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STUDY OF SEICHE IN LAKE BIWA-KO (II) —ON A NUMERICAL EXPERIMENT BY NONLINEAR TWO-DIMENSIONAL MODEL—

By

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Abstract

A nonlinear two-dimensional numerical experiment has been carried out by using a one-layer model to study a seiche in Lake Biwa-ko. The periods of the longitudinal seiches from this experiment are 255. 5, 79.8 and 69.1 minutes, and the oscillations with these periods are the uni-, bi- and tri-nodal seiche in Lake Biwa-ko, respectively. These periods from the two-dimensional experiment are larger than those from the Defant's method in the former paper, but the longitudinal distributions of the amplitude and velocity from both experiments agree well with each other. An amplitude of the bi-nodal seiche is very small in the north basin where the tri-nodal seiche dominates, but in the south basin both seiches have nearly the same value of amplitude regardless of wind direction. Nonlinear effects to the seiche motion are negligible in the deep areas in the north basin, but are not negligible in the shallow areas. The experimental results agree well with the observation, and the comparisons between them seem to show that the spatial distributions of the wind over the lake have an important effect to the seiche.

1. Introduction

In the former paper of the same title (Imasato [1970]), it was shown from the one-dimensional numerical calculation by the Defant's method that the possible periods of the longitudinal seiches are 212.0, 71.3 and 61.0 minutes and these are the uni-, bi- and tri-nodal seiche respectively. Profiles of the vertical displacements of water surface and the horizontal velocity were also presented.

It will be easily understood that the phenomena in a field such as Lake Biwa-ko where the shape of the basin is very complicated will be too much intricate to understand their nature from the one-dimensional approach which excludes the effects of bottom stress, inertia term and so on, and can not give two-dimensional distributions of the amplitude and velocity of a seiche as a function of the wind speed. Therefore, in order to get more precise information about the seiche in Lake Biwa-ko, a numerical experiment has been carried out by using a non-linear two-dimensional one-layer model. In this paper, some results of experimentation on the longitudinal seiches which are generated by the uniform wind of SW, SSE, ESE and NW and of the speed 5 m/sec are discussed, and some comparisons with an observation in the south basin will be given.

2. Fundamental equations, and boundary and initial conditions

In a shallow area of the lake, bottom stress and inertia term may be unable to be neglected, and the deflecting force of the earth's rotation may be effective for a seiche of a long period.

So the equations of motion and continuity are as follow,

$$\frac{\partial Q_x}{\partial t} = - g(h+\eta)\frac{\partial \eta}{\partial x} + A_x, \tag{1}$$

$$\frac{\partial Q_y}{\partial t} = - g(h+\eta)\frac{\partial \eta}{\partial y} + A_y, \tag{2}$$

$$\frac{\partial \eta}{\partial t} = - \frac{\partial Q_x}{\partial x} - \frac{\partial Q_y}{\partial y}, \tag{3}$$

and

$$A_x = \frac{\tau_{sx}}{\rho_w} - \frac{\tau_{bx}}{\rho_w} + f\cdot Q_y + \varepsilon(h+\eta)\left(\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y}\right), \tag{4}$$

$$A_y = \frac{\tau_{sy}}{\rho_w} - \frac{\tau_{by}}{\rho_w} - f\cdot Q_x + \varepsilon(h+\eta)\left(\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y}\right), \tag{5}$$

where x, y are the horizontal axes, and the vertical axis z is positive downwards, and u and v are the x- and y-component of velocity of water. Q_x and Q_y are the quantities defined by the equation

$$Q_x = \int_{-\eta}^{h} udz, \qquad Q_y = \int_{-\eta}^{h} vdz, \tag{6}$$

where h=h (x,y) is the depth, $\eta = \eta(x,y,t)$ the water surface elevation, τ_s the wind stress at the water surface, τ_b the bottom stress, f the parameter of the deflecting force of the earth's rotation, and ε the coefficient of the inertia term.

The surface wind stress τ_s is given by the equation

$$\tau_s = \rho_a \gamma_a^2 W^2, \tag{7}$$

where ρ_a is the density of air (0.0012g/cm^3), W the wind speed at the anemometer height over the water surface, and γ_a^2 the drag coefficient of air. It is said that the drag coefficient γ_a^2 is a function of wind speed, and also the results of our experiments in wind tunnels (Kunishi [1963], Kunishi and Imasato [1966]) have supported this statement. In this experiment, the drag coefficient γ_a^2 is set to be 0.0013 after Kunishi [1963].

