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ON THE TRANSPORT AND DISTRIBUTION OF GIANT SEA-SALT PARTICLES OVER LAND [I]

THEORETICAL MODEL

by

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Abstract

The transport and distribution of giant sea-salt particles inland from the coast are studied on the basis of the diffusion theory. The wind speed and the eddy diffusivity are assumed as constant. It is presumed that sea-salt particles originate only on the sea surface, and that vertical distribution of the particles at the coast is given in an exponential form. On land, a ground sink caused by sedimentation plus the impaction by trees and other obstacles on the ground surface is considered.

The outstanding result is that a maximum value in the vertical distribution of the number concentration of the giant sea-salt particles always appears at some height above the land, in agreement with the observational fact of the vertical distribution of the particles inland. Also, the number concentration on the ground surface inland decreases with the increasing magnitude of the impaction effect.

The effect of the impaction by trees and other ground obstacles is expressed by the impaction-sedimentation ratio. A preliminary estimate of the ratio, obtained by an application of the present model to a few observational data, indicates a value of the order of magnitude of 10 to 100.

1. Introduction

The sea-salt particles in the atmosphere are produced on the sea surface and transported inland by wind and tubulent diffusion. Over the ocean, observations of giant sea-salt particles were made by Woodcock [1953, 1957, etc.], Lodge [1955], Durbin and White [1961] and some other investigators. Toba [1965] discussed the average distribution of sea-salt particles over the ocean by synthesizing these observational data. According to him, the general feature of the vertical distribution of the number concentration of giant particles decreases exponentially with height, when the particles are being produced on the sea surface.

Over the land, the exponential vertical distribution at the coast changes into a more complicated feature with distance from the coast. Few studies have been made on this problem. Toba [1961] studied theoretically the transport of sea-salt particles from the coast inland and the distribution over the land on the basis of the diffusion equation. In that study, however, he presumed that the sink of particles on the ground surface consisted only of the sedimentation due to gravitational force. Therefore in Toba's 1961 model, a maximum in the vertical distribution of particle concentration lies always on the ground surface. Toba and Tanaka [1963] made an observation of the sedimentation of giant sea-salt particles at several stations along a transversal cross-section of Japan, in the direction of the winter monsoon flow during a few days. Applying Toba's 1961 theory to the observed horizontal distribution of the particles, they obtained a value of vertical eddy diffusivity in the atmosphere of the order of 10^2 to 10^3 cm^2 sec^{-1}. This too small value seemed to indicate that there was some discrepancy between Toba's 1961 model and the actual distribution of sea-salt particles.

In fact, according to the observation of the vertical distribution of the number concentration of the giant sea-salt particles made by Byers et al. [1957] in the central American continent, a striking characteristic of the vertical distribution over the continent is that the particle concentration has a maximum at some level between a few hundred meters and two kilometers or so, and that the maximum concentration is larger than that on the ground surface by a factor of 10 to 100. Also, at the height of maximum concentration, the number concentration changes little with distance from the coast. On the other hand, below the height of maximum concentration, there is a sharp decrease of the concentration towards the ground. Junge and Gustafson [1957], considering a stronger convection over the land, explained the fairly uniform vertical distribution by a non-steady diffusion equation. Their theory, however, can not explain the sharp reduction of the particle concentration at the lower level either.

Also, Toba and Tanaka [1965] observed the sedimentation of giant sea-salt particles by use of a dry-fallout gauge in Kyoto for three years from 1963 to 1965. In addition, for most of the first year, they observed the particle concentration near the ground by using an impactor once every afternoon. From those observations, it was concluded that the sedimentation of sea-salt particles obtained by the dry-fallout gauge represented the product of the terminal velocity, w, and the number concentration, θ_0, of the particles near the ground, $w\theta_0$, and that the total sedimentation of sea salt in Kyoto was 1.7×10^{-5} gm cm^{-2} year^{-1}. This total value is of the order of one-tenth of the total ground sink of sea-salt particles estimated from the usual salt concentrations of rain water and river water.

These facts concerning the sharp reduction in the concentration of giant sea-salt particles at the lowest level, and the small value of the sedimentation of sea salt, suggest that the ground sink consists not only of sedimentation, but also of the impaction by trees and other obstacles, and moreover, that the magnitude of impaction is much larger than that of the sedimentation.

