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OBSERVATIONS OF THE EFFECT ON THE TIDAL STRAIN OF TORSIONAL TYPE DEFORMATION

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Abstract

The formulas for the observable quantities of the tidal strains of the torsional type are set out in this paper.

It is also shown that through observations of the phase lags of the tidal strain, the torsional type deformation can be detected.

1. Introduction

The observable quantity of tidal deformation usually has been discussed as being deformation of the spheroidal type only, on the assumptions that the earth's shape is a complete sphere and that the physical properties of the earth's material are distributed symmetrically in respect to the earth's center. Also included these assumptions is that there is no moment in the tide generating force.

The real earth, however, has a non-symmetrical shape and the distribution of elasticity in the earth is not a function of central-distance only but also of latitude and longitude. Consequently, its tide generating force has some moment components.

The author (Ozawa [1966]) has obtained observable strain components of deformation of the spheroidal type in the earth tide. In this paper he discusses those of the torsional component.

According to his investigations, by superposing torsional type deformation onto the spheroidal type the deviation of amplitude is not large, but phase shift is most effective. Therefore, the observation of phase lag can be utilized to detect the exsistence of torsional type deformation in the earth tide.

2. Theoretical calculations

Let us consider one kind of observation of the earth tide, for example, tilting, gravity-acceleration change, extensioning and so on. Using one kind of observation, we are able to classify the tidal components only periodically as

fortnightly, diurnal, semi-diurnal and so on. There components of the potential of the tide generating force, W, may be approximately summarized as follow:

$$W=k'M[A_0\{P_i(\cos\delta)\cdot P_i(\cos\theta)\} \\ +A_1\sin 2\delta\sin 2\theta\cos(t+\phi)+\sum_{i=2}A_i\cos^i\delta\sin^i\theta\cos i(t+\phi)] \quad \cdots\cdots\cdots\cdots(1)$$

where $A_0\propto(r^2/c^3)F_0$, $A_1\propto(r^2/c^3)F_1$, $A_i\propto(r^i/c^{i+1})F_i$, $P_i(\cos\delta)$ and $P_i(\cos\theta)$ are the Legerndre functions of i degrees in respect to $\cos\delta$ and $\cos\theta$, respectively. F, F_1 and F_i are constants in respect to the component-tides, and M, k', δ, θ, t, ϕ, r and c are the mass of the heavenly body, for example the moon or the sun, the gravitational constant, the declination, the colatitude, the hour angle of the pending heavenly body at the observatory, the eastern longitude of the observatory, the distance from the earth's center to the observatory, and the distance from the center of the heavenly body to the earth's center, respectively.

The general expressions of the displacement components u_r, u_θ and u_ϕ of the earth's deformation are shown as (Alterman et al., [1959]),

$$\left.\begin{aligned} u_r&=\sum_i\frac{H_i(r)}{g}W_i,\\ u_\theta&=\sum_i\left\{\frac{L_i(r)}{g}\frac{\partial W_i}{\partial\theta}+\frac{N_i(r)}{g\sin\theta}\frac{\partial W_i}{\partial\phi}\right\},\\ u_\phi&=\sum_i\left\{\frac{L_i(r)}{g\sin\theta}\frac{\partial W_i}{\partial\phi}-\frac{N_i(r)}{g}\frac{\partial W_i}{\partial\theta}\right\},\end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(2)$$

where $H_i(r)$, $L_i(r)$ and $N_i(r)$ are functions of r only, and g is the mean value of the acceleration of gravity at the earth's surface.

Using (1), (2) and the relations between the strain elements, e_{rr}, $e_{\theta\theta}$, $e_{\phi\phi}$, $e_{\theta\phi}$, and the rotational elements ω_r, ω_θ, ω_ϕ, of the components of torsional type deformation as follows,

i) Boundary condition at the earth's surface

$$(i-1)N_i(r)+r\frac{\partial N_i(r)}{\partial r}=0. \quad \cdots\cdots\cdots\cdots(3)$$

ii) Strain components at the earth's surface,

a) long period's tide (case of $i=2$)

$$\left.\begin{aligned} &e_{rr}=e_{\theta\theta}=e_{\phi\phi}=0,\\ &e_{\theta\phi}=-\frac{3}{ag}k'MA_0P_2(\cos\delta)N_2(a)\sin^2\theta,\\ &2\omega_r=\frac{1}{ag}k'MA_0P_2(\cos\delta)N_2(a)(3\cos^2\theta-1),\\ &2\omega_\theta=-\frac{3}{2ag}k'MA_0P_2(\cos\delta)N_2(a)\sin 2\theta,\\ &2\omega_\phi=0,\end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(3)$$

