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A DIRECT MEASUREMENT OF VERTICAL TRANSPORT OF TURBULENT KINETIC ENERGY IN THE AIR LAYER NEAR THE GROUND WITH SONIC ANEMOMETERS

By

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Abstract

Direct measurements of the vertical flux of turbulent kinetic energy near the ground by sonic anemometers were made, and by their results its divergence was estimated. According to the preliminary results obtained in this experiment, it might be concluded that the vertical divergence of tulbulent kinetic energy should not be neglected in the discussion of energy balance equation.

1. Introduction

Generation and transportation of kinetic energy are the main controlling factors of the field of the atmospheric motion. However, their direct measurements are rather troublesome, especially, in the turbulent flow near the ground. And the observational verification of the energy balance in the atmospheric boundary layer has been one of the important but less troden problems in micrometeorology.

In this paper, the preliminary results of an attempt of direct measurement of kinetic energy and its generation and transport by the use of the recently developed sonic anemometers are discussed.

2. Kinetic energy, its generation and transport

Kinetic energy of a unit density of air, E is shown as

$$E=\frac{1}{2}\{(\bar{u}+u')^2+v'^2+w'^2\}, \qquad (1)$$

where u is the wind velocity component along the mean wind direction, v the lateral wind velocity component and w the vertical wind velocity component, respectively. And the bar denotes the time mean value and the prime the deviation from the time mean value.

The vertical transport of kinetic energy, F is then given by

$$F=\frac{1}{2}\overline{(\rho w)'\{(\bar{u}+u')^2+v'^2+w'^2\}}, \tag{2}$$

where ρ is density of the air. Expanding the Eq. (2), the following relation is obtained:

$$F=\bar{u}\overline{(\rho w)'u'}+\frac{1}{2}\overline{(\rho w)'(u'^2+v'^2+w'^2)}. \tag{3}$$

The first term of right hand side is the kinetic energy flux of the mean flow, which is generally downward near the surface and the second term is the flux of turbulent kinetic energy, e_3, defined by

$$e_3=\frac{1}{2}(u'^2+v'^2+w'^2). \tag{4}$$

Unlike other physical entities such as momentum, sensible heat and water vapor, turbulent kinetic energy is not always expected to show constancy of flux within surface boundary layer but produced or dissipated in every layer.

The production of turbulent kinetic energy, e_3, in the atmosphere is not only caused by the mechanical actions but also by thermal and other causes. The mean rate of local change of turbulent kinetic energy per unit density has been investigated by Calder [1949], whose conclusion is as follows,

$$\frac{\partial e_3}{\partial t}=-\overline{u'w'}\frac{\partial \bar{u}}{\partial z}+\frac{g}{\bar{T}}\overline{w'T'}-\frac{\partial \overline{e_3 w'}}{\partial z}-\frac{1}{\bar{\rho}}\frac{\partial \overline{p'w'}}{\partial z}-\varepsilon, \tag{5}$$

where $\bar{T}$ and $\bar{\rho}$ are the mean values of temperature and density of the air respectively, T' and p' the fluctuation from mean state of temperature and atmospheric pressure, ε the rate of viscous energy dissipation.

The first term of the right hand side is a transformation from kinetic energy of mean motion to that of the turbulent motion. The second term is the work of the gravitational field on the turbulent flux of mass. The third term is the vertical divergence of turbulent kinetic energy. The fourth term represents the mean rate of the work done by the stresses on turbulent motion. The last term, ε, is the dissipation rate of energy by molecular viscosity.

The order of the magnitude of the terms on the right hand side of Eq. (5) was roughly checked by R. J. Taylor [1952] and he concluded that in neutral conditions, the first and the last term are fairly large and that other terms can be neglected. But recent studies by Panofsky [1962] and Cramer [1962] showed that the second and third terms may become large and can not be neglected especially in higher levels (i.e. 20 m to 100 m). But the fundamental studies on the estimation of these terms are lacking.

