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# ON THE DEFORMATION OF FROZEN SOIL

By

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**Abstract**

It has been reported by Y. Fukuo that, in order to examine the deformation of frozen soil, a compressional experiment was carried out and its result was analysed with reference to the Murayama and Shibata theory for the rheological characters of unfrozen clay.

However, at that time, the correspondence of strain to various compressive loads could not be examined. So, using frozen pieces moulded from the same soil block, this correspondence was examined. The result of the examination showed that the compressional strain was not proportional to the compressive load although it was considered that the load was smaller than the upper yield value.

## 1. Introduction

It is well known that the mechanical strength of soil is extremely dependent on the cohesion between soil particle and water. When the soil is frozen, its cohesion will be highly increased by the cementation of particles due to ice formation. However, water in very small pore, which is probably adsorbed on the particle surfaces, is not frozen easily and remains unfrozen (Williams [1964]). So, it is expected that unfrozen water has a significant effect on the deformation of frozen soil by its fluidity. Recently, in our country, the soil freezing method has been used in engineering projects (Takashi, *et al.* [1961]). Unfrozen water plays an important role in frost heaving (Jackson, *et al.* [1965]).

In construction work, the phenomena of soil freezing must be understood exactly and fully for safety reasons (Scheidegger [1961]). From this point of view, the deformation of frozen soil has been examined by tensile, compressional and flexural experiments. The result of the compressional experiment will be reported here.

## 2. Soil samples

A block of earth soil was sampled from the alluvial layer in Tokyo. The soil mechanical properties and particle size distribution of this block are shown in Table 1 and Fig. 1 respectively.

Table 1. Physical properties of undisturbed soil sample

| Properties of soil sample | Values |
|---|---|
| Bulk specific gravity | 1.92 gr/cm$^3$ |
| Specific gravity of soil particle | 2.69 gr/cm$^3$ |
| Water content in weight | 30.6 % |
| Particle content in volume | 55 % |
| Water content in volume | 45 % |
| Air content in volume | 0 % |
| Liquid limit | 40.4 % |
| Plastic limit | 21.8 % |

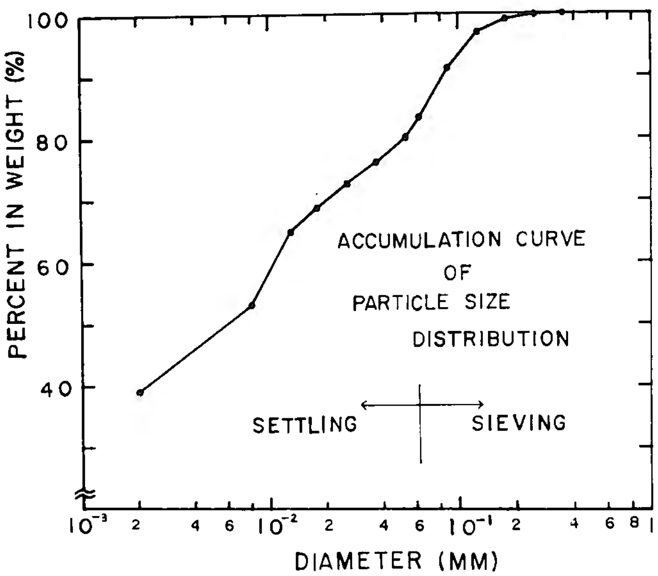


Fig. 1. Accumulation curve of particle size distribution. Diameters were determined by sieving mesh for diameter larger than 62$\mu$ and by settling velocity for diameter smaller than 62$\mu$.

The block was divided into several pieces with a wire saw. Each piece was moulded with a trimmer into cylindrical form 5 cm in diameter and 9 cm in height. A moulded piece was put in a case which had a cylindrical hole with the same diameter and 12 cm in height. This case was immersed in a brine box kept at a temperature of $-15\pm0.5$°C. After exactly five days of immersion, a frozen piece was taken out of the case. The piece expanded about 3 mm through freezing in an axial direction only because of casing restraint in the radial direction. Then, one end of it was scraped off so that the piece was again 9 cm in height. Soon after the end had been scraped off, the piece was coated with grease all over its surface and was set on the base of an axial compressive device, as seen in Fig. 2.

### 3. Instrument and method of experiment

The schema of the axial compressive device is shown in Fig. 2. This device was placed in a freezing box.
The weight ($W$) was hung on the end of a wire rope which was stretched out through the wall of the freezing box. The test piece was compressed in its axial direction by weight ($W$) multiplied thirty five fold through pulleys I, II (diameter ratio five) and lever arm ($B$) (arm length ratio seven).

