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SOME PHYSICAL PROPERTIES OF THE STEAM DISCHARGED FROM GEOTHERMAL GROUND

By

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Abstract

There is a large quantity of natural steam discharged from fumaroles or drill-holes in geothermal areas. The flow of steam in underground layers is characterized by the condition of conservation of enthalpy, so far as it is assumed thermally insulated to the surrounding medium. Therefore, as to the steam originating from deep thermal water, a state of superheat can be observed at each pressure level as it rises to ground surface. The highest possible limit of its temperature is 163°C at 1 *atm*.

Velocity of fluid ejected from a pipe is known to be unable to exceed the local velocity of sound at the outlet. This limitation is adapted to the flow in a well ejecting steam or boiling water. The sound velocity in the two-phase system of saturated steam and water are calculated at various pressures and mixing ratios. The result shows nearly linear relation of pressure and discharge rate for the flow, the velocity of which is equal to that of sound. The fluid pressure at the orifice is considered to be nearly always equal to the external atmospheric pressure for the well from which the ejecting velocity is below the sound velocity under atmospheric pressure but it becomes higher and is governed by the discharge rate through the well which has higher velocity of ejection than the above. Discharge rates from such boiling wells are then restricted.

The well ejecting superheated steam must has a different limit of discharge to preserve its superheated temperature. This limit is obtained by the continuity of energy flux and is shown to be lower than that given by the sound velocity for most of superheated range. Such states are compared with observations.

1. Natural steam in hydrothermal field

It was a typical circumstance of hydrothermal area in Japan in former times that hot ground water flows out in the lower part of the area and fumaroles discharged steam in hilly part. Such a state led us to the explanation that the thermal activity was continuously supplied by magmatic steam, a part of which ascended to the fumaroles through fissures of rocks and the other part of which mixed to the ground water in the aquifer. This hypothesis was partly supported by observations in which temperatures at the outlets of fumaroles were often higher than 100°C. Some researchers maintained that superheated steam could not originate from liquid water but from magmatic activity because the flow of steam must be in the state of adiabatic change (entropy is constant) following considerable lowering of the temperature till it reached the ground surface. It suggested the existence of extremely superheated steam beneath the steaming ground.

Distributions of deep drill-holes have lately extended to the mountain areas owing to the progressive demands for resorts or electric powers. Many of the holes tap the aquifers of liquid thermal water even they are drilled in steaming grounds. They yield the mixture of steam and boiling water which is rich in Cl^- content and usually called sodium-chloride type. Such states give us a different kind of representation for hydrothermal structure, especially well adapted to Beppu Hot Springs. It is shown in Fig. 1. Thermal water of sodium-chloride type flows in deep layer

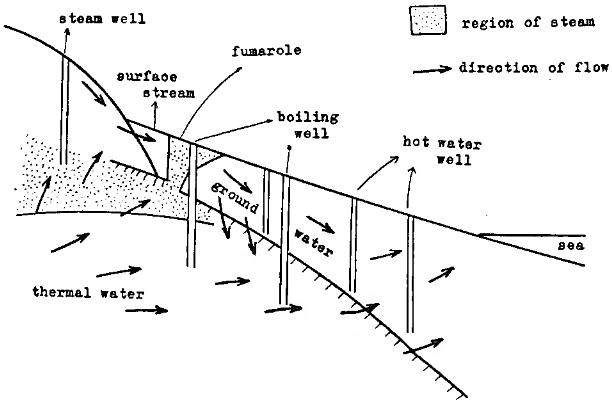

Fig. 1. Representative diagram of hydrothermal structure in Beppu.

