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# DYNAMIC RESPONSE OF THE HYGROMETER USING FINE THERMOCOUPLE PSYCHROMETER

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**Abstract**

A new method to obtain the frequency response character of the hygrometer using a fine thermocouple psychrometer by introducing the new concept, that the internal energy of the environment is constant, is presented with a practical example.

## 1. Introduction

It is the purpose of this paper to discuss the dynamic response character of the hygrometer using a fine thermocouple psychrometer to the changes of humidity. The humidity sensor with fast response character is imperatively needed for the study of turbulent structure of boundary layer, especially for direct measurement of vertical eddy transport of water vapor. The hygrometer using psychcrometer with fine wire sensor is the only practical instrument for this purpose for the present. However, no complete study has been made of the dynamic response character of the fine wire psychrometric hygrometer.

Difficulties in the study of the dynamic response of the hygrometer of this type come from the facts that it does not measure humidity directly but dry- and wet bulb temperature and humidity is computed from these temperarures by the use of the psychrometric equation. Moreover, it is difficult to make arificial rapid changes of humidity even in a chamber, which prevents the experimental approach of this problem.

A method of approach to this problem is to study the response characters of dry- and wet-bulb thermometer to the changes of dry- and wet-bulb temperature respectively first and then to combine both by some assumptions on the environmental condition. Most of the studies of the dynamic response are on this way but only the former half is discussed except the one by Taylor [1963], who has discussed the overall response of psychrometric hygrometer combining the response characters of dry- and wet-bulb thermometer. But he has assumed that the water vapor content fluctuation is accompanied by the air temperature fluctuation in the same waveform and in the same phase. This assumption was derived from his observational experience in the field measurements but it is not always true in the atmospheric boundary layer nor appropriate in the course of the hygrometer response study to consider the environmental parameter fluctuation other than humidity itself.

Therefore, in this study, a new concept of the psychrometric hygrometer response to humidity changes in the constant environmental conditions is introduced. And the conclusion is applied to a practical example which is the case of the hygrometer using the fine thermocouple psychrometer under use by the present authors.

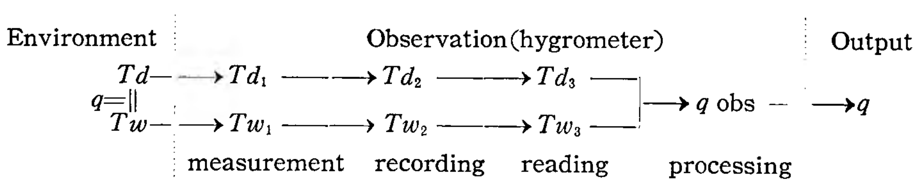
## 2. Basic concept

The psychrometric hygrometer measures directly dry- and wet-bulb temperature instead of humidity itself. Dry- and wet-bulb temperatur are related to humidity by the psychrometric equation,

$$q=q_s(Tw)-(c_p/L)(Td-Tw), \qquad \cdots\cdots\cdots\cdots(1)$$

where $q$ is specific humidity, $q_s(Tw)$ saturated specific humidity at wet-bulb temperature $Tw$, $c_p$ specific heat of air at constant pressure, $L$ latent heat of water, $Td$ dry-bulb temperature respectively. Wet-bulb temperature in the environment, which is a hypothetical meteorological parameter, is also defined by Eq. (1) and can be measured by a wet-bulb thermometer. As is clear from Eq. (1), wet-bulb temperature is a function of air temperature ($Td$) as well as specific humidity. This is the cause of the confusion in the treatment of the dynamic response of psychrometric hygrometer, because we cannot define wet-bulb temperature fluctuations from humidity fluctuation uniquely without some assumptions on the air temperature changes. Another difficulties come from the fact that we really use two sensors, dry- and wet-bulb thermometer to measure only one parameter of humidity.

The procedure of measurement of humidity by psychrometric hygrometer can be summarrized as follows,




First, dry-, and wet-bulb temperature of environment are measured by dry- and wet-bulb thermometer, whose outputs are $Td_1$ and $Tw_1$. And then they are recorded, read out and finally processed obtaining humidity output. Sometimes the second and third process are omitted and directly processed. If we define a hygrometer as an instrument which sends out humidity signals the whole steps from measurement to processing in the above diagram should be inclued into one system of a hygrometer. Therefore we must discuss the overall response including dry- and wet-bulb thermometer, measurement, recording, reading and processing processes. And humidity output signals should be compared with input environmental humidity changes.

