DIURNAL VARIATIONS OF THE SIZE OF THE MAGNETOSPHERE

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Abstract

Diurnal variations of the size of the magnetosphere are estimated from simple consideration on the pressure balance at the subsolar point of the magnetosphere. Associated geomagnetic variations on the surface of the earth are calculated.

1. Introduction

Ness et al. (1964) pointed out that the position of the magnetospheric boundary must be sensitive to the geomagnetic latitude of its subsolar point, $\theta_s$. Since the magnetic pressure at the subsolar point of the magnetopause is determined by $\theta_s$ and $R_s$, the distance from the earth's center to the subsolar point of the magnetopause, $R_s$ is required to vary by the change of $\theta_s$ in order to provide a constant pressure which should be balanced by the solar wind pressure. Using a factor $(1+3 \sin^2 \theta_s)^{1/6}$ they normalized magnetopause distances from the earth's center which were observed at various positions of the magnetospheric boundary during the flight of IMP-1 satellite. The same normalizing factor was used by Patel and Dessler (1966) to investigate relations between geomagnetic activity and the size of the magnetosphere. Their results show that at most 12% change of $R_s$ is caused by the variation of $\theta_s$.

If it is assumed that $R_s$ can be accepted as a measure of the size of the magnetosphere, diurnal and seasonal variations of the geomagnetic field intensity might be caused at the surface of the earth by the change of $\theta_s$. This is the main subject of this paper.

2. Results

The boundary condition which should be satisfied at the subsolar point of magnetopause is

$$\frac{B_s^2}{8\pi} = 2\rho v_s^2$$  \hspace{1cm} (1)

1 Now at the Ionosphere Research Laboratory.
where \( \rho \) and \( v \) are mass density and velocity of the solar wind and \( B \) is the magnetic field intensity tangential to the magnetospheric boundary. The field \( B \) can be approximately replaced by twice of the dipole magnetic field intensity

\[
B = 2B_0 \left( \frac{R_e}{R_0} \right)^3 (1 + 3 \sin^2 \theta) \sqrt{\frac{B_0}{4\pi \rho v^2}}
\]

where \( B_0 \) is magnetic field intensity at the earth's equator and \( R_0 \) is earth's radius. From (1) and (2), the relation

\[
R_s = R_0 \frac{B_0 (1 + 3 \sin^2 \theta)}{4\pi \rho v^2}
\]

is obtained, which shows that \( R_s \) increases with increase of \( \theta \). Since the rotational axis of the earth makes an angle of about 66.5° to the ecliptic plane and the geomagnetic dipole axis is tilted about 11.5° from the rotational axis, the angle \( \theta \) makes diurnal and seasonal variations. It varies diurnally from \((23.5+11.5)^\circ N\) to \((23.5-11.5)^\circ N\) in the June solstice, from \(11.5^\circ N\) to \(11.5^\circ S\) in the equinox and from \((23.5-11.5)^\circ S\) to \((23.5+11.5)^\circ S\) in December solstice and shown by the dotted line in Fig. 1.

Diurnal variations of \( R_s \) due to diurnal variation of \( \theta \), are calculated from Eq. (3) and presented in Fig. 1 by solid line. In the June solstice, \( \theta \) takes its

![Fig. 1. Solid line; diurnal variation of \( \theta_s \) (geomagnetic latitude of the subsolar point of the magnetopause). Dotted line; diurnal variation of \( R_s \) (geocentric distance to the subsolar point of the magnetopause). (a); June solstice, (c); spring or autumnal equinox, (e); December solstice, (b); intermedeate state between (a) and (c), (d); intermedeate state between (c) and (e).](image-url)
maximum value (35°N) at 16.6 hours UT, because the geomagnetic north pole lies on the longitude line of about 69°W (4.6 hours west from the zero longitude). At the same time $R$, takes its maximum value which is larger by 12% than that for $\theta_t=0$ (Fig. 1a). The minimum of $R$, occurs at about 4.6 hours UT which is about 4% larger than that for $\theta_t=0$. Therefore diurnal variation of about 8% is expected for $R$,. After the June solstice the curve of diurnal variation of $R$, comes down to the base line (which shows the value of $R$, for $\theta_t=0$) without any change of its shape, but as soon as its minimum touches the base line two peaks appear as shown in Fig. 1b. In the equinox two peaks take the same value and the diurnal variation of $R$, is about 4% at this time (Fig. 1c). After the autumnal equinox the peak at 4.6 hours UT becomes larger (Fig. 1d) and becomes largest in the December solstice (Fig. 1e).

If the distance $R$, may be accepted as a measure of the magnetospheric size in the solar side of the earth, UT variations same as for $R$, is expected for the size of the magnetosphere.

Mead (1964) showed that total magnetic field variation $\Delta H$ at earth's surface due to the change of $R$, is given as follows,

$$\Delta H \text{(gamma)} = \frac{75000}{R^3_t} \Delta r$$

(4)

where $\Delta r$ is change of $R$, . If $R$, is taken to be 10$R_e$ (earth radii) as usual, $\Delta r$ becomes 0.8$R_e$ in the solstice and 0.4$R_e$ in the equinox. From the relation (4) these values correspond to values of $6\gamma$ and $3\gamma$ of $\Delta H$. After all we can expect the diurnal UT variations of geomagnetic field which have properties presented in Table 1.

<table>
<thead>
<tr>
<th>Time of maximum (UT)</th>
<th>Time of maximum (UT)</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>June solstice</td>
<td>16.6h</td>
<td>4.6h</td>
</tr>
<tr>
<td>Spring and autumnal</td>
<td>4.6h</td>
<td>10.6h</td>
</tr>
<tr>
<td>equinox</td>
<td>16.6h</td>
<td>22.6h</td>
</tr>
<tr>
<td>December solstice</td>
<td>4.6h</td>
<td>16.6h</td>
</tr>
</tbody>
</table>

3. Discussion

In Eq. (2), $B_t$, the magnetic field intensity tangential to the magnetospheric boundary, is approximately taken to be twice of the dipole magnetic field intensity. Rigorously, $B_t$ should be the sum of the dipole magnetic field, $B_d$, and the magnetic field due to currents flowing on the magnetopause, $B_c$, so that

$$B_t = B_d + B_c.$$
The field $B_e$ varies with $\theta$, but it is not assured that $B_e$ varies in the same way as $B_0$. In order to know behaviour of $B_0$, it is necessary to carry out much more complex calculation such as done by Spreiter and Briggs (1963). Our purpose is to estimate geomagnetic response to variations of $\theta$, in rather simple way and the basis of the argument is a belief that our results would not be greatly altered by more detailed calculations.

Even if $R$, could be accepted as a measure of the size of the magnetosphere, it is so only for the solar side of the magnetosphere. For the tail side it is required to take into consideration effects of such as tail currents and its peculiar shape. Therefore the geomagnetic field variations derived here would be realized when a observing station is in the dayside of the earth.

Ionospheric dynamo currents would have also components varying with universal time because of tilting of the magnetic dipole axis. Geomagnetic field observed at the surface of the earth involves UT variations due to these ionospheric currents as well as those due to mechanisms mentioned above. Separation of these variations from ordinary $S_q$ is very difficult and remains as a future problem.

4. Conclusion

From simple consideration on the pressure balance at the subsolar point of the magnetopause, it is deduced that the geomagnetic field at the dayside of the earth's surface makes UT variations with amplitude of 3–6 $\gamma$ depending on seasons. More detailed calculation is needed to derive the field pattern through all of the day, but the results obtained here may be thought to be approximately correct for the dayside of the earth.

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References