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ON THE SPACING OF CONVECTIVE CLOUD BANDS

By

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Abstract

A theoretical investigation is made on the spacing of convective cloud bands aligned parallel to the general wind. The primary concern to be discussed here deals with the ratio of the spacing of the bands to their depth as they are frequently observed in the atmosphere, which is taken here to be much larger than the one depending on Rayleigh's theory of thermal convection.

A preferred mode of convective cloud bands is determined so as to maximize the upward heat transport, based on the cellular cumulus model proposed by Asai [1967] for a thermal convection in a conditionally unstable atmosphere which is unstable for moist ascending motion and stable for dry descending motion. The result obtained suggests that the spacing of bands being 10 times or more larger than the depth may be realized in the atmosphere as well as that comparable to the depth.

1. Introduction

A streaked or banded structure of clouds has been often observed on the atmosphere. Recent observations from aircrafts and by radar and meteorological satellites make it possible to analyze the distribution of clouds quantitatively in a large area of the atmosphere.

Of the various types of clouds we are here concerned only with convective clouds which are likely to form in a conditionally unstable atmospheric layer heated from below. Quite frequently photographs taken from meteorological satellites off the east coast of the continents under the influence of the winter monsoon have shown a remarkable band structure of clouds.

The observations made so far indicate that convective clouds are aligned parallel to the vertical wind shear (e. g., Asai [1966], Miyazawa [1965], Tsuchiya and Fujita [1967]). This conclusion agrees with the laboratory experiment (cf. Brunt [1951]) and the theory (Kuo [1963], Asai [1964], Asai and Nakasuji [1968]) which show the Bénard convection cells in the fluid layer heated from below are replaced by longitudinal rolls parallel to the flow in which shear is introduced.

With regard to the spacing of cloud bands, no definite conclusion has been obtained. The convection theory established by Rayleigh [1916] results in the spacing of convection rolls two to three times larger than the depth of the convection layer. This is also supported by the laboratory experiment mentioned above. The spacing of convective cloud bands observed in the atmosphere, however, ranges from a spacing of the same order to one order of magnitude larger than their depth. A possible explanation for this will be presented in this paper and will be based on the cellular cumulus convection model proposed by Asai [1967].

2. Model and governing equations

Basing his conclusions on the perturbation theory of thermal convection in a conditionally unstable atmosphere, Kuo [1961, 1965] suggested that the horizontal scale of a convection cell might be one order larger than the vertical scale. A definite size ratio, however, could not be obtained by relying on the perturbation theory.

In this paper the cellular cumulus model proposed by Asai [1967] will be applied to convective cloud bands in the atmosphere, and will be restricted to the spacing of the cloud bands without regard to their direction with respect to a prevailing wind.

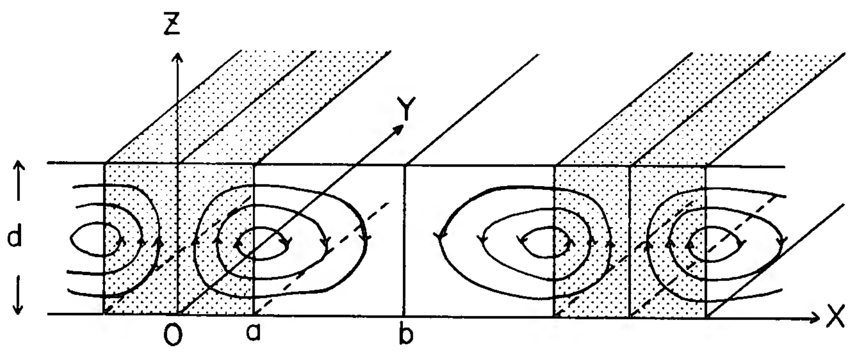

Fig. 1. A simplified model of convective cloud bands parallel to y-axis. The solid lines with arrows indicate streamlines in x-z plane. The stippled area represents the ascending region in which clouds are assumed to form.