The most convenient presentation of response character of the sensor is the frequency response character, the ratio of amplitudes and phase lag relation between the input and output sinusoidal humidity signals. As is mentioned before, it is required to place an environmental condition about air temperature as well as humidity to define dry- and wet-bulb temperature in the case of psychrometric hygrometer. For this purpose any parameter other than humidity should not be changed. Taylor's [1963] treatment is not appropriate in this point. In this paper, constancy of internal energy of the environmental air, that is, constancy of virtual temperature, $Tv$ is assumed. Virtual temperature is defined as follows,

$$Tv=(1+0.608\,q)Td. \qquad \cdots\cdots\cdots\cdots(2)$$

By this relation dry- and wet-bulb temperature of the environment are defined uniquely as followings. As $Td$, $Tw$ and $Tv$ are not so largely different each other, Eq. (1) is approximated as

$$q=q_s(Tv)+\frac{dq_s}{dT}\bigg|_{Tv}(Tw-Tv)-\frac{c_p}{L}(Td-Tw). \qquad \cdots\cdots\cdots\cdots(3)$$

While Eq. (2) becomes, assuming $q$ is small,

$$Td=(1-0.608\,q)Tv. \qquad \cdots\cdots\cdots\cdots(4)$$

From Eqs. (3) and (4),

$$Tw=\frac{1}{s+a}\{(1-0.608\,aTv)\,q+A\}, \qquad \cdots\cdots\cdots\cdots(5)$$

where $s=(dq_s/dT)_{Tv}$, $a=c_p/L$ and $A=(s+a)Tv-q_s(Tv)$. These equations give the relation between $Td$ and $Tw$, and humidity. Separating variables into constant part and fluctuating part,

$$q=\bar{q}+q' \qquad \cdots\cdots\cdots\cdots(6)$$

$$Td=\bar{T}d+Td'=\{(1-0.608\,\bar{q})Tv\}+(-0.608\,Tvq') \qquad \cdots\cdots\cdots\cdots(7)$$

$$Tw=\bar{T}w+Tw'=\left[\frac{1}{s+a}\{(1-0.608\,aTv)\bar{q}+A\}\right]$$
$$+\left(\frac{1-0.608\,aTv}{s+a}q'\right). \qquad \cdots\cdots\cdots\cdots(8)$$

By these relations we can derive dry- and wet-bulb temperature fluctuations from humidity fluctuation, and can discuss the response character of dry- and wet-bulb thermometer to these fluctuations and the overall response of the hygrometer. In this treatment humidity is the only independent variable, and the relation between the environmental change of humidity and hygrometer output can be discussed free from other meteorological parameters.

### 3. Response of dry- and wet-bulb thermometer

The time rate of temperature indication change of the dry-bulb thermometer can be written as

$$dTd_1/dt = H/k_d, \qquad \cdots\cdots\cdots\cdots(9)$$

where $Td_1$ is the indication of the dry-bulb thermometer, $k_d$ heat capacity of the thermometer per unit length, $H$ the rate of heat transport from surrounding air to the thermometer per unit length which is the function of temperature difference between the thermometer and surrounding air $(Td - Td_1)$.

The rate of heat transport from fine hot wire to the air was studied by King [1914] for wide range of Reynolds number. His conclusion may be applied to heat exchange between the thermometer and the environment including the cases of heating and cooling of the wire. King's formula is written in terms of Reynolds number as follows,

$$H = \{K + (2\pi K c_v \mu Re)^{1/2}\}(Td - Td_1), \qquad \cdots\cdots\cdots\cdots(10)$$

where $K$ is thermal conductivity of air, $c_v$ specific heat of air at constant volume, $\mu$ dynamic viscosity of air and $Re$ Reynolds number respectively. Fig. 1 shows

