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THE FLUCTUATION OF THE LEVEL OF THE WATER TABLE DUE TO BAROMETRIC CHANGE

By

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Abstract

Fluctuations of the water level in a water table well were observed in which the diurnal and the semi-diurnal were most remarkable. Such fluctuations have a close relationship to those of barometric pressure. The water level fluctuations seem to occur somewhat in advance of barometric changes. It has been found that the ratios of amplitudes become smaller, and the phase advances increase when fluctuations have longer periods. Those phenomena are explained on the basis of the theory of pressure propagation through soil air, in which the permeability of the underground and its effective porosity are estimated as about 4.8 Darcy and 0.3 respectively.

1. Introduction

For the purpose of investigating the mechanism of the infiltration in an alluvial cone, a well about 31 m deep was drilled at the Geophysical Research Station, Kyoto University in Beppu to reach the ground water table and the profile of the soil-water contents around the well were observed with a neutron moisture probe. In order to understand the relationship between the depth of the ground water table and the profile of soil-water contents, the water level was observed once a day until November 1968, and after that continuous observation was carried out by self-recording equipment.

The water level shows seasonal variation under the influence of precipitation. In addition to this, it is recognized that fluctuations having small amplitudes always superpose on the curve. Our attention was attracted by the fact that the latter fluctuations had nearly semi-diurnal periods occuring every day throughout the period of observation. Some possible causes which would have an effect upon such fluctuations have been investigated, and finally the conclusion has been reached that the most probable cause seems to be changes in the barometric pressure.

2. Field conditions around the observation well

The location of the observation well (31 m deep) is about 2 km distant from the sea shore line and about 80 m above sea level. Some wells tapping the deep artesian aquifers of hot ground water are found here and there, the nearest of which is more than 150 m away from this well. There is, however, no well taking upper unconfined waters of low temperature.

The ground surface around this well is almost flat and bare. According to core samples obtained during the drilling work, the underground layer is composed of sands and gravels to the bottom of the well involving fairly large rocks in places. An unperforated casing pipe was inserted to the bottom of the 30 m drilling.

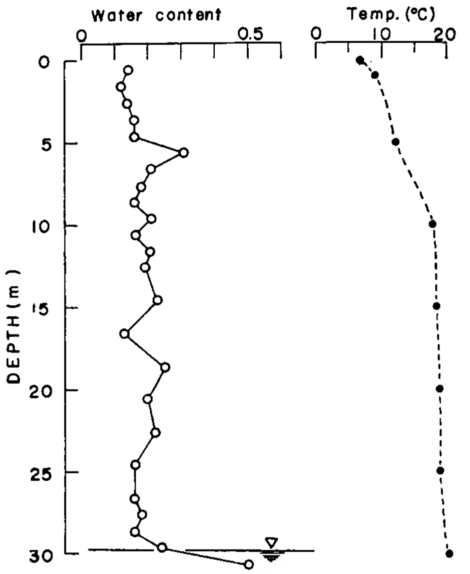

Fig. 1. Profiles of water content and temperature observed in Jan. 9, 1968. The water contents (volume fraction) were obtained from the measurements by the neutron moisture probe.

The vertical profiles of the soil-water content and temperature are shown in Fig. 1. The soil-water contents were determined from measurements by a neutron moisture probe. A thin layer at about 5 m depth is characterized by high water content, but a water level is not found in this layer. It almost corresponds to the place occupied by a layer of hornblende andesite rock about 40 cm thick. Below this, however, the layer composed of sands and gravels reappears, and is similar to conditions found near the ground surface. Therefore, it is thought that this isolated rock may contribute little to the macroscopic continuous passage of water or air. The high water content at this depth probably results from water being held on the upper side of the rock. Except for this, the water content is almost uniform from the surface to the water table, being around 0.2 in volume fraction. Then, going down nearer the water level, the water contents sharply increase to reach their highest value of about 0.5. The water contents in the submerged and unsubmerged layers are always shown as about 0.5 and 0.2 irrespective of the level of the water table during the whole period of observation.

It is reported that the general distribution of the water content in the unsaturated zone shows gradual decrease in accordance with the distance from the water table and, in addition, a saturation zone is sometimes found in some thickness on the water table. However, the observed distribution of water content in this field differs from the general feature outlined above, and characteristically indicates that the water content of the upper and lower sides of the water table are quite different from each other. Thus, it is considered that the soil-air extends continuously from ground surface to a level very close to that of the water table. This region is called the air continuous zone, and is an area in which air is able to flow freely (see Fig. 7). The porosity in this field is taken as 0.5 equal to the water content in the saturated zone below the water table. The difference between the above mentioned porosity and the water content in the upper side of the water table is 0.3, which is assumed as the effective porosity contributing to the flow of soil-air through the air continuous zone.