As is shown in Fig. 1 Cartesian coordinates (x, y, z) are designated so that the convective cloud bands are parallel to y-axis. Thus we deal with a convective motion in a x-z plane averaged along the y axis which is confined in a rectangular domain with the depth d and the width b. The equations of motion, the equation of mass continuity and the first law of the thermodynamics may be expressed as follows:

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=-\frac{\partial}{\partial z}\left(\frac{p^*}{\rho_0}\right)+\frac{g}{\Theta}\theta^*+F_w, \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{\partial}{\partial x}\left(\frac{p^*}{\rho_0}\right)+F_u, \tag{2.2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \tag{2.3}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+w\frac{\partial \theta}{\partial z}=\frac{L\theta}{C_pT}M+F_\theta, \tag{2.4}$$

where

$$F_w=-\frac{\partial \widetilde{w'u'}}{\partial x}-\frac{\partial \widetilde{w'w'}}{\partial z}, \tag{2.5}$$

$$F_u=-\frac{\partial \widetilde{u'u'}}{\partial x}-\frac{\partial \widetilde{u'w'}}{\partial z}, \tag{2.6}$$

$$F_\theta=-\frac{\partial \widetilde{\theta'u'}}{\partial x}-\frac{\partial \widetilde{\theta'w'}}{\partial z}. \tag{2.7}$$

Here w and u are the vertical and the horizontal (normal to the band) components of the velocity averaged longitudinally. T and θ are the temperature and the potential temperature averaged longitudinally. Θ is the constant standard value of potential temperature, g is the acceleration due to gravity, and ρ_0 denotes a mean density of air defined by the hydrostatic equation,

$$\frac{\partial p_0}{\partial z}+\rho_0 g=0, \tag{2.8}$$

where p_0 denotes a horizontally averaged pressure. Deviations of pressure and potential temperature from their horizontal averages are represented by p^* and θ^* respectively. C_p denotes the specific heat of dry air at constant pressure; L is the latent heat of condensation; and M is the rate of condensation of water vapor. The definitions (2.5) and (2.6) for F_w and F_u express the vertical and horizontal Reynolds dissipations, while F_θ denotes the rate of heating due to eddy motion. Tilde and prime notations indicate longitudinal average and the deviation from it, respectively. We assume that the velocity components normal to the boundaries vanish, i. e.,

$$\left.\begin{aligned} w&=0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=d,\\ u&=0 \quad \text{at} \quad x=0 \quad \text{and} \quad x=b. \end{aligned}\right\} \tag{2.9}$$

Furthermore no eddy transport of momentum and heat through the boundaries is assumed.

$$\left.\begin{aligned} \widetilde{A'w'}&=0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=d,\\ \widetilde{A'u'}&=0 \quad \text{at} \quad x=0 \quad \text{and} \quad x=b. \end{aligned}\right\} \tag{2.10}$$

where A' stands for one of variables w', u' and θ'.

If we follow the method used by Asai [1967] and apply the circulation theorem to a unit cell of convective motion in x-z plane, we may reduce the equations of motion (2.1) and (2.2) with the aid of the equation of mass continuity (2.3) and the boundary conditions (2.9) and (2.10) to the following:

$$\frac{\partial}{\partial t}\langle w_a\rangle = k_1\langle \Delta\theta\rangle - k_2\langle w_a\rangle^2. \tag{2.11}$$

Here $\langle w_a\rangle$ and $\langle \Delta\theta\rangle$ represent the average vertical velocity of the updraft region and the average excess of the potential temperature in the updraft region from that in the descending region.

$$k_1 = \frac{g}{\Theta}(1-\sigma)\left\{1+\frac{(1-\sigma)}{\mu(1-\mu)\sigma}\left(\frac{a}{d}\right)^2\right\}^{-1},$$

$$k_2 = \frac{\alpha}{a^2(1-\sigma)^2}\left\{1+\frac{(1-\sigma)^3}{\mu^3(1-\mu)^3\sigma}\left(\frac{a}{d}\right)^4\right\}\left\{1+\frac{(1-\sigma)}{\mu(1-\mu)\sigma}\left(\frac{a}{d}\right)^2\right\}^{-1}.$$

Here $\sigma = a/b$ which is the ratio of the width of the ascending region to that of the cell, while $\mu = c/d$ which is the ratio of the thickness of the inflow layer to that of the whole layer. α^2 is a kind of entrainment constant. The detailed derivation of (2.11) is referred to Appendix A in this paper.