The efficiency of impaction by ground obstacles was first introduced analytically by Toba [1965]. According to him, the total ground sink, F, in general, consists of the term, $w\theta_0$: the sedimentation caused by gravitational force, and the term, $\left(D\frac{\partial\theta}{\partial z}\right)_{z=0}$: the downward flux due to the vertical gradient of the particle concentration, where D represents the eddy diffusivity, i. e.,

$$F=w\theta_0+\left(D\frac{\partial\theta}{\partial z}\right)_{z=0}.$$

Considering that the second term is caused by the impaction by ground obstacles, he introduced a concept of the efficiency of impaction λ, defined by

$$\lambda u\theta_0=\left(D\frac{\partial\theta}{\partial z}\right)_{z=0},$$

where u represents the uniform wind speed.

In Toba's 1965 paper, the meaning of the facter λ was not mentioned clearly, but it may be explained in the following way. Consider a case where the wind blows through a layer of trees of the height, a, on the ground surface, and the trees have a surface area of leaves, s', per unit volume of space. It is considered that the efficiency of impaction is proportional to the product of the wind speed, u', the area of leaves, s', the coefficient of impaction, λ', and the number concentration of the particles near the ground, θ_0. In this case, the efficiency of impaction per unit land area will be expressed by something like

$$\int_0^a \lambda' s' u' \theta_0 dz.$$

The values of λ', s' and u' may change with time and space, respectively, and λ is considered a factor which includes all effects of λ', s' and u'. It may be meaningless or impossible to determine the impaction factor λ for a certain forest or for a building, but it must be possible to determine the average value of λ for a large scale ground structure such as an average Japanese one.

Toba [1965] discussed the vertical distribution of giant sea-salt particles far inland by taking account of this effect of ground impaction. His paper, however, gave only an approximate expression. In this paper is given an exact solution of the diffusion equation under the boundary conditions including λ as an arbitrary constant. It seems that the results of the calculation, which will be discribed in the following sections, are in good agreement with actual distribution obtained by observations over land. Moreover, making a comparison with the actual distribution, we must be able to determine the value of the impaction factor λ.

2. The equation of the diffusion of sea-salt particles and its solution

The characteristics of the field in which the diffusion process of giant sea-salt particles takes place are assumed as follows. (1) The sea-salt particles treated here are giant particles larger than 10^{-12} gm in the mass of salt; therefore the sedimentation velocity of the particles can not be disregarded. (2) The region treated here is a few thousand kilometers in the horizontal plane, from the coast inland, and few kilometers in the vertical. (3) The sea-salt particles which have been held in an equilibrium distribution over the ocean are transported inland by a uniform horizontal wind and vertical diffusion. (4) There is no source of sea-salt particles on land. Moreover there is no cloud, front, rain or snow in the atmosphere which either provides a source or a sink of the particles. (5) The eddy diffusivity and the wind speed are constant. (6) The ground surface is treated as uniform from the coast inland. The effect of the change in the ground structure might be included in the term of sink caused by the impaction by trees and other ground obstacles.

Take the coordinate origin at the coast, the x-axis in the direction of the wind, and the z-axis in the positive, upward direction. The number concentration of the particles of a certain class of weight per unit volume of the air, θ, is expressed, in a stationary condition, by

$$u\frac{\partial\theta}{\partial x}=w\frac{\partial\theta}{\partial z}+D\frac{\partial^2\theta}{\partial z^2}, \tag{1}$$

where u is the wind speed, w is the downward velocity of the particles and D the eddy diffusivity in the atmosphere. Boundary conditions may be written as,

$$\left.\begin{array}{ll}\theta=\theta_{00}\exp\left(-\frac{w}{D}z\right) & \text{at } x=0\\ D\frac{\partial\theta}{\partial z}=\lambda u\theta & \text{at } z=0\\ \theta=0 & \text{at } z=\infty\end{array}\right\}, \tag{2}$$

where θ_{00} represents the value at $z=0$ and $x=0$. For the sake of simplicity, we introduce dimensionless parameters, Θ, ξ, ζ and γ, instead of θ, x, z and λ, which are defined by

$$\Theta=\frac{\theta}{\theta_{00}},\quad \xi=\frac{w^2x}{4Du},\quad \zeta=\frac{wz}{2D}\quad\text{and}\quad \gamma=\frac{\lambda u}{w}.$$

It should be noted here that γ is expressed as the ratio between the ground sink due to the impaction by ground obstacles and that due to the sedimentation of the particles, and we will term γ the impaction-sedimentation ratio.