Bottom stress may be written after Miyazaki [1961],

$$\tau_b = \rho_w \gamma_b^2 \bar{V} | \bar{V} | - 0.5 \frac{\gamma_b^2}{\gamma_a^2} \tau_s, \tag{8}$$

where ρ_w is the density of water, $\bar{V}$ the velocity vector of water, and γ_b^2 the drag coefficient of water at the bottom. The value of γ_b^2 has been given as 0.0026 by Hansen [1956], and this value has been used in a numerical experiment of tidal current or storm surge in the ocean (Miyazaki, Ueno, and Unoki [1961]). It must be examined through comparisons with the observational results whether this value is suitable or not in the present experiment on Lake Biwa-ko, and in this paper, is temporarily chosen to be 0.0026 as a preliminary step.

Effects of the inertia term will not be so remarkable in the seiche excepting in shallow parts of the lake such as in the south basin. Therefore the coefficient ε of the inertia term is set to be 1.0, and the Coriolis' parameter f is 0.00008 sec^{-1}.

While the computation is being carried out, the wind field over the lake is one of the important but troublesome problems. It is said that the wind over Lake Biwa-ko changes in a complicated way, but there is very little information about the spatial distributions of the wind speed and direction over Lake Biwa-ko. One of our present purposes is to discover the nature of the free oscillation of water surface in Lake Biwa-ko. So we adopted a very simple model for the wind field as the wind blows uniformely and continuously over the lake. Meteolorogical records in Hikone indicate that the mean wind direction is SSE in summer, and NW in winter, and that the mean wind speed is about 3 m/sec, and that a high wind over 10 m/sec blows frequently.

So the computations were carried out in four cases in which the wind directions are SW, SSE, ESE and NW, and the wind speed is 5 m/sec.

The initial condition is that the lake is rest, $\eta=u=v=0$ everywhere.

3. Difference equations and computational process

Various methods of a numerical calculation of the equations (1)–(5) have been presented (Miyazaki, Ueno, and Unoki [1961], the 2nd Regional Harbor Construction Bureau [1964]) in the case of the numerical experiment of a tidal current and a storm surge in the ocean. Unoki's method (The 2nd Regional Harbor Construction Bureau [1964]) seems to be most suitable for a numerical experiment of a water motion in a lake which is surrounded by land, because perpendicular components of velocity can be set to be zero in his method. Fig. 1 shows an arrangement of calculation points of water surface elevations (mark ●) and water velocity (mark ↑), and of depth points (mark ○) at each mesh.

In this arrangement in Fig. 1, the difference equations of the equations (1)–(5) are given by the equations

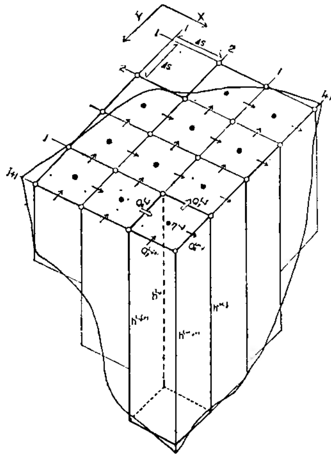

Fig. 1. Arrangement of mesh points.

$$S_x^{i,j}(t+\Delta t) = S_x^{i,j}(t) - \frac{g}{2}\left(\frac{\Delta t}{\Delta s}\right)^2 \left\{\eta^{i,j}\left(t+\frac{\Delta t}{2}\right) + \eta^{i-1,j}\left(t+\frac{\Delta t}{2}\right) + h^{i,j} + h^{i,j+1}\right\}\left\{\eta^{i,j}\left(t+\frac{\Delta t}{2}\right) - \eta^{i-1,j}\left(t+\frac{\Delta t}{2}\right)\right\} + A_x^{i,j}, \tag{9}$$

$$S_y^{i,j}(t+\Delta t) = S_y^{i,j}(t) - \frac{g}{2}\left(\frac{\Delta t}{\Delta s}\right)^2 \left\{\eta^{i,j-1}\left(t+\frac{\Delta t}{2}\right) + \eta^{i,j}\left(t+\frac{\Delta t}{2}\right) + h^{i,j} + h^{i+1,j}\right\}\left\{\eta^{i,j-1}\left(t+\frac{\Delta t}{2}\right) - \eta^{i,j}\left(t+\frac{\Delta t}{2}\right)\right\} + A_y^{i,j}, \tag{10}$$