b) diurnal tide

$$\left.\begin{aligned}
&e_{rr}=0,\\
&e_{\theta\theta}=\frac{2}{ag}k'MA_1N_2(a)\sin 2\delta\sin\theta\sin(t+\phi),\\
&e_{\phi\phi}=-\frac{2}{ag}k'MA_1N_2(a)\sin 2\delta\sin\theta\sin(t+\phi),\\
&e_{\theta\phi}=-\frac{2}{ag}k'MA_1N_2(a)\sin 2\delta\sin 2\theta\cos(t+\phi),\\
&2\omega_r=\frac{4k'M}{ag\sin\theta}A_1N_2(a)\sin 2\delta(\cos 2\theta-\cos\theta)\cos(t+\phi),\\
&2\omega_\theta=0,\\
&2\omega_\phi=0,
\end{aligned}\right\}\quad\cdots\cdots\cdots(4\text{-}2)$$

c) semi-diurnal tide

$$\left.\begin{aligned}
&e_{rr}=0,\\
&e_{\theta\theta}=-\frac{2k'M}{ag}A_2N_2(a)\cos^2\delta\cos\theta\sin 2(t+\phi)\\
&e_{\phi\phi}=\frac{2k'M}{ag}A_2N_2(a)\cos^2\delta\cos\theta\sin 2(t+\phi),\\
&e_{\theta\phi}=\frac{2k'M}{ag}A_2N_2(a)\cos^2\delta\sin^2\theta\cos 2(t+\phi),\\
&2\omega_r=-\frac{2k'M}{ag\sin\theta}A_2N_2(a)\cos^2\delta(3\cos^2\theta-2\cos\theta-1)\cos 2(t+\phi),\\
&2\omega_\theta=-\frac{4k'M}{ag}A_2N_2(a)\cos^2\delta\cos\theta\cos 2(t+\phi),\\
&2\omega_\phi=\frac{k'M}{ag}A_2N_2(a)\cos^2\delta\sin\theta\sin 2(t+\phi),
\end{aligned}\right\}\quad\cdots\cdots(4\text{-}3)$$

d) i-th diurnal tide

$$\left.\begin{aligned}
&e_{rr}=0,\\
&e_{\theta\theta}=-i(i-1)\frac{k'M}{ag}A_i\cos^i\delta\sin^{i-2}\theta\cos\theta\sin i(t+\phi),\\
&e_{\phi\phi}=i(i-1)\frac{k'M}{ag}A_i\cos^i\delta\sin^{i-2}\theta\cos\theta\sin i(t+\phi),\\
&e_{\theta\phi}=\{i^2\sin^{i-2}\cos^2\theta+i\sin^i\theta-i^2\sin^{i-2}\theta\}\\
&\qquad\times\frac{k'M}{ag}\cos^i\delta A_iN_i(a)\cos i(t+\phi),\\
&2\omega_r=-\{i^2\sin^{i-2}\theta\cos^2\theta-i\sin^i\theta+i^2\sin^{i-1}\theta\}\\
&\qquad\times\frac{k'M}{ag}A_iN_i(a)\cos^i\delta\cos i(t+\phi),\\
&2\omega_\theta=2i\frac{k'M}{ag}A_iN_i(a)\cos^i\delta\sin^{i-1}\theta\cos\theta\cos i(t+\phi),\\
&2\omega_\phi=-2i\frac{k'M}{ag}A_iN_i(a)\cos^i\delta\sin^{i-1}\theta\sin i(t+\phi),
\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(4\text{-}4)$$

iii) Tidal change of latitude. The tidal change of latitude consists of the component, $d\theta_1$, due to the change of the vertical, and that of $d\theta_2$ due to horizontal displacement in the spheroidal type of deformation. However, it consists only of the component $d\theta_2$, in the torsional type of deformation as follows:

$$d\theta_2 = \frac{u_\theta}{a} = \sum_i \frac{N_i(a)}{ag \sin\theta} \frac{\partial W^i}{\partial \phi}. \qquad \cdots\cdots\cdots\cdots(5)$$

a) long period's change (case of $i=2$)

$$d\theta_2 = 0, \qquad \cdots\cdots\cdots\cdots(5\text{-}1)$$

b) diurnal tide

$$d\theta_2 = -\frac{k' N_2(a)}{ag} MA_1 \sin 2\delta \sin 2\theta \sin(t+\phi), \qquad \cdots\cdots\cdots\cdots(5\text{-}2)$$

c) semi-diurnal tide

$$d\theta_2 = -\frac{2k' N_2(a)}{ag} MA_2 \cos^2\delta \sin^2\theta \sin 2(t+\phi), \qquad \cdots\cdots\cdots\cdots(5\text{-}3)$$

d) i-th-diurnal tide

$$d\theta_2 = -\frac{ik' N_i(a)}{ag} MA_i \cos^i\delta \sin^i\theta \sin i(t+\phi). \qquad \cdots\cdots\cdots\cdots(5\text{-}4)$$

According to these formulas and those of the spheroidal type deformation (Ozawa [1957], [1966]), we find that the phase differences between the phase angles of the torsional deformation and those of the spheroidal one are $\pm 90°$. Now, the amplitude of torsional deformation is far smaller than that of spheroidal one, and so the phase shift by superposing the torsional deformation onto the spheroidal one is approximatly largest.