The solution of Eq. (1) under the boundary conditions (2) is expressed by

$$\Theta = e^{-2\zeta} - \int_0^{\xi}\Big[\frac{2(1+\gamma)}{\sqrt{\pi\xi}}e^{-\left(\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\right)^2} \\ -2(1+\gamma)(1+2\gamma)e^{2\gamma\{\zeta+2(1+\gamma)\xi\}}\operatorname{erfc}\Big\{(1+2\gamma)\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\Big\}\Big]d\xi, \tag{3}$$

where

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-u^2}du.$$

For the case $\gamma=0$, Eq. (3) is the same as Toba's 1961 solution. By carrying out the integration in Eq. (3), we finally obtain the following solutions. For the case $\gamma \neq 0$,

$$\Theta = \frac{1}{2}\operatorname{erfc}\Big(\sqrt{\xi}-\frac{\zeta}{2\sqrt{\xi}}\Big)e^{-2\zeta} - \frac{1}{2}\Big(1+\frac{1}{\gamma}\Big)\operatorname{erfc}\Big(\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\Big) \\ +\Big(1+\frac{1}{2\gamma}\Big)e^{2\gamma\{\zeta+2(1+\gamma)\xi\}}\operatorname{erfc}\Big\{(1+2\gamma)\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\Big\}. \tag{4}$$

When the value of $(1+2\gamma)\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}$ is large, Eq. (4) can be approximately expressed by

$$\Theta \cong \frac{1}{2}\operatorname{erfc}\Big(\sqrt{\xi}-\frac{\zeta}{2\sqrt{\xi}}\Big)e^{-2\zeta} - \frac{1}{2}\Big(1+\frac{1}{\gamma}\Big)\operatorname{erfc}\Big(\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\Big) \\ + \frac{(2\gamma+1)}{2\sqrt{\pi}\gamma\Big\{(1+2\gamma)\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\Big\}}e^{-\left(\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\right)^2}. \tag{5}$$

If $(1+2\gamma)\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}>2$, the error contained in Eq. (5) is within 1/64, and if $(1+2\gamma)\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}>1.5$, the error is within 1/15. For the case $\gamma=0$,

$$\Theta = \frac{1}{2}\operatorname{erfc}\Big(\sqrt{\xi}-\frac{\zeta}{2\sqrt{\xi}}\Big)e^{-2\zeta} + \Big(\frac{1}{2}+\zeta+2\xi\Big)\operatorname{erfc}\Big(\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\Big) \\ -\frac{2}{\sqrt{\pi}}\sqrt{\xi}\,e^{-\left(\sqrt{\xi}+\frac{\zeta}{2\sqrt{\xi}}\right)^2}. \tag{6}$$

3. Horizontal distribution of the particle concentration near the ground surface

Since ground observation is much easier to make than aerial observation, and more data are available concerning the particle concentration near the ground surface, it would be of special importance to study the horizontal distribution of the particle concentration on the ground by the present model.

Number concentration of the particles on the ground $\zeta=0$ is obtained from Eqs. (4), (5) and (6), as follows. For $\gamma \neq 0$

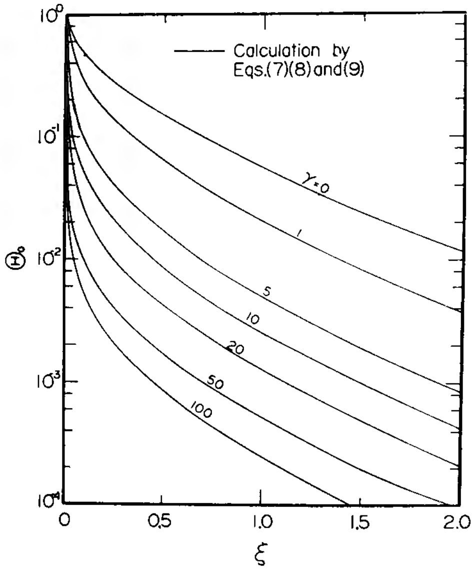

Fig. 1. Some examples of the horizontal distribution of Θ_0 for several values of γ, calculated from Eqs. (7), (8) and (9).