$$\eta^{i,j}(t+\Delta t) = \eta^{i,j}(t) - S_x^{i+1,j}\left(t+\frac{\Delta t}{2}\right) + S_x^{i,j}\left(t+\frac{\Delta t}{2}\right) - S_y^{i,j}\left(t+\frac{\Delta t}{2}\right) + S_y^{i,j+1}\left(t+\frac{\Delta t}{2}\right), \tag{11}$$

$$\begin{aligned} A_x^{i,j} \equiv\ & \frac{\rho_a}{\rho_w}\gamma_a^2(1+\beta)\frac{(\Delta t)^2}{\Delta s}\sqrt{(W_x^{i,j})^2 + (W_y^{i,j})^2}\, W_x^{i,j} \\ & - \gamma_b^2 \frac{(\Delta t)^2}{\Delta s} u^{i,j}\sqrt{(u^{i,j})^2 + (v^{i,j} + v^{i-1,j} + v^{i-1,j+1} + v^{i,j+1})^2/16} \\ & + f\frac{\Delta t}{4}(S_y^{i,j} + S_y^{i-1,j} + S_y^{i-1,j+1} + S_y^{i,j+1}) \\ & - \frac{\varepsilon}{8}\left(\frac{\Delta t}{\Delta s}\right)^2 (\eta^{i,j} + \eta^{i-1,j} + h^{i,j} + h^{i,j+1})\,\{(u^{i+1,j} - u^{i-1,j})(u^{i+1,j} \\ & + 2u^{i,j} + u^{i-1,j}) + (u^{i,j-1} + u^{i,j})(v^{i,j} + v^{i-1,j}) \\ & - (u^{i,j} + u^{i,j+1})(v^{i,j+1} + v^{i-1,j+1})\}, \end{aligned} \tag{12}$$

$$A_y^{i,j} \equiv \frac{\rho_a}{\rho_w} \gamma_a^2 (1+\beta) \frac{(\Delta t)^2}{\Delta s} \sqrt{(W_x^{i,j})^2 + (W_y^{i,j})^2}\, W_y^{i,j}$$
$$- \gamma_b^2 \frac{(\Delta t)^2}{\Delta s} v^{i,j} \sqrt{(v^{i,j})^2 + (u^{i+1,j-1} + u^{i,j-1} + u^{i,j} + u^{i+1,j})^2/16}$$
$$- f(\Delta t)(S_x^{i+1,j-1} + S_x^{i,j-1} + S_x^{i,j} + S_x^{i+1,j})/4$$
$$- \frac{\varepsilon}{8}\left(\frac{\Delta t}{\Delta s}\right)^2 (\eta^{i,j-1} + \eta^{i,j} + h^{i,j} + h^{i+1,j}) \{(u^{i+1,j-1}$$
$$+ u^{i+1,j})(v^{i+1,j} + v^{i,j}) - (u^{i,j-1} + u^{i,j})(v^{i,j} + v^{i-1,j})$$
$$+ (v^{i,j-1} - v^{i,j+1})(v^{i,j-1} + 2v^{i,j} + v^{i,j+1})\}, \tag{13}$$

where

$$S_x \equiv Q_x \frac{\Delta t}{\Delta s} = (h+\eta)\, u \frac{\Delta t}{\Delta s}, \tag{14}$$

$$S_y \equiv Q_y \frac{\Delta t}{\Delta s} = (h+\eta)\, v \frac{\Delta t}{\Delta s}, \tag{15}$$

and Δs, Δt are mesh and time intervals, respectively. In this paper, Δs is 1 km, and Δt 10 seconds. Therefore, Lake Biwa-ko is divided into square meshes of 25×62 which are shown in Fig. 2, where the black points indicate the calculated elevation points at each mesh. $A_x^{i,j}$ and $A_y^{i,j}$ in equations (12) and (13) are computed at each

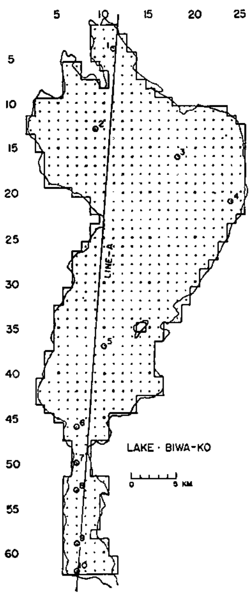

Fig. 2. Computed area of Lake Biwa-ko and arrangement of elevation points (black points).

