

$$\frac{\partial^2\Phi_p}{\partial\xi^2}+\frac{\partial^2\Phi_p}{\partial\eta^2}+\frac{\partial^2\Phi_p}{\partial\zeta^2}=0 \qquad \cdots\cdots\cdots\cdots(9)$$

and the solution of this equation is given by

$$\Phi_p=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(A_n{}^m+\frac{B_n{}^m}{l^n}\right)P_n{}^m(\theta)\begin{Bmatrix}\cos m\phi\\ \sin m\phi\end{Bmatrix} \qquad \cdots\cdots\cdots\cdots(10)$$

in polar coordinates. Coefficients ($A_m{}^n$'s and $B_m{}^n$'s) are unknown and these are decided from the following boundary conditions: i) Φ_p is finite at infinity ($l=0$ and $l=\infty$), ii) Φ_p is continuous at the surface ($l=L$) of the anomaly, and iii) the normal component of the total current density $\vec{J}$ is continuous at the surface of the anomaly. The approximation that $\exp(KL\sin\theta\sin\phi)$ is equal to unity is employed. If we take $L=10$ km and $\omega=2\pi/60$, for an example, we have

$$0.97\leqq\exp(KL\sin\theta\sin\phi)\leqq1.03$$

though this approximation might be somewhat rough when the anomaly is an ellipsoid of revolution as will be mentioned later.

Coefficients being obtained, the potential are given by

$$\begin{aligned} \Phi_{p1} &= \frac{L^3}{l^2} A_1^0 \cos\theta \qquad \text{(out of the anomaly)} \\ \Phi_{p2} &= A_1^0 l \cos\theta \qquad \text{(in the anomaly)} \end{aligned} \qquad \cdots\cdots\cdots\cdots(11)$$

where $A_1^0=(\lambda-1)/(\lambda+2)E_g\exp(-KZ)$, and λ is the ratio σ_2/σ_1. From (8) and (11) the current dnsity $\vec{J}_p$ caused by the anomaly are obtained, and the total current density becomes

$$\vec{J}=\begin{cases} J_{p1}+\sigma_1\vec{E} & \text{(out of the sphere)} \\ J_{p2}+\sigma_2\vec{E} & \text{(in the sphere)} \end{cases}$$

Then Biot-Savart's law gives the magnetic effect observed on the ground just above the anomaly, i.e.,

$$\vec{B}=\vec{H}=\int_0^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\vec{J}\times\vec{R}}{R^3}d\xi d\eta d\zeta \qquad \cdots\cdots\cdots\cdots(12)$$

where the distance from the point (ξ, η, ζ) in the earth to the point (x, y) on the ground are denoted by $\vec{R}$, its magnitude being R.

This calculation was carried out for anomalous (proper) currents by use of a high-speed computer, and the results are listed in Table 1, where the effect of the anomaly is normalized by the effect where no anomaly exists.

Table 1. Magnetic effect of the anomaly which is a sphere of radius 10 km and is located at depth 50 km. The internal field when no anomaly exists is supposed to be unity and the period $T=60$ sec.

At the point	λ	X	Y	Z
$x=0$, $y=0$	10	0.000	0.001	0.000
$x=0$, $y=0$	10^2	0.000	0.002	0.000
$x=20$ km, $y=20$ km	10	0.000	~0.001	0.000

Table 2. Magnetic effect at the point (0, 0) on the earth's surface when the anomaly is an ellipsoid of revolution and is located at depth 50 km. The internal field when no anomaly exists is supposed to be unity and $T=60$ sec. Case A; major axis is in the ξ direction. Case B; major axis is in the η direction.

Case	λ	X	Y	Z
A	10	0.000	0.018	0.000
A	10^2	0.000	~0.16	0.000
B	10	0.000	~0.004	0.000
B	10^2	0.000	~0.05	0.000

Table 3. Magnetic effect at the point (0, 0) on the earth's surface when the anomaly is an ellipsoid of revolution. The internal field when no anomaly exists is supposed to be unity and $T=1$ sec. Case C; the depth of the center of the anomaly is 50 km. Case D; the depth of the center of the anomaly is 30 km.

Case	λ	X	Y	Z
A & C	10	0.000	0.061	0.000
A & D	10	0.000	$\sim$0.17	0.000

Although the present argument is confined to the case when the anomaly is a sphere, it is easy to extend into the case of an ellipsoid of revolution. There are two cases; when the major axis is along the ξ direction, and when it is along the η direction. These results are also listed in Table 2 and Table 3.

In the above argument the ratio λ of σ_2 to σ_1 was taken as 10 or 100. This conductivity gap is thought to be probable as mentioned in a recent paper by Akimoto and Fujisawa [1965] in which they suggested an abrupt ascent of conductivity when Fe_2SiO_4 changes its phase from olivine to spinel in a heating process.

4. Concluding Remarks

Above estimate may suggest the difficulty to catch the effect of change in the underground electrical state by use of geomagnetic variations. However, this is only a model calculation, and therefore the possibility is never abandoned, because it is expected that the effect would be larger when the anomaly exists at depth shallower than 30 km, e.g., as in volcanic cases. Another possibility may arise when some electromotive forces are produced in the earth by different mechanisms, such as a chemical reaction. In this case, the associated electric current may directly be reflected in the magnetic field observed on the ground.

Finally, it should be pointed out that even if we cannot detect any effect of anomalous behaviour of the underground electrical state, the result of this observation would be very useful for the study of short-period geomagnetic variations of external origin, because most of discussions about these problems are still based on the result of the total (external plus internal) field.

References

Akimoto, S. and Fujisawa, H., 1965 ; Demonstration of the electrical conductivity jump produced by the olivine-spinel transition, J. Geophys. Res., 70, 443–450.

Gauss, C. F., 1838 ; Allgemeine Theorie des Erdmagnetismus, Resultate magn. Verein ; also Werke, 5, 121–193.

Price, A. T. 1965 ; Effects of induced earth currents on low-frequency electromagnetic oscillations, Radio Sci. J. Res. NBS/USNC-URSI, **69D**, 1161–1168.

Rikitake, T., 1959 ; The anomalous behaviour of geomagnetic variations of short period in Japan and its relation to the subterranean structure, The 9th report, Bull. Earthq. Res. Inst. Univ. Tokyo, **37**, 545–570.

Rikitake, T., 1962 ; Possibility of detecting the mantle low-velocity layer by geomagnetic deep sounding, Bull. Earthq. Res. Inst. Univ. Tokyo, **40**, 495–509.

Siebert, M. and Kertz, W., 1957 ; Zur Zerlegung eines lokalen erdmagnetischen Feldes in äusseren und inneren Anteil, Nachr. Akad. Wiss. Göttingen, Math-Phys., Kl. Abt. **IIa**, 87–112.

Vestine, E. H., 1941 ; On the analysis of surface magnetic fields by integrals, Part I, Terr. Magn. Atmos. Elect., **46**, 27–41.

Weaver, J. T., 1964 ; On the separation of local geomagnetic fields into external and internal parts, Zeitsch. f. Geophys., **30**, 29–36.

Note added in proof. Numerical coefficients in equation (6) are too small as compared with the results hitherto obtained by means of spherical harmonic analysis. The reason is that the numerical integration of equation (4) is made, as an example, for a limited area ($r \leq 4a$), instead of a wide area ($r \to \infty$), so that the coefficients should, in practical use, be decided experimentally after much observation are made.