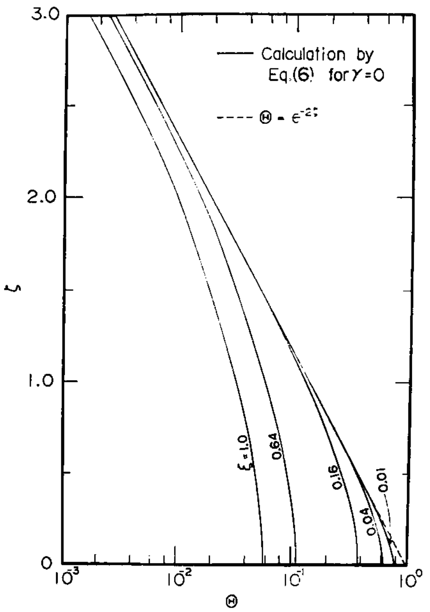

Fig. 2a. Changes in the vertical distribution of Θ with ξ for $\gamma=0$ (no impaction), calculated from Eqs. (4), (5) and (6). The broken line : $\Theta=\exp(-2\zeta)$ expresses the distribution at the coast.

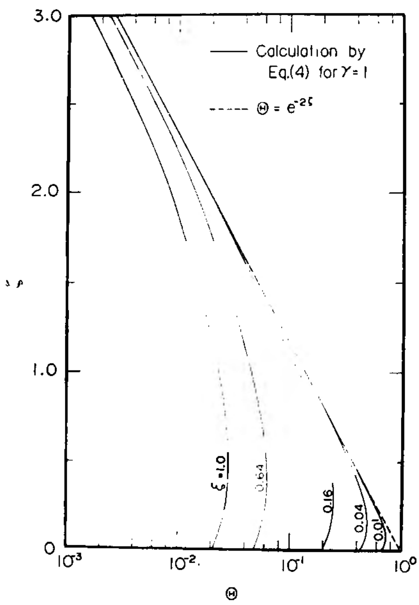

Fig. 2b. Ditto, $\gamma=1$.

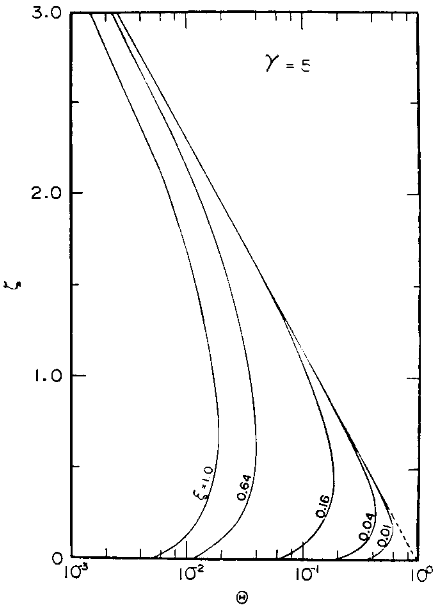

Fig. 2c. Ditto, $\gamma=5$.

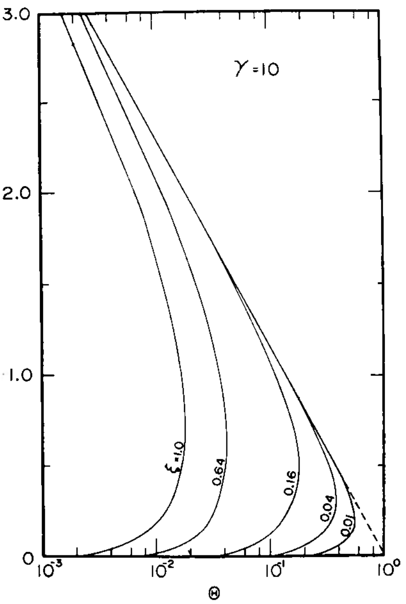

Fig. 2d. Ditto, $\gamma=10$.

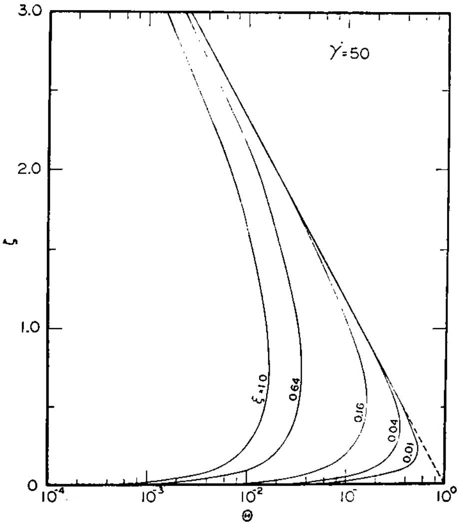

Fig. 2e. Ditto, $\gamma=50$.