from mountain area toward the coast. The deep layer is composed of the older volcanic rock and overlaid by the later volcanic rock or its sediments. A layer of tuff placed between these two layers somewhat restricts the hydrological communication between thermal water and the upper ground water. The piezometric head of the thermal water is generally kept lower than that in the upper aquifer except in a relatively narrow area near the coast. The difference of both levels increases upstream and exceeds 100 *m* near the foot of the mountain. Recharge of the ground water to the deeper aquifer, then, occurs owing to the difference of piezometric levels in both aquifers and dilutes the thermal water so as to make a proportional relation between Cl^- content and temperature in the water of hot springs. Beneath the hilly area, steam occupies the limited region on the thermal water and resists the downward recharge of the upper ground water. It ascends through fissures along the fault in the foot of the mountain. Ground water diluting the deep thermal water in Beppu is characterized by its higher temperature and HCO_3^- content than that in other usual areas. These properties show that heat and CO_2 gas are supplied with flow of steam from the thermal water to the ground water in the above-mentioned region of steam.

Such a state as the above representation makes the prior hypothesis quite doubtful in which the superheated steam actually appearing on the ground surface may have a magmatic origin. It may be that the steam originates from the underground boiling of liquid thermal water. The flow of steam coming from the saturated state with thermal water is studied in the next section and its thermal property under atmospheric pressure is presented.

2. Flow of steam in underground layers

The flow of fluid in porous media or narrow fissures of rock is generally treated as that of viscous fluid in which three forces owing to friction, pressure gradient and gravity are assumed to be always kept in equilibrium. Its representative formulation is Darcy's law widely adapted to the flow in underground layers.

$$w = -\frac{k}{\mu}\frac{dp}{dz},$$

where w is volume flux of fluid per unit cross section of the medium, p is pressure, k and μ are coefficients of permeability and viscosity. The effect of gravity is neglected for simplicity's sake throughout this paper. The above equation is transformed to show the equilibrium of forces to fluid contained in a unit volume of porous medium, porosity of which is θ.

$$\theta\frac{\mu}{k}w = -\theta\frac{dp}{dz}.$$

As the term of the left-hand side corresponds to the work done by friction along a unit length of the flow, it is transformed to the heat such as

$$\rho\,\theta\frac{dq}{dz} = A\,\theta\frac{\mu}{k}w,$$

where q is the heat supply to a unit mass of the fluid and A is the reciprocal of mechanical equivalent of heat. The fluid is everywhere supplied the heat and entropy of the flowing fluid is continuously increasing under the assumption of thermal insulation to surrounding media.

$$T\frac{ds}{dz} = A\frac{\mu}{k}v\,w > 0,$$

where T is temperature, s is entropy of a unit mass of the fluid and v is specific volume. Combining this with the thermodynamical relation as $di = Tds + Avdp$, in which i is enthalpy of a unit mass of the fluid, we can gain the next result.

$$\frac{di}{dz} = A\left(\frac{\mu}{k}v\,w + v\frac{dp}{dz}\right) = 0\ .$$

This shows that enthalpy of the fluid is conserved throughout the flow in underground layers so far as the loss of heat by conduction is quite negligible. Such a flow would be under isothermal condition if the fluid was assumed to be ideal gas. Hence, it is not a rigorous treatment for the flow of gas in porous media to theoretically combine the isentropic condition of gas with Darcy's equation as described by Muscat [1937]. Underground steam is not an ideal gas but it would be expected to become drier or more superheated according to the lowering of pressure in the upward flow to the ground surface if above assumptions to conserve the enthalpy were satisfied.

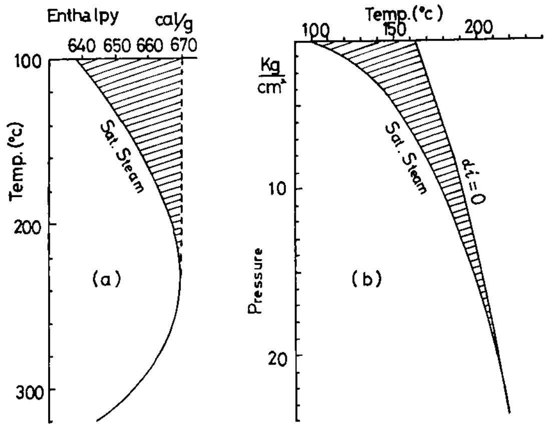

Fig. 2. Possible ranges in thermal properties of steam originating from thermal ground water.
(a) Enthalpy of a unit mass of steam.
(b) Temperature of steam.