The temperatures shown in Fig. 1 were observed in winter, and they increase with depth. They show daily and seasonal variations, but the values below 5 m depth, especially below 10 m depth, are maintained at an almost constant value of 20°C.

3. Results of the observation

Though many examples were reported in which the level of the water table is apparently affected by precipitation, the effect does not appear immediately, but is only very slowly noticeable, taking as long as ten days in this well.

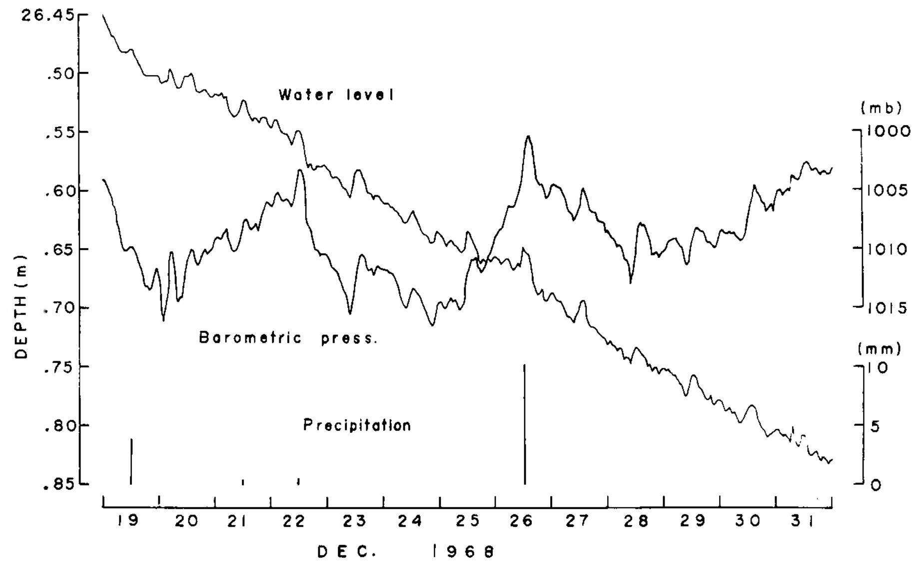

Fig. 2. An example of the water level record.

The water table, thus, gradually rises and falls throughout the year, and it seems that the superposing effects of different precipitations are remarkable only as a seasonal variation. Throughout such variations, it reaches its highest level (about 26 m below ground surface) in early November, then descends almost linearly below the bottom of the well around May.

An example of a record of water level is shown in Fig. 2 with daily precipitation. This interval already belongs to the depressed period of the water table, which is gradually falling at a rate of about 3 cm a day. It is interesting to find that fluctuations of short periods are superposing on the depression curve. In addition, these fluctuations occur every day, rain or shine. Consequently, they do not seem to be attributable to the precipitation. Watching them carefully, it is evident that they have approximately semi-diurnal periods, i.e. the maximum levels occur at about 0 and 13 o'clock and the minimum levels at about 9 and 18 o'clock respectively.

It is well-known that the barometric pressure fluctuates with semi-diurnal and diurnal periods, and it is noted that the water level fluctuation in this observation well behaves in a very similar way.

Therefore, in Fig. 2 the barometric pressure data observed at the Geophysical Research Station, Kyoto University in Beppu are plotted and compared to the water level. According to this figure, the approximately semi-diurnal fluctuations of the barometric pressure are also recognized, superposing on the fluctuations over a period of several days. We can, then, see the close relationship between both the semi-diurnal fluctuations of barometric pressure and the water table; the rise and fall of the water level are found to correspond to the fall and rise of the barometric pressure. It is also recognized that there seems to be some relationship between both fluctuations whose periods are several days long.