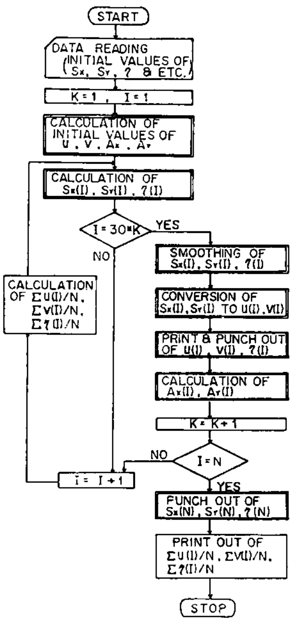

Fig. 3. Flow chart of the computation program.

five minutes (each 30 time steps), and assumed to be constant during these 30 time steps. A flow chart of the computing program is shown in Fig. 3.

4. Results of numerical calculation

Fig. 4 shows ten examples of the calculated water surface elevations (mark ●), and x- and y-components of the water velocity (marks ⊃ and >) at each 5 minutes for about 21 hours at the mesh points indicated by the double circles in Fig. 2. It is seen from the figure that the oscillation with the period of about 70 minutes dominates in the north basin, and the oscillations of about 255 and 70 minutes dominate in the south basin. These oscillations are the seiches in the lake, and oscillate arround the set up of water surface by the wind, making their amplitudes decrease gradually with time. Their frequency spectra are calculated by Fourier transform (FFT method)

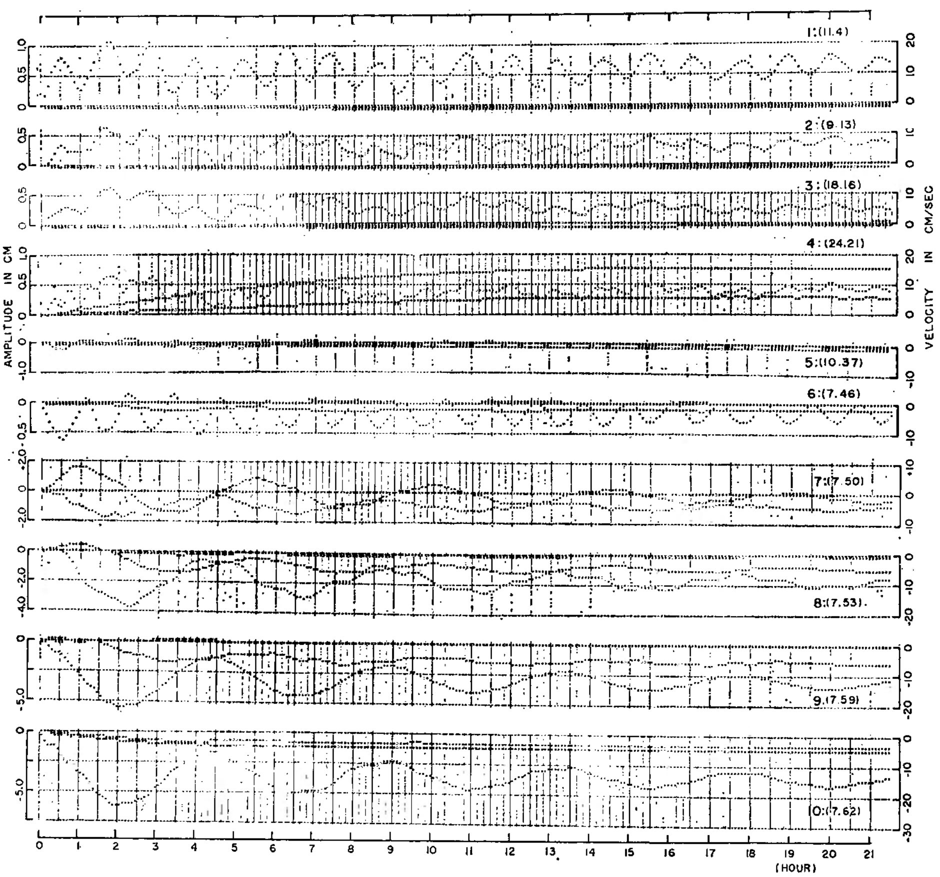

Fig. 4. Computed variations of vertical displacement of water surface (mark ●). x- and y-components of velocity (marks ⊃ and >).