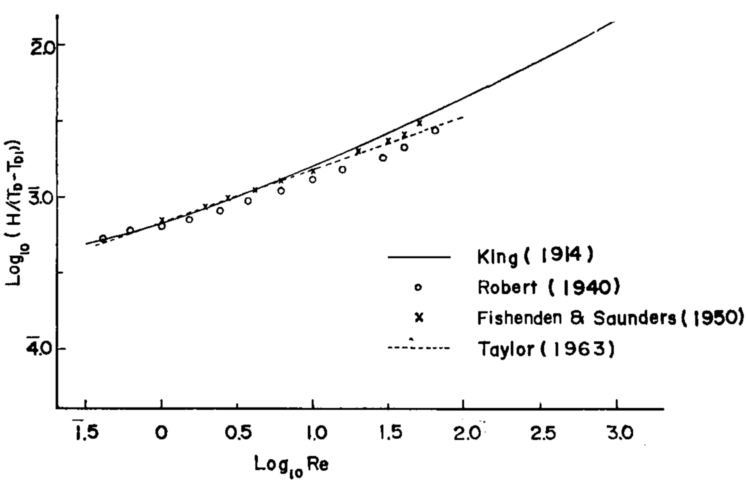



Fig. 1. Heat transfer rate (watt cm$^{-1}$ °C$^{-1}$) from fine wire as a function of Reynolds number.

the variation of $H$ with Reynolds number at air temperature of 20°C. In this figure recent experimental data on heat exchange and experimental formula by Taylor [1963] are plotted at the same time. These show good agreement with King's formula.

By the use of this relation, Eq. (9) becomes,

$$dTd_1/dt=\{f(Re)/k_d\}(Td-Td_1), \qquad \cdots\cdots\cdots\cdots(11)$$

where $f(Re)=K+(2\pi Kc_v\mu Re)^{1/2}$. This equation is a linear first order response equation and the characteristics of the solution is described by the time constant, $\tau_d$ which is given as,

$$\tau_d=k_d/f(Re). \qquad \cdots\cdots\cdots\cdots(12)$$

While, the rate of time change of the wet-bulb thermometer indication, $Tw_1$ is given as following,

$$dTw_1/dt=(Hw-LE)/k_w, \qquad \cdots\cdots\cdots\cdots(13)$$

where $Hw$ is sensible heat transfered into the wet-bulb from surrounding air per unit length, which is given by the same relation as Eq. (10), $E$ the rate of evaporation from the wet-bulb per unit length and $k_w$ heat capacity of the wet-bulb per unit length respectively. In the equilibrium state, when $Tw_1=Tw$, $Hw$ should be equal to $LE$, therefore

$$LE=f(Re)(Td-Tw), \qquad \cdots\cdots\cdots\cdots(14)$$

and by psychrometric equation (1)

$$LE=f(Re)\{q_s(Tw)-q\}/a. \qquad \cdots\cdots\cdots\cdots(15)$$

If we assume that the rate of evaporation in the transient state is given by the same form of Eq. (15), the latent heat release by evaporation is given by

$$LE=f(Re)\{q_s(Tw_1)-q\}/a. \qquad \cdots\cdots\cdots\cdots(16)$$

Thus Eq. (13) becomes

$$dTw_1/dt=\{f(Re)/k_w\}[(Td-Tw_1)-\{q_s(Tw_1)-q\}/a] \qquad \cdots\cdots\cdots\cdots(17)$$

again by the use of the psychrometric equation

$$\begin{aligned} dTw_1/dt&=\{f(Re)/k_w\}[(Tw-Tw_1)+\{q_s(Tw)-q_s(Tw_1)\}/a] \\ &=\{(a+s)f(Re)/ak_w\}(Tw-Tw_1), \qquad \cdots\cdots\cdots\cdots(18) \end{aligned}$$

where $q_s(Tw)-q_s(Tw_1)=(dq_s/dT)_{Tw}(Tw-Tw_1)$ and $(dq_s/dT)_{Tw}\doteqdot(dq_s/dT)_{Tv}$. This is also a linear first order response equation and the time constant, $\tau_w$ is given by

$$\tau_w=ak_w/(a+s)f(Re), \qquad \cdots\cdots\cdots\cdots(19)$$

where $a$ $(=c_p/L)$ is nearly constant with temperature $(4.1\times10^{-4}$ at 20°C) and $s(=dq_s/dT)$ changes with temperature as shown in Fig. 2.