In order to draw out the fluctuations whose periods are shorter than 24 hours, procedures of 24 hours running mean were carried out on hourly values of the barometric pressure and the water level, and then the differences between the calculated results and the observation data were obtained. Fig. 3 shows an example of the results from the above procedure. As shown in Fig. 3, approximately semi-diurnal fluctuations appear distinctly, and a positive relation is well established between both fluctuations, though the water level fluctuates slightly in advance of the barometric prressure. Such a relationship continues almost regularly throughout the observation period. Next, by gathering statistics over any period of several successive days, we can obtain patterns of both fluctuations whose periods are shorter than 24 hours. For example, using the modified data shown in Fig. 3, the patterns of barometric change and water

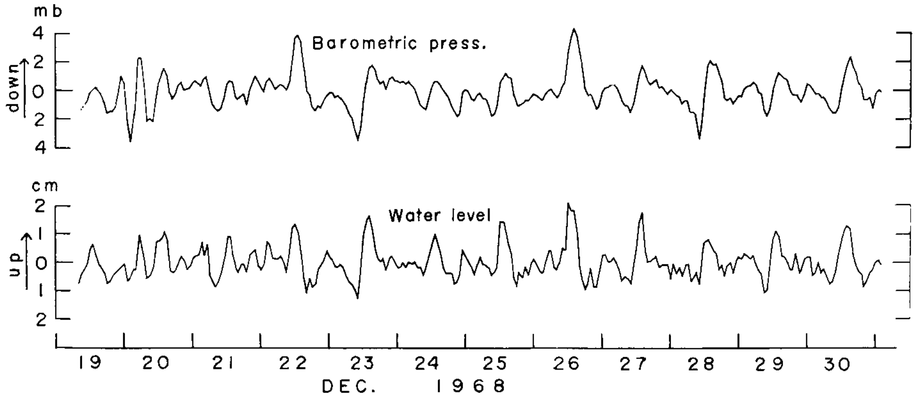

Fig. 3. Fluctuations of the barometric pressure and the water table, whose periods are shorter than 24 hours.

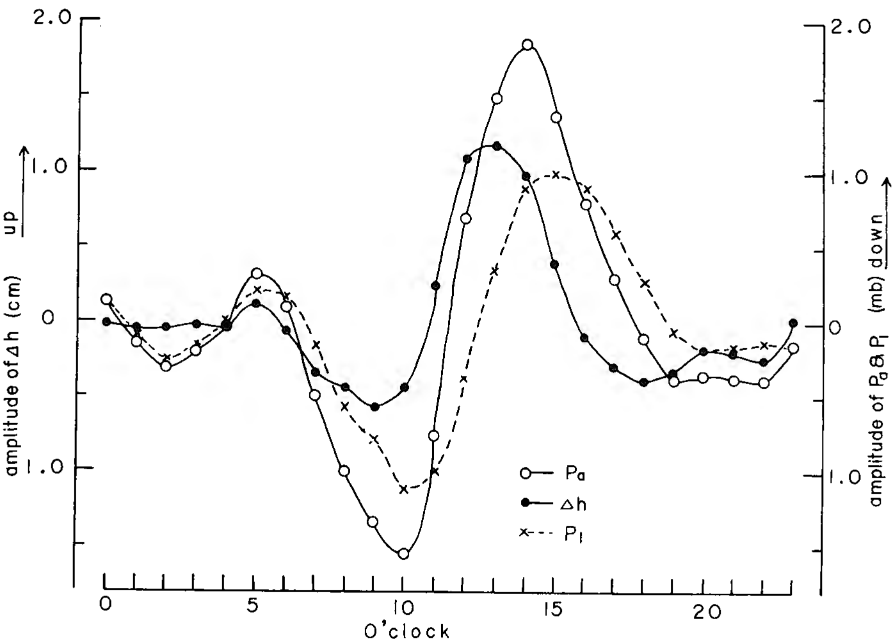

Fig. 4. Patterns of Δh, p_a and p_l in Dec. 20~29, 1968.

level fluctuation from December 20 to 29, 1968 are obtained and shown in Fig. 4 as p_a and Δh respectively.

The results of the harmonic analyses of p_a and Δh in Fig. 4 show that the diurnal and semi-diurnal are predominant in both fluctuations and the amplitude ratios ($\Delta h/p_a$) are 0.46 in diurnal and 0.54 in semi-diurnal, and the phase advances are 0.834 and 0.670 (radian) respectively.