We try to obtain the highest limit of superheated temperature brought on by the steam ascending from the layer occupied by liquid thermal water. The highest value of enthalpy of the saturated steam is 669.7 *cal/g* at the temperature between 230° and 240°C, when the effect of dissolved matter in thermal water is neglected. The highest enthalpy of the underground steam can be achieved in the case as in Fig. 2 (a) when perfectly dry steam separated from the thermal water by underground boiling or evaporation in the above temperature range flows upwards under the condition of isoenthalpy. Variation of the superheated temperature of the steam having the above highest enthalpy is given with that of pressure in Fig. 2 (b). The temperature of such steam is shown to be 163°C at 1 *atm*. It is considered to be the highest on the ground surface among temperatures brought by the steam ascending from deep thermal water. Observations have never reported a higher temperature than this at the outlets of fumaroles or wells in various hydrothermal areas except in some of the latest volcanic activities. It is, then, possible to say that the underground steam often found in the hilly part of hot spring area originates from liquid thermal water in deep layer even if it is in superheated state.

3. Flow of steam in a pipe

The fundamantal natures of the flow of viscous gas in a pipe are introduced prior to study the flow of steam in a well. Let us consider the steady flow of gas in a pipe of constant cross section and thermally insulated. Continuities of mass and energy fluxes are given as follows, denoting the velocity of gas by w and mass flux per unit section by q.

$$q = w/v = \text{constant.}$$
$$i + Aw^2/2 = \text{constant.}$$

If z is the co-ordinate along the pipe with z increasing downstream, the above is rewritten as

$$\frac{d}{dz}\left(i + \frac{A}{2}\,w^2\right) = 0 \qquad \cdots\cdots (1)$$

The entropy of a unit mass of gas must increase with the flow along the pipe because of the internal friction and reaches its maximum value at the point where velocity is equal to that of sound. We know about such a flow that, if the gas velocity at the entrance of the pipe is less than that of sound, the flow remains subsonic everywhere in the pipe. The highest velocity of the flow is attained when it becomes equal to the local velocity of sound only at the outlet of the pipe. Denoting the velocity of sound by c and the mass flux per unit cross section in that case by q_c, they are obtained by the next relations.

$$c^2 = -\left(v^2 \frac{dp}{dv}\right)_{ds=0}, \qquad \cdots\cdots (2)$$

$$q_c{}^2 = -\left(\frac{dp}{dv}\right)_{ds=0}. \qquad \cdots\cdots (3)$$

Table 1. Possible highest limits in discharge-rate of superheated steam under 1 *atm*. q_c; Calculated from sound velocity. q_m; Calculated from continuity of energy flux.

Temperature	q_c	q_m
100°C	28.3 g/cm^2·sec	30.3 g/cm^2·sec
110	27.5	26.1
120	27.2	22.9
130	26.9	19.5
140	26.5	15.8
150	26.2	11.8
160	25.8	6.1

Values of q_c for dry steam at 1 *atm* are calculated for various temperatures as given in Table 1 assuming the steam as ideal gas.

4. Flushing flow in boiling well

There are many boiling wells ejecting large quantities of mixtures of steam and water in hydrothermal areas. A boiling well means a drill-hole in which thermal fluid rapidly increases its volume flux under the condition of continuous boiling throughout the flow with pressure being upwards lowered. Steam and water in it are assumed to be in equilibrium state everywhere after boiling of thermal water begins. Supposing that the mixture of steam and water is a nearly homogeneous system and that the pipe is thermally insulated to the surrounding media, we can make similar relations as shown for the flow of gas in Section 3. Let us define that suffixes 1 and 2 indicate the properties of thermal water and steam in equilibrium state and x is the coefficient of dryness of the fluid given by the ratio of mass of the saturated steam in the mixed system. Thermodynamical properties of the mixed fluid are then determined by the next relations under various values of temperature and pressure.

