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ON THE DILUTION AREA OF EFFLUENT IN THE SEA

By

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Abstract

The dilution area of effluent is sometimes identified by its colored water spread on the sea surface. On the basis of the number of observations, the relationship between the volume of effluent and its dilution area had been introduced as the empirical formula (Nitta [1963]). The author discussed the formation of dilution area of effluent from the viewpoint of the entry mixing. Consequently, a formula explaining the relationship between the volume of effluent and its dilution area, was obtained and compared with the empirical one.

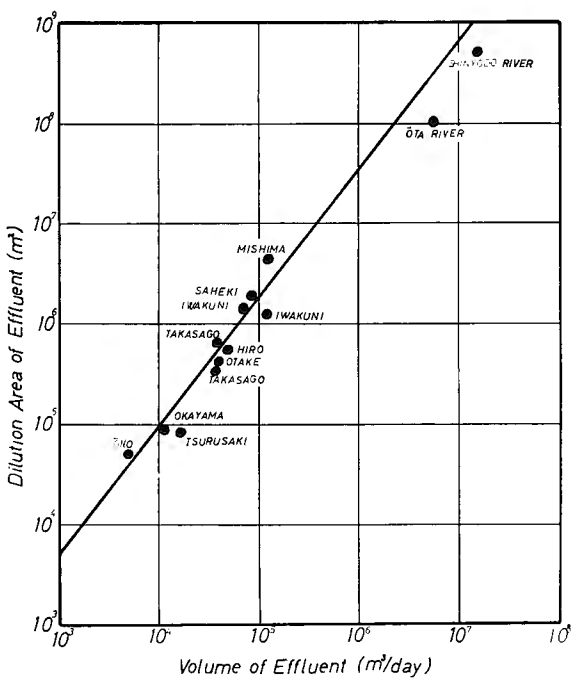

Fig. 1. Relationship between the volume of effluent and its dilution area (after Nitta, 1963).

1. Introduction

The effluent from industrial plants generally forms two layers system with the sea water in the sea and the primary dilution area of effluent is sometimes identified by its colored water spread on the sea surface. The extent of this colored water has been named the influenced sea area (or the dilution area) of the effluent by Nitta [1963]. On the basis of the number of observations, he showed that there exists the relationship between the volume of efflu-

ent, x (m^3/day), and the influenced sea area, y (m^2), as shown in Fig. 1, and its empirical formula as follows :

$$\log y = 1.2261 \log x + 0.0855.$$

He also stated that inside the vicinity of the front of influenced sea area, the effluent is usually diluted with 60~100 times of sea water.

For the estimation of distributions of effluent diluted in the sea, mixing processes of effluent have been commonly explained by the diffusion equations adequately modified according to the geographic conditions of the sea, discharging conditions of effluent, tidal condition, etc.. It seems that these explanations have considerably contributed to practical use. Nevertheless, some evidences obtained by the observations are not easily interpreted by advections and simple eddy diffusions. For instance, most of the cases, under the layer of dilution area of effluent, the sea water is little mixed with the efluent even in the neighborhood of the discharging mouth as shown in Fig. 2. On the other hand, it has been

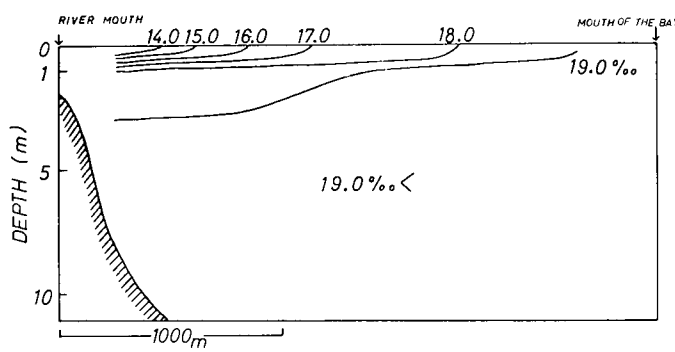

Fig. 2. A chlorinity distribution in the vertical section of the Nagashima Bay, Mie Prefecture.

observed that the effluent concentration decreases abruptly in the same region. (Hirano and Sugiura [1958]). It is suggested from these facts that an intensive mixing takes place between the effluent and the sea water until reaching a certain stage just after the effluent is discharged, associating with an upwelling of the sea water from the lower layer. This process seems to mainly contribute to the primary dilution of effluent. The structure of vertical circulation near the river mouth associated with the river water flushing has been proposed (Tully [1949, 1958], Cameron and Prichard [1963]). The existence of entrainment or intrusion from the lower layer near the discharging mouth was also ascertained by a simple model experiment (Hirano, *et al.* [1964]). Recently, the mixing near the mouth associating with the entrainment is sometimes called the

entry mixing.

