Special Contributions, Geophysical Institute, Kyoto University, No. 10, 1970, 59-74

THE MEASUREMENT OF THE ENERGY DISSIPATION RATE IN THE SURFACE BOUNDARY LAYER

By

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(Received November 16, 1970)

Abstract

Estimations of the rate of energy dissipation in the surface boundary layer from spectra of wind speed fluctuations in high frequency region were made. Measurements of wind fluctuations were made over sea and land surface by a hot-wire anemometer. The friction velocity and the drag coefficient derived from the energy dissipation rate show fairly good agreement with the values obtained by the other method of measurement. This shows that this method of drag estimation is a reasonable one for the measurement on a moving platform such as a ship or an aircraft.

1. Introduction

The rate of dissipation of kinetic energy, which is the amount of turbulent energy being converted into heat by work done against viscous stress, is of fundamental importance in the study of atmospheric turbulence. It plays an important role in modern turbulence theory which was put forward by Kolmogoroff [1941a, 1941b]. The spectrum of the atmospheric turbulence in the large wave number region is characterized by this energy dissipation rate. Also, vertical eddy flux of momentum can be estimated from this rate on the basis of some assumptions.

As fluctuations of the wind velocity in low frequency regions hardly contribute to the energy dissipation rate, it is possible to obtain the energy dissipation rate only from the high frequency part of the spectrum. Hence it is advantageous to use this method for the study of air-sea interaction processes from a moving ship free from the effects of the ship's motion, as Deacon [1959] has suggested, from a flying plane (as shown by MacCready [1962], Payne and Lumley [1965]), or from a tethered balloon.

Our research group in Kyoto University is trying to establish a direct method of measurement of the turbulent flux of momentum, heat and water vapor by the eddy correlation method as a part of GARP-Severe Rainstorm Research

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Project of Japan (Mitsuta, Hanafusa and Maitani [1969, 1970]). Though the eddy correlation method is a promising one for obtaining these fluxes without any assumptions, wind velocity measured on a ship must be corrected for anemometer swaying by the use of various informations about ship motion, which makes the procedure of measurement complicated and liable to error.

The objective of the analysis of the wind velocity fluctuations in the high frequency region measured by a hot-wire anemometer on a moving ship is to obtain the momentum flux estimations as an independent check of the values measured by the eddy correlation method on the ship at the same time.

2. Rate of energy dissipation

In isotropic turbulence, the rate of energy dissipation of kinetic energy per unit mass is given by the following equation (Hinze [1959]),

$$\varepsilon=15\nu\int_0^\infty k^2\phi(k)dk, \tag{1}$$

where ν is the kinetic viscosity, k the angular wave number and $\phi(k)$ the one-dimensional energy spectrum. $k^2\phi(k)$ is referred to as the dissipation spectrum. It is possible to evaluate the energy dissipation rate by the Eq. (1), when the energy spectrum $\phi(k)$ is obtained.

According to Kolmogoroff's theory, the energy spectral density $\phi(k)$ is proportional to the $-5/3$rd power of the wave number in the inertial subrange, which has been confirmed by numerous experiments (Lumley and Panofsky [1964]). Therefore, in the inertial subrange, the dissipation spectrum $k^2\phi(k)$ increases with the wave number. On the other hand, it is known that the energy spectral density decreases rapidly with increasing wave number in the wave number range larger than the inertial subrange. Inoue [1952], Hinze [1959], Pao [1965] and others have studied the energy spectrum, $\phi(k)$, in this large wave number region theoretically. In either case, dissipation spectral density tends to approach zero rapidly in the large wave number region. Consequently an approximate value for the energy dissipation rate will be obtained, if the velocity fluctuations can be measured in the wave number region from the inertial subrange to a number large enough to regard the dissipation spectral density as zero.

Experimentally the measurements are made as a time series at a fixed point. In the high frequency range, the time scale can be transformed into the space scale by the use of the Taylor's hypothesis,

$$k=\frac{2\pi n}{U}, \tag{2}$$

where U is the mean wind speed and n the frequency. Therefore k in Eq. (1)

can be replaced by n.

So far as the authors know, the work done by Pond *et al.* [1963] is the first experiment to estimate the energy dissipation in the atmosphere over sea directly by the method shown above. They measured the wind velocity fluctuations by a hot-wire anemometer, mounted at a height of 1 m above the mean water level, when the mean wind speed was about 3 m/sec. According to their results, the angular wave number of the peak of the energy dissipation spectrum is about 1.5 cm^{-1} and the wave number at which the energy dissipation spectral density decreases to one tenth of the peak value is about 8 cm^{-1}. Thus in the measurement of the energy dissipation rate, it is estimated to be necessary to measure the wind velocity fluctuations up to a few hundred cycles per second in a wind of a few meters per second.