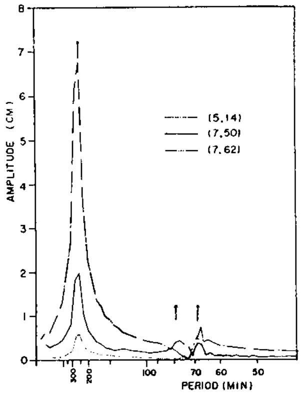

Fig. 5. Amplitude spectrum.

of these time series, which consist of 296 digital members of the calculated water surface elevation and velocity at every 5 minutes. The results of the amplitude spectrum are shown in Fig. 5. From these amplitude spectra, it will be seen that in Lake Biwa-ko there exist the seiches with the period of 255.5, 79.8, 69.1 minutes and so on. Contour curves of the amplitude of each component of seiches may be given from the amplitude and phase spectra at every mesh. The results for the seiches with the period of 255.5, 79.8 and 69.1 minutes are shown in Fig. 6, where full curves show the nodal lines. It is clearly seen from Fig. 6 that the seiche of period 255.5 minutes has one node at the neibourhood of the mouth of the south basin, that of

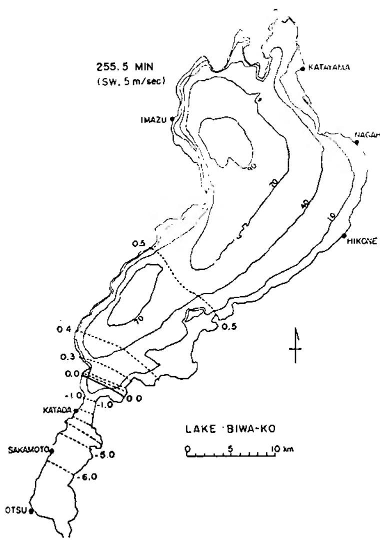

Fig. 6(a). Horizontal distribution of the amplitude of the uni-nodal seiche.

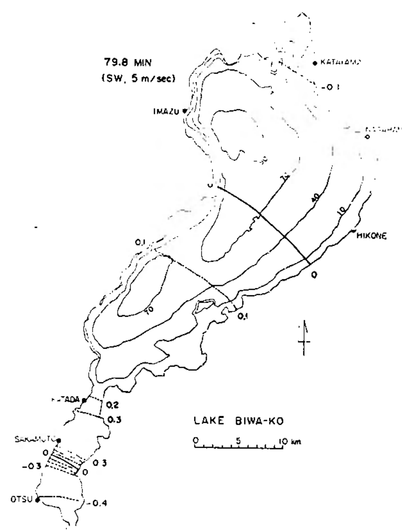

Fig. 6(b). Horizontal distribution of the amplitude of the bi-nodal seiche.

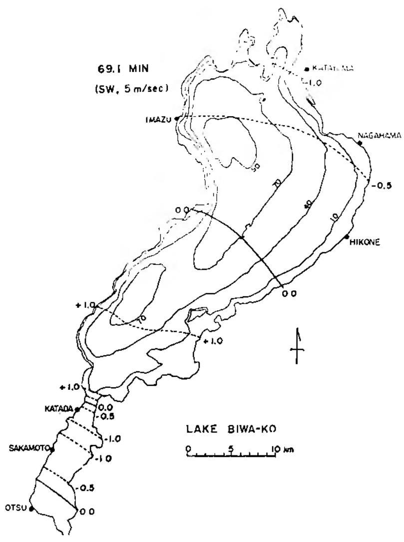

Fig. 6(c). Horizontal distribution of the amplitude of the tri-nodal seiche.