$$\Theta_0=\left(1+\frac{1}{2\gamma}\right)e^{4\gamma(1+\gamma)\xi}\mathrm{erfc}\{(1+2\gamma)\sqrt{\xi}\}-\frac{1}{2\gamma}\mathrm{erfc}\,(\sqrt{\xi}). \tag{7}$$

When the value of $(1+2\gamma)\sqrt{\xi}$ is large, we may use an approximate expression as in Eq. (5):

$$\Theta_0\cong\frac{1}{2\gamma}\left\{\frac{e^{-\xi}}{\sqrt{\pi\xi}}-\mathrm{erfc}\,(\sqrt{\xi})\right\}. \tag{8}$$

For the case $\gamma=0$,

$$\Theta_0=(1+2\xi)\mathrm{erfc}(\sqrt{\xi})-\frac{2}{\sqrt{\pi}}\sqrt{\xi}\,e^{-\xi}. \tag{9}$$

In Fig. 1 is shown the relation between Θ_0 and ξ for several values of γ calculated from Eqs. (7), (8) and (9). The ordinate of this figure is shown in a logarithmic scale. As seen from Fig. 1, Θ_0 decreases as ξ increases or as γ increases, namely, as the distance from the coast, x, increases or the effect of the impaction increases for the same conditions of u, D and w. Θ_0 decreases strikingly near the coast especially when γ is large.

Eq. (8) indicates that the particle concentration on the ground far inland is inversely proprotional to γ.

4. Vertical distribution of the particle concentration

In Figs. 2a through 2e are illustrated the vertical distributions of the number concentration of the particles over land, calculated from Eqs. (4) and (6), for various values of ξ for several values of γ. In Fig. 3 are shown the vertical distributions for several values of γ for $\xi=0.16$. The scale for Θ is logarithmic. Broken lines express the vertical distribution at the coast: $\Theta=\exp(-2\zeta)$. As seen from Figs. 2a through 3, the following items are made clear, for the condition of $\gamma\neq 0$, namely, when there is an effect of impaction.

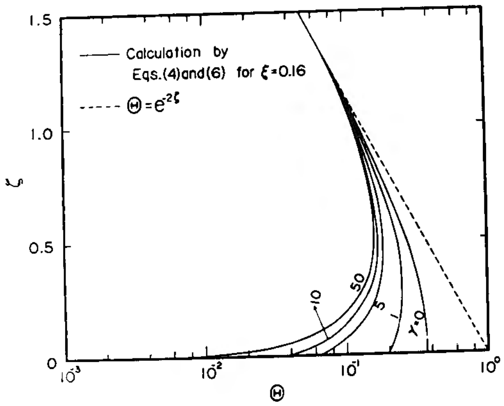

Fig. 3. Some examples of changes in the vertical distribution of Θ with γ for a given value of ξ $(=0.16)$, calculated from Eqs. (4) and (6). The broken line: $\Theta=\exp(-2\zeta)$ expresses the distribution at the coast.

(1) The height, ζ_m, where the maximum concentration lies, appears always at some height, although it exists on the ground surface in the case of $\gamma=0$.

(2) Above the height ζ_m, the vertical gradient of the particle concentration is small especially for the larger ξ.

(3) The particle concentration decreases sharply with decreasing height near the ground surface.

(4) The height of maximum concentration, ζ_m, becomes higher with increasing ξ, and with increasing γ, and it seems to tend to a certain height.

(5) As a general feature, the distribution of particle concentration is vertically fairly constant far inland, except near the ground. In other words, the shape of the vertical distribution far inland approximates to an inverse "L" shape, as a result of the lowering of the particle concentration near the ground due to the effect of the impaction.

5. Discussion

In order to make further comparison with actual observational data easier, the results of this model are now discussed by actual scales of u, D, w, x, and z. Let us use values of $u=10^3$ cm sec^{-1}, $D=10^5$ cm^2 sec^{-1} and take w as the terminal velocity of the particles in the atmosphere of 85% in the relative humidity, as an example. An example of the calculated values of $\xi=\frac{w^2x}{4Du}$ and $\zeta=\frac{wz}{2D}$ is shown in Table 1. In Table 2 is tabulated the change in Θ_0 with γ for two cases: $x=100$ km and $x=500$ km (cf. Fig. 1). From Table 2, it is found that the number concentration of sea-salt particles smaller than about 10^{-11} gm does not change from the coast far inland, if the impaction-sedimentation ratio

γ is small ; on the other hand, the number concentration of the particles of about 10^{-8} gm at the distance of 500 km from the coast is only about 0.1% of that at the coast, even if there is no sink due to the effect of impaction. Furthermore, if the sink due to impaction is much larger, the particle concentration decreases to about one-tenth or less of that for $\gamma=0$.

