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A METHOD TO ANALYSE THE EFFECT OF PRECIPITATION ON THE GROUND WATER STREAM

By

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Abstract

The variation of the level of the ground water table involves the total effect of hydrological elements necessary to determine the budget of water balance under the ground. It then appears that the value of each element can be found by analysing the state of variation in the water level. Long time records of the water level in a well in Beppu City are analysed according to the approximate theory concerning the effect of precipitation on the ground water stream under water table conditions. The result is used to estimate the water budget in the basin. A part of precipitation does not have an effect on the ground water stream and its rate is obtained by the method of analysing the declining state of the water level as being about one-third of the precipitation. This value shows good agreement with the observed effect of precipitation. A part of the supply to the lower confined ground water in the upstream part of the basin does not come back to the upper unconfined aquifer owing to artificial withdrawals from the lower aquifer. Its rate is estimated and compared with the rate of withdrawals of hot water in the city. They recently show a clear tendency to be increased.

1. Introduction

It is well-known that water level in a well shows gradual lowering during the period without recharge and rises under the effect of precipitation. Such observed data were often analysed so as to be expressed by the following formula.

$$dh/dt=\lambda(h_0-h)-cP, \tag{1}$$

where h is the height of water level taken downward from the ground surface, P is precipitation and other symbols, λ, h_0 and c, are coefficients obtained to satisfy the observed condition. λ is called as a coefficient of decay and h_0 is a base level, both of which are considered as being characteristics of the aquifer. Contrary to such opinions, both values, especially the value of h_0, have often been found as variables during different periods of observation in the same well. It means that such coefficients are not determined by geometry only but also by hydrological properties in the basin, though the hydrological meanings of such variables have not been made clear because Eq. (1) has never been theoretically presented but only empirically.

Another problem in Eq. (1) is that it often shows fairly good agreement with the observed trends of water level not only in the unconfined aquifer but in artsian wells. It has as its supposition that the effect of precipitation on artesian ground water must be related to that on the ground water table. Infiltration from ground surface first reaches the water table and makes it rise. The pressure rise of water, then, propagates through confining stratum to the lower aquifer and forces the rise of the piezometric level, which is laterally transmitted through the artesian aquifer. It may be possible that the flow of ground water follows such changing vertical gradients of water pressure through semi-impervious stratum. Some expressions concerning the possible effects of precipitation are, then, presented according to the model of ground water basin after Tóth [1962]. Effect of precipitation promotes the supply to the artesian aquifer in a so-called recharge area, being the upstream part of the basin, and increases the discharge to the upper unconfined aquifer through a semi-impervious layer or to the ground surface through bore holes in a so-called discharge area, being the downstream part of the basin. This is a process of ground water circulation between ground surface and deep aquifer. The changing state of the level of the water table as expressed by Eq. (1) must, therefore, involve at least three of the important factors governing water circulation under the ground.

(1) Relation between precipitation and recharge across the water table. (2) Loss from unconfined ground water stream by supplying the lower confined water. (3) Hydrological character included in the lateral flow through the unconfined aquifer.

It would be possible to find such conditions by estimating the value of λ, h_0, or c empirically obtained, if theoretical deviation of the form as Eq. (1) is possible.

2. An approximate theory of ground water flow under water table conditions

Ground water stream is assumed only laterally in the unconfined aquifer and Dupuit's approximation is adapted to simplify the unsteady flow as follows.

$$S\frac{\partial z}{\partial t} \doteqdot kz\frac{\partial^2 z}{\partial x^2}+R, \tag{2}$$

where z is the height of the water table from the bed of the aquifer, R is the rate of recharge assumed uniform on the whole aquifer and S and k are coefficients of storage and permeability also assumed uniform. Separating z and R to the mean and variable parts respectively, such as $z=z_m+z'$, $R=R_m+R'$, the mean state of flow is given as follows.