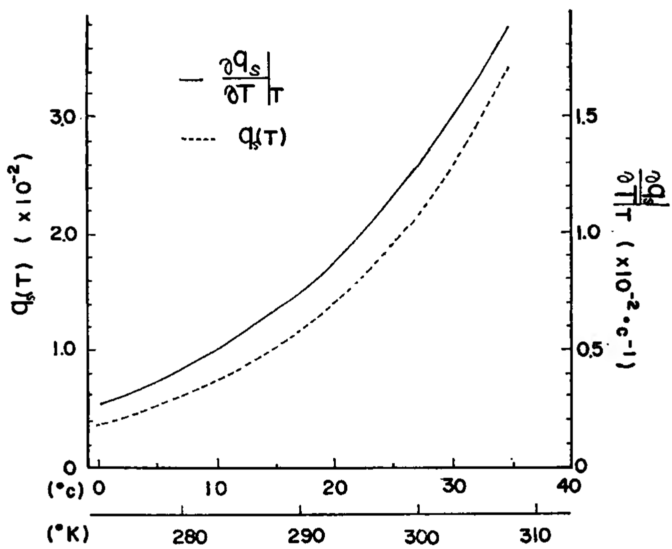


Fig. 2. Saturated specific humidity, $q_s$ and its gradient as the function of temperature.

## 3. Overall response of psychrometric hygrometer

In this section the overall frequency response character of psychrometric hygrometer is discussed using the time constants of dry- and wet-bulb thermometer in the preceeding section. Here the fluctuation of humidity is assumed to be sinusoidal form as

$$q=\bar{q}+p\sin 2\pi nt, \qquad \cdots\cdots\cdots\cdots(20)$$

where $n$ is the frequency of the fluctuation. The response of the recorder and reader is ignored to be ideally good compared to psychrometer or as in the case of direct processing.

The environmental input of dry- and wet-bulb temperature, $Td$ and $Tw$ are given by Eqs. (7), (8) and (20),

$$Td=\bar{T}d+g_d p\sin 2\pi nt \qquad \cdots\cdots\cdots\cdots(21)$$

$$Tw=\bar{T}w+g_w p\sin 2\pi nt, \qquad \cdots\cdots\cdots\cdots(22)$$

where $g_d=-0.608\,Tv$ and $g_w=(1-0.608\,aTv)/(s+a)$.

By the use of the solution of the first order response equation (e. g. MacCready and Jex [1963]), the indication of dry- and wet-bulb thermometer become as follows,

$$Td_1=\bar{T}d+Md_1 g_d p\sin(2\pi nt+\delta d_1) \qquad \cdots\cdots\cdots\cdots(23)$$

$$Tw_1=\bar{T}w+Mw_1 g_w p\sin(2\pi nt+\delta w_1), \qquad \cdots\cdots\cdots\cdots(24)$$

where $Md_1$ and $Mw_1$ are amplitude gains of dry- and wet-bulb thermoneter and $\delta d_1$ aod $\delta w_1$ are phase lags, which are shown as,

$$Md_1 = 1/\{1+(2\pi n\tau_d)^2\}^{1/2},$$
$$Mw_1 = 1/\{1+(2\pi n\tau_w)^2\}^{1/2},$$
$$\delta d_1 = \tan^{-1}(2\pi n\tau_d),$$
$$\delta w_1 = \tan^{-1}(2\pi n\tau_w). \qquad \cdots\cdots\cdots\cdots(25)$$

The humidity output is then deduced from these measured values of dry- and wet-bulb thermometer by psychrometric equation Eq. (1),

$$q_{out} = q_s(Tw_1) - a(Td_1 - Tw_1). \qquad \cdots\cdots\cdots\cdots(26)$$

This can be written as follows by the approximation of Eq. (3),

$$q_{out} = q + (a+s)Tw_1' - aTd_1'. \qquad \cdots\cdots\cdots\cdots(27)$$

Thus the fluctuation of humidity output will be also sinusoidal and by Eqs. (23), (24) and (25)

$$q'_{out} = Mp \sin(2\pi nt + \delta), \qquad \cdots\cdots\cdots\cdots(28)$$

where $M$ is the amplitude gain of the psychromrtric hygrometer and $\delta$ is phase lag and,

$$M = \{(a+s)^2 Mw_1^2 g_w^2 + a^2 Md_1^2 g_d^2 - 2a(a+s) Md g_d Mw g_w \cos(\delta d_1 - \delta w_1\}^{1/2}, \cdots(29)$$

$$\delta = \tan^{-1}\frac{(a+s)Mw_1 g_w \sin\delta w_1 - aMd_1 g_d \sin\delta d_1}{(a+s)Mw_1 g_w \cos\delta w_1 - aMd_1 g_d \cos\delta d_1}. \qquad \cdots\cdots\cdots\cdots(30)$$

When dry- and wet-bulb thermometer time contants are the same, the overall time constant is the same as the unit thermometer and response character is the same.