In steady state, the equation of energy balance is shown as follows,

$$u_*^2\frac{\partial u}{\partial z}=\varepsilon+g\frac{\overline{w'T'}}{T}+\frac{1}{\rho}\frac{\partial}{\partial z}\overline{p'w'}+\frac{\partial \overline{E'w'}}{\partial z}, \tag{3}$$

where u_* is the friction velocity, ρ the density of the air, g the acceleration of the gravity and E the turbulent kinetic energy. The order of magnitude of the last three terms on the right hand side of Eq. (3) in a near neutral condition is thought to be so small compared with the energy dissipation rate in the first term that the following equation holds good without serious errors (Panofsky [1962]),

$$u_*^2\frac{\partial u}{\partial z}=\varepsilon. \tag{4}$$

From this equation, friction velocity u_* can be obtained by measuring the energy dissipation rate and the wind shear. In a neutral condition,

$$\frac{\partial u}{\partial z}=\frac{u_*}{kz}, \tag{5}$$

where k is von Karman's constant ($\fallingdotseq 0.4$), therefore, Eq. (4) becomes

$$u_*=(kz\varepsilon)^{1/3}. \tag{6}$$

Then the drag coefficient C_d is written as follows,

$$C_d=\frac{(kz\varepsilon)^{2/3}}{U^2}. \tag{7}$$

Hence the drag coefficient C_d can be approximately estimated only from the energy dissipation rate ε, the mean wind speed U and the height from the surface z.

3. Method of measurement

The hot-wire anemometer used in this study is a constant temperature type (supplied by Nihon Kagaku Kogyo Co., Kanomax Model 28-3111) with a hot-wire of tungsten 5 μ in diameter and 1 mm in length heated to a temperature of about 150°C. The electrical circuit of this instrument has enough response character to wind changes from direct current to 50 kc.

As the hot-wire has a finite length, the line averaging character of the wire attenuates the amplitude of the fluctuations with wave length comparable to or less than the length of the wire.

Mitsuta [1966] and Silverman [1968] have studied the attenuating effect of the line averaging character of the sonic anemometer probe. Further, Kaimal [1968] has studied the effect of line averaging character of the platinum wire thermometer and confirmed that the attenuation character of the thermometer is the same as that of a sonic anemometer in vertical velocity measurement. From these results it may be said that the attenuating effect of the hot-wire is also the same as the case of vertical velocity measurement by a sonic anemometer. According to Silverman, the 90 % power cut off limit of a wire of length s is

$$K_m = 2\pi \times 0.17/s, \tag{8}$$

where K_m is the maximum limit of the angular wave number which is measured without reduction in amplitude. As the length of the hot-wire is 0.1 cm, the limit of the angular wave number is about 8 cm^{-1}, whch is the same as the wave number, at which the dissipation spectral density decreases to a tenth of the peak value, in the results of the study by Pond *et al.* Thus this hot-wire has enough geometrical response character for the present purpose.

Because the energy spectral density decreases rapidly in a high frequency region, the amplitude of the large wave number component of wind fluctuation becomes small so rapidly that the ordinary method of recording and reproducing cannot be applied in dissipation measurement. Therefore the output signal of the hot-wire anemometer is divided into three frequency range signals by band pass filters, which are recorded on the different channels of the data recorder.

The filters are solid state active filters specially made for this study, the block diagram of which is shown in Fig. 1. The lower channel (hereafter shown as L) is a low pass filter which passes d.c. to 10 cps in 90 % cut-off in amplitude. The medium channel (M) is a band pass filter which passes from 1.7 to 45 cps and the high channel (H) passes from 15 to 6000 cps. The three kinds of output are amplified or attenuated to the same level in order to attain good signal-to-noise ratio in recording on a data tape recorder (supplied by TEAC Co., R-200). Frequency responses of these filters are shown in Fig. 2.

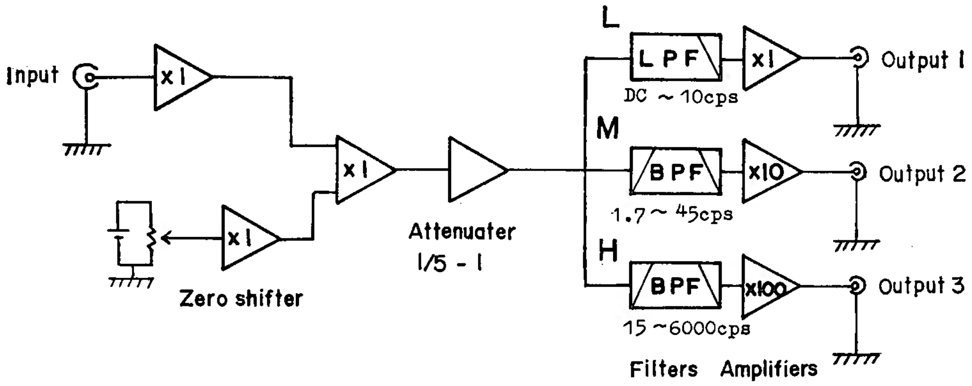

Fig. 1. Block diagram of the active filters.

