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EFFECTS OF IONS ON IONOSPHERIC REFLECTION COEFFICIENTS FOR VLF RADIO WAVES

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Abstract

Effects of positive ions on the ionospheric reflection coefficients for very low frequency radio waves are examined with special attention to the lower hybrid resonance in which ions play the principal role. The ionosphere is assumed to be uniform and sharply bounded and impressed with a horizontal static magnetic field. The reflection coefficients are calculated for VLF radio waves which are incident perpendicularly to the static magnetic field with their electric field in the plane of incidence. It is shown that ions should be taken into consideration on the nighttime reflection coefficients for grazing incidence.

1. Introduction

Round, Eckersley, Tremellen and Lunnon (1925) examined the results of the field strength measurements of VLF radio waves propagating in the earth-ionosphere wave-guide and suggested that VLF radio waves suffered less attenuation if they were propagated from west to east instead of from east to west. Crombie (1958) re-examined the results of Round et al. in addition to his own observations and concluded that the evidence strongly supported their suggestion.

In order to interpret this non-reciprocal property of VLF radio wave propagation, Barber and Crombie (1959) calculated the reflection coefficient for VLF radio waves which are incident perpendicularly to the horizontal magnetic field on a sharply bounded ionosphere and found that the reflection coefficient for waves incident from the west, is greater than that for waves incident from the east, when the angle of incidence is large and the wave electric field is in the plane of incidence. Though there are several kinds of ions in the ionosphere Barber et al. (1959) considered electrons only as charged particles. It is well known (Stix, 1960) that for waves propagating perpendicularly to the magnetic field in the plasma the lower hybrid resonance occurs for which ions play an essential role. Though this resonance would be smeared because of collisions with neutral particles dominant in the lower ionosphere from which VLF radio waves are reflected, it is important to estimate the effects of this resonance on VLF reflection coefficients.

In section 2 of this paper, general expression for conductivity tensor is derived taking into consideration the presence of electrons, ions, and neutral particles and their mutual collisions. By the use of this general conductivity tensor, the reflection coefficients from sharply bounded ionosphere are calculated for VLF radio waves propagating perpendicularly to the horizontal magnetic field in section **3**. The results of numerical calculation of reflection coefficients are presented in section **4** and discussed in section **5**.

2. Conductivity Tensor in Partly Ionized Plasma

We consider a partly ionized cold plasma which is composed of electrons of charge $-e$, positive ions of charge Ze and neutral particles and immersed in the static magnetic fields $\boldsymbol{B}_0$. Equations of motion for three particles are (in CGS Gaussian Unit)

$$\frac{d\boldsymbol{V}_e}{dt} = -\frac{e}{m_e}\left(\boldsymbol{E}+\frac{\boldsymbol{V}_e}{c}\wedge \boldsymbol{B}_0\right) - \nu_{ei}(\boldsymbol{V}_e - \boldsymbol{V}_i) - \nu_{en}(\boldsymbol{V}_e - \boldsymbol{V}_n),$$

$$\frac{d\boldsymbol{V}_i}{dt} = \frac{Ze}{m_i}\left(\boldsymbol{E}+\frac{\boldsymbol{V}_i}{c}\wedge \boldsymbol{B}_0\right) - \nu_{ie}(\boldsymbol{V}_i - \boldsymbol{V}_e) - \nu_{in}(\boldsymbol{V}_i - \boldsymbol{V}_n), \tag{1}$$

$$\frac{d\boldsymbol{V}_n}{dt} = -\nu_{ni}(\boldsymbol{V}_n - \boldsymbol{V}_i) - \nu_{ne}(\boldsymbol{V}_n - \boldsymbol{V}_e),$$

where $\boldsymbol{V}_{e,i,n}$ and $m_{e,i,n}$ are velocity and mass of the three particles and **E** is the electric field and $\nu_{\alpha\beta}$ is effective collision frequency for momentum transfer of particle α to particle β. Since there is no loss of total momentum in collisions among the particles,