$$kz_m\frac{\partial^2 z_m}{\partial x^2}+R_m=0.$$

$$\therefore \quad k\frac{\partial^2 z_m}{\partial x^2}=-\frac{R_m}{z_m}. \tag{3}$$

When we take a semi-infinite aquifer, a boundary of which is constant at $x=0$, it can be explained by well-tried analysis that the farther the distance from the boundary, the smaller the variable part of gradient of the water level becomes. Then, the value of $\partial^2 z'/\partial x^2$ seems to be negligibly small compared with that of $\partial^2 z_m/\partial x^2$ in the region far enough from the boundary such as the coast or river valley. In such a region, Eq. (2) approximates to

$$\begin{aligned}\frac{\partial z}{\partial t}&=\frac{k}{S}z\frac{\partial^2 z_m}{\partial x^2}+\frac{R}{S}\\&=-\frac{R_m}{Sz_m}z+\frac{R}{S}.\end{aligned} \tag{4}$$

The rate of natural recharge is considered to be the sum of two effects, one of which is directly brought about by infiltration owing to rainfall and the other of which is supplied by leaking from the lower confined aquifer. The latter probably gives a negative value of R from a view-point of the water budget in a whole basin, because artificial withdrawals from the confined aquifer disturb the natural balance of water circulation through semi-confining stratum and reduce the rate of leakage. We, then, put the rate of recharge to the unconfined aquifer as the next.

$$R=P-\theta_1-\theta_2. \tag{5}$$

θ_1 means a part of precipitation not entering into the ground water stream, and is probably represented as the sum of the effects of surface run-off and evapotranspiration. The latter implies the process, in which water once preserved in a subterranean unsaturated zone goes back into the atmosphere after a rain. θ_2 is the rate of loss taken away from ground water streams under water table conditions. A part of it is direct withdrawal from that aquifer

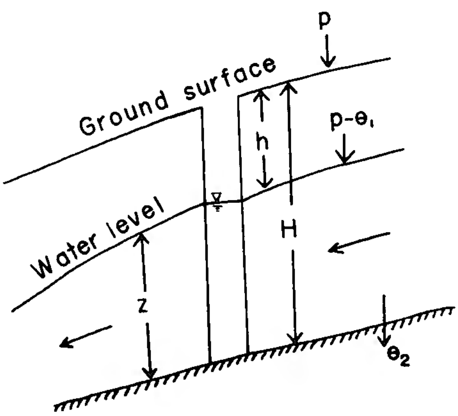

Fig. 1. Schematic diagram of the unconfined aquifer.

and the other shows a portion of supply to the lower confined aquifer which does not come back owing to the artificial withdrawal or natural flow out through the bed of the sea or river.

Representing the height of ground surface from the bed of the unconfined aquifer by H, it follows that

$$z=H-h. \tag{6}$$

Substituting relations of (5) and (6), Eq. (4) is transformed to the next equation which is quite similar to empirical formula (1).

$$dh/dt=R_m(H-h)/Sz_m-(P-\theta_1-\theta_2)/S.$$

$$\left.\begin{aligned} \therefore\ & dh/dt=\lambda(h_0-h)-P/S. \\ & \lambda=R_m/Sz_m. \\ & h_0=H+(\theta_1+\theta_2)/S\lambda. \end{aligned}\right\} \tag{7}$$

It is noted that, among the coefficients in (7), those determined by geometric property of the aquifer are only S and H and others would take different values with the time periods of observation used for analysis. This agrees with the experience in which the values of λ or h_0 are often found as variablee in observations at the same well as mentioned in Section **1**. Eq. (7) gives the interesting information on the feature included in h_0, the so-called base level. The value of h_0 obtained from the declining water level in a period of no rain must differ from that for a rising state in or soon after a rain, because it is expected that θ_1 would approach zero and only θ_2 would be effective when $P=0$. Therefore, the method often used up till now has given us a possibility of mistaking the precipitation effect, to which the same value of h_0 was applied during the rainfall as was obtained from data in the period of no rain.