In this paper, the discussion will be held on the formation of dilution area of effluent from the viewpoint of the entry mixing mechanism.

2. The concept on formation of dilution area of effluent

First of all, the effluent, whose density is smaller than that of the sea water, is supposedly discharged with a considerable speed initially at the mouth. Just after discharging, the effluent in the sea offers the turbulent flow and mixes with the sea water. As the density of this mixed water is still smaller than that of the water, the mixed water rises to the surface and flows forming a certain thickness of the layer. Consequently, the volume of mixed water increases more than the discharged one by the volume of the sea water added by the entry mixing. Therefore, it is expected that the mixed water will still continue to spread over the sea surface with a considerable speed to a certain extent. While the mixed water is spreading, the dilution of effluent may be mainly advanced by the sea water intruding from the lower layer, and by the usual horizontal eddy diffusion. At the primary stage of dilution, the entrainment is considered to predominate. However, amounts of the entrainment must be closely related with the stability of mixed water, which depends upon the velocity of the mixed water, the density difference between the sea water in the lower layer and the mixed water, etc.. In general, this stability of the mixed water can be well interpreted by the Froude Number F shown in the following formula :

$$F=\frac{u}{\sqrt{g\frac{\Delta\rho}{\rho}h}}\gtrless 1,$$

where u is the velocity of the mixed water, g the acceleration due to gravity, $\Delta\rho$ the density difference between the sea water in the lower layer and the mixed water, ρ the density of the sea water and h the thickness of the mixed water. Accordingly, when u decreases to the value near, or less than the original velocity of the surface water in the sea region, though $\Delta\rho$ may be fairly small, the turbulent state of the sea itself comes to contribute to the mixing as the eddy difusion, more than to the entrainment as a result of the instability of mixed water. Therefore, at this stage of mixing, it is expected that the dilution of effluent becomes much accelerated. On the other hand, before this stage of mixing, even if u is still quite large, the mixed water might veer from an unstable state to the stable one, in which the entrainment would not happen, and accordingly no more dilution would be advanced, depending on the value of F. Estimating from these considerations, it may be expected that the edge of the primary dilution area of effluent locates in the neighborhood of the sea region where the entry

mixing changes into the eddy diffusion process.

Now, let us consider the dilution process of effluent from one place (or one point) along the flat coast to the sea where no current and no tide essentially exist. It may be assumed as the first approximation that the effluent spreads making like a half circle with a certain thickness, and its center may be designated as the discharging mouth. If we assume that the dilution can be only advanced by the entry mixing as mentioned above, the velocity of mixed water, u_n, at an arbitrary place, will be shown as follows :

$$u_n = \frac{nV}{\pi L_n h}, \tag{1}$$

where n is the reciprocal of dilution rate of effluent, V the volume of effluent (L^3T^{-1}), L_n the distance from the discharging mouth to the place where the effluent was diluted with n times of the sea water, and h the thickness of mixed water.

In this case, it is easily understood that distributions of effluent concentration show cocentric circles, but the distribution pattern for the radius direction remains to be unknown because it depends on the entrainment of the sea water from the lower layer. However, we know empirically, with reference to the observations of distribution of waste concentration in the industrial water pollutions and others, that the value of n increases approximately in proportion to the distance L_n, as shown in the following equation,

$$L_n = \frac{n-1}{a-1}(L_a - L_1) + L_1 \approx \frac{n-1}{a-1} L_a, \tag{2}$$

where a is the another value of n, $L_1 \ll L_a$ and $n > 1$. The value n, also, is given as the following relation :

$$n = \frac{\rho - \rho'}{\Delta\rho} = \frac{\alpha}{\Delta\rho}, \tag{3}$$

where ρ' is the density of the effluent and $\alpha = \rho - \rho'$ is usually known.