$$\rho_\alpha \nu_{\alpha\beta} = \rho_\beta \nu_{\beta\alpha}. \tag{2}$$

Here the static magnetic field $\boldsymbol{B}_0$ is taken to be parallel to x-axis and harmonic time variation $e^{-i\omega t}$ is assumed. After linearization of equation (1) and elimination of $\boldsymbol{V}_n$, we reach following equations,

$$C_1 \boldsymbol{V}_i - \frac{Zm_e}{m_i} C_3 \boldsymbol{V}_e = \frac{Ze}{m_i}\boldsymbol{E} + \Omega_i \boldsymbol{V}_i \wedge \boldsymbol{e}_x,$$

$$C_3 \boldsymbol{V}_i - C_2 \boldsymbol{V}_e = \frac{e}{m_e}\boldsymbol{E} + \Omega_e \boldsymbol{V}_e \wedge \boldsymbol{e}_x, \tag{3}$$

where $\boldsymbol{e}_x$ is a unit vector of x-direction and

$$C_1 = \frac{\gamma_i \gamma_n - \nu_{in}\nu_{ni}}{\gamma_n},$$

$$C_2 = \frac{\gamma_e \gamma_n - \nu_{en}\nu_{ne}}{\gamma_n}, \tag{4}$$

$$C_3 = \frac{\gamma_n \gamma_e + \nu_{ni} \nu_{en}}{\gamma_n},$$

$$\left(\begin{array}{ll} \gamma_e = \nu_{ei} + \nu_{en} - i\omega, & \Omega_e = \dfrac{eB_o}{cm_e}, \\ \gamma_i = \nu_{ie} + \nu_{in} - i\omega, & \\ \gamma_n = \nu_{ni} + \nu_{ne} - i\omega, & \Omega_i = \dfrac{ZeB_o}{cm_i}, \end{array} \right).$$

In the derivation of equations (3) and (4), equation (2) and a relation of charge neutrality, $n_e = Zn_i$ are used. From equation (3), three component of $\boldsymbol{V}_e$ and $\boldsymbol{V}_i$ are derived as a function of $\boldsymbol{E}$, and current $\boldsymbol{J}$ is calculated as follows;

$$J_x = en_e (V_{ix} - V_{ex}) = en_e \cdot \frac{ZD_6 - D_4}{D_1} E_x ,$$

$$J_y = en_e (V_{iy} - V_{ey}) = en_e \cdot$$
$$\frac{[(D_1+D_3)(ZD_6-D_4)+D_2 D_5 (Z-1)] E_y + [D_5 (D_1+D_3)(Z-1) - D_2 (ZD_6 - D_4)] E_z}{\Delta},$$

$$J_z = en_e (V_{iz} - V_{ez}) = en_e \cdot \tag{5}$$
$$\frac{[-D_5(D_1+D_3)(Z-1)+D_2(ZD_6-D_4)] E_y + [(D_1+D_3)(ZD_6-D_4)+D_2 D_5(Z-1)] E_z}{\Delta}$$

where

$$D_1 = \frac{C_1 C_2 - ZmC_3^2}{C_3},$$

$$D_2 = \frac{C_1 \Omega_e - C_2 \Omega_i}{C_3},$$

$$D_3 = \frac{\Omega_e \Omega_i}{C_3},$$

$$D_4 = e \frac{Zm_e C_3 - m_i C_1}{m_i m_e C_3}, \tag{6}$$

$$D_5 = \frac{e\Omega_i}{C_3 m_e}, \qquad \Delta = (D_1 + D_3)^2 + D_2^2,$$

$$D_6 = e \frac{C_2 - C_3}{m_i C_3}, \qquad m = m_e / m_i.$$

Generally current $\boldsymbol{J}$ is given by a product of a conductivity tensor $[\sigma]$ and an electric field vector $\boldsymbol{E}$

$$\boldsymbol{J} = [\sigma] \cdot \boldsymbol{E} = \begin{pmatrix} \sigma_{\parallel}, 0, 0 \\ 0, \sigma_{\perp}, \sigma_H \\ 0, -\sigma_H, \sigma_{\perp} \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} . \tag{7}$$