3. Application of the data obtained in Beppu

The southern region of Beppu City is one of the most famous hot spring districts in Japan. A number of holes are bored to tap the artesian aquifer and a total rate of flow out of hot ground water is estimated at about 16,000 m^3/day. Depths of holes have gradually increased in the last forty years from the mean depth of 47 m in 1924 to 99 m in 1959. Progressive demand to get hot water has not only increased the rate of withdrawal from the deeper aquifer but lowered the temperature in the shallower aquifer. Such a tendency is clearly found in a water table well continuously observed from 1925 to 1964, in which bottom temperature at about 9 m depth has undergone a gradual or relatively sudden lowering from about 50°C to 22°C during the whole period of observation. This may originate from the reduction of upward leakage of deeper hot water in these years owing to the increase of withdrawal from deeper aquifer. The position of this well is represented as No. 970 in Fig. 2 and is about 1 km apart from the coast. The level of the reference point of water level observation on the ground surface is 32.8 m higher than the mean sea level and the depth to the bottom is 8.8 m.

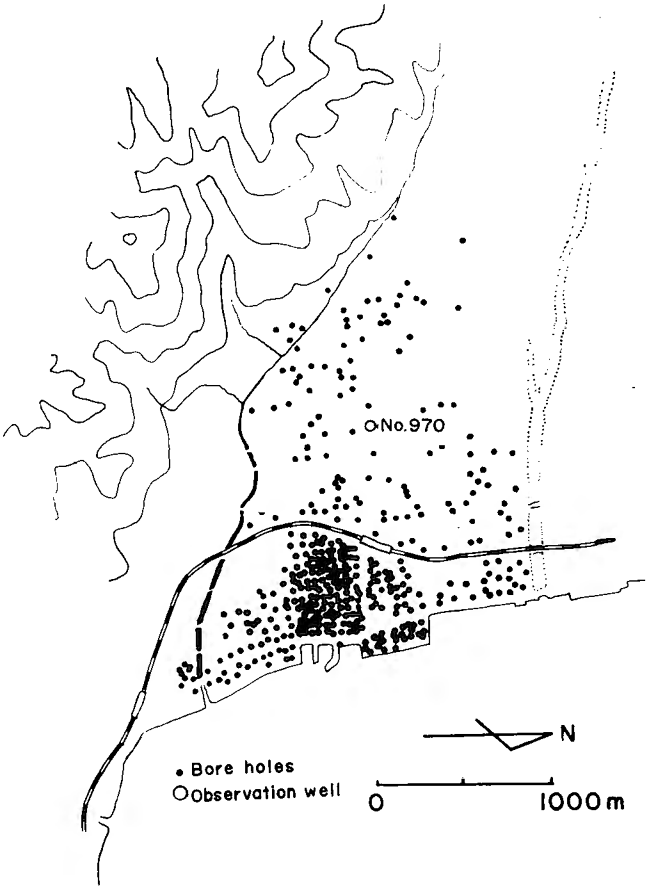

Fig. 2. Position of the observation well and distribution of bore holes tapping artesian hot water.

We will try to analyse the precipitation effect in this well using the theory for the unconfined ground water far enough from the coast as described in the above section, because the water level seems to be primarily affected by the precipitation.

(*a*) *Annual variation*

Monthly mean of the water level shows almost regular annual variation in which the highest is during September or October and the lowest is during April or May as given in Fig. 3. A pattern of annual variation of the water