$$\left.\begin{aligned} i &= (1-x)\,i_1 + x i_2, \\ s &= (1-x)\,s_1 + x s_2, \\ v &= (1-x)\,v_1 + x v_2. \end{aligned}\right\} \qquad \cdots\cdots (4)$$

When dryness, temperature and discharge-rate were observed at the orifice of a boiling well, we can easily ascertain the value of energy flux through the well by Eq. (1) and then deduce the temperature of the thermal water before boiling.

As the velocity of sound was shown to give the limitation to the discharge rate of fluid through the well, the value of c or q_c must be determined so as to be adapted to a homogeneous two-phase system consisting of steam and water in boiling well. The next result was attained after some thermodynamical studies as to the system satisfying the Clapayron's relation.

$$-\left(\frac{dv}{dp}\right)_{ds=0} = x\,v_2\left\{\frac{1}{p} - \frac{Av_2}{l}\left(2-\frac{c_pT}{l}\right)\right\} + (1-x)\frac{A\,c_pT\,v_2^2}{l^2} \quad \cdots\cdots (5)$$

Giving values of T, v_2 and l (latent heat) with various values of p from the table for saturated steam, values of $-\left(\frac{dv}{dp}\right)_{ds=0}$ in the above equation are calculated and inserted to Eqs. (2) and (3). An example of the result shows that the value of c varies from 9.8 to 450 *m/sec* and q_c from 104 to 27 $g/cm^2 \cdot sec$ according to the variation of x from 0.005 to 1 under the pressure of 1 *atm*. In that case, heat flux discharged through a unit area is determined by iq_c and shown as being in the range from 10.6 to 17 $kcal/cm^2 \cdot sec$.

Those values of q_c are multiplied by the cross sectional area of the well having the diameter of 75 *mm* which is the most common type among boiling wells in Beppu, and symbolized by Q_c. Ranges of variation of Q_c and iQ_c are respectively shown by lines drawn on the vertical axis against the temperature of 100°C in Fig. 3. They are realized as the ranges of mass and heat fluxes through boiling wells in Beppu, being subjected to the limitation given by sound velocity under the condition of atmospheric pressure.

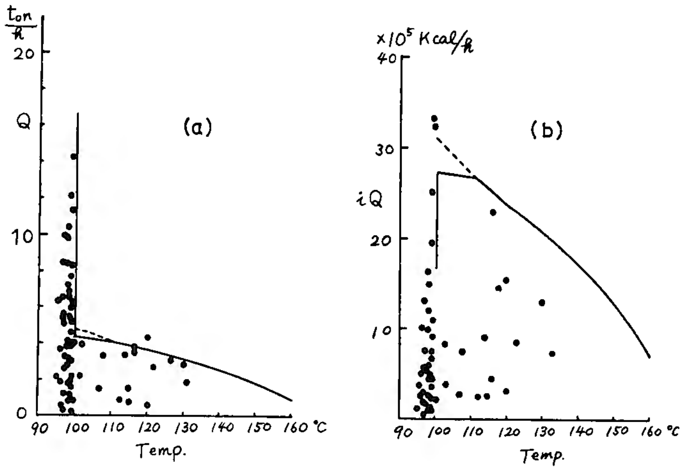

Fig. 3. Comparisons of observed fluxes through boiling and steam wells in Beppu (Yuhara [1965]) and the highest limits theoretically expected under the pressure of 1 *atm*.
(a) Mass flux. (b) Heat flux.