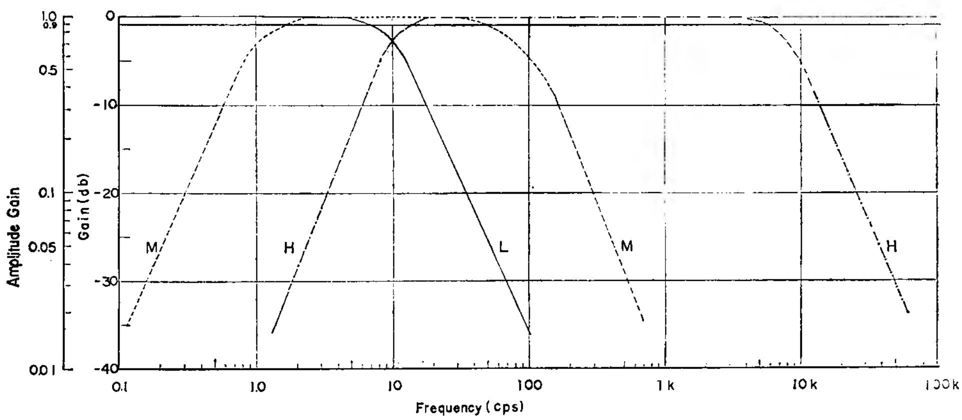

Fig. 2. Frequency response of the active filters.

In processing the data, the reproduced signals were digitized by a data analyzer (supplied by TEAC Co., DP-300) and power spectra were computed by Tukey's method by the electronic computer of Kyoto University (KDC-II) for each channel. However, the maximum sampling speed is not large enough, so the very high frequency part of the H channel was processed again reducing the tape speed by one tenth, which is shown as VH channel hereafter.

4. Observation on the sea

The observation was made on the R. V. Hakuho-Maru of the University of Tokyo anchored in the central part of the East China Sea as a part of the GARP-Severe Rainstorm Research Project (from 3rd to 8th July, 1966 (Fig. 3)). The hot-wire probe was installed on the top or in the middle of the foremast (about 22 or 15 m from the sea surface). Direct measurement of turbulent flux using a sonic anemometer was made on the top of the foremast at the same time (Mitsuta, Hanafusa and Maitani [1969b, 1970]).

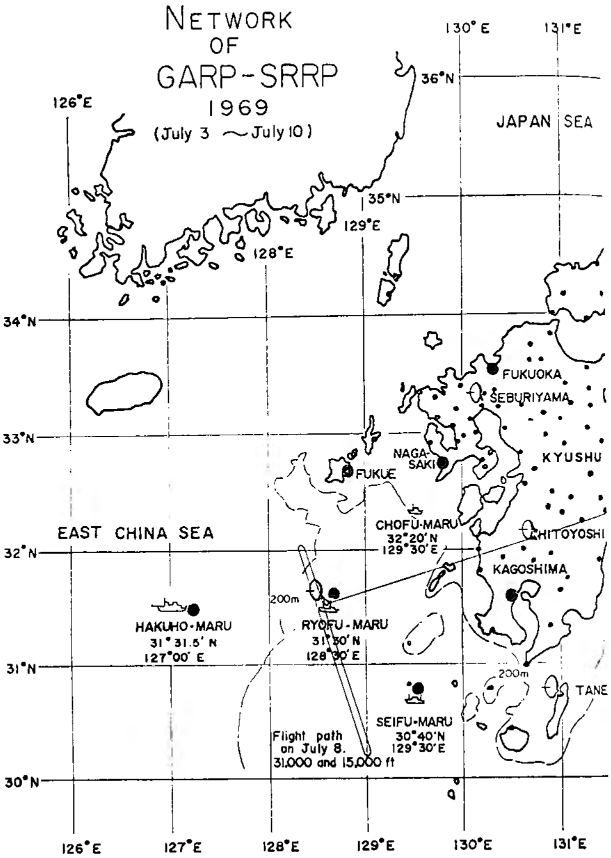

Fig. 3. Map indicating the position of the Hakuho-Maru.

Table 1. Details of the observations

Run No.	H-2	H-5	AH-3
Position	East China Sea	East China Sea	Shionomisaki
Surface condition	over open sea	over open sea	over bare land
Date	July 7, 1969	July 7~8, 1969	Aug. 19, 1969
Time	16 : 04~16 : 30	23 : 35~00 : 30	16 : 10~16 : 57
Weather	overcast	overcast	blue sky
Height of measurement	15m	22m	1.5m
Wind direction	SSW (left 25° from the bow)	SSW (left 25° from the bow)	W
Wind speed (m/s)	6.0*m/s	13.0*m/s	2.5 m/s
Dry-bulb temperature	24.1°C	23.1°C	27.7°C
Sea surface temperature	21.7°C	22.4°C	—

* Obtained by the aerovane on the foremast.