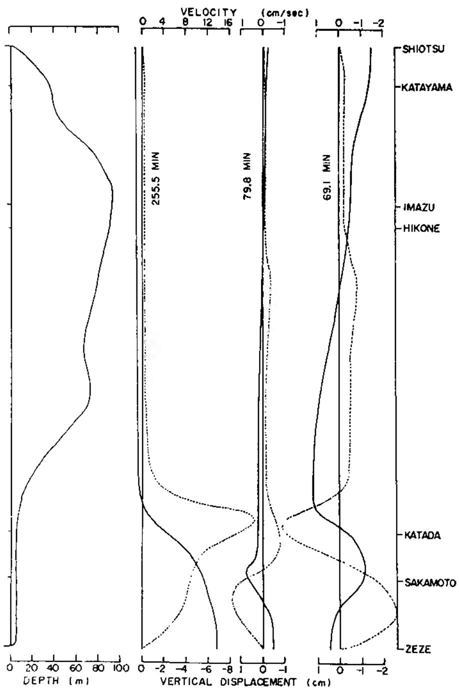

Fig. 7. Distributions of vertical displacement (full curve) and maximum velocity (dashed curve) of each component of the seiches.

79.8 minutes two nodes, and that of 69.1 minutes three nodes, and they are longitudinal oscillations in Lake Biwa-ko.

The distributions of the amplitude and velocity of these seiches along the line A in Fig. 2 are shown in Fig. 7, where the full curves show the distribution of the amplitude and the dashed curves those of the velocity. The seiche of the period 255.5 minutes has a maximum amplitude at the southern end of the lake, and the amplitude in the north basin is only one-tenth of that in the south basin. Amplitudes of the bi- and tri-nodal seiche at the southern end of the lake are nearly equal, but that of the tri-nodal seiche at the northern end reaches to about 7 times of that of the bi-nodal seiche. It must be noted that a phase difference between the uni- and tri-nodal seiche is about 41°, and that between the bi- and tri-nodal seiche is about 64° at the southern end of the lake. The maximum velocity with these oscillations appears at the nodal lines, especially at those in the south basin, and their values are about 19.9, 1.1, and 2.6 cm/sec for the uni-, bi- and tri-nodal seiche respectively.

The period of a seiche which is a free oscillation in a lake will not change by a shift of the wind direction, but amplitude of any component of the seiche may increase or decrease according to the wind direction. To examine this problem, similar numerical experiments for three cases of the wind direction, SSE, ESE and NW have been carried out under the same conditions excluding wind direction. The results are summarized in Table 1, where the amplitude of each seiche at the southern end of the lake and its normalized amplitude in the case of south-west winds are shown. It is clearly seen from this table that the longitudinal seiche develops most fully with the longitudinal wind (SW), and is substantially less affected by the lateral wind (ESE).

Table 1. Change of the amplitude of seiche with the wind direction

		WIND DIRECTION				PERIOD (MIN)
		SW	SSE	ESE	NW	
AMPLITUDE (CM)	A_1	6.76	6.09	0.13	5.78	T_1: 255.5
	A_2	0.40	0.39	0.16	0.37	T_2: 79.8
	A_3	0.45	0.36	0.06	0.34	T_3: 69.1
RATIO OF AMPLITUDE	A_1	1.0	0.904	0.021	0.855	
	A_2	1.0	0.940	0.339	0.973	
	A_3	1.0	0.780	0.080	0.700	

Other components of the seiches with shorter periods than those mentioned above seem to have a very complicated nature, and to need a closer examination in the future. Therefore, discussions concerning these are left inevitably for a later paper.

To examine the effects of each term in the equations of motion to the seiche, they have been evaluated at five minute intervals. Some results in the case of the south-west wind have been summarized in Table 2. It will be easily seen from this table that the time change of velocity is in the same order as the pressure gradient as might have been expected. In the deep areas in the north basin, the bottom stress term and inertia term are less than 1/400 of the pressure gradient term, and therefore the nonlinear effects on the seiche are negligible there. And also, the amplitude and velocity of the uni-nodal seiche are very small in these areas, and therefore the effect of the Coriolis' term is also small.