On the other hand to find the excess potential temperature when condensation of water vapor is assumed to take place only in the ascending motion and no evaporation is taken into account, the thermodynamic equation (2.4) is transformed to the following equation.

$$\frac{\partial}{\partial t}\langle \Delta\theta\rangle = k_3\langle w_a\rangle - k_4\langle w_a\rangle\langle \Delta\theta\rangle, \tag{2.12}$$

where

$$k_3 = S_b\left(\delta - \frac{\sigma}{1-\sigma}\right),$$

$$k_4 = \frac{\alpha^2}{a(1-\sigma)^2}\left\{1+\frac{2(1-\sigma)}{\alpha^2}\frac{a}{d}\right\},$$

$$S_b = \frac{\partial\theta_0}{\partial z},\quad S_a = -\frac{\partial\theta_{e0}}{\partial z}\quad \text{and}\quad \delta = \frac{S_a}{S_b}.$$

The derivation of (2.12) is also shown in Appendix B.

3. Results

The set of equations (2.11) and (2.12) has a steady state solution as follows:

$$\left.\begin{aligned}\langle w_a\rangle &= \left(\frac{k_1k_3}{k_2k_4}\right)^{1/2},\\ \langle \Delta\theta\rangle &= \frac{k_3}{k_4}.\end{aligned}\right\} \tag{3.1}$$

The other solution, $\langle w_a\rangle = \langle \Delta\theta\rangle = 0$, is trivial and is not considered here. Since upward heat flux through a horizontal unit area per unit time, H, can be expressed by

$$H = C_p \rho_0 \sigma \langle w_a \rangle \langle \Delta\theta \rangle . \tag{3.2}$$

substitution of the solution (3.1) into (3.2) leads to the equation

$$H = C_p \rho_0 \sigma \left(\frac{k_1}{k_2}\right)^{1/2} \left(\frac{k_3}{k_4}\right)^{3/2} . \tag{3.3}$$

In the following we employ the selection hypothesis that convection realizes so as to maximize the rate of upward heat transport. Thus one can determine the preferred mode of convection which would prevail in a conditionally unstable layer.

Fig. 2 shows the variation of the upward heat flux with the spacing of the convective cloud bands, represented by the ratio of the spacing $2b$ to the thickness of the convection layer d, for different values of δ. The parameter δ defined in Section 2 as the ratio of the static stability for the moist-adiabatic ascending motion to that for the dry-adiabatic descending motion is appropriate for dealing with cumulus convection in a conditionally unstable atmosphere. As the static stability decreases from the moist-adiabatic lapse rate to the dry-adiabatic one, the value of δ increases from zero to infinity. The value of geometrical parameter a/d is assumed here to be 1/2, since it is regarded as the most efficient mode of a convection cell to transport heat upward for the case of $\alpha^2 \fallingdotseq 0.1$ which is adopted throughout the present work (cf. Asai [1967]). As is seen in Fig. 2, there exists a preferred spacing of convective cloud bands to maximize the upward heat transport for each value of δ. It is also shown that the spacing of the bands decreases with an increase in the value of δ. The circumstances are summarized in Fig. 3 which illustrates the dependence of the preferred spacing $2b/d$ upon the static stability ratio δ. The ratio of the spacing to the depth approaches a value of 2~3, as the static stability ratio increases and this value is nearly equal to that of the Rayleigh convection cell in an absolutely unstable layer.