Mass and heat fluxes through many of boiling and steam wells in Beppu were observed as given in Fig. 3 by Yuhara [1965], in which observations with temperatures below 100°C correspond to those in boiling wells. Most of them are shown to be within or lower than the range of Q_c or iQ_c as above-mentioned. Therefore, it may be possible to say that fluid pressure at the outlets of boreholes are mostly kept nearly equal to the atmospheric pressure at the orifices of the boiling wells in Beppu. It is however noticed that a few of observed rates of discharge exceeds that limitation under atmospheric pressure. Such high rates of discharge may be understood by a supposition that fluid pressures at the outlets are kept quite higher than the external atmospheric pressure. Results calculated from Eqs. (3) and (5) are shown to be fairly approximated by the next relation in the range of pressure lower than 10 *atm*, which is obtained by assuming the steam as ideal gas and the value of T/l as 0.8 in Eq. (5).

$$q_c \fallingdotseq \frac{p}{\sqrt{RT}(0.85x+0.07)} \quad .$$

More approximation is made neglecting terms less than the second order in the mathematical expansion of $1/\sqrt{T}$.

$$q_c = \frac{10^{-4} p}{4\sqrt{x+0.08}} \quad . \qquad \cdots\cdots (6)$$

This linear relation between q_c and p tells us that the limitation in discharge-rate by sound velocity is extended with the increase of orifice pressure.

Though the value of x can be determined by both of the temperature at the entrance and outlet of the drill-hole, the discharge-rate does not directly concern the pressure at the outlet but only that at the entrance. It essentially refers to the drawdown of the hydrological head in the aquifer. This induces the possibility that the fluid pressure at the outlet is determined so as to satisfy the given value of flux through the well. It means that the relation between the pressure and the discharge-rate at the orifice is governed by that of p and q_c in Eq. (6). Such a condition is a-dapted only to the wells through which ejecting velocities exceed the sound velocity expected under the pressure of external medium. It is summarized for wells opening directly to the atmosphere as follows. The pressure at the orifice is kept nearly equal to that of the atmosphere so far as the velocity of ejection is lower than that of the sound expected under the atmospheric pressure but it proportionally increases with discharge-rate in the well from which ejecting velocity exceeds the above limitation. In the latter well, a sudden drop of the fluid pressure occurs immediately beyond the outlet owing to the rapid expansion of the fluid, possiblly approximated by the process of adiabatic change ($ds=0$). Proper installation is practically needed on the ground for such a well to protect the violent noise and vibration of the pipe by making so high pressure on the well that the flow rate becomes lower than the value expected from Eq. (6) under that artificial pressure.

Some tests were carried out to obtain the relation of discharge-rate to the pressure of the orifice in some of boiling wells in Kuzyu for the use of electric power. Results are compared with the limiting values, Q_c, in Fig. 4. Q is the mass flux through each well having diameter of 20 *cm* and then Q_c is also denoted by the product of q_c and cross sectional area of the well.

Though there are some possibilities involving errors in the process of calculation to gain the discharge rates of thermal water and steam under orifice pressure from the data separately observed after issues from the separater, they are estimated to be not so large as to give essential discrepancy to the state discussed as follows. The discharge-rate and dryness increase with lowering of orifice pressure as expected, though their changing states are differnt according to the properties of wells (Yamashita [1970]). Well No. 8 discharges almost dry steam and its rate never reach the sound velocity limit. However, in Well No. 7, the value of x becomes 0.15—0.17 and Q approaches the line showing the relation of Q_c and p when the orifice pressure decreases below 5 *atm*. It appears that in this well, especially in the tests 3 and 4,