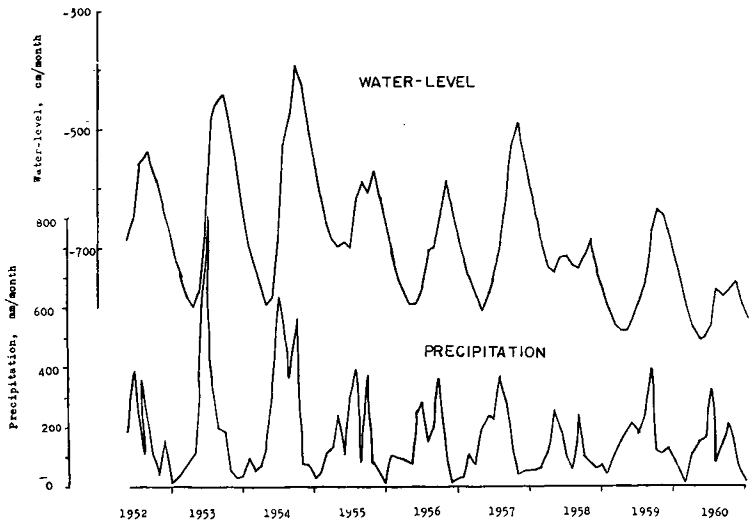

Fig. 3. Monthly mean of the water level and the monthly precipitation.

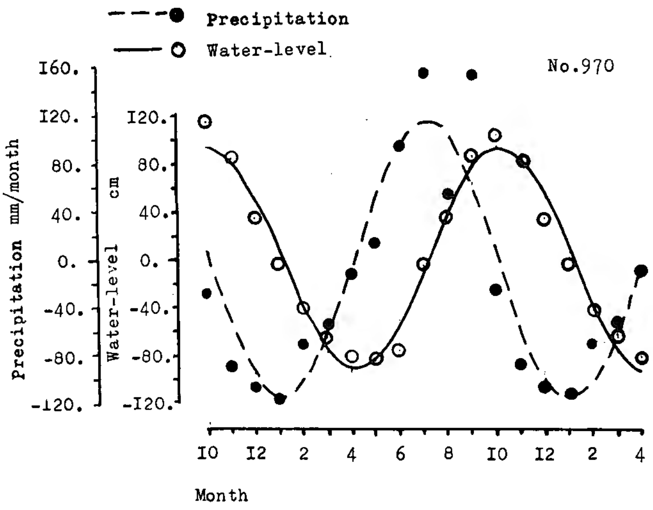

Fig. 4. Annual variations of the water level and the precipitation.

level was derived from the mean value for each month in the period between 1925 and 1960 and is related in Fig. 4 with that of precipitation similarly calculated. Curves in this figure give the results of each harmonic analysis for a period of one year. They show fairly distinct characteristics of annual variation in water level being three months behind that of precipitation. Considering that the lag of three months is equivalent to that of $\pi/2$ for a period of one year, the next formula is presented as closely approximating to the above relation.

$$dh/dt = -4.26(P - P_m). \quad \text{(cm/month)} \tag{8}$$

Assuming that the value of θ_1 or θ_2 in Eq. (7) would be admitted to be taken as each mean value in the case of analysing the annual variation, Eq. (7) is transformed to satisfy the relation between deviations from the mean values of water level and precipitation.

$$dh/dt = -\lambda(h - h_m) - (P - P_m)/S.$$

Stating that

$$P - P_m = P' \sin \omega t,$$

the solution for h is deduced as being

$$h = h_m - \frac{P'}{S\sqrt{\lambda^2 + \omega^2}} \sin\left(\omega t - \tan^{-1}\frac{\omega}{\lambda}\right).$$

When the value of ω is more than 2.5 times larger than the value of λ,

$$1.08 > \frac{\sqrt{\lambda^2 + \omega^2}}{\omega} \geqq 1, \quad 1.2 < \tan^{-1}\frac{\omega}{\lambda} \leqq \frac{\pi}{2}.$$

Therefore, if $\lambda \leqq 0.2$ (month^{-1}) in the annual variation, it is possible that it can be approximated by the next equation to within an error of 10% in amplitude and a half month in phase lag.

$$h = h_m - \frac{P'}{S\omega} \sin\left(\omega t - \frac{\pi}{2}\right).$$

This is equivalent to the next differential equation.

$$dh/dt = -(P - P_m)/S.$$

We can thus estimate the value of S as about 0.24 from the agreement between the above equation and the empirical formula (8). This value of S is noted to be nearly equal to the value of effective porosity obtained at a relatively near part of this well using a neutron moisture probe by Yusa [1969].