Table 2. Evaluation of effects of nonlinear terms and Coriolis' term on the seiches

POSITION NO	POSITION MESH NO	DEPTH (M)	COM-PONENT	BOTTOM STRESS	INERTIA TERM	CORIOLIS' TERM	VELOCITY TIME CHANGE	PRESSURE GRADIENT
2	9,13	84.7	X	0.00002	0.0002	0.022	0.141	0.490
			Y	0.00006	0.00007	0.012	0.229	0.600
3	18,16	59.3	X	0.0001	0.0012	0.024	0.182	0.500
			Y	0.0002	0.0005	0.013	0.167	0.390
4	24,21	11.9	X	0.002	0.047	0.007	0.088	0.280
			Y	0.010	0.048	0.011	0.069	0.220
6	7,46	19.1	X	0.001	0.005	0.065	0.044	0.160
			Y	0.006	0.025	0.020	0.218	0.620
9	7,59	5.1	X	0.001	0.002	0.046	0.029	0.120
			Y	0.047	0.020	0.011	0.347	0.580

On the other hand, the bottom stress and the inertia term can not be neglected in the shallow areas of the lake. In the south basin where the seiche of period 255.5 minutes dominates, the effects of the Coriolis' term can not be neglected, too. Another nonlinear term $g\eta \frac{\partial \eta}{\partial x}$ or $g\eta \frac{\partial \eta}{\partial y}$ is very small in comparison with $gh \frac{\partial \eta}{\partial x}$ or $gh \frac{\partial \eta}{\partial y}$, though their values are not summarized in the table. The effect of non-linearity to the period of the seiche is not mentioned in this paper to be left as an important problem in the future.

5. Discussion and comparison with observation

The distributions in Fig. 7 agree well with the results of the one-dimensional calculation in the former paper, although the periods from the Defant's method are shorter a little than those from the present two-dimensional experiment. The amplitude of the bi-nodal seiche is very small in the north basin, and the oscillation of the period of 69.1 minutes will dominate there. This is why all the previous investigators have considered the oscillations of the period of about 70 minutes as the uni-nodal seiche only in the north basin, and overlooked that they consist of the two oscillations of the different mode (Imasato [1970]). On the other hand, the bi- and tri-nodal seiche have the same value of the amplitude in the south basin, especially at the southern end of the lake. These interesting facts have been anticipated to be real from a very simple observation in the former paper.

Some observations of seiche have been carried out in Lake Biwa-ko for about two weeks in February in 1970 in order to compare the results with those of the numerical experiment. Fig. 8 shows the four pieces of observational records in Otsu, where is at the southern end of Lake Biwa-ko, and the meteorological records about the wind in Hikone are also shown in the figure as a measure of the meteorological condition over Lake Biwa-ko.

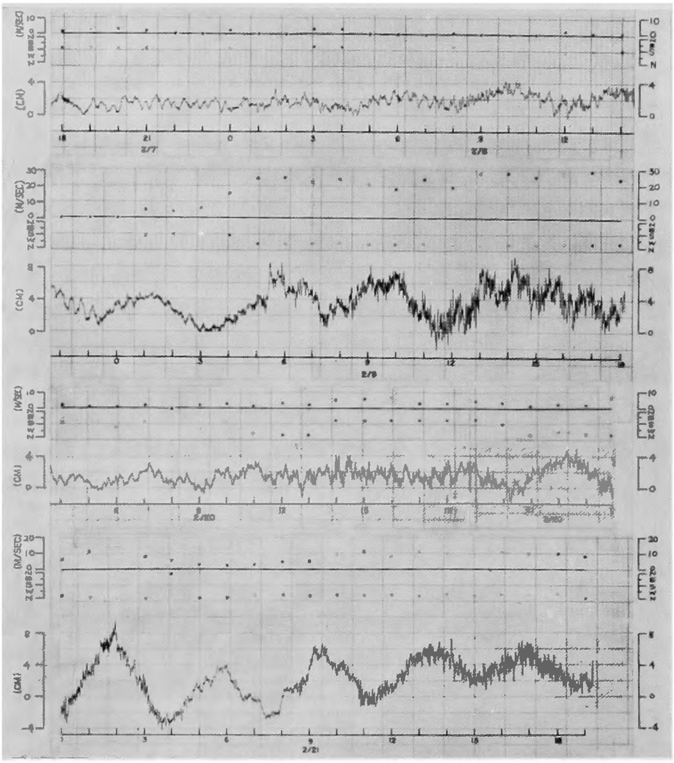

Fig. 8. Observational records of seiche in Otsu in February, 1970. Meteorological data are those in Hikone.