The control unit of the hot wire anemometer was installed just near the hot-wire and the output signal was sent by a coax-cable to the research room where it was filtered and recorded.

Six runs of the record which are 30 or 60 min. in length were obtained throughout the expedition. The details of the conditions of the two of these runs which are analyzed and discussed in this paper are summarized in Table 1. Run H-2 was a case of moderate wind of 6 m/sec and Run H-5 was in strong wind conditions with 3 m swells. An example of the record of Run H-5 is shown in Fig. 4. The trace denoted by H means the output of the high channel filter and M and L are the outputs of medium and low channels.

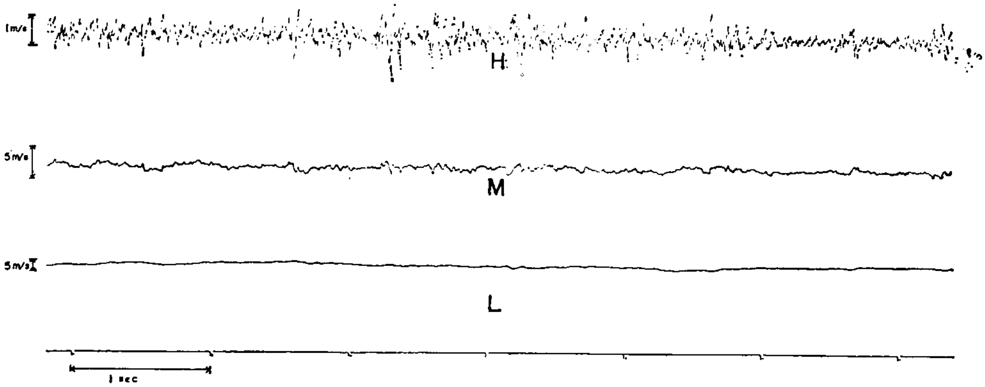

Fig. 4. An example of the trace measured with hot-wire anemometer.

The records were analyzed by the method shown in the preceeding section. Fig. 5 shows the wide range spectrum of wind fluctuations over sea obtained by composing the results of the spectral analysis of the record of each channel of Run H-5 and the traces of aerovane type anemometer on the mast for a longer period (Mitsuta, Hanafusa and Maitani [1970]). Energy concentrations are seen at about 2×10^{-2} cps, which is the so-called micro-turbulence peak, and at about 2×10^{-1}, which corresponds to the frequency of ship swaying or the waves. The energy density decreases in proportion to $-5/3$rd power of frequency in the frequency range from the second peak of wave frequency up to about 200 cps, above which density decreases more rapidly.

From this, it is concluded that the frequency range which contributes to dissipation of the eddy energy is between about 10 cps and about 1000 cps. As this range is higher than the frequency of ship swaying, the measurement of the dissipation can be made independent of ship motion without any correction on the output of the anemometer, which is the advantage of this method in air-sea interaction research from a moving platform.