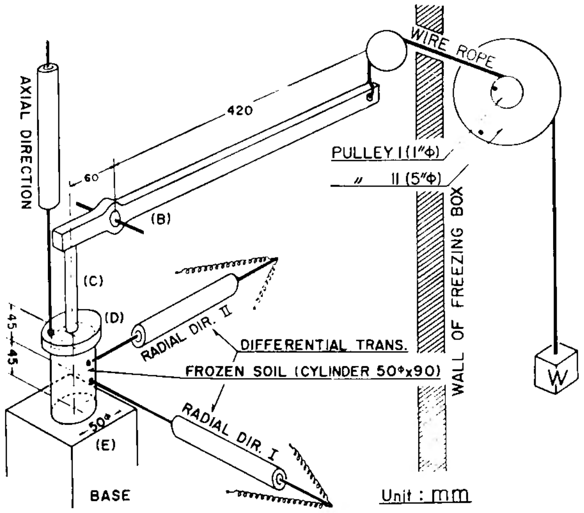


Fig. 2. The schema of axial compressive device.

The freezing box had a net capacity of 0.7 m in width, 0.8 m in depth and 0.9 m in height. The temperature in the freezing box was lowered by an electric refrigerator and was kept at −15°C by a thermostat and a stirring fan with an accuracy of ±0.5°C.

The deformation of the piece in the axial direction and the radial directions were converted to D. C. voltage through linear differential transformers and were detected on the dotting auto-recorder with an accuracy of ±0.02 mm.

### 4. Result of the experiment and some considerations

The result of the experiment is shown in Fig. 3. It is seen that the test piece underwent creeping deformation, that is, the strain increased gradually with time under constant load after the initial contraction at the instant of loading.

It was previously reported by Fukuo [1966a, 1966b] that, using another soil sampled in Tokyo, the deformation of frozen soil was measured and its result was examined with reference to the theory of Murayama and Shibata [1956] on the creeping phenomenon of unfrozen clay.

According to their theory, compressional strain $\varepsilon_1$ is represented by the equation

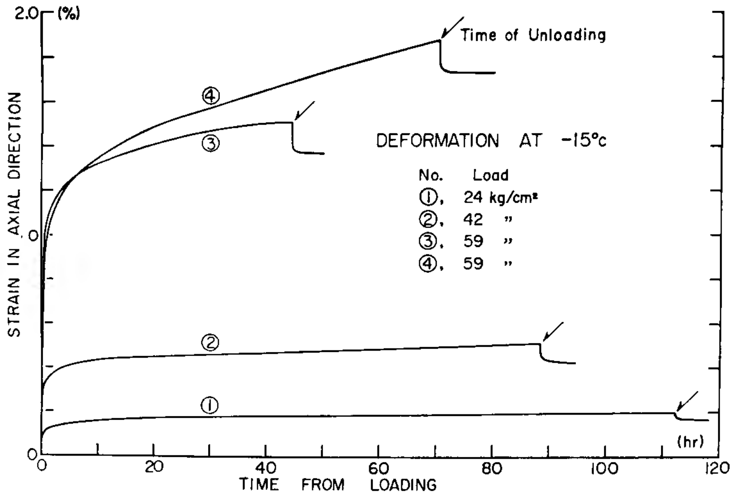


Fig. 3. Deformations of cylindrical frozen pieces moulded from undisturbed soil block.

$$\varepsilon_1=\frac{\sigma}{E_1}+\frac{\sigma-\sigma_0}{E_2}-\frac{\sigma-\sigma_0}{B_2E_2}\tanh^{-1}\left[\exp\left(-A_2B_2E_2t_1\right)\tanh\frac{B_2}{2}\right] \tag{1}$$

under constant compressive load $\sigma$ smaller than the upper yield value $\sigma_u$, where $E_1$ and $E_2$ are Young's moduluses, $A_2$ and $B_2$ are proper constants of clay depending on temperature, $\sigma_0$ is the lower yield value of clay and $t_1$ is the time of loading. When

$$\frac{B_2}{2}\gg 1 \text{ and } \frac{\sigma}{E_1}<\varepsilon_1<\frac{\sigma}{E_1}+\frac{\sigma-\sigma_0}{2B_2E_2}(2B_2-1),$$

eq. (1) can be represented approximately by

$$\varepsilon_1=a_1+b_1\log_e t_1$$

$$\begin{cases} a_1\equiv\dfrac{\sigma}{E_1}+\dfrac{\sigma-\sigma_0}{E_2}+\dfrac{\sigma-\sigma_0}{B_2E_2}\log_e\dfrac{1}{2}A_2B_2E_2 \\ b_1\equiv\dfrac{\sigma-\sigma_0}{B_2E_2}\,. \end{cases} \tag{2}$$

If constant load $\sigma$ exceeds the upper yield value $\sigma_u$, the strain $\varepsilon_1$ will increase along a curve concaved upward with the logarithm of time because the increasing rate of strain does not tend to zero at time infinity.