Amplitude spectra have been calculated from these continuous records by Fourier transform, and the results are summarized in Fig. 9. It is shown from this figure that mean periods of the longitudinal seiche are 243.9, 74.1 and 65.1 minutes. The observational results of these three components of the seiche are summarized in Table 3, and the experimental results at mesh point (7,62), which is in the neighbourhood of the observational station in Otsu, are summarized in Table 4. In both

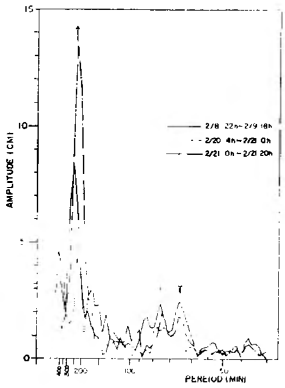

Fig. 9. Amplitude spectra of seiche observed in Otsu.

Table 3. Observational results of seiche in Otsu in February, 1970

CASE NO.		3	1	2	4	MEAN
DATE (1970)		2/20 4^h~ 2/21 0^h	2/7 18^h~ 2/8 14^h	2/8 22^h~ 2/9 18^h	2/21 0^h~ 2/21 21^h	
MEAN WIND SPEED (HIKONE)		2.70	1.10	18.55	8.33	
MEAN WIND DIRECTION (HIKONE)		SW	SSE	WNW	NNW	
PERIOD (MIN)	T_1	232.3	255.5	255.5	232.3	243.9
	T_2	73.0	73.0	75.2	75.2	74.1
	T_3	65.5	63.9	65.5	65.5	65.1
AMPLITUDE (CM)	A_1	4.33	2.78	8.25	13.41	
	A_2	1.36	1.23	2.33	1.96	
	A_3	0.99	1.20	2.47	1.78	
RATIO OF AMPLITUDE	A_2/A_3	1.370	1.025	0.942	1.098	
	A_2/A_1	0.314	0.442	0.282	0.146	
	A_3/A_1	0.229	0.432	0.299	0.133	
PHASE DIFFERENCE (DEGREE)	$\theta_1 \sim \theta_2$	36	45	105	25	
	$\theta_2 \sim \theta_3$	58	46	5	11	

tables, T shows a period of seiche, A an amplitude, and θ a phase, and the numerical subscripts are the numbers of the node of the seiche.

It is shown from Table 3 that the ratio of the amplitude of the bi-nodal seiche to that of the tri-nodal seiche is nearly constant and independent of the wind direction, and that the phase differences between the bi- and tri-nodal seiche seem to change slightly with the wind direction. These observational results agree well with the experimental results in Table 4. The observational ratios A_2/A_1 and A_3/A_1 are very larger than the ratios of the numerical experiments. This disagreement between the observations and the experiments may suggest that the wind field over Lake

Table 4. Experimental results at the mesh point (7, 62)

WIND SPEED (M/SEC)		5.00	5.00	5.00	5.00	PERIOD (MIN)
WIND DIRECTION		SW	SSE	ESE	NW	
AMPLITUDE (CM)	A_1	6.76	6.09	0.13	5.78	T_1: 255.5
	A_2	0.40	0.39	0.16	0.37	T_2: 79.8
	A_3	0.45	0.36	0.06	0.34	T_3: 69.1
RATIO OF AMPLITUDE	A_2/A_3	0.900	1.080	2.453	1.113	
	A_2/A_1	0.060	0.065	1.189	0.065	
	A_3/A_1	0.066	0.060	0.485	0.058	
PHASE DIFFERENCE (DEGREE)	$\theta_1 \sim \theta_2$	23	2	73	8	
	$\theta_2 \sim \theta_3$	64	42	109	37	

Biwa-ko is not uniform and that the spatial distribution of the wind has an important role of the generation and maintenance of seiche oscillations.

It is shown from Table 3 that the observational ratios A_2/A_1 and A_3/A_1 decrease as the amplitude of the uni-nodal seiche increases. The wind speed may not be the cause as far as considered from Table 3. And also, the wind direction may not, because it is shown from Table 3 that there is difference in the experimental ratios in the case of the wind direction SW, SSE and NW except in the case of ESE. If a value of an amplitude of the uni-nodal seiche is the cause, the nonlinear interaction among the components of seiche must be investigated, and if the spatial distribution of the wind is the cause, a new numerical experiment must be carried out under the boundary condition of a variable wind field. The last two may be the reasons why the observational periods of the components of seiche change a little with time, and they may be important problems to be investigated in the future.

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