As mentioned previously, the fluctuations over a sevaral day period may be deduced from the data shown in Fig. 2. In order to clarify them, a 150 hours running mean was carried out on the results of a 24 hours running mean, and then the parts of deviation were obtained. The results are shown in Fig. 5. The close relationship between both fluctuations is also recognized, but the

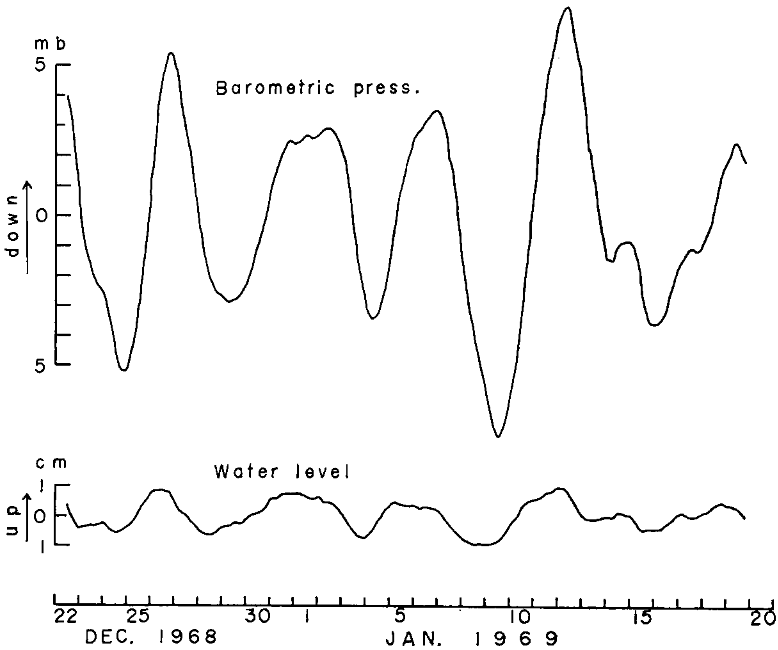

Fig. 5. Long-period fluctuations of the barometric pressure and the water table.

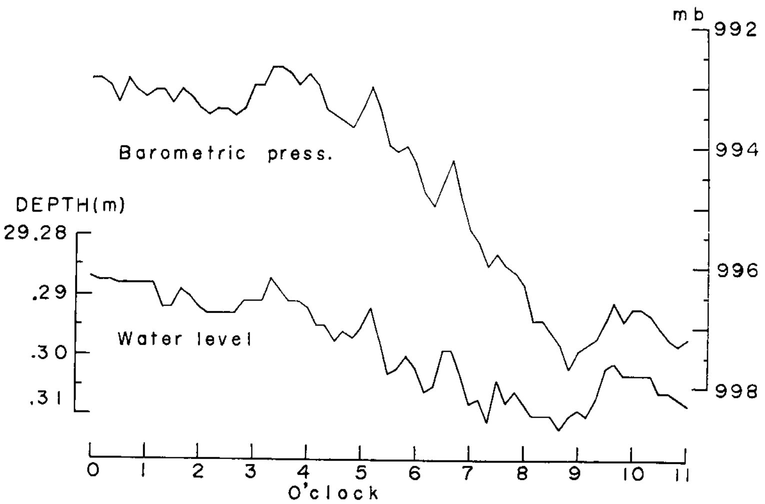

Fig. 6. Short-period fluctuations of the barometric pressure and the water table, observed on March 21, 1969.

amplitude ratios, the mean value of which is about 0.15, are smaller than those of diurnal or semi-diurnal ones. The preceding phases are larger than those of the shorter period fluctuations, in which advances are about 6~12 hours.

Fig. 6, then, indicates the micro-variations having very short periods (a few hours) which were observed on 21th March, 1969. The amplitudes and phases of water level fluctuations are nearly equal to those of the barometric pressure. (The pressure of 1 mb is nearly equal to 1 cm of water column) Summarizing the phenomena, we can make the following observations:

(1) Barometric effect on the water table in this observation well is clearly evident for the variations having periods shorter than a few days.

(2) The longer the periods are, the smaller the amplitude ratios become.

(3) The longer the periods are, the greater the phase advances become.

It is generally explained that the water level in a water table well is not affected by barometric change. In Fig. 7, p_l is the variable part of the pressure just on the lower limit of the air continuous zone, p_a is that of the barometric pressure not only on the ground surface but also on the water level in a well, and Δh is the variation of the level of the water table. (The water table is generally defined as the surface on which the pressure of water is equal to the barometric pressure, and consequently it is represented by the water level in a well. p_l and p_a are designated by the heights of the water column.)