(*b*) *Declining state of the water level*

An example of observed variation of the water level is presented in Fig. 5. The water level in this well shows gradual lowering after the sudden rise owing to precipitation. Such lowerings are relatively speedy compared with those reported in other aquifers, and appear almost linearly during several ten day periods. Twenty one intervals are picked out from the whole period of observation, in which precipitation was not distinct in periods of over one month. Each interval is again divided into short intervals of ten days. The value of dh/dt is calculated according to the difference of water levels during each short interval and is related to the mean value of h in the same interval. It is found from Fig. 6 thus obtained, that the next linear relation is approxi-

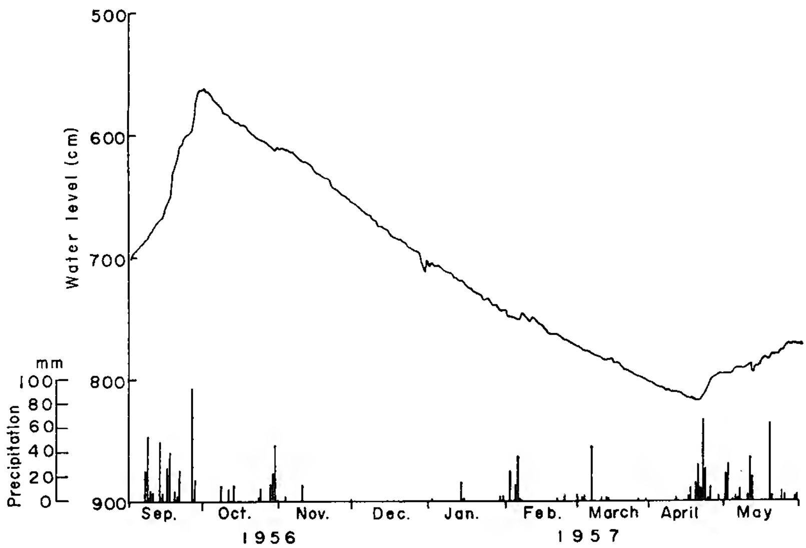

Fig. 5. An example on the variation of the water level.

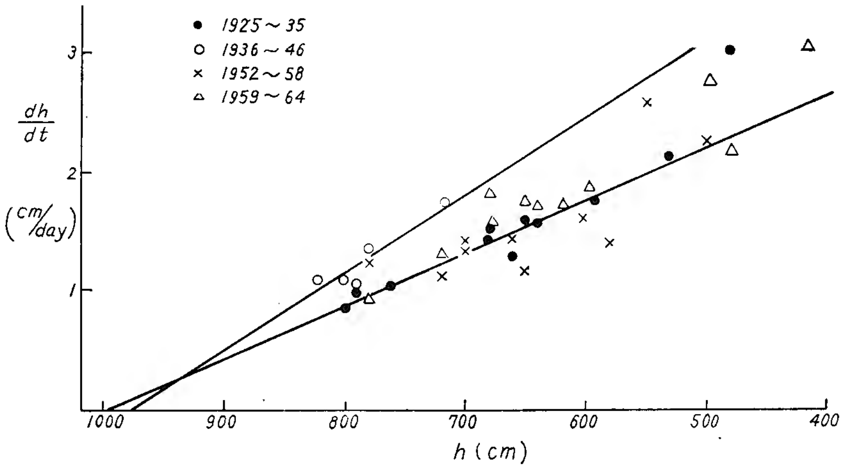

Fig. 6. Relation between values of *dh*/*dt* and *h* in each period of observation.

mately shown between h and dh/dt though some scattering discrepancies remain.