By comparison of equation (5) and (7), elements of the conductivity tensor are given by

$$\sigma_{\parallel} = en_e \frac{ZD_6 - D_4}{D_1},$$
$$\sigma_{\perp} = en_e \frac{(D_1 + D_3)(ZD_6 - D_4) + D_2 D_5 (Z - 1)}{\Delta}, \tag{8}$$
$$\sigma_H = en_e \frac{D_5 (D_1 + D_3)(Z - 1) - D_2 (ZD_6 - D_4)}{\Delta}.$$

3. Reflection Coefficients.

We consider a situation that in a half space $z>0$, there is a uniform ionosphere having a horizontal magnetic field in x-direction. This ionosphere is characterized by the conductivity tensor (7). We derive a reflection coefficient for a plane electromagnetic wave incident from below perpendicularly to the magnetic field on a sharp boundary at $z=0$. A set of governing equations for this problem is

$$\nabla \wedge \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t},$$
$$\nabla \wedge \boldsymbol{B} = 4\pi \boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}, \tag{10}$$
$$\boldsymbol{J} = \begin{pmatrix} \sigma_{\parallel}, 0, 0 \\ 0, \sigma_{\perp}, \sigma_H \\ 0, -\sigma_H, \sigma_{\perp} \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}.$$

The incident and reflected waves below the boundary ($z=0$) and the transmitted waves above it can all be assumed to vary with space and time in a manner

$$\exp\ (k_y y + k_z z - \omega t). \tag{11}$$

By substituting equation (11) into (10), the following component equations are obtained,

$$k_{yt} E_z - k_{zt} E_z = \frac{\omega}{c} B_x , \tag{12-1}$$

$$k_{zt} E_x = \frac{\omega}{c} B_y , \tag{12-2}$$

$$-k_{yt} E_x = \frac{\omega}{c} B_z , \tag{12-3}$$

$$k_{yt} B_z - k_{zt} B_y = -\frac{\omega}{c} CE_x , \tag{12-4}$$

$$k_{zt} B_x = -\frac{\omega}{c} AE_y - \frac{\omega}{c} BE_z , \tag{12-5}$$

$$-k_{yt}B_x = \frac{\omega}{c}BE_y - \frac{\omega}{c}AE_z \ , \tag{12-6}$$

where $$A = i\frac{4\pi c\sigma_\perp}{\omega} + 1, \qquad B = i\frac{4\pi c\sigma_H}{\omega},$$

$$C = i\frac{4\pi c\sigma_\parallel}{\omega} + 1,$$

and suffix t denote a transmitted waves. It is evident from equations (12–1)—(12–6) that two modes of waves are de-coupled in this case. Equations (12–1), (12–5) and (12–6) show waves with a transverse magnetic field (TM-wave) and equations (12–2), (12–3) and (12–4) show waves with a transverse electric field. The latter is independent of the static magnetic field and excluded from present consideration. From the former, refractive index in the ionosphere is obtained,

$$n^2 = \frac{c^2(k_{yt}^2 + k_{zt}^2)}{\omega^2} = \frac{A^2 + B^2}{A}. \tag{13}$$

Electromagnetic field in a half space $z<0$ is determined by equation (10) putting $\sigma_\parallel = \sigma_\perp = \sigma_H = 0$. If suffices i and r are used to express incident and reflected waves, bounddary conditions at $z=0$ are given by

$$\begin{aligned} E_{yi} + E_{yr} &= E_{yt}, \\ B_{xi} + B_{xr} &= B_{xt}. \end{aligned} \tag{14}$$

By the use of equation (10), (12), (14) and relation $k_{yi}=k_{yr}=k_{yt}$ and $k_{zr}=-k_{zi}$, the reflection coefficient at the boundary $z=0$ for TM-wave is obtained as

$$_\parallel R_\parallel = \frac{B_{xr}}{B_{xi}} = -\frac{(An^2 - B)k_{yi} + An^2 k_{zi} - k_{zt}A}{(An^2 - B)k_{yi} - An^2 k_{zi} - k_{zt}A} \tag{15}$$

For a given angle of incidence θ_i and frequency ω, k_{yi}, k_{zi} and k_{zt} are determined by

$$\begin{aligned} k_{yi} &= \frac{\omega}{c}\sin\theta_i \ , \\ k_{zi} &= \frac{\omega}{c}\cos\theta_i \ , \\ k_{zt} &= \left(\frac{\omega^2}{c^2}n^2 - k_{yi}^2\right)^{1/2}. \end{aligned} \tag{16}$$