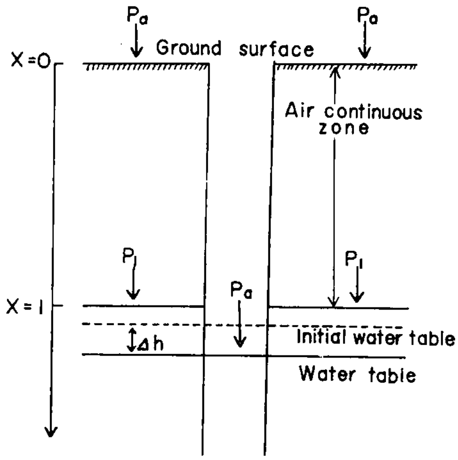

Fig. 7. Schematic model of the water table fluctuation due to the barometric change.

When the diameter of the well is so small that the water volume going in and out of the well is almost negligible compared with the water table fluctuation, the following equation for the pressure balance is established.

$$p_l = p_a - \Delta h, \tag{1}$$

Since the pressure in the air continuous zone does not usually differ so much from the barometric pressure; i.e. $p_l \fallingdotseq p_a$, Δh is nearly equal to 0. This is the cause that the water table is generally regarded as not being affected by barometric change.

It it considered, however, that the pressure change (p_l) which is propagated through porous materials would tend to decrease owing to the compressible character of air. The difference between p_l and p_a, then, becomes a distinct one as the distance of propagation becomes greater. It may be possible that the level of the water table would be changed owing to such a difference, as was seen in the field where the depth of the ground water table was large enough to be observed. Therefore, the problem of water table fluctuation must be solved by consideration of the mechanism by which pressure is propagated from the surface to the water table.

Applying Eq. (1) to the records of water level fluctuations over several periods, we have next summarized relationships between p_l and p_a.

(1)′ Amplitude of p_l is always smaller than that of p_a.

(2)′ The shorter the period of time, the smaller the amplitude ratios (p_l/p_a) become.

(3)′ The shorter the period of time, the greater the delay of the phases of p_l.

Now, let us look at the pattern of p_l whose periods are shorter than 24 hours. An example of its pattern is drawn with a dashed line in Fig. 4, which has been calculated from Eq. (1) using values of Δh and p_a which are shown in the same figure. The amplitude of p_l is smaller than that of p_a, and the phase is delayed. The procedure of harmonic analysis was also performed for p_l, and the result was compared with that of p_a. According to that analysis, the amplitude ratios (p_l/p_a) of diurnal and semi-diurnal fluctuations are 0.78 and 0.67, and the phase lags of p_l are 0.454 and 0.530 (radian) respectively.

4. Propagation of pressure through soil air

It is frequently stated, as has already been pointed out in the previous section, the water level in a water table well is not affected by the barometric change. Nevertheless, some observations of the barometric effect on the water table have been previously reported. Tolman [1939] gives an instance, in his '*Ground Water*', that the fluctuation of the water table due to barometric change was observed only when the ground surface was wet, i.e. when the communication between the atmosphere and the water table was estranged. J. von Eimern [1950] reported the phenomenon that a water level decline in the water table well amounting to 1 cm of water column occurred accompanying an increase in barometric pressure of 4 cm of mercury column. He accounted for this phenomenon by basing his explanation on the theory of Lucke and Rose [1938], which is deduced from the fundamental equation for gas flow through porous materials.

We will interpret the results of our observations on the basis of analysing the propagation of the sinusoidal train of barometric change by an approximating theory from generalized Darcy's law.

The equation of motion for air flow through porous materials is expressed as follows.

$$v=-\frac{k}{\mu}\frac{\partial P}{\partial x}, \tag{2}$$

where v is velocity, k is permeability, μ is viscosity of air, and P is pressure. In the above equation, the flow is assumed to be one-dimensional, and the directions of x-axis and v are downwardly positive. The effect of gravity is neglected.

The continuity equation is

$$\phi\frac{\partial\rho}{\partial t}=-\frac{\partial(\rho v)}{\partial x}, \tag{3}$$

where ρ is density of air, and ϕ is effective porosity defined in Section 2 and assumed as constant.

Since the ground temperature is almost uniform at about 20°C except near the ground surface, we assume that the flow is isothermal and moreover that air is an ideal gas. Consequently, the equation of state is expressed as follows.

$$P/\rho=P_0/\rho_0=\text{const.} \tag{4}$$

where suffix 0 represents the standard state, in this case 1 atm and 20°C.