Table 1. Some examples of the values of ξ and ζ calculated from $\xi=\frac{w^2x}{4Du}$ and $\zeta=\frac{wz}{2D}$, for the case $u=10^3$ cm sec^{-1} and $D=10^5$ cm^2 sec^{-1}

Weight of dry particle (gm)	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}
Terminal velocity w for 85% RH (cm sec^{-1})	0.0125	0.0581	0.269	1.25	5.81
x=100km	$\xi=3.9\times10^{-6}$	8.4×10^{-5}	1.8×10^{-3}	3.9×10^{-2}	8.4×10^{-1}
500	2.0×10^{-5}	4.2×10^{-4}	9.1×10^{-3}	2.0×10^{-1}	4.2×10^{0}
z=1km	$\zeta=6.3\times10^{-3}$	2.9×10^{-2}	1.3×10^{-1}	6.3×10^{-1}	2.9×10^{0}

Table 2. Some examples of the relations between Θ_0, x and γ, obtained from Fig. 1, for the case $u=10^3$ cm sec^{-1} and $D=10^5$ cm^2 sec^{-1}

Weight of dry particle(gm)		10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}
x=100km	$\gamma=0$	$\Theta_0=1.0$	0.98	0.91	0.62	0.073
	10	0.95	0.80	0.43	0.098	0.0035
	100	0.58	0.24	0.062	0.011	0.00035
x=500km	$\gamma=0$	$\Theta_0=0.99$	0.96	0.80	0.33	0.0010
	10	0.90	0.64	0.22	0.027	0.000
	100	0.48	0.12	0.025	0.0027	0.000

In order to know the height of maximum concentration, the values of ζ_m and $\Theta_m/\Theta_0(=\theta_m/\theta_0)$ are calculated from Fig. 2 and tabulated in Table 3 for some interesting cases, where Θ_m/Θ_0 is the ratio of the maximum concentration to the concentration on the ground surface. If we take the example of a particle of 10^{-9} gm in the mass of salt, a distance near the coast of $\xi=0.01$ corresponding to $x=25$ km, and the effect of impaction of $\gamma=50$, then $\zeta_m=0.22$ and $\Theta_m/\Theta_0=11$ are obtained from Table 3, indicating that the maximum concentration appears at 350 m and the maximum concentration is eleven times larger than the concentration on the ground surface. Furthermore, if we take the example of a distance far inland of $\xi=1.0$ corresponding to $x=2500$ km, and $\zeta_m=0.75$ indicating $z_m=1.2$ km, and $\Theta_m/\Theta_0=36$ are obtained.

These results, namely that a maximum concentration appears at a height from a few hundred meters to a few kilometers, and that the ratio of the maximum concentration to the concentration on the ground surface is about 40, are in good agreement with the observational evidence synthesized by Toba [1965] based

Table 3. Some examples of the relations between ζ_m, Θ_m/Θ_0, γ and ξ obtained from Fig. 2, for the case $u=10^3$ cm sec^{-1} and $D=10^5$ cm^2 sec^{-1}

ξ	0.01	0.04	0.16	0.64	1.0
$\gamma=5$	$\zeta_m=0.18$	0.25	0.42	0.58	0.68
10	0.21	0.32	0.50	0.65	0.71
50	0.22	0.37	0.55	0.70	0.75
$\gamma=5$	$\Theta_m/\Theta_0=1.7$	2.5	3.1	3.7	3.9
10	2.2	3.8	5.6	7.2	9.5
50	11	12	21	33	35

on observations by Byers et al. [1957], Reitan and Braham [1954], Podzimek and Černoch [1961] and Mészaros [1964]. We have no knowledge *a priori* about the magnitude of the impaction-sedimentation ratio, γ. The problem of determining a more definite value of γ from observations of the vertical and horizontal distribution of the particle concentration is left for the future. However, the above-mentioned agreement of the calculation for $\gamma=50$ with the observational evidence seems to imply that the magnitude of γ is something around 10 to 100. Also from the data of the decrease of the particle concentration on the ground with distance inland obtained by Toba and Tanaka [1963], we may get the value of γ of the order of 50 to 100, in agreement with the above-mentioned value, if values of $w=1$ cm sec^{-1}, $D=10^5$ cm^2 sec^{-1} and $u=10^3$ cm sec^{-1} are used again.

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