4. Numerical Results

We consider NO^+ as positive ions and the static magnetic field is taken to be 0.3 Gauss with the value at the equator in mind. In the lower ionosphere, electron-ion (or ion-electron) collision can be neglected in comparison with electron-neutral (or ion-neutral) collision because of dominant neutral particles. For the same reason, neutral particles can be considered to be unmoved. Therefore, ν_{ei}, ν_{ie}, ν_{ne} and ν_{ni} may be put to zero.

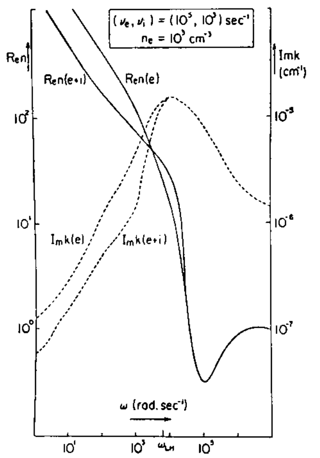

Fig. 1. Frequency dependence of the real part of the refractive index (solid line) and the imaginary part of wave number (dotted line) for a wave propagating perpendicularly to a static magnetic field in a partly ionized plasma. The wave electric field is assumed to be perpendicular to the static magnetic field. Letters, e and e+i, in the bracket show that effects of electrons only and of electrons and ions are taken into consideration, respectively.

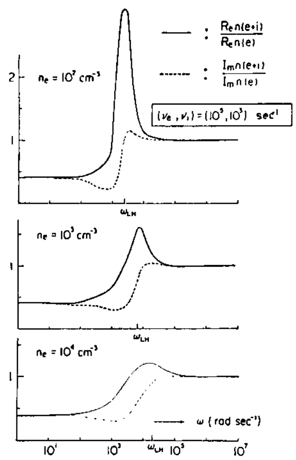

Fig. 2. Frequency dependence of a ratio of the refractive index with consideration for ions ($n(e+i)$) to that without ions ($n(e)$). The lower hybrid resonance frequency is shown by ω_{LH} on the abscissa.

Calculated results of the real part of the refractive index and the imaginary part of the wave number in the ionosphere are shown in Figure 1. The collision frequency is taken to be 10^5 $\sec^{-1}$ for electrons and 10^3 $\sec^{-1}$ for ions and electron density to be 10^3 cm^{-3}. Figure 2 shows a frequency dependence of a ratio of the refractive index with consideration for ions, $n(e+i)$, to that without ions, $n(e)$. It is evident that the ratio for the real part of the refractive index has a clear peak at the lower hybrid resonance frequency and this peak becomes sharp as electron density decreases. Figure 2 also shows a general tendency that ions reduce the refractive index at lower frequency, though they do not play an important role in higher frequency.