If we assume that k and μ are independent of x, we have the next fundamental equation by combining Eqs. (2), (3) and (4).

$$\begin{aligned}\phi\frac{\partial P}{\partial t}&=\frac{k}{\mu}\frac{\partial}{\partial x}\left(P\frac{\partial P}{\partial x}\right)\\&=\frac{k}{\mu}\left\{P\frac{\partial^2 P}{\partial x^2}+\left(\frac{\partial P}{\partial x}\right)^2\right\},\end{aligned} \tag{5}$$

Neglecting the square term $(\partial P/\partial x)^2$ and moreover assuming that ρ is nearly equal to ρ_0, the following linearized equation is approximately obtained.

$$\left.\begin{aligned}&\frac{\partial P}{\partial t}=\kappa\frac{\partial^2 P}{\partial x^2},\\&\kappa=\frac{kP_0}{\mu\phi}\end{aligned}\right\} \tag{6}$$

Putting $P=\bar{p}+p$, where $\bar{p}$ and p correspond to the mean part and the variable part of P respectively, it follows

$$\frac{\partial p}{\partial t}=\kappa\frac{\partial^2 p}{\partial x^2}, \tag{7}$$

because $\bar{p}$ is regarded as constant over the whole region of the air continuous zone.

Then we assume that the barometric change at the ground surface ($x=0$) is periodical (sinusoidal), and that the flow of air does not occur at the level of the lower limit of the air continuous zone ($x=l$, see Fig. 7). We assume, for simplicity of treatment, that the level of $x=l$ is maintained almost constant during the interval of one day or so. Thus, the boundary conditions are expressed as follows:

$$\left.\begin{aligned} p_{x=0} &\equiv p_a = A\sin\omega t, && \text{at} \quad x=0 \\ \frac{\partial p}{\partial x} &= 0, && \text{at} \quad x=l \end{aligned}\right\} \tag{8}$$

where A is the amplitude of the barometric fluctuation, and $\omega=2\pi/T$; T is the period.

From Eqs. (7) and (8), we have a solution at $x=l$,

$$\left.\begin{aligned} & p_{x=l}\equiv p_l=\frac{A}{\sqrt{D}}\sin(\omega t-\delta), \\ & D=\cos^2 \alpha l\cdot\cosh^2 \alpha l+\sin^2 \alpha l\cdot\sinh^2 \alpha l, \\ & \delta=\tan^{-1}(\tan \alpha l\cdot\tanh \alpha l), \\ & \alpha=\sqrt{\frac{\omega}{2\kappa}}, \end{aligned}\right\} \tag{9}$$

Using the above solution, the amplitude ratios, $p_l/p_a=1/\sqrt{D}$, and the phase lags δ are calculated for various values of αl, and the results are shown in Fig. 8, in which it is easily understood that the shorter the periods are (i.e. the larger the ω are), the smaller the amplitude ratios become and the larger the phase lags are. They correspond with conditions already described in (1)′, (2)′

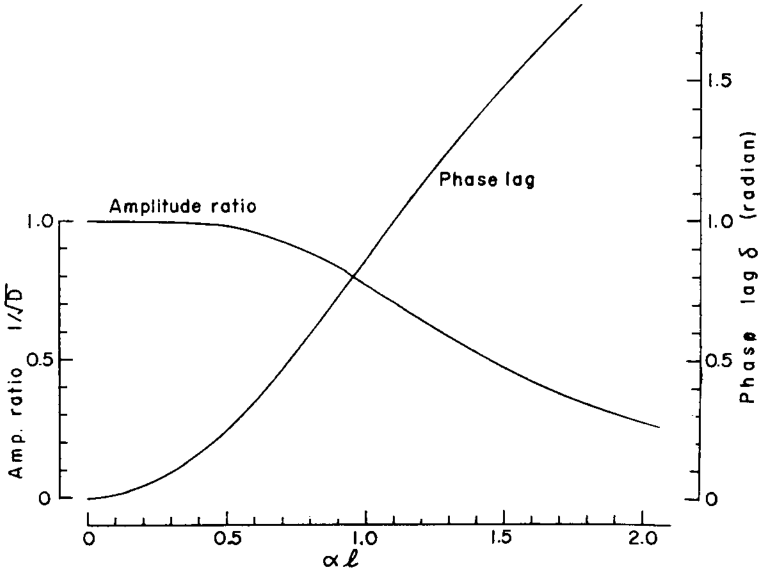

Fig. 8. Relationships between the amplitude ratio or the phase lag and αl calculated from Eq. (9).

and (3)′.