3. Considerations

We should estimate not only the direct effects caused by astronomical tides, but also the indirect effects caused by oceanic tides and atmospheric tides on observed tidal strains. We can easily eliminate the effects (Ozawa [1967]) caused by atmospheric tides, but we still have many difficulties in eliminating the effects caused by oceanic tides. The reasons why elimination of the effects caused by oceanic tides is difficult, are that we still have no precise data for oceanic tides which are extremely complex, and that the load tide of the real earth is not homogeneous within its concentric shells as has been calculated by some researchers (Takeuchi et al. [1965]). It is the most regrettable thing that we cannot obtain observations at places much farther away from the ocean. If tidal observations were performed on the moon's surface we could easily solve this problem of the moon tide because there is neither air nor water on

the moon. Putting aside such day dreams, however, observations made at some places where the ocean is more than a few hundred miles away give interesting data which can be used to evaluate the components of torsional type deformation, constants N_t and so forth. We also know very well that observed tidal components of gravity acceleration, of tilting and of cubical dilatation have some phase lags (cf. Note). The tidal components of gravity acceleration, tilting and cubical dilatation are not connected with the torsional type of deformation of the sphere which has symmetrical distribution of the elastic constants in respect to its center. But it is not hard to estimate that the systematic phase lags of these tidal changes are caused by the torsional deformation on the real earth. Fortunately, according to formulas (4) and (5), the phase differences between the phase angles of the deformation of torsional type and those of spheroidal type are $\pm 90°$. Therefore, the observations of the phase lags can be utilized to detect the existence of the torsional deformation.

Note

Example of observational results of the earth tide.

Cubical dilatation at Osakayama (Ozawa [1966]),

M_2-tide $=1.109\times10^{-8}\cos(2t-1.6°)$, O_1-tide $=0.675\times10^{-8}\cos(t-353.3°)$.

Gravity variation at Kyoto University (Nakagawa et al. [1966]),

M_2-tide $=58.189\cos(2t-189.1°)$, O_1-tide $=33.494\cos(t-178.85°)\mu$gal.

Tilting at Sclaigneaux (Melchior [1966]),

Diminishing factor $1+k-h$ for M_2-tide $=0.977$ in NS-component,
$=0.927$ in EW-component,

Phase lag for M_2-tide $=2.09°$ in NS-component,
$=5.04°$ in EW-component.

Deviation of latitude at Mizusawa (VZT) (Sugawa [1961]),

Correcting factor $1+k-l=1.34$, phase lag $=15°$.

References

Alterman, Z., H. Jarosh and C. L. Pekeris, 1959; *Oscillations of the Earth*, Proc. Roy. Soc. London, *A*. 252, 80–85.

Melchior, P., 1966; *The Earth Tides*, Pergamon Press.

Nakagawa, I., T. Mikumo and T. Tanaka, 1966; *Spectral Strucrure of the Earth Tides and Related Phenomena —Gravimetric Record—*. Special Contributions, Geophys. Inst., Kyoto Univ. 6, 225–232.

Ozawa, I., 1957; *Study on Elastic Strain of the Ground in Earth Tide*, Disaster Prevention Res. Inst., Kyoto Univ., Bull., 15, 1–37.

Ozawa, I., 1966; *On the Observations of the Tidal Strains at Osakayama Observatory*, Special Contributions, Geophys., Inst., Kyoto Univ., 6, 233–246.

Ozawa, I., 1967; *Observation of the Atmospheric Tide Effect on the Earth's Deformations*, Special Contributions Geophys., Inst., Kyoto Univ., 7, 133–142.

Sugawa, C., 1961; *Determination of* $1+k-l$ *from latitude observations made at Mizusawa*, Communications de l'Observatoire Royal de Belgique N°188, Série Geóphysique N°58, Quatriéme Symposium International sur les Marées Terrestres, 76–77.

Takeuchi, H. and L. E. Alsop, 1965; *Comparison of Theoretical and Observational Expression for Elastic Strain of the Earth*, Bull., Seis., Soc., Amer., 55, 153–163.