As the third and fourth term of the Eq. (5) are quite difficult to evaluate in field measurement, most of the reasoning are based on the assumption in any forms. Thus, to discuss the kinetic energy balance, we should have exact knowledge on all of these terms. But if we know one of two, the rest can be estimaied assuming stationarity from Eq. (5). From these point of view, the direct estimation of the magnitude of the third term was attempted in this study.

3. The method of observation

The observation were made over sea at the Shirahama Oceanographic Tower Station in summer of 1966 and over land at the Shionomisaki Wind Effects Laboratory of Kyoto University in the end of 1966.

We measured the fluctuation of wind components by three sonic anemometers in various ways to obtain the desired quantities which, were usually mounted at the height of 1.5 m or 3 m on a portable tower. Simultaniously, with the fluctations, mast profile observations were carried out, which included measurement of wind speed and temperature in the layer up to a height of 6 m. These instruments were installed as shown in Fig. 1.

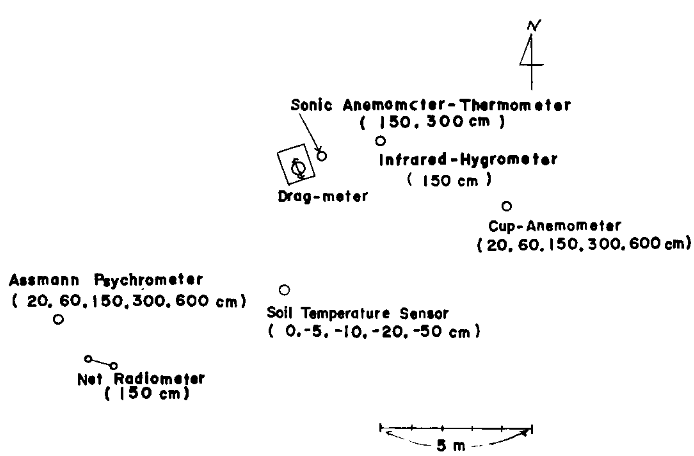

Fig. 1. Map showing the instollation of the instruments. Height of instruments is shown in brackets.

4. Results and the discussion

Thirteen cases out of eleven runs were analyzed. Most of the data were obtained from the measurements at Shionomisaki during the relatively strong north-westerly winds in winter except two cases at Shirahama (Run 4A, B). Selection was made 0.2 seconds as averaging time and 5 minutes or 3 minutes as a sampling duration in the process of the analysis.

Weather conditions during the experiment are summarized in Table 1. The thermal stratifications were ranged from slight lapse to slight inversion, but