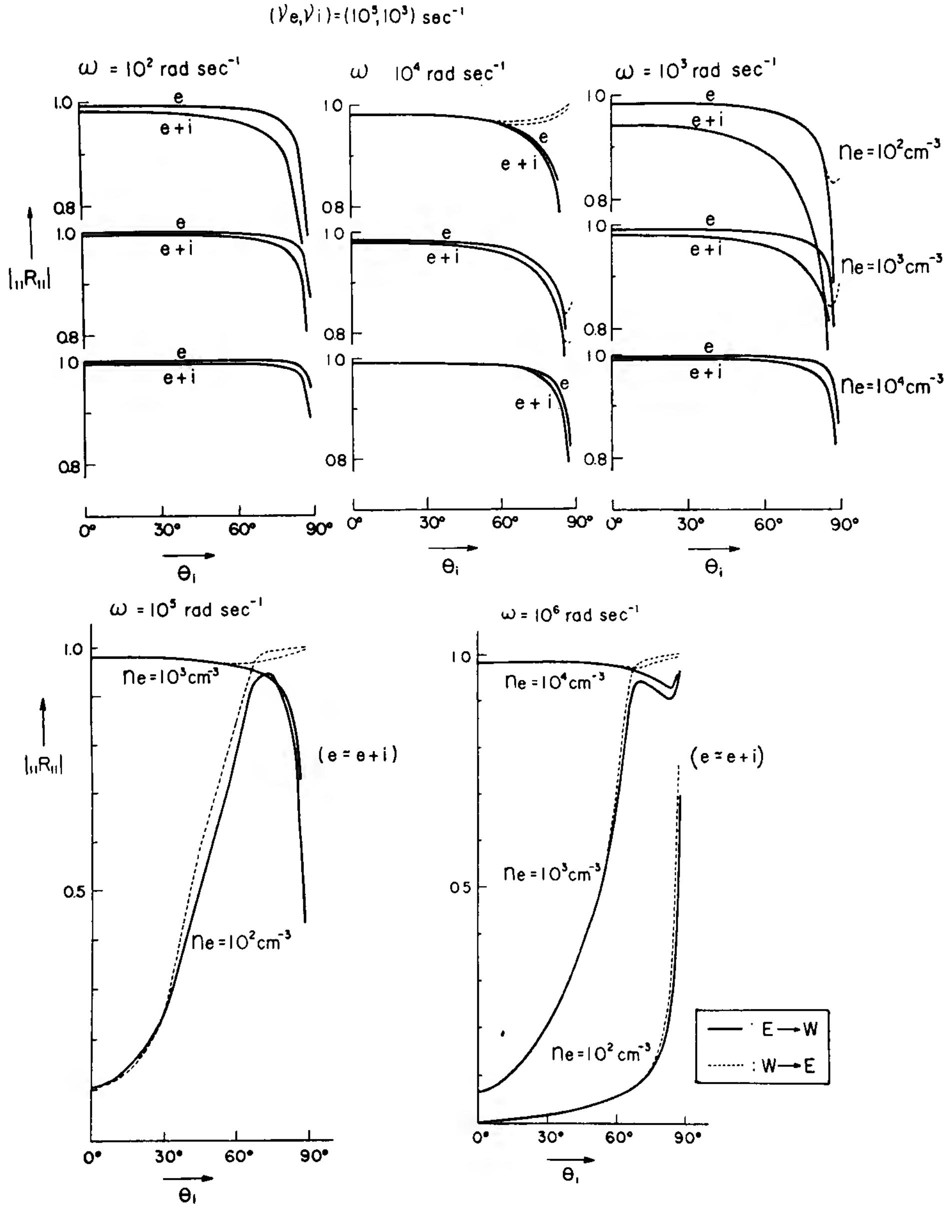

Fig. 3. Modulus of the reflection coefficients for the angle of incidence. The solid line shows the case that the waves incident on the sharply bounded ionosphere with a horizontal static magnetic field from east to west and the dotted line shows incidence from west to east.

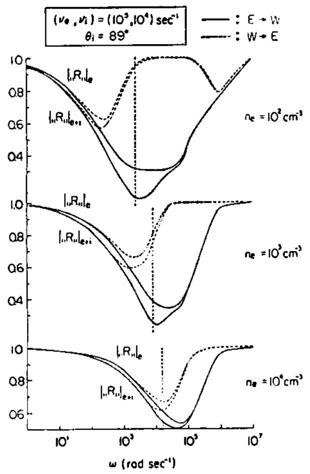

Fig. 4. Frequency dependence of the modulus of the reflection coefficients for the grazing incidence. The solid line shows reflection from east to west and the dotted line the reflection from west to east. The vertical dotted line shows the lower hybrid resonance frequency.

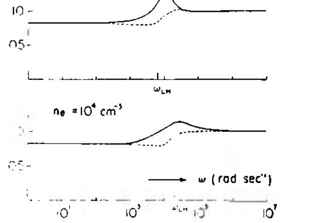

Fig. 5. Frequency dependence of a ratio of a refractive index with consideration for ions ($n(e+i)$) to that without ions ($n(e)$). The lower hybrid resonance frequency is shown by ω_{LH} on the abscissa.