We, then, obtain the next expression for Δh from Eqs. (1), (8) and (9).

$$\left.\begin{aligned}\Delta h&=p_a-p_l\\&=A\sqrt{1-\frac{2\cos\alpha l\cdot\cosh\alpha l}{D}+\frac{1}{D}}\sin(\omega t+\theta),\\\theta&=\tan^{-1}\left(\frac{\sin\alpha l\cdot\sinh\alpha l}{D-\cos\alpha l\cdot\cosh\alpha l}\right),\end{aligned}\right\}\tag{10}$$

It is found in the above equations that the amplitude ratio between the fluctuations of the water table and the barometric pressure, $\Delta h/p_a=\sqrt{1-\frac{2\cos\alpha l\cdot\cosh\alpha l}{D}+\frac{1}{D}}$, is diminished, and the value of θ becomes large (always positive), as the value of αl decreases (i.e. the period becomes longer.). Therefore, Eq. (10) can also explain the features of barometric effect on the water table fluctuation expressed by the remarks (1), (2) and (3).

5. Discussions and conclusions

It has been understood that relatively short period fluctuations of the water level in this well can be explained for the most part by the supposition that they are due to propagation of pressure through soil air. Consequently, it is felt that the data of water level fluctuation offers useful information about the pressure propagation through soil air and about the permeable structure of the underground layer.

Let us look at the relationship between the amplitude ratios p_l/p_a and the phase lags δ. An example of them has already been shown in Section 3. In addition, the procedure of harmonic analysis was performed upon other successive-ten-day statistical data as well as upon that shown in Fig. 4, and some amplitude ratios and phase lags were obtained. The results are plotted in Fig. 9.

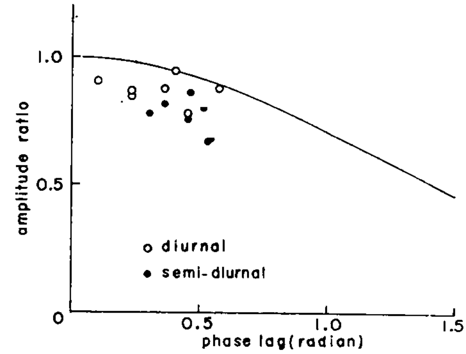

Fig. 9. Relationships between the amplitude ratios and the phase lags. The solid curve indicates the theoretical relationship calculated from Eq. (9).

The hollow circles and black circles correspond to the diurnal and semi-diurnal fluctuations respectively. The solid curve in the same figure is calculated from Eq. (9). The observation values for each statistical period are changing irregulary. However, the amplitude ratios of semi-diurnal fluctuations are essentially smaller than those of

Table 1. The mean values of amplitude ratios and phase lags of p_l for diurnal and semi-diurnal fluctuations.

Period	Amplitude rario	Phase lag (radian)
Diurnal	0.873	0.337
Semi-diurnal	0.797	0.451

diurnal fluctuations, and the phase lags of the former are larger than those of the latter. The mean values for each fluctuation are tabulated in Table 1.

In Fig. 9, it is most interesting that all the circles obtained from the observation data are situated on or at the lower side of the theoretical line, which seems to mean the upper limit of experience values. This may suggest that there is a diminishing of the amplitude without phase lags when the barometric change is propagated through soil air from the ground surface. Such phenomena may possibly result from the effect of a thin layer, which cause only the diminishing of the amplitude, at or near the ground surface. And when such a thin layer is not formed, the observation data may be situated just on the theoretical line. The occurrence of such a layer is, perhaps, closely related to the ground surface conditions; e.g. whether the ground surface is wet or not, etc. However, we were not able to get information about these conditions because of the shortness of the observation period. Moreover, we have assumed that the temperature is constant over the whole range of the air continuous zone, but actually it fluctuates near the ground surface during the diurnal period. So, there is a possibility that the temperature fluctuation affects on the pattern of pressure fluctuation as a condition at ground surface.

Next, let us estimate the permeability of the underground layer. Using the amplitude ratios or the phase lags given in Table 1, the values of k/ϕ can be estimated from Fig. 8, and the results are shown in Table 2 with remarks for estimation. The values of other properties necessary for calculation are taken as follows: P_0 is 1 atm (1.03×10^6 c.g.s), μ is 1.81×10^{-4} c.g.s (at 20°C) and l is 2.78×10^3 cm. This value of l is given from the mean depth of the water table during the observation period by assuming that the level of the lower limit of the air continuous zone is nearly equal to that of the water table based on the

Table 2. The values of k/ϕ calculated from the data shown in Table 1.