$$dh/dt = 4.8 \times 10^{-3}(990 - h). \quad \text{(cm/day)}$$

A more detailed examination suggests that different forms of relation would be satisfied with different periods of observation. Two straight lines are then drawn in Fig. 6 respectively to fit the features in the periods from 1925 to '35 and from 1959 to '64. Each line corresponds to the next relation.

$$1925\sim'35\,;\ dh/dt = 4.5 \times 10^{-3}(994 - h). \quad \text{(cm/day)}$$

$$1959\sim'64\,;\ dh/dt = 6.3 \times 10^{-3}(982 - h). \quad \text{(cm/day)}$$

Theoretical equation adapted only to the declining water table in the period of no recharge is easily gained by making $\theta_1=0$ in Eq. (7).

$$\left.\begin{aligned} dh/dt &= \lambda(h_0'-h). \\ \lambda &= R_m/Sz_m. \\ h_0' &= H+\theta_2/S\lambda. \end{aligned}\right\} \tag{9}$$

This corresponds to the relations obtained above and values of λ and h_0' for each period are tabulated in Table 1 with other observed values. It is noted that values of λ are always within the range in which annual variation of the water level is closely approximated by (8).

Table 1. Variations of hydrological elements with different periods of observation

Period	λ cm/month	h_0' cm	h_m cm	P_m cm/month	Temperature °C	θ_1 cm/month
1925～'64	14.4×10^{-2}	990	690	15.3	36.4	5.15
1925～'35	13.5	994	690	14.04	42.3	4.43
1959～'64	18.9	982	750	15.6	28.9	5.32

Eq. (9) gives us interesting results under the assumption that the value of θ_2 remains constant throughout each period now considered. Putting $R_m=P_m-\theta_1-\theta_2$, where θ_1 and θ_2 are the mean values in each period, Eq. (9) is rewritten as follows.

$$P_m-\theta_1-\theta_2=\lambda S(H-h_m). \tag{10}$$

$$\theta_2=\lambda S(h_0'-H). \tag{11}$$

Adding (10) and (11), substituting values of P_m, λ, h_0' and h_m in Table 1 and taking the value of S as 0.24, we gain the value of θ_1 in each period as mentioned in Table 1.

Those values of θ_1 are about one-third of the precipitation and is similar to the rate of evapotranspiration expected from conditions such as are found in Beppu. Kawanishi [1966] reported the daily mean of the amount of evapotranspiration being 3 to 4 mm/day in the warm season and 1.0 to 1.5 mm/day in the cold season from his observation during fine days at Ōita, which is in the neighbourhood of Beppu. It is then to be expected that almost all of θ_1 is attributed to the rate of evapotranspiration removing the water once retained in the subsurface unsaturated zone after rain. It is also to be supposed that the reason surface run-off may be quite small is that this is an area where river streams seldom appear other than in short periods after heavy rains. Direct transpiration from the ground water stream must be negligibly small because the water table in this aquifer is deep enough when compared with other districts.

It is an interesting problem that the possibility of estimating the mean rate of evapotranspiration from a basin is presented by the relatively simple method of analysing the declining state of the level of the ground water table.

(c) The rising state under the effect of precipitation

Considering that the values of θ_1 are given as monthly means in Table 1, let us examine those values first by analysing the variation in a unit of a month. The value of dh/dt is obtained as the difference between water levels during the last days of each neighbouring month and h is taken as the mean value of the previous month. Then, using the values of h_0' and λ in each period of observation in Table 1, the calculated value of $-dh/dt+\lambda(h_0'-h)$ for each month is compared with precipitation in that month because Eq. (7) is easily rewritten as follows.

$$-dh/dt+\lambda(h_0'-h)=(P-\theta_1)/S.$$

Fig. 7, thus obtained for the period of 1959 to 1964, shows fairly good agreement with that theoretically expected which is given by a straight line in the figure taking values of S and θ_1 as 0.24 and 5.3 respectively. This agreement supports the certainty of the method of gaining the value of θ_1 by an approximate theory as already described.

