Contributions, Geophysical Institute, Kyoto University, No. 12, 1972, 7-14

ON THE FLR EFFECT ON KELVIN-HELMHOLTZ INSTABILITY OF THE MAGNETOPAUSE

By

Hiroshi NAGANO and Takayoshi INOUE

(Received September 20, 1972)

Abstract

The effect of finite ion Larmor radius on the Kelvin-Helmholtz instability of the magnetopause is investigated, making use of the equations introduced by Thompson, in the case of incompressible plasma. It is found that the effect of finite ion Larmor radius tends to stabilize perturbations with a shorter wavelength.

1. Introduction

Quasi-periodically oscillatory motions of the earth's magnetopause, which have periods ranging from several to ten and a few minutes, were observed by many satellites, IMP-2, Explorer 12, OGO-1, OGO-5 and so on (Konradi and Kaufmann [1965], Heppner et al. [1967], Anderson et al. [1968], Kaufmann and Konradi [1969], Aubry et al. [1971]). These oscillatory motions have been regarded as those due to hydromagnetic surface waves on the magnetopause, and Kelvin-Helmholtz instability has been considered one of the factors for a generating mechanism of such surface waves (Anderson et al. [1968], Aubry et al. [1971], Ershkovich and Nusinov [1972]). This instability was also introduced as the origin of the viscous interaction between the solar wind and the magnetosphere (Fejer [1964]), and the ultra low frequency micropulsation phenomena (Dungey and Southwood [1970]). The Kelvin-Helmholtz instability at the boundary layer as the solar wind plasma flows over the magnetopause has been investigated theoretically by a number of researchers (Fejer [1964], Sen [1963, 1964, 1965], Talwar [1964], Lerche [1966], Southwood [1968], McKenzie [1970], Ong and Roderick [1972]).

Lerche [1966] discussed the validity of the usual MHD approach to the Kelvin-Helmholtz instability of the magnetospheric boundary and showed that the waves with the shortest wavelength are the most unstable. Thus, he concluded that the usual MHD formulation is inapplicable to the stability problem of the magnetospheric boundary, and that the finite thickness of the boundary layer and the finite radius of particle's gyration should be taken into account.

Ong and Roderick [1972] took into account the finite thickness of the magnetospheric boundary layer and found that the boundary layer was stabilized by the thickness of the layer with respect to short wavelength perturbations.

Rosenbluth, Krall and Rostoker [1962] first studied the effect of the finite Larmor radius (FLR) on the gravitational instability of plasma. Nayyar and Trehan

[1971] considered such effects on Rayleigh-Taylor instability. Singh and Hans [1966], and Kalra [1967] discussed the stability of superposed fluids, taking the FLR effect into account. The former treated only the mode propagating perpendicular to a magnetic field, the latter both the perpendicular and parallel modes. However, Kalra did not give the quantitative results on the Kelvin-Helmholtz instability.

In this paper we discuss the effects of the finite ion Larmor radius on the Kelvin-Helmholtz instability of the magnetopause. The mode propagating parallel to the magnetic field is considered, and an incompressible plasma and a simple configuration of a model are assumed.

2. Dispersion relation

The FLR equations for an infinitely conducting plasma introduced by Thompson [1961, 1962] are used, assuming the isotropic pressure and incompressibility

$$\nabla\cdot\boldsymbol{v}=0, \tag{1}$$

$$\rho\left\{\frac{\partial\boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right\}=-\nabla p-\nabla\cdot\overleftrightarrow{\Pi}+\frac{1}{4\pi}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}, \tag{2}$$

$$\boldsymbol{E}+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}=0, \tag{3}$$

$$\nabla\times\boldsymbol{E}=-\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t}, \tag{4}$$

$$\nabla\cdot\boldsymbol{B}=0, \tag{5}$$

where ρ, $\boldsymbol{v}$, p, $\boldsymbol{B}$ and $\boldsymbol{E}$ are the mass density, the velocity, the pressure, the magnetic field and the electric field respectively, and c is the speed of light. The magnetic viscosity tensor $\overleftrightarrow{\Pi}$ arises from the FLR effects of ions, and if the magnetic field is assumed to be along the z-axis in a system of Cartesian co-ordinates (x, y, z), then each component is given as

$$\Pi_{xy}=\Pi_{yx}=\rho\nu\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right),$$

$$\Pi_{xz}=\Pi_{zx}=-2\rho\nu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right),$$

$$\Pi_{yz}=\Pi_{zy}=2\rho\nu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right),$$

$$\Pi_{xx}=-\Pi_{yy}=-\rho\nu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),$$

$$\Pi_{zz}=0.$$

Here $\boldsymbol{v}=(u, v, w)$ and $\nu=\frac{1}{4}R_{Li}^2\Omega_{Li}$, where R_{Li} and Ω_{Li} denote the ion Larmor radius and the ion Larmor frequency respectively.

We take the y direction to be vertical as shown in Fig. 1, and both the directions of flow and magnetic field, in the magnetosphere (Region I) and in the solar wind (Region II), to be parallel to the interface. The plasma in either region is assumed to be incompressible. In equilibrium (denoted by the subscript 0), the pressure balance is given as

$$p_{01}+\frac{B_{01}^2}{8\pi}=p_{02}+\frac{B_{02}^2}{8\pi}, \tag{6}$$

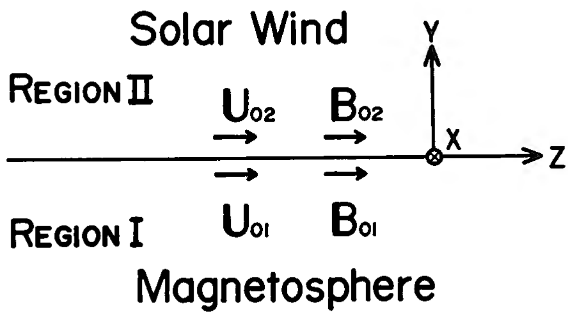

Fig. 1. The equilibrium configuration.

where the subscripts 1 and 2 represent the values in the region I and II respectively.

To investigate the stability of the equilibrium state, we now impose a small amplitude disturbance. Then the velocity, magnetic field, pressure and mass density on either side of the interface may be written as

$$\begin{aligned}
\boldsymbol{v}&=\boldsymbol{U}_0+(u, v, w),