Table 1. Gross weather conditions and gross turbulent statistics

Run No.	9A	9B	10	11	12	13	22	23	25	26	29	4A	4B	Mean
Time	09.45–09.48	09.45–09.48	11.51–11.54	12.27–12.32	13.14–13.19	14.01–14.06	12.26–12.46	15.46–16.01	12.21–12.15	23.28–23.31	15.23–15.26	17.29–17.34	17.29–17.34	
Height (m)	3.0	1.5	3.0	1.5	3.0	1.5	3.0	3.0	3.0	3.0	3.0	2.0	9.0	
Aver time in sec	0.2	0.2	0.2	0.2	0.2	0.2	0.8	0.8	0.2	0.2	0.2	0.2	0.2	
Weather	3.0	3.0	3.6	3.8	4.0	4.4	4.8	4.6	4.7	4.7	7.0	2.4	2.4	
Wind speed (m/sec)	3.4	3.4	4.5	4.7	4.8	5.5	6.0	5.7	5.7	5.7	7.8	2.5	2.5	
Temperature difference $T_{6.0m}-T_{1.5m}$ (c)	0.04	0.04					−0.31	−0.12	−0.21	0.30	0.06			
Richardson number	0.00	0.00					−0.05	−0.02	−0.02	0.04	0.01			
u_* (cm/sec)	29.2	29.2	44.9	32.8	33.7	40.2		58.1	77.5	50.8	51.7			
σ_u (cm/sec)	104.9		101.5	112.2	75.9	110.4		140.7	237.4	133.9	139.8			
σ_v (cm/sec)								130.8	158.4	103.9	120.5			
σ_w (cm/sec)	44.4		45.8	32.8	35.4	41.4	87.2	89.6	114.5	54.1	74.5			
σ_u/u_*	3.59		2.26	3.42	2.25	2.75		2.42	3.07	2.48	2.72			2.77
σ_v/u_*								2.23	2.04	2.00	2.33			2.15
σ_w/u_*	1.52		1.02	1.00	1.05	1.03		1.54	1.48	1.07	1.47			1.24
Skewness	−0.40		0.22	−0.37	0.13	−0.03	−0.44	0.22	0.23	0.14	0.21	0.45	0.01	0.22
Flatness	5.46		2.55	3.80	4.54	3.85	4.24	3.67	3.62	1.25	3.40	3.95	3.81	3.7
e_3/u_*^2	11.2		4.6	9.6	4.6	6.5		6.7	7.1	5.6	7.4			7.0
$\overline{we_1}$ (cm³/sec³)	−0.174	−0.054	0.106	−0.067	0.028	0.011	1.463	0.781	1.684		0.425	0.0014	0.0006	
$\overline{we_3}$ (cm³/sec³)	−0.687*		0.069*	−0.266*	0.049*	−0.312*		1.741	0.436		0.965			
$\overline{we_3}/-u^2\bar{u}_1$	0.25		−0.02	0.06	−0.01	0.04		−0.10	−0.04		−0.08			
$\overline{we_3}/\overline{we_1}$	4.1		0.6	4.0	1.8	−29.0		2.2	0.6		3.8			2.4

* Two dimensional ones.

in most case, they could be regarded almost near neutral because of the strong winds.

Gross turbulent statistics are also shown in Table 1. These results were computed by the digital computer of Kyoto University, KDC-2. As for the dimensionless quantities σ_u/u_*, σ_v/u_*, σ_w/u_*, they are fairly scattered but their mean values can be summarized as follows, respectively.

$$\sigma_u/u_*=2.8, \quad \sigma_v/u_*=2.2, \quad \sigma_w/u_*=1.2$$

They agree well with the mean values at the various places in neutral conditions 2.5, 1.9 and 1.25 respectively (Lumley and Panofsky [1964]). This shows that the turbulent state over this site is similar to other places. It is reported that e_3/u_*^2 was 8.5 at the height of 12 m, at O'Neill, Nebraska. In the present analysis, the mean value of this quantity is 7.0 at the height of 3 m.

The fluxes of turbulent kinetic energy shown as $\overline{we_3}$ are 1/10–1/30 of the fluxes of mean flow energy ($\bar{u}$, $\overline{u'w'}=-\bar{u}\cdot u_*^2$). These values are somewhat higher than R. J. Tayler's estimation about at the same height. The direction of fluxes of turbulent kinetic energy were not definite in this case. But in unstable conditions, the fluxes were positive and the signs of $\overline{w'^3}$ were also positive. This fact agrees with the results that the skewness of distributions of vertical wind components are positive (Deacon [1955] and Gurvich [1960]). The magnitude of skewness $\overline{w'^3}/\sigma_w^3$ varies over the range from 0 to 1/2 and the mean value of them is about 0.2. The distribution of vertical velocity components, w, in unstable condition is shown in Fig. 2. In this case, the skewness was 0.44.

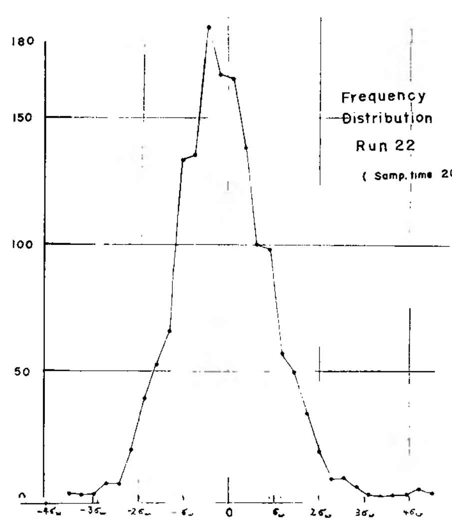

Fig. 2. Frequency distribution of vertical velocity comp)nent.