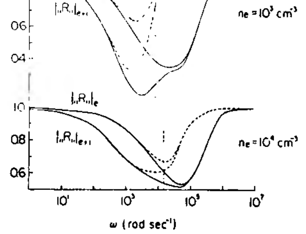

Fig. 6. Frequency dependence of the modulus of the reflection coefficients for the grazing incidence. The solid line shows reflection from east to west and the dotted line shows the reflection from west to east. The vertical dotted line shows the lower hybrid resonance frequency ω_{LH}.

Figure 3 shows the modulus of the reflection coefficients for the TM-wave, $|_{\parallel}R_{\parallel}|$, calculated from eq. (15). From this figure, we can see that ions make $|_{\parallel}R_{\parallel}|$ smaller than when electrons only is taken into consideration. This tendency is notable for the grazing incidence in the neighbourhood of the lower hydrid resonance frequency. The frequency dependence of $|_{\parallel}R_{\parallel}|$ for the grazing incidence ($\theta_i=89°$) are shown in Figure 4. It should be noted that a difference between $|_{\parallel}R_{\parallel}|_e$ and $|_{\parallel}R_{\parallel}|_{e+i}$ becomes marked at frequencies lower than the lower hybrid resonance frequency.

The ratio $n(e+i)/n(e)$ for $(\nu_e, \nu_i)=(10^5, 10^4)$ sec^{-1} is shown in Figure 5. In this case the peak of $Re\ n(e+i)/Re\ n(e)$ broadens because the lower hybrid resonance is smeared by increased collisions for ions. Therefore, effects of ions on reflection coefficients become small except for the grazing incidence. The modulus of the reflection coefficients is plotted versus frequencies in Figure 6. It is interesting that $|_{\parallel}R_{\parallel}|_{e+i}$ is different from $|_{\parallel}R_{\parallel}|_e$ at the both sides of the lower hybrid frequency in contrast to the previous case (Fig. 4).

5. Discussions

VLF radio waves propagating obliquely in the earth-ionosphere wave-guide is considered to be reflected at the height of 70km in daytime and 90km in nighttime. The collision frequency at 70 km height is of the order of 10^6 for electrons and 10^5 for ions (for example, Belrose et al. (1964)). Calculations for these values of collision frequency show that the refractive index $n(e+i)$ is nearly equal to $n(e)$ at any frequencies. So it may be said that the presence of ions does not affect daytime propagation of VLF radio waves. At nighttime reflection level (90km), the order of the collision frequency is 10^5 for electrons and 10^4 for ions. In this case, presence of ions change the values of the reflection coefficients at least for the grazing incidence at the neighbourhood of the lower hybrid resonance frequency as evident from figure 6.

ELF radio waves which have longer wave-length may penetrate deep into the ionosphere without fatal attenuation where collision frequency is smaller than the lower region. Figure 1–6 suggest that the frequency range in which ions play an important role extends to lower frequency as the collision frequency becomes smaller. Therefore, ELF radio waves propagating perpendicularly to the geomagnetic meridian might be affected by ions but assumptions used here (sharp boundary and homogenous ionosphere) limit further discussion. In order to make consideration for waves with greater wave-length, it is necessary to solve a wave equation in an inhomogeneous ionosphere.

In conclusion it is said that ions should be taken into consideration on the nighttime reflection coefficients for grazing incidence of VLF radio waves.

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References

Barber, N. F. and D. D. Crombie, 1959; V. L. F. reflections from the ionosphere in the presence of a transverse magnetic field, J. Atmosph. Terr. Phys., **16**, 37–45.

Belrose, J. S. and M. J. Burke, 1964; Study of the lower ionosphere using partial reflection, J. Geophys. Res., **69**, 2799–2818.

Crombie, D. D., 1958; Differences between the east-west and west-east propagation of VLF signals over long distances, J. Atmosph. Terr. Phys., **12**, 110–117.

Round, H. J., T. L. Eckersley, K. Tremellen and F. C. Lunnon, 1925; Report on measurements made on signal strength at great distances during 1922 and 1923 by an expedition sent to Australia, J. Inst. Elec. Engrs, **63**, 933–1011.

Stix, T. H., 1962; *The theory of plasma waves*, McGraw-Hill, New York, 283 pp.