As mentioned above, if it can be considered that the dilution of effluent is mainly advanced by the entry mixing, the following formula will be satisfied in the dilution area of effluent which has been proposed by Nitta :

$$F = \frac{u}{\sqrt{g\frac{\Delta\rho}{\rho}h}} \geqq 1. \tag{4}$$

Then, from (1), (2) and (3), the formula (4) is shown as follows :

$$F = \frac{nV(a-1)}{\pi L_a (n-1) h}\left(\frac{n\rho}{\alpha g h}\right)^{1/2} \geqq 1$$

or

$$\frac{\rho^{1/2}(a-1)}{\pi(\alpha g)^{1/2}h^{3/2}}\frac{n^{3/2}}{(n-1)}V \geqq L_a. \tag{5}$$

If at the edge of dilution area, $a=60\sim100$ is accepted according to the Nitta's report, the formula (5) will show the relationship between the distance to the edge, L_a and the volume of effluent, V. In addition, the formula (5) must be satisfied with any value of $1<n<a$. Then, n in the formula can be estimated 3 for practical use, because $\frac{n^{3/2}}{n-1}$ is the minimum in $n=3$.

Considering the extent of dilution area $A_a=\frac{1}{2}\pi L^2{}_a$, the following formula can be introduced from the formula (5),

$$A_a \leqq \frac{(a-1)^2\rho}{2\pi\alpha g h^3}\frac{n^3}{(n-1)^2}V^2$$

$$=\frac{27}{4}\frac{(a-1)^2\rho}{2\pi\alpha g h^3}V^2. \tag{6}$$

3. Comparison with the Nitta's formula

For the comparison with the Nitta's empirical formula stated above, we put $a=100$, $\rho\fallingdotseq1$, $g=9.8\,\mathrm{m/sec^2}$, and $\alpha=23\times10^{-3}$, assuming that the density of effluent is almost same with fresh water. Assuming $h=1\,\mathrm{m}$, the following formula is obtained,

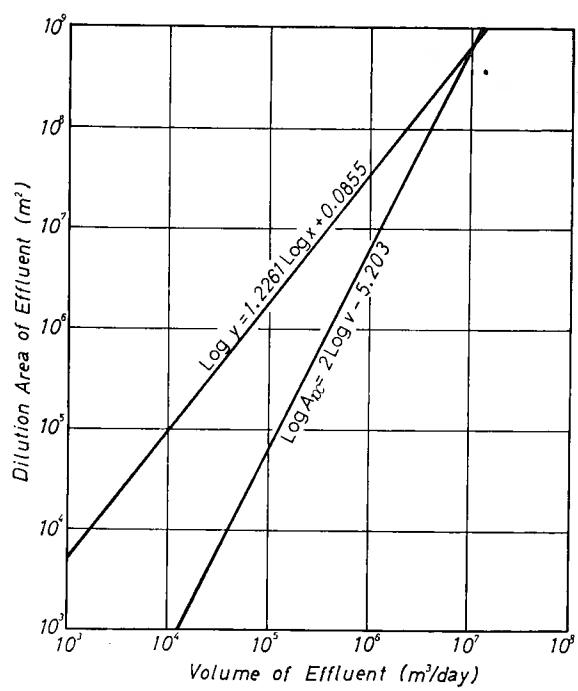

Fig. 3. Comparison between the formula (7) and the Nitta's empirical formula.

$$\log A_{100} \leqq 2 \log v - 5.203, \tag{7}$$

where A_{100} is the dilution area of effluent (m^2) and v the volume of effluent (m^3/day). The formula (7) is shown in Fig. 3 with the Nitta's empirical formula.

In the formula (5) or (6), it may be a problem how to estimate the thickness of mixed water, h. It should be deeply related with the volume of effluent and also might not be independent from the mixing process. This problem will remain to be researched in future as a very important one on the analysis of the mixing process of effluent. Moreover, some of the assumptions adopted in this treatment should be examined again and then the result obtained here was not so enough for explanations on the relationship between the volume of effluent and its dilution area observed as shown in Figs. 1 and 3. Nevertheless, the comparison between the formula (7) and Fig. 1 was considered to support the concept that the entry mixing played a important part at the primary stage of the dilution of effluent in the sea.

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