The values of flatness, $\overline{w^4}/\sigma_w^4$, were somewhat larger than 3 which is expected in the normal distribution and the mean value of them was about 3.7.

The flux of one dimensional turbulent component, $\overline{we_1}\left(e_1=\frac{1}{2}w'^2\right)$were also computed and compared with three dimensional one, $\overline{we_3}$. By this ratio, we can estimate the total turbulent kinetic energy flux from the fluctuation character of vertical components.

Spectral characteristics of turbulent structure and turbulent kinetic energy

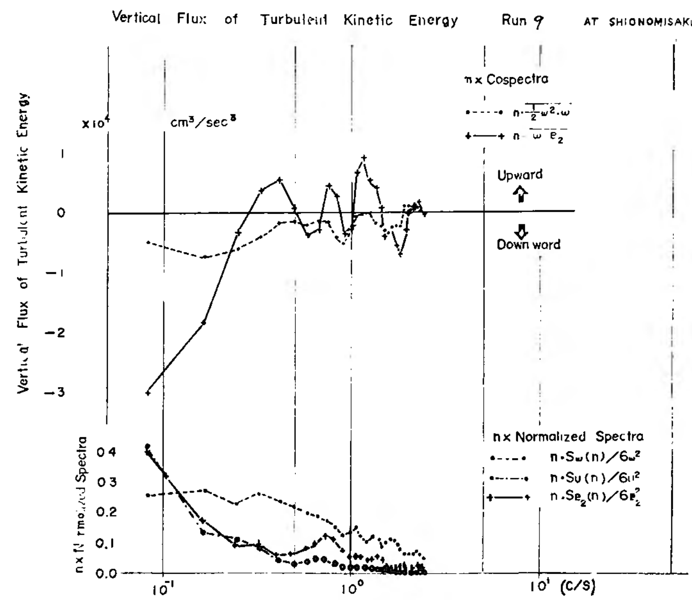

Fig. 3. Vertical flux of turbulent kinetic energy and other spectra at 3 m (Run 9A at Shionomisaki).

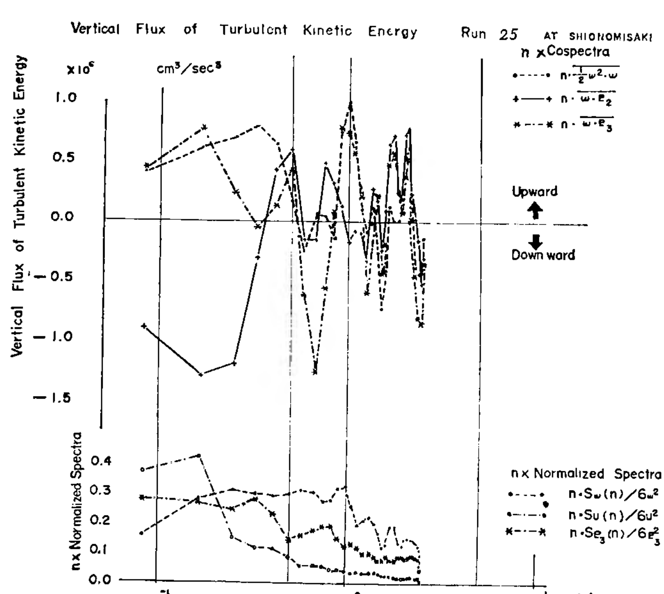

Fig. 4. Vertical flux of turbulent kinetic energy and other spectra at 3 m (Run 25 at Shionomisaki).