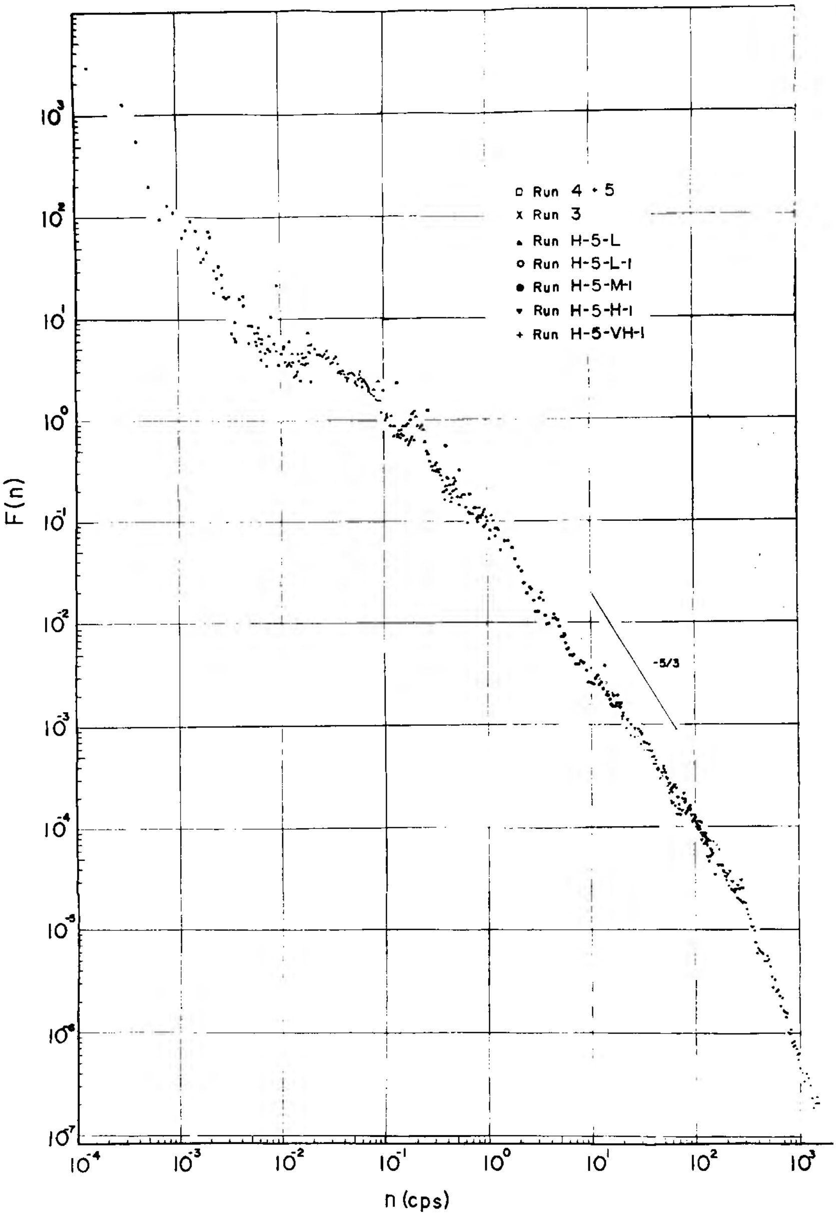

Fig. 5. Power spectrum of wind speed over the sea.

The high frequency part of the spectrum, which makes a large contribution to energy dissipation, is shown in Fig. 6. This is based on the results of the analysis of the M, H and VH channels of the hot-wire anemometer recorder. The 90 % limit of the hot-wire response discussed above is at 1600 cps in this

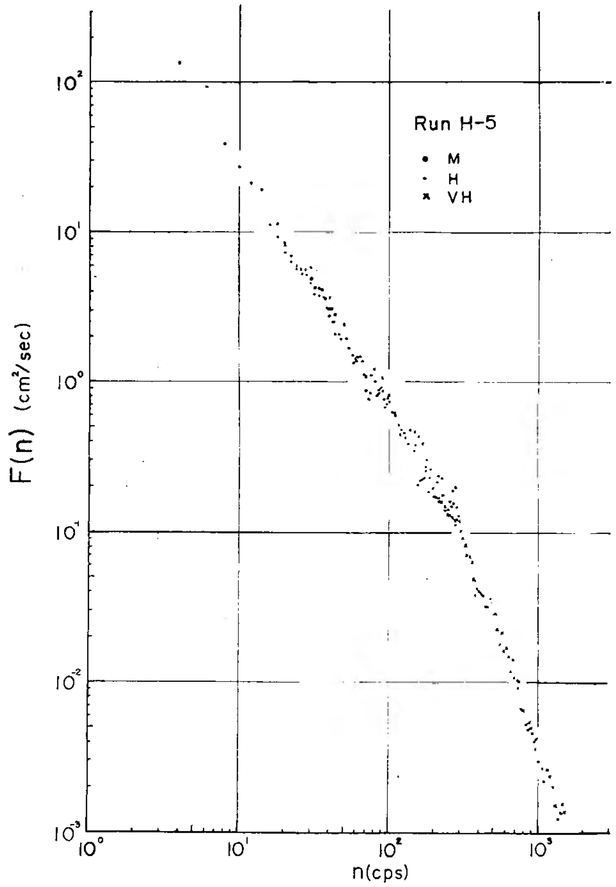

Fig. 6. Energy spectrum of Run H-5.

case. However, the amplitude of the frequency component near 1000 cps is almost the same as the noise level of the tape recorder, therefore, there is less confidence in the high frequency end of the spectrum. It is clearly seen from this figure that the spectrum follows $-5/3$rd law up to about 200 or 300 cps.

The dissipation spectrum is shown in Fig. 7. The dissipation spectrum has a peak at about 230 cps and decreases gradually with the frequency range. However it does not tend to zero in the frequency range analyzed in this case, but decreases down to about 1/3 of the peak value and can be exterpolated to zero as shown in this figure.

The energy dissipation rate can be integrated following Eq. (1), the spectral curve fitted by eye shown in the figure and converting the frequency into an angular wave number, following Eq. (2), in which the mean wind speed shown in Table 1 is used. The value of the energy dissipation thus obtained is 3.8×10^2 cm²/sec³.

The rate of the energy dissipation in Run H-2 is also obtained by the same procedure as Run H-5 shown above, and the result is 1.0×10^2 cm^2/sec^3.