## 4. An example of a fine thermocouple psychrometric hygrometer

The present authors are using a fine thermocouple psychrometer, which is

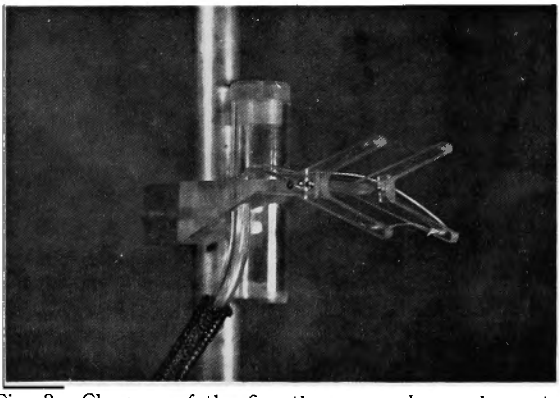


Fig. 3. Close-up of the fine thermocouple psychrometer.

shown in Fig. 3, to measure humidity fluctions. The dynamic response character of this hygrnmeter is shown as a practical example. The thermocouples are made of copper and constantan wire of 0.12 mm in diameter. The wet-bulb is covered by fine gauze thread wound at 5 pitches per 1 cm. The amount of water on the wet-bulb, which is the main factor to determine the time constant, was tested experimentally. The average water amount on the wet-bulb in normal condition of observation was about 0.64 mg/cm, which was determined as the weight difference of the wet-bulb in normal state and after drying up. Therefore, the effective diameter of the wet-bulb is 0.31 mm, assuming that water on wet-bulb uniformly covers the wet-bulb wire.

The time constants of these thermometers calculated by Eqs. (12) and (19) are shown in Table 1 as the function of temperature and wind speed. The time constant of wet-bulb thermometer is two or three times larger than that of dry-bulb in cold condition, but the difference decreases with temperature and

Table 1. Time constants of dry- and wet-bulb thermometer, in sec.

| Wind Speed / Temperature | 0.5 m/sec | | 1.0 m/sec | | 5.0 m/sec | | 10 m/sec | |
|---|---|---|---|---|---|---|---|---|
| | dry | wet | dry | wet | dry | wet | dry | wet |
| 0°C | 0.34 | 1.10 | 0.26 | 0.80 | 0.13 | 0.38 | 0.09 | 0.27 |
| 10°C | 0.34 | 0.79 | 0.26 | 0.59 | 0.13 | 0.28 | 0.09 | 0.20 |
| 20°C | 0.34 | 0.57 | 0.26 | 0.42 | 0.13 | 0.20 | 0.09 | 0.14 |
| 30°C | 0.35 | 0.39 | 0.26 | 0.29 | 0.13 | 0.14 | 0.09 | 0.10 |

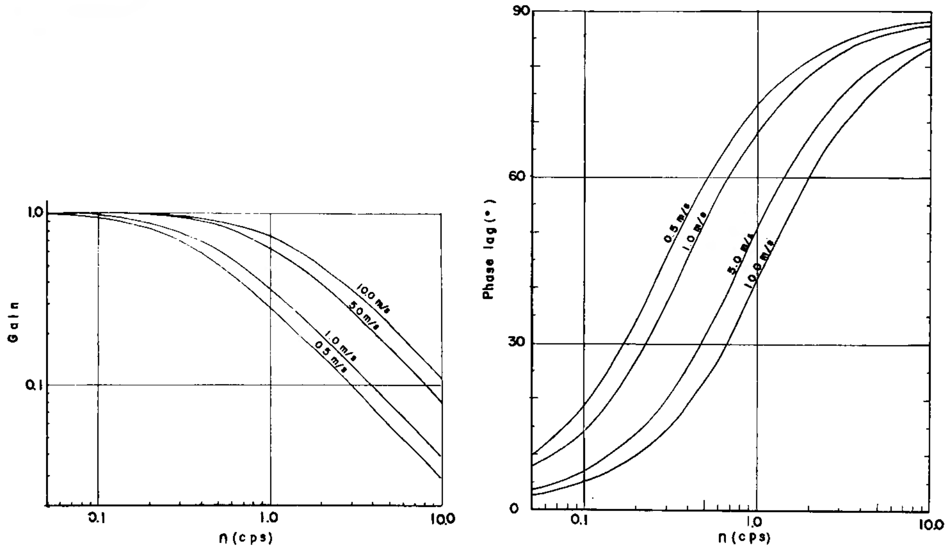



Fig. 4. The frequency response of the hygrometer using the fine thermocouple psychrometer of 0.12 mm in diameter at 20°C.