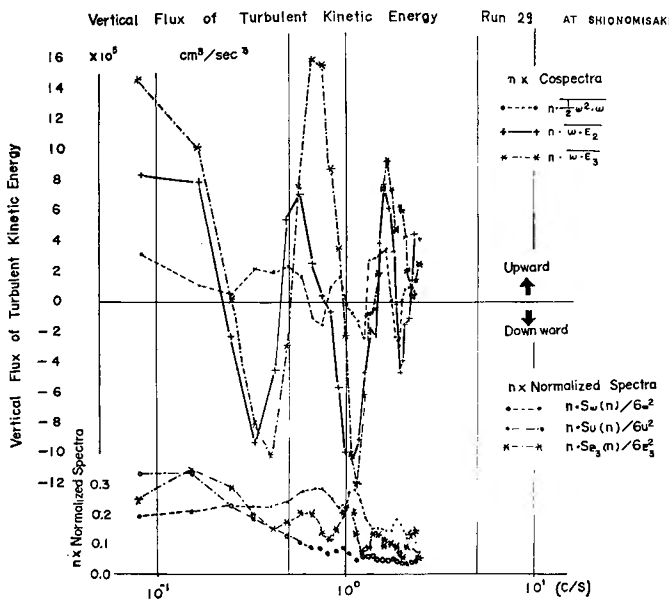

Fig. 5. Vertical flux of turbulent kinetic energy and other spectra at 3 m (Run 29 at Shionomisaki).

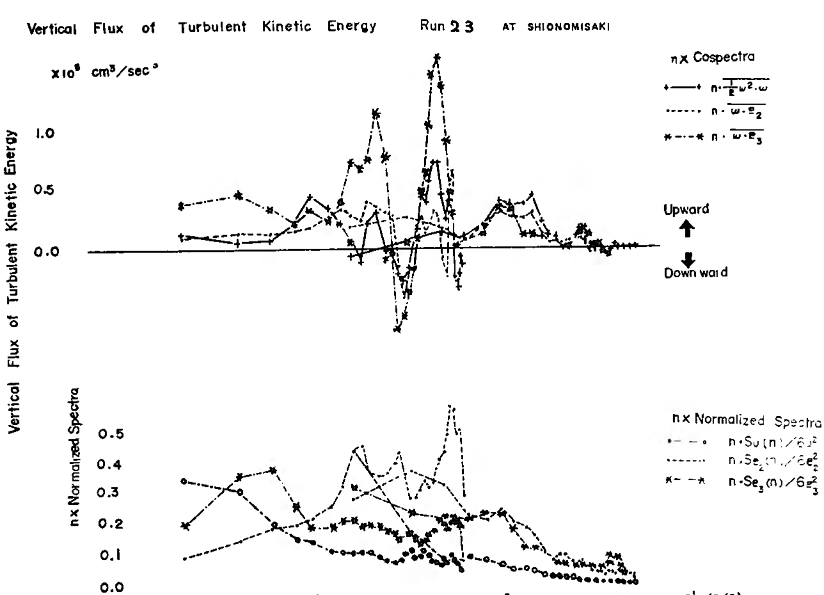

Fig. 6. Vertical flux of turbulent kinetic energy and other spectra at 3 m (Run 23 at Shionomisaki).

flux are shown in Fig. 3-6. In general, cospectral density, $n \cdot Co\overline{we}(n)$ decreases with frequency over the range of the analysis. In most cases, contribution of high frequency eddies to energy transport is quite small and even the direction of the transport is not definite. Such frequency range is clearly in the inertial subrange which is expected to have no contribution to transport process.

The spectral sphapes of $nCo\overline{we_3}(n)$, $nCo\overline{we_2}(n)$ and $nCo\overline{we_1}(n)$ have usually similar shapes with each other. The peak of energy flux, is at lower frequency than the peak of the power spectrum of w, $nSw(n)$ and is at higher frequency than the peak of the power spectrum of u, $nSu(n)$ or e_2, $nSe_2(n)$.