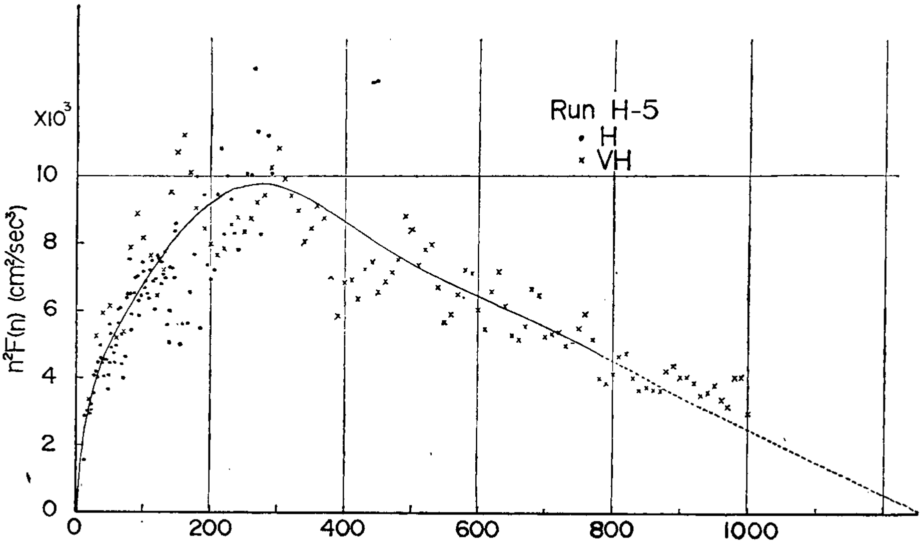

Fig. 7. Dissipation spectrum of Run H-5.

5. Observation over the bare ground

For the comparison to the observation over sea, an observation was made over land. This observation was made over the bare ground at a height of 1.5 m at the Shionomisaki Wind Effect Laboratory of Kyoto University in August 1969. The details of the related conditions are shown in Table 1 as Run AH-3.

The methods of the observation and data analysis are the same as those over the ocean. The energy spectrum obtained is shown in Fig. 8. The composition of the frequency bands is not so well done as in the case of Run H-5. The frequency limit of hot-wire response and the frequency at which the amplitude decreases to the noise level of the recorder are both at about 300 cps.

The dissipation spectrum of Run AH-3 over land is shown in Fig. 9. The discrepancy of the end of each channel is clearly seen in this spectrum. The energy dissipation rate is computed from the spectral curve shown in the figure mainly obtained from the result of the VH channel which covers the whole frequency range, contributing to dissipation. The result is 7.2×10^2 cm^2/sec^3, which is clearly larger than the values obtained over sea.

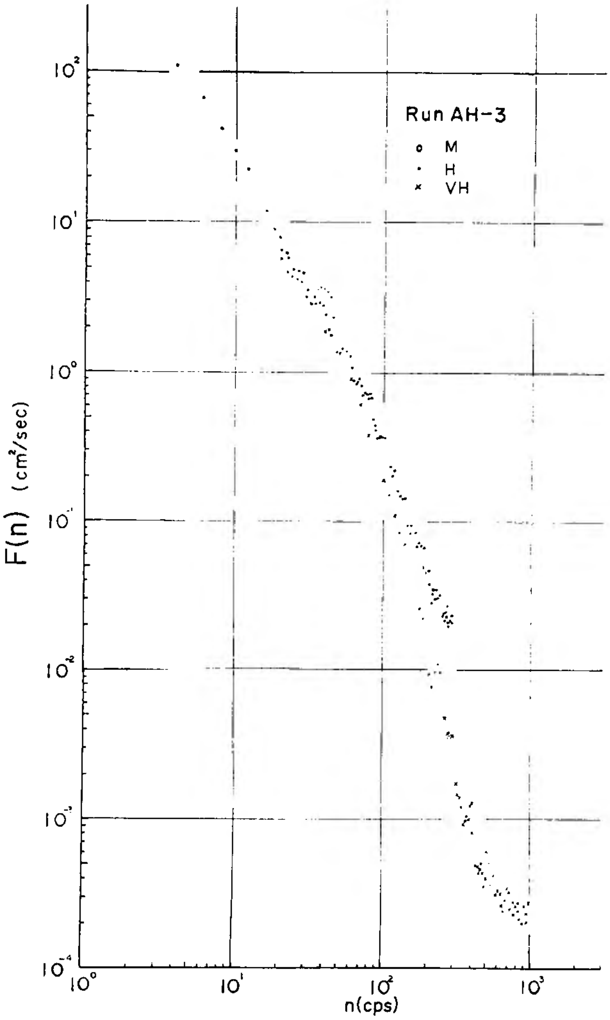

Fig. 8. Energy spectrum of Run AH-3.