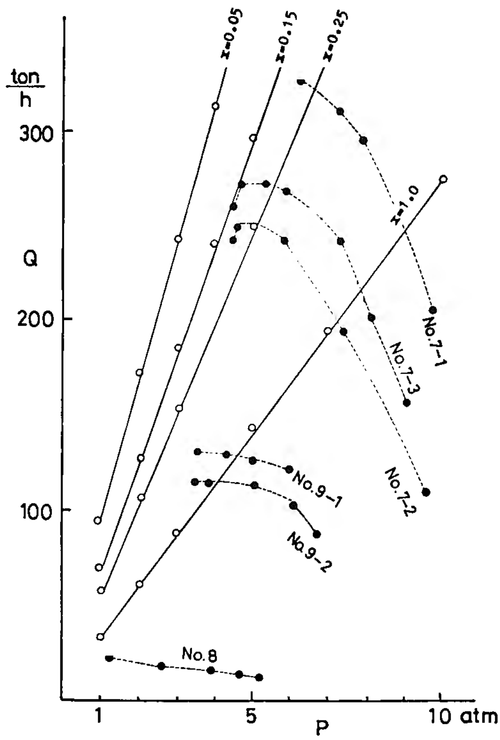

Fig. 4. Relations between Q and p in boiling wells in Kuzyu. Circles are the values of Q_c calculated from Eq. (3), straight lines show the approximate relations of Q_c and p for various values of x and points and dotted lines show observed results.

disturbance of flow begins to occur near the limiting line and discharge-rate cannot increase in spite of more enlargement of the orifice by controlling the valve. These experiments show that the highest limit of mass flux through a boiling well is attained when it corresponds to that calculated from sound velocity in a homogeneous two-phase system as before mentioned. This limit is adapted only to the well through which discharge-rate exceeds a definite value.

5. Flow of superheated steam through a well

Fig. 3 involves the range of superheated steam discharging from steams well in Beppu. Though the limitation in discharge-rate due to the sound velocity was already given as q_c in Table 1, a different kind of the highest limit in it is possibly expected to preserve the superheated temperature. It is led from the continuity of energy flux shown in Eq. (1). Integrating it,

$$i_o - i + \frac{A}{2}(w_o^2 - w^2) = 0,$$

where i or w is the value at the bottom of the well and i_o or w_o is that at the orifice. Combining the continuity of mass flux with this and denoting the specific volume at the orifice by v_o,

$$q^2 = \frac{2(i - i_o)}{A(v_o^2 - v^2)} \quad .$$

It is clear that the value of q depends on values of i and v as i_o and v_o are determined owing to the condition given at the orifice. Supposing that the superheated steam originates from thermal water as before described, the maximum of i is 669.7 *cal/g* and the value of v in that steam becomes so much smaller than v_o that the value of v^2 can be neglected. Then,

$$q^2 = \frac{2(669.7 - i_o)}{A\ v_o^2} \doteqdot \frac{670 - i_o}{1.2\ v_o^2} \times 10^8 \quad ,$$

$$q = \frac{0.91 \times 10^4}{v_o} \sqrt{670 - i_o} \quad .$$

Values of q are calculated at various temperatures under the pressure of 1 *atm* and shown as q_m in Table 1. Comparison of q_c and q_m shows that q_m is smaller than q_c in the range of higher temperature than 106°C, where the discharge-rate of steam cannot exceed the value of q_m at each temperature. Distribution of the product of q_m and cross sectional area of the well is drawn by a line in the above temperature range in Fig. 3 (a) so as to define the highest limit of mass flux. In the range between 100° and 106°C where q_c is less than q_m, the highest limit is shown by the line of Q_c calculated by q_c and distribution of Q_m is given by dotted line for comparison. The highest limit in heat flux is similarly given in Fig. 3 (b). Observed values of Q and iQ for superheated steam are shown to be almost within the limitations. Then, it can be said that the fluid pressure at the orifices are kept nearly equal to the atmospheric pressure in most of boiling and steam wells in Beppu.

5. Conclusion

Steam discharging in hydrothermal area can be considered as that originating from thermal ground water whether it is saturated or supersaturated. Therefore, various types of hydrothermal activities in the area of hot springs can be explained as phenomena originating only from thermal water of a sodium-chloride type in deep aquifer.

Some boiling wells have peculiar limitations of orifice pressures so as to be never lowered to the pressure of external media. They originate the restriction that the ejecting velocity cannot exceed the sound velocity. Because of violent noises and vibrations of pipes in those wells, proper installations are needed to make fluid pressure in the well so high that the flow is kept below the sound velocity.

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