Fig. 6 shows cospectrum of the flux of turbulent kinetic energy over wider frequency range. From this figure, we can know the behavior of the flux of turbulent kinetic energy fairly well. In this case, the flux was positive (upward

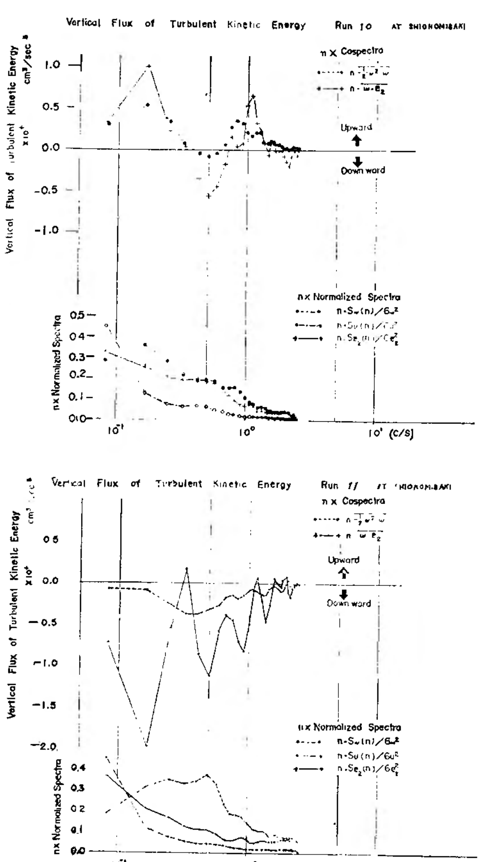

Fig. 7. Vertical flux of turbulent kinetic energy and other spectra at 3 m (Run 10) and at 1.5 m (Run 11) at Shionomisaki.

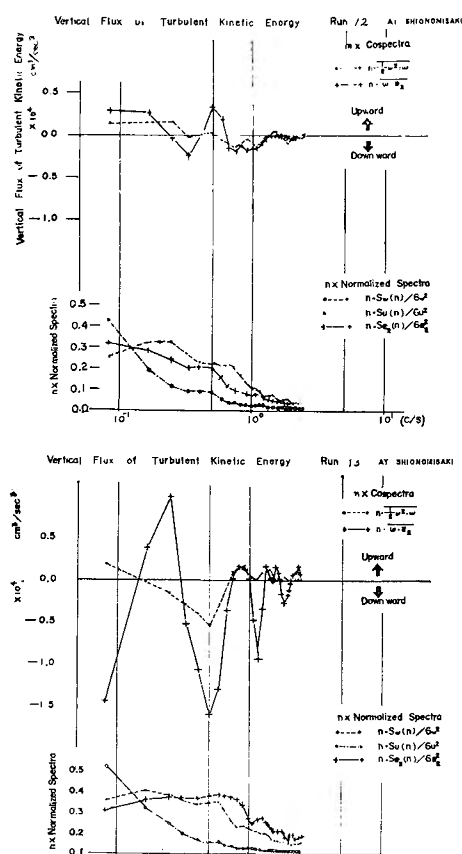

Fig. 8. Vertical flux of turbulent kinetic energy and other spectra at 3 m (Run 12) and at 1.5 m (Run 13) at Shionomisaki.

directed). As mentioned before, the peak of $nCow\bar{e}_3(n)$ is at somewhat lower frequency than the peak of $n{\cdot}Sw(n)$. Furthermore, it can be seen that the frequency range is in the range from 10^{-1} (c/s) to 10^0 (c/s).

One of the remarkable characteristics which can be seen from this figure is that the variation of the flux of turbulent energy, $nCowe_3(n)$, agrees with that of the power spectrum of w. From this, we can expect that there is a close relation between the vertical velocity and the flux of turbulent kinetic energy as Panofsky noted [1962].