becomes negligible at high temperature of 30°C or so.

The overall frequency response of this hygrometer, which is calculated at temperature of 20°C by Eqs. (28), (29) and (30), is shown in Fig. 4. As is clear from this figure, response character depends on wind speed largely as well as temperature. The 90 % amplitude gain of this hygrometer is at about 0.4 cps in wind of 5.0 m/sec and about 0.2 cps at 1.0 m/sec (at 20°C). Fig. 5 shows the 90, 80 and 70 % amplitude gain and 10, 20 and 30° phase lag isopleths in relation to wind speed and temperature.

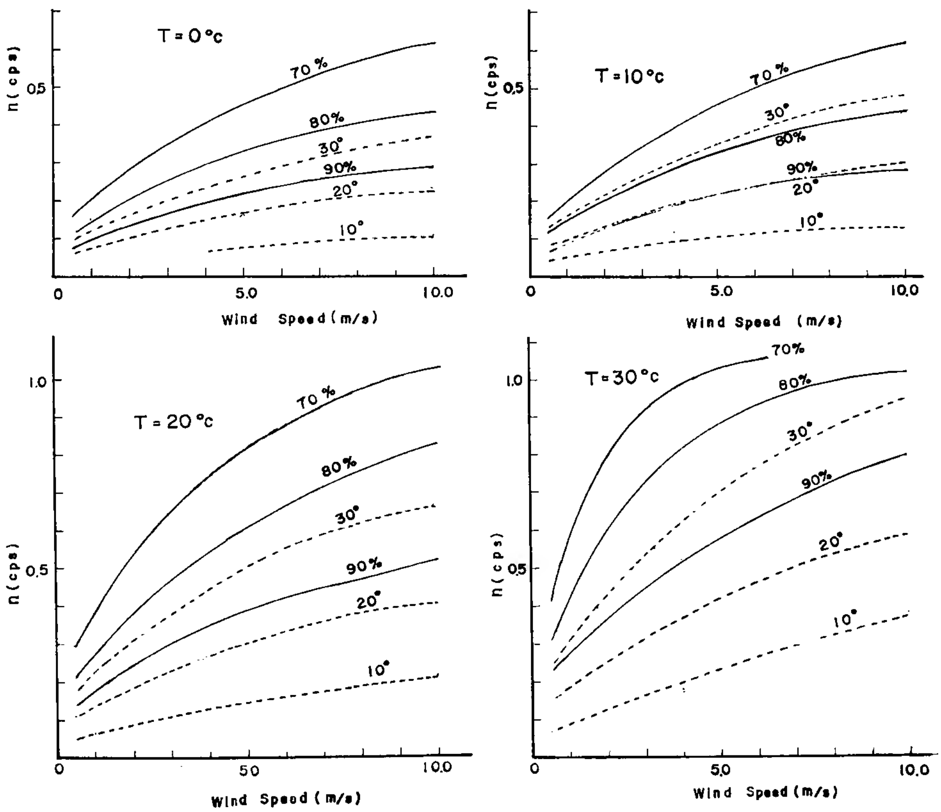



Fig. 5. The frequency response of the hygrometer using the fine thermocouple psychrometer as the functions of wind speed and temperature.
—— Amplitude gain, …… Phase lag.

## 5. Conclusion

A new method of evaluation of the dynamic response character of the hygrometer using a fine thermocouple psychrometer is presented. The response character of the hygrometer is described by linear first order response function and is largely affected by wind speed and air temperature.

The 90% amplitude gain in frequency response of a fine thermocouple psychrometric hygrometer (wire diameter being 0.12 mm) is about 0.4 cps at 5.0 m/sec and 20°C but about 0.2 cps at 5.0 m/sec and 10°C.

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