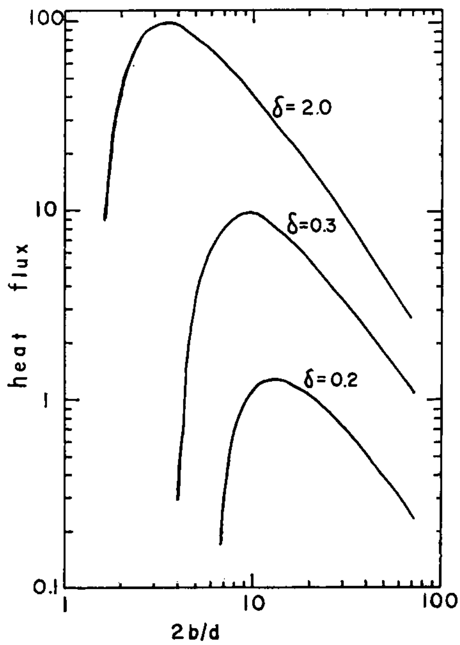

Fig. 2. Upward heat flux, $H/C_p\rho_0$ in unit of deg cm sec^{-1} against $2b/d$ for $\delta=0.2$, 0.3 and 2.0.

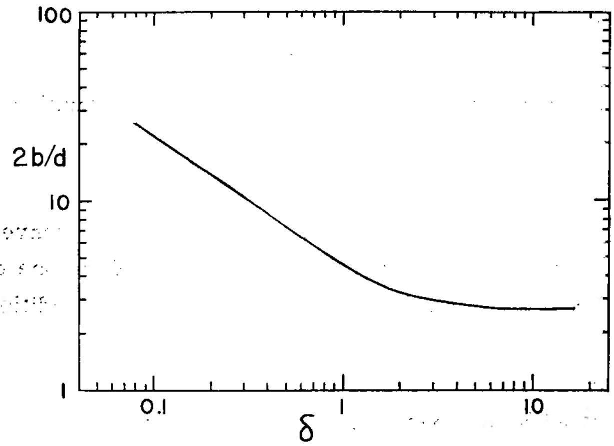

Fig. 3. Dependence of a preferred ratio of the spacing of convective cloud bands to their depth upon the static stability ratio.

On the other hand, spacing increases as the stability ratio decreases. For instance, the value of $2b/d$ is around 20 for $\delta=0.1$. This means a spacing of 20 km for a depth of convection layer of 1 km. It has been observed that the static stability ratio in the atmosphere ranges mostly from 0.1 to 2. Therefore we can conclude that a preferred mode of cumulus convection in a conditionally unstable atmosphere may bring about a spacing of convective cloud bands even one order of magnitude larger than their depth.

Acknowledgments

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APPENDIX A

The derivation of the equation (2.11) will be made in the following. Eliminating the pressure term from (2.1) and (2.2) by cross differentiation and using the continuity equation (2.3), we obtain the equation,

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}(\eta u)+\frac{\partial}{\partial z}(\eta w)=-\frac{g}{\Theta}\frac{\partial \theta^*}{\partial x}-\frac{\partial F_w}{\partial x}+\frac{\partial F_u}{\partial z}, \tag{A.1}$$

where $\eta=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}$, i.e. the y-component of vorticity. Integrating (A.1) over a unit cell from $x=0$ to $x=b$ and from $z=0$ to $z=d$, (A.1) may be reduced to

$$\frac{\partial}{\partial t}\oint v_l dl = -\frac{g}{\Theta}\int_0^d [\theta^*]_{x=0}^{x=b} dz - \int_0^d [F_w]_{x=0}^{x=b} dz + \int_0^b [F_u]_{z=0}^{z=d} dx, \tag{A.2}$$

with the aid of the boundary conditions (2.9). And v_l is the velocity component parallel to the boundary of the cell. In order to arrange (A.2) we need to derive some kinematical relationship among the velocity components and the geometrical parameters relevant to convection roll under consideration.