Finally, the divergence of the flux of turbulent kinetic energy was studied. In order to measure the fluxes at the two heights of 3 m and 1.5 m, we moved two sonic anemometers up and down between the two heights (Run 10-Run 13). In the period of these observations, the wind had a tendency to increase slightly. The fluxes were positive at 3 m (Run 10 and Run 13). It is noted that the shapes of spectra are similar at each height in spite of measurements at different time (Fig. 7–Fig. 8). These features are shown schematically in Fig. 9. Therefore, we can think that there was the divergence in the layer between 1.5 m and 3.0 m. In addition, it is noted that σ_u, σ_u and u_* were irregularly fluctuating during these observations. But this method to estimate the divergence comes into question in the case which there is not sequrity for the stationarity or in the point of the troubles.

So, we attemped to estimate the divergence from we_1 at two heights. Here, it is assumed that the total flux of turbulent kinetic energy we_3, was proportional to we_1 and that the proportional constants was 2.4. The divergence of Run 9 (at Shionomisaki) and Run 4 (at Shirahama) were estimated by this method. The results obtained are shown schematically in Fig. 9. In Run 9, the fluxes were negative both at 3 m and 1.5 m. These values were -0.174×10^5 (cm^3/sec^3) and -0.054×10^5 (cm^3/sec^3) respectively. Therefore, there exsisted the convergence of turbulent kinetic energy and its magnitude 0.120×10^5 (cm^3/sec^3) in the layer between 1.5 m and 3 m. This value is comparable with the convergence of the flux of mean flow kinetic energy. While the data of over sea observation at Shirahama show upward turbulent kinetic energy fluxes at both heights 2 m and 9 m. These

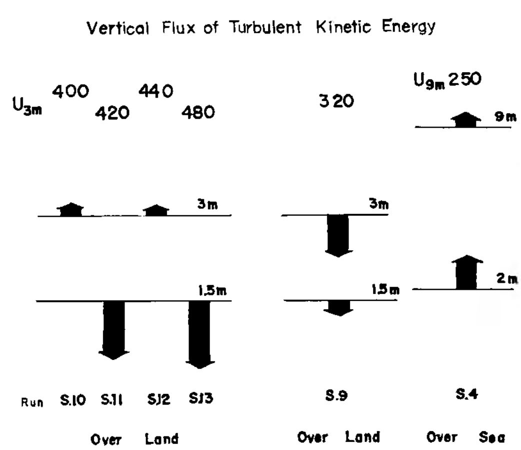

Fig. 9. Schematic behavior of turbulent kinetic energy flux over land (at Shionomisaki) and over sea (at Shirahama).

schematic behaviors are shown in Fig. 9.

5. Conclusion

Kinetic energy balance equation is one of the fundamental equations to describe the nature of turbulent field and is used in many ways. Some terms of this equation have hardly been estimated because of difficulties of their measurements. In this paper, we tried to evaluate the vertical divergence of turbulent kinetic energy of this equation by the direct measurement using sonic anemometers. This term has been generally considered negligible near the surface.

The preliminary results obtained in these studies are as follows.

1. The direction of turbulent kinetic energy fluxes were not usually defined except unstable conditions in which the fluxes were almost upward.

2. The fluxes of turbulent kinetic energy were 1/10-1/30 in magnitude compared with the flux of mean flow energy.

3. The ratio of the magnitude of the flux of turbulent kinetic energy to the approximated value of that estimated only from vertical wind component was about 2.4 on average.

4. The peak of cospectrum between turbulent kinetic energy and vertical wind component, $nCo_{we_3}(n)$ was in the frequency range from 10^{-1} (cps) to 10^{0} (cps) at 3 m. This frequency range may contribute largely to vertical transport of turbulent kinetic energy. On the whole, the behavior of cospectrum is similar to the spectrum of vertical velocity component.

5. The divergence of turbulent kinetic energy was evaluated. The divergence was as large as the energy production from mean wind (the first term of Eq. (5)). Furthermore, the ratios of these terms were not also constant. Therefore, it may be necessary to pay attention to this term in discussing the kinetic energy balance.

The results shown in this paper are only the case studies and the preliminary ones for the future detailed studies. However, the facts shown here suggest that the traditional concept of the turbulent kinetic energy transport should be changed through the studies by the direct measurement of it.

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