6. Discussions

The energy dissipation rates obtained in the foregoing sections are 1.0×10^2 cm^2/sec^3 (Run H-2) and 3.8×10^2 (Run H-5) over sea and 7.2×10^2 (Run AH-3) over land respectively. Though wind velocity is weaker over land, the dissipation rate is larger than that over sea.

Even though the two runs over the sea are in stable stratification, a rough estimation of drag coefficient by Eq. (7) using the energy dissipation rate might be possible. The results are shown in Table 2 together with the results of observation over land and the results of Pond *et al.* The results over sea are

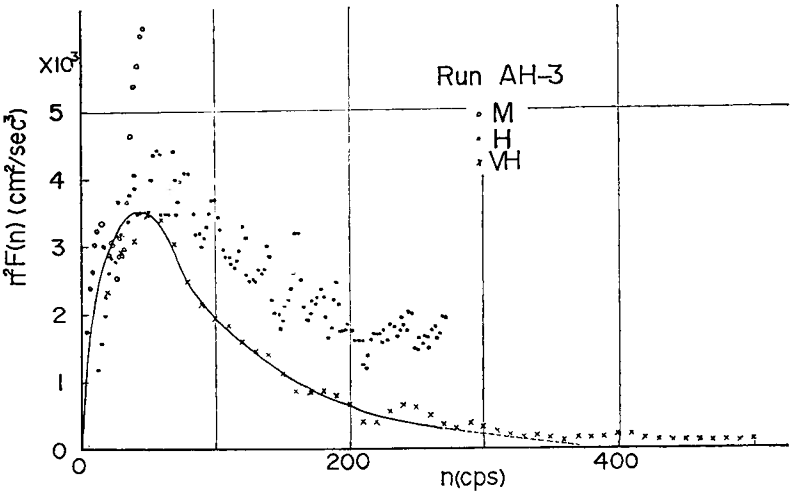

Fig. 9. Dissipation spectrum of Run AH-3.

near the value obtained by Pond *et al.*, but the value of Run H-2 is a little larger. This may be caused by the assumption of neutral stability in spite of clearly stable stratification in this case as shown in Table 1.

Table 2. The results and related quantities

Run No.	Height of measurement Z (m)	Mean wind speed U (m/s)	Energy dissipation ($\times 10^2$ cm²/s³)	Friction velocity u_* (cm/s)	Drag coefficient Cd ($\times 10^{-3}$)	Kolmogoroff's wave number K_k (cm⁻¹)	Normalized peak K_p/K_k	Kolmogoroff's constant A
H-2	15	6	1.0	39	4.3	13	0.13	0.36
H-5	22	13	3.8	69	2.9	18	0.08	0.61
AH-3	1.5	2.4	7.2	35	21	21	0.07	0.79
Pond *et al.* [1963]	1.0	3.0	0.85	15	2.5	11	0.10	0.48

The result of direct measurement of eddy transport by the use of a sonic anemometer made in July of 1968 same on the sea shows that the value of C_d is about 1.2×10^{-3} (Mitsuta, Hanafusa and Maitani [1969a]). To discuss the difference of the magnitude obtained by direct measurement and by dissipation measurement, more data are required and constitute a future problem.

At a sufficiently large wave number range, it was suggested by Kolmogoroff [1941a] that spectra generally should be functions only of the rate of energy dissipation ε and kinetic viscosity ν. Thus it is expected that the energy spectrum would be a function only of ε and ν and is written in normalized form using Kolmogoroff's wave number, K_k defined as

$$K_k=\left(\frac{\varepsilon}{\nu^3}\right)^{1/4}, \tag{9}$$

at which inertial and viscous forces have the same magnitude.

The values of K_k for each run of this experiment is 13 cm^{-1} for Run H-2, 18 cm^{-1} for H-5 and 21 cm^{-1} for AH-3 respectively.

The normalized spectrum derived by the use of these values of K_k is shown in Fig. 10, where normalized spectral density is defined as

$$F\left(\frac{K}{K_k}\right)=\frac{1}{(\varepsilon\nu^5)^{1/4}}\phi\left(\frac{K}{K_k}\right). \tag{10}$$