Moreover, at the recovery stage, the strain $\varepsilon_2$ is represented by the equation

$$\varepsilon_2=\frac{\sigma_0}{E_2}+\frac{2\sigma_0}{B_2E_2}\tanh^{-1}\left[\exp(-A_2B_2E_2t_2)\tanh\frac{B_2}{2\sigma_0}(\varepsilon_aE_2-\sigma_0)\right] \tag{3}$$

where $t_2$ is time of unloading and $\varepsilon_a$ is the strain $\left(\varepsilon_1-\frac{\sigma}{E_1}\right)$ at $t_2=0$. When

$$\frac{B_2}{2} \gg 1 \text{ and } \varepsilon_a > \varepsilon_2 > \frac{\sigma_0}{2B_2 E_2}(2B_2+1),$$

eq. (3) can be represented approximately by

$$\varepsilon_2 = a_2 - b_2 \log_e t_2$$

$$\left\{ \begin{array}{l} a_2 \equiv \dfrac{\sigma_0}{E_2} \\ b_2 \equiv \dfrac{\sigma_0}{B_2 E_2} \end{array} \right. \qquad (4)$$

In the previous examination, it was clearly seen that the strain had linear relation to the logarithm of time. Unfortunately, at that time, because there was only one undisturbed piece for each sampling depth, the correspondence of the strain to various constant loads could not be examined. Now we can see that correspondence. Test pieces Nos. 1 to 4 were moulded from the same block and so may be regarded as frozen soil samples having the same properties. In the same manner as in the previous examination, the results were plotted on a semi-logarithmic graph as shown in Fig. 4. It is found in the present examination that the strain increased along respective straight lines with logarithmic time for all pieces expect No. 4, and that all recovery strain decreased along respective straight lines. Using these results, the values of $a_1$, $b_1$ $a_2$ and $b_2$ in eqs. (2), (4) were determined by the least squares method for respective straight lines in Fig. 4 and were tabulated in Table 2.

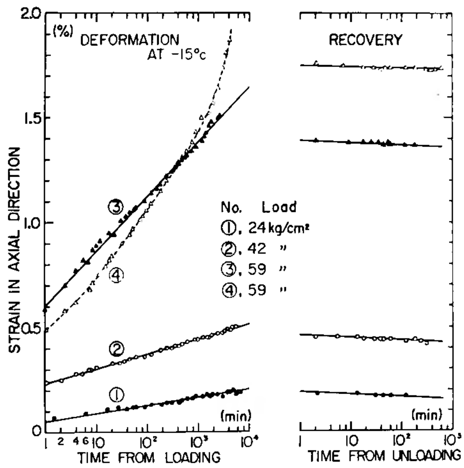

Fig. 4. Deformations of frozen pieces at temperature −15°C.

According to eqs. (2) and (4), the value of $a_1$ and $b_1$ will be proportional to the stress if the lower yield value $\sigma_0$ is neglisible compared with compressive load $\sigma$ and the value of $b_2$ will be constant in any recovery stage. Table 2 indicates that the values of $a_1$ and $b_1$ were not proportional to the compressive load, while the values of $b_2$ were the same for the piece No. 1 to 3.

In the piece No. 4, the strain increased along a curve concaved upward with the logarithm of time in spite of having the same load as No. 3. The compressive load 59 kg./cm² may exceed the upper yield value $\sigma_u$ for that piece and, owing to the recovery after this excessive load, the value of $b_2$ for No. 4 may be different from that for the other pieces.

Table 2. Calculated values of coefficients from the straight lines shown in Fig. 4

| Reference number | Compressive load $\sigma$ (kg/cm$^2$) | Loading | | Unloading | |
|---|---|---|---|---|---|
| | | $a_1 \times 10^3$ | $b_1 \times 10^4 \log_e 10$ | $a_2 \times 10_3$ | $b_2 \times 10^4 \log_e 10$ |
| ① | 24 | 0.48 | 3.9 | 1.9 | 1.2 |
| ② | 42 | 2.34 | 7.0 | 4.6 | 1.1 |
| ③ | 59 | 6.04 | 26.0 | 13.9 | 1.1 |
| ④ | 59 | — | — | 17.5 | 0.5 |

According to Vyalov [1955], the deformation of frozen soil is closely related to the behavior of unfrozen water. When frozen soil is compressed, concentration of stress originates at contact portions between the particle and ice and results in the fusion of ice through the change of its thermodynamic equilibrium condition. Successive compression squeezes this fused water from the film layer of unfrozen water and cements the particles again to ice. Through such continuous processes, frozen soil is deformed irreversibly. The values of $a_1$ and $b_1$ may be determined as the result of these complicated processes. Nevertheless, it is interesting that the strain has linear relation with the logarithm of time.

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