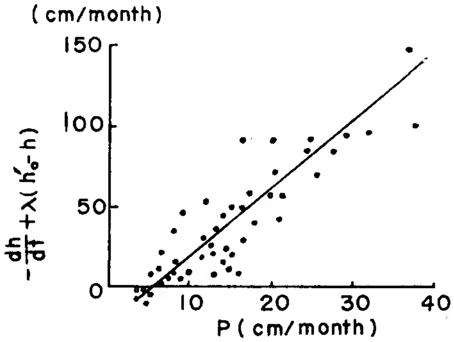

Fig. 7. Relation between monthly difference of the water level and the precipitation.

Let us, then, investigate the rising state of the water table under the effect of a continuous rain. Forty two examples are picked out, in which total precipitations exceed 30 mm and rises of water level are quite apparent. The daily mean of dh/dt is calculated from the difference between the water level before rain and the highest level after the rain. The value of h is approximated by the mean during this period. This approximation is probably admitted because the difference in the calculated value of λh according to that in h is quite small compared with the value of dh/dt owing to the small value of λ. Therefore, the daily value of $-dh/dt+\lambda(h_0'-h)$ is related in Fig. 8 with the precipitation represented as the daily mean. It is noted that a line satisfying the approximate linear relation cuts P-axis at about 7 mm/day. This means that precipitation below 7 mm/day has little influence on the ground water stream. Attempts are made as follows to compare this limiting value with the monthly mean value of θ_1 in Table 1.

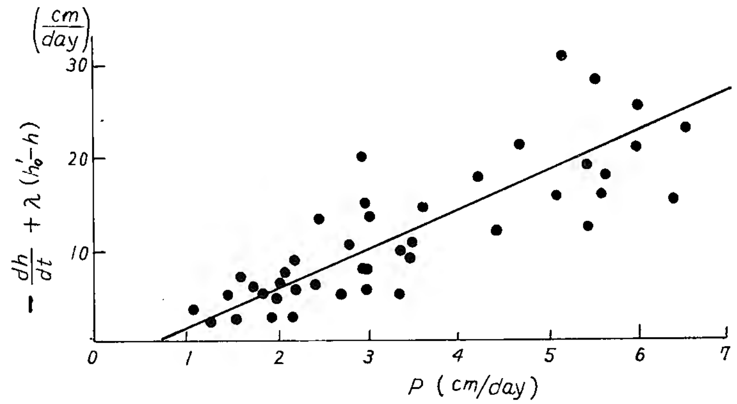

Fig. 8. Relation between rise of water table and precipitation.

Table 2. Mean values of modified precipitation

Period	P_m' cm/month	$P_m-\theta_1$ cm/month
1925～'64	10.56	10.14
1925～'35	9.24	9.61
1959～'64	10.23	10.28

Calculation to gain the mean value of precipitation is modified by substracting 7 mm from observed value for each day of more of precipitation and taking a day originally below 7 mm as being of no precipitation. The mean value of the modified precipitation, thus obtained, is designated by P_m' and tabulated in Table 2, in which modified values of precipitation show good correspondences to the value of $P_m-\theta_1$. It is therefore possible to explain that a part of precipitation, equivalent to 7 mm/day is preserved in the unsaturated layer above the water table and consumed by the evapotranspiration. Average rate of such losses may be equal to the value of θ_1 in Table 1.

We can safely explain by the above investigations that the theoretical result of Eq. (7) is satisfactorily adapted to the observed variations in the level of the ground water table under the effect of precipitation.

4. Estimation of the water budget in the basin

The value of θ_2 is still left unknown among the elements connected with the recharge rate of the unconfined aquifer. It is assumed that it can be easily determined by giving the value of H in (10) or (11). However, an accurate value of H is usually very difficult to ascertain because the discontinuous surface indicating the bed of the aquifer is not clearly distinguishable. In this aquifer, only the tendency of diminishing the permeable structure appears near the bottom of the observation well.