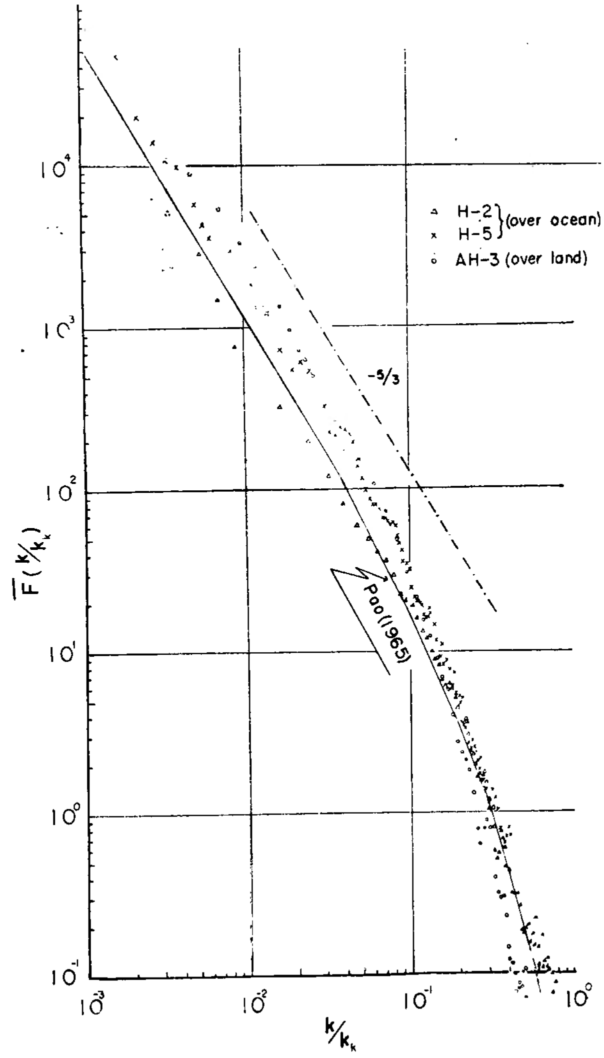

Fig. 10. Normalized energy spectrum.

Though the points are scattered a little, three spectra are normalized on the same curve. The normalized energy spectra obtained by Pond *et al.* [1963] are shown in Fig. 11. Both spectra coincide well in shape, though the density is a little larger in the present case than in that of Pond *et al.*

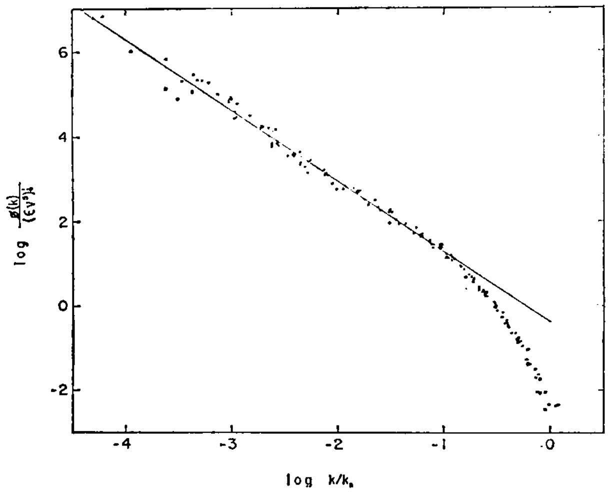

Fig. 11. Normalized energy spectra (after Pond *et al.* [1963]).

The present normalized spectrum is well approximated by Pao's formula (Pao [1965]) with the arbitrary constant, α, adjusted to be 1.70 which is presented by Pao to give good agreement with the results of the tidal channel experiment by Grant *et al.* [1962].

The normalized wave numbers of the peaks of the dissipation spectra (in Figs. 7 and 9) are 0.13 for Run H-2, 0.08 for Run H-5 and 0.07 for AH-3. These values are also in good agreement with Pond's results of about 0.1 and Pao's of 0.08.

The spectral density in the inertial subrange can be written in form shown as

$$\phi(K) = A\varepsilon^{2/3}K^{-5/3}, \tag{11}$$

where A is the Kolmogoroff's constant. The values of A obtained in this experiment are 0.39 (Run H-2), 0.61 (Run H-5) and 0.79 (Run AH-3).

7. Conclusions

In this paper the results of energy dissipation measurement over sea and land are presented. The knowledge of the energy dissipation rate gives a good momentum transport estimate as well as yielding fundamental knowledge of turbulent structures. As the dissipation rate can be estimated only from the high frequency part of the wind fluctuations, the measurement can be done easily on a moving platform such as a ship, which will be a powerful technique for use in air-sea interaction studies. As the careful measurement of energy dissipation will supply a great deal of information on atmospheric turbulence, further experiments on this subject will be needed.

Acknowledgements

The research was undertaken as a part of GARP-Severe Rainstorm Research Project. The authors wish to express their sincere thanks to Prof. R. Yamamoto, Geophysical Institute, Kyoto University for his kind advice and guidance throughout the course of this study. The authors extend their sincere thanks also to Prof. Ogura and others who gave us the opportunity to work on board the R. V. Hakuho-Maru of the Ocean Research Institute, University of Tokyo, and would like to express their thanks to Captain T. Shirasawa and all the crew of the R. V. Hakuho-Maru. This study was sponsored by a Grant in Aid for Fundamental Scientific Research from the Ministry of Education of Japan.

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