Period	From amplitude ratio	From phase lag
Diurnal	7.91×10^{-8} cm^2	2.04×10^{-7} cm^2
Semi-diurnal	1.03×10^{-7} 〃	2.49×10^{-7} 〃

profile of the soil-water contents shown in Fig. 1.

The values of k/ϕ from diurnal fluctuation are smaller than those from semi-diurnal fluctuation, and the values from phase lags are about 2~2.5 times as large as those from amplitudes ratios. For the present it has not been determined which is the most reliable value of the actual permeability, because the agreement between observed and theoretical results were not completely saticfactory, as previously mentioned in Fig. 9. However, they are useful estimating the approximate value of permeability. If we assume that the effective porosity ϕ is about 0.3 from the soil-water content data, then the coefficients of permeability, k, are within the relatively narrow range of $2.37\sim7.47\times10^{-8}$ cm^2. Taking their mean value, it is given as 4.8×10^{-8} cm^2, which corresponds to about 4.8 Darcy. This value is reasonable as the permeability of an underground layer composed of sands and gravels (Todd [1959]). It may be useful when we investigate the infiltration process of this field in the future.

Therefore, if the value of k/ϕ is constant throughout the observation period, the amplitude of p_l will decline to a greater degree and the phase lags will become larger with the lowering of the water table because of the increased value of αl. Actually, such tendencies were not found. It is thought that the properties of ground condition such as k and μ etc. may vary when a difference in the level of water table is quite apparent. However, such a variation of field properties has not yet been discovered.

Using the mean value of k/ϕ and l, the ratio of the amplitude of the water level fluctuation to that of the barometric change having one week period is calculated as 0.04 and the phase advance becomes 1.53 radian from Eq. (10), while the observation values of the amplitude ratio and the phase advance shown in Fig. 5 are about 0.15 and 0.5 radian respectively. Thus, there are some discrepancies between the theory and the observation. The precise interpretation of some details have been left as problems for the future. However, it has become clear that one of the main causes of the water table fluctuation in this field is the effect of damping the barometric change through propagation in soil layer.

Of course, water table fluctuations over relatively short periods may be influenced by other factors, such as ocean tides and artificial pumpings near the well. If these are the effects of tides, fluctuations having periods of about 12.5 hours or 25 hours should appear, but no such periods were observed. In regard to the artificial withdrawal of ground water, probable effects may be considered by pumpings from artesian wells crowded together in the city rather than from other water table wells, because there is no water table well near the observation well and the shallow unconfined aquifers are not entirely isolated from the deep artesian aquifers. Since the pumpings are being performed at almost regular times every day, the effects resulting from them will also appear regularly. However, a clear example of barometric effect is obtained, as shown in Fig. 10. It shows that on days when the barometric change shows quite a different pattern from usual (i.e. semi-diurnal fluctuation is not found, and the time of the maximum is about 8 o'clock), the water table fluctuation shows a pattern similar to that of the barometric change. It is therefore concluded that water table fluctuation having relatively short periods occur only because of barometric change, and other effects may be negligible in comparison.

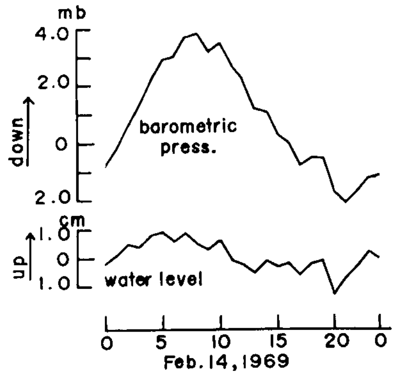

Fig. 10. A distinct example of the barometric effect on the water table fluctuation.

In spite of this, why have such phenomena rarely been observed in other fields? The reason can probably be attributed to the fact that the depths of the water table in other fields are usually much shallower, often by several meters. If the depth in this field had been half of its actual depth, it would have been difficult to detect such diurnal or semi-diurnal fluctuations except for the very short period fluctuation, as can easily be deduced from Eq. (9) or (10). Therefore, as shown in Tolman's '*Ground Water*' [1937], barometric effect on the water table could be observed only when the layer depressing the permeable character of soil appears at or near the ground surface and the water table is relatively intercepted from the atmosphere.

If the above condition is established after a severe precipitation in this field, the water table fluctuation will show different features. However, there was no severe precipitation completely covering the ground surface with water, nor snow film during the observation period.

Acknowledgements

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