The theoretical point of view in this paper is to distinguish the unconfined aquifer from other layers on the assumption that vertical movement of the ground water is almost negligible in this aquifer. Temperature in this observation well is known to have been lowered from 50°C to 20°C as previously described. Such a state shows that the supply of hot water from the lower layer has diminished over these years and suggests that the bottom of this well may be situated near the bed of the aquifer. Therefore, one can make the assumption that the bed of the aquifer can be taken as the level of the bottom of this well. The value of H is thus assumed to be 880 cm. This value is substituted in Eq. (11) and the value of θ_2 are calculated as in Table 3. Suitabilities of such values are examined as follows.

Table 3. Elements concerned to the water budget in this region

Period	P_m cm/month	θ_1 cm/month	θ_2 cm/month	R_m cm/month	$\theta_2 \times 9$ km^2 m^3/day
1925～'64	15.3	5.16	3.72	6.4	11,160
1925～'35	14.04	4.43	3.59	6.0	10,800
1959～'64	15.6	5.32	4.52	5.8	13,510

Recharge rate was assumed in this paper to be uniform all over the aquifer. Then, assuming that the values of θ_2 obtained above are the averages throughout the alluvial plain in this region, an area of about 9 km^2, the total rate of θ_2 through alluvial stratum is calculated to be in the range of 10,000～14,000 m^3/day summarized in Table 3.

Total rate of withdrawal of hot ground water from this region is about 16,000 m^3/day. The large amount of heat and the high salt content of this water is known to be chiefly supported by thermal water of the sodium-chloride type flowing out from the deep layer of the mountain area. Hot ground water streaming through the artesian aquifer is formed by the mixing of such thermal water with the infiltrating ground water of the usual type. Yamashita [1965] estimated the mixing ratio of thermal water in this region at about 20%. It can thus be roughly stated that 80% of the hot water withdrawn in this region is supplied by the unconfined ground water. That rate is about 13,000 m^3/day and is noted to be nearly equal to that calculated from the value of θ_2. It is also interesting that the value of $\theta_2 \times 9$ km^2 in Table 3 shows an apparent increase in recent years so as to correspond with that of withdrawals. It is thus concluded that by assuming the value of θ_2 there is no large discrepancy in the procedure, and that it is therefore admissable. Rates of the elements thus obtained are summarized in Table 3. They are all concerned with the water budget in this region.

5. Conclusion

An approximate equation was derived theoretically to analyse the variation in the level of ground water table far enough from a boundary of semi-infinite aquifer. It evidently corresponds with the empirical formula generally employed. Then the hydrological meanings of the coefficients contained in it are established and utilized to deduce the factors contributing to the process of water circulation under the ground. Recharge rate to the unconfined aquifer is given by $P-\theta_1-\theta_2$. Estimations of θ_1 and θ_2 are achieved as to their mean values over relatively long periods of observation. It is interesting that the ratio of θ_1 and precipitation remains relatively constant as about one-third in spite of the difference of the periods though the value of θ_2 shows a tendency to increase in recent years. Such a discrepancy is understood from the reason that θ_1 indicates the rate of natural evapotranspiration but θ_2 is the supply to the lower artesian aquifer to compensate the artificial withdrawal of hot water. Then, the water budget in the southern region in Beppu is approximately expressed as follows. As to the mean state in the period from 1925 to '64, about 5 cm/month is consumed by evapotranspiration, about 4 cm/month is utilized as hot water and about 6 cm/month streams through unconfined aquifer.

Some correspondences have been observed between variations of the level of ground water table and the piezometric level of artesian water. It is therefore expected that the value of θ_2 is not preserved as the mean value as given in Table 3 but continuously changes according to the variation of the water level. Such a process as to the variation of the rate of supply to the artesian aquifer will be investigated further in the near future by a more precisely arranged analysis.

Acknowledgements

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