Averaging the continuity equation (2.3) over the horizontal and vertical cross-sections respectively with the aid of the boundary conditions (2.9), we obtain the equation

$$\sigma w_a + (1-\sigma) w_b = 0, \tag{A.3}$$

$$\mu u_c + (1-\mu) u_d = 0, \tag{A.4}$$

where $\sigma \equiv a/b$ and $\mu \equiv c/d$. w_a denotes the horizontal average of vertical velocity for the ascending region and w_b for the descending region, while u_c denotes the vertical average of horizontal velocity for the lower inflow layer and u_d for the upper outflow layer. For the sake of simplicity a top hat model is assumed here in which both ascending and the descending velocities are independent of the x-coordinate. Hence the following relationship may be derived:

$$<w_a> = \frac{1}{2} w_{a,\ z=c}, \tag{A.5}$$

$$\bar{u}_c = -\frac{a}{c} <w_a> = -\frac{a}{\mu d} <w_a>. \tag{A.6}$$

Here $\bar{u}_c$ denotes the average of u_c from $x=0$ to $x=b$ with respect to x-coordinate. Using the relations (A.3), (A.4), (A.5) and (A.6) in addition to the boundary conditions (2.10), each term of (A.2) will be transformed in the following.

$$\oint_l v_l dl = (<w_a> - <w_b>)d + (\bar{u}_d - \bar{u}_c)b$$
$$= \frac{d}{1-\sigma}\left\{1 + \frac{(1-\sigma)}{\sigma\mu(1-\mu)}\left(\frac{a}{d}\right)^2\right\} <w_a>, \tag{A.7}$$

$$\int_0^d [\theta^*]_{x=0}^{x=b} dz = (<\theta^*_a> - <\theta^*_b>)d$$
$$= d <\Delta\theta>. \tag{A.8}$$

$$\int_0^d [F_w]_{x=0}^{x=b} dz = \left\{\frac{1}{a}\int_0^a <\frac{\partial \widetilde{w'u'}}{\partial x}> dx - \frac{1}{b-a}\int_a^b <\frac{\partial \widetilde{w'u'}}{\partial x}> dx\right\} d$$
$$= \frac{d}{a(1-\sigma)} <\widetilde{w'u'}>_{x=a}, \tag{A.9}$$

$$\int_0^b [F_u]_{z=0}^{z=d} dx = \left\{\frac{1}{c}\int_0^c \frac{\partial \overline{\widetilde{u'w'}}}{\partial z} dz - \frac{1}{d-c}\int_c^d \frac{\partial \overline{\widetilde{u'w'}}}{\partial z} dz\right\} b$$
$$= \frac{a}{\sigma\mu(1-\mu)d} (\overline{\widetilde{u'w'}})_{z=c}. \tag{A.10}$$

Here we are applying the mixing-length hypothesis to momentum exchange in (A.9) and (A.10), i. e.,

$$<\widetilde{w'u'}>_{x=a}=-\left(l_x^2\left|\frac{\partial<w>}{\partial x}\right|\frac{\partial<w>}{\partial x}\right)_{x=a}$$

$$=\left(\frac{l_x}{a}\right)^2\frac{1}{(1-\sigma)^2}<w_a>^2, \tag{A.11}$$

$$(\widetilde{u'w'})_{z=c}=-\left(l_z^2\left|\frac{\partial\tilde{u}}{\partial z}\right|\frac{\partial\tilde{u}}{\partial z}\right)_{z=c}$$

$$=-\left(\frac{2l_z}{d}\right)^2(\tilde{u}_d-\tilde{u}_c)^2$$

$$=-\left(\frac{2l_z}{d}\right)^2\frac{1}{\mu^2(1-\mu)^2}\left(\frac{a}{d}\right)^2<w_a>^2, \tag{A.12}$$

where l_x and l_z denote the mixing lengths in the x and the z directions, respectively. Depending on Asai [1967], we assume

$$l_x=\alpha a \text{ and } l_z=\alpha\frac{d}{2}, \tag{A.13}$$

where α^2 is regarded as an entrainment constant. Using (A.11), (A.12) and (A.13), (A.9) and (A.10) will be rewritten as follows:

$$\int_0^d[F_w]_{x=0}^{x=b}dz=\frac{\alpha^2}{(1-\sigma)^3}\frac{d}{a}<w_a>^2, \tag{A.14}$$

$$\int_0^b[F_u]_{z=0}^{z=d}dx=-\frac{\alpha^2}{\sigma\mu^3(1-\mu)^3}\left(\frac{a}{d}\right)^3<w_a>^2. \tag{A.15}$$

By substituting (A.7), (A.8), (A.14) and (A.15) into (A.2), we may obtain the equation (2.11).

APPENDIX B

The equation (2.12) evolves from the following. By averaging the thermodynamic equation (2.4) over the ascending region and the descending region respectively, and by making use of the boundary conditions (2.9) and (2.10) we obtain the equation

$$\frac{\partial}{\partial t}<\theta_a>+\frac{1}{a}\left\{<\theta u>+<\widetilde{\theta' u'}>\right\}_{x=a}=\frac{L}{C_p}<\frac{\theta}{T}M>_a, \tag{B.1}$$

$$\frac{\partial}{\partial t}<\theta_b>-\frac{1}{b-a}\left\{<\theta u>+<\widetilde{\theta' u'}>\right\}_{x=a}=\frac{L}{C_p}<\frac{\theta}{T}M>_b. \tag{B.2}$$

By subtracting (B.2) from (B.1), we obtain the equation

$$\frac{\partial}{\partial t}<\Delta\theta>=-\frac{1}{a(1-\sigma)}\left\{<\theta u>+<\widetilde{\theta' u'}>\right\}_{x=a}$$

$$+\frac{L}{C_p}\left\langle\left(\frac{\theta}{T}M\right)_a-\left(\frac{\theta}{T}M\right)_b\right\rangle. \tag{B.3}$$

As in (A.11) the eddy exchange of heat through the lateral cloud boundary may be expressed as

$$<\widetilde{\theta' u'}>_{x=a}=-l_r^2\left(\left|\frac{\partial<w>}{\partial x}\right|\frac{\partial<\theta>}{\partial x}\right)_{x=a}=\frac{\alpha^2}{1-\sigma}<w_a><\Delta\theta>. \tag{B.4}$$

And

$$\begin{aligned}<\theta u>_{x=a}&=\mu\theta_{bc}u_{c,\ x=a}+(1-\mu)\theta_{ad}u_{d,\ x=a}\\&=\frac{2a}{d}<w_a>(\theta_{ad}-\theta_{bc})\\&=\frac{2a}{d}<w_a>\left\{<\Delta\theta>+\frac{d}{2}\frac{\partial\theta_0}{\partial z}\right\},\end{aligned} \tag{B.5}$$

where θ_{ad} denotes the average potential temperature for the ascending current in the upper outflow layer and θ_{bc} for the descending current in the lower inflow layer. No variation in the static stability is taken into account. The non-adiabatic heating is assumed to be due only to condensation of water vapor associated with the ascending current in the cloud band and evaporation which may occur in the cloudless region is not considered, that is,

$$M_a=-\left(w\frac{\partial q_s}{\partial z}\right)_a \text{ and } M_b=0, \tag{B.6}$$

where q_s is the saturation specific humidity. Using the definition of the equivalent potential temperature θ_e,

$$\frac{L}{C_pT}\frac{\partial q_s}{\partial z}=\frac{1}{\theta_e}\frac{\partial\theta_e}{\partial z}-\frac{1}{\theta}\frac{\partial\theta}{\partial z}, \tag{B.7}$$

and hence

$$\begin{aligned}\frac{L}{C_p}\left\langle\left(\frac{\theta}{T}M\right)_a-\left(\frac{\theta}{T}M\right)_b\right\rangle&=\left\langle w_a\left(\frac{\partial\theta_a}{\partial z}-\frac{\partial\theta_{ea}}{\partial z}\right)\right\rangle\\&=<w_a>\left(\frac{\partial\theta_0}{\partial z}-\frac{\partial\theta_{e0}}{\partial z}\right),\end{aligned} \tag{B.8}$$

Substituting (B.4), (B.5) and (B.8) into (B.3), we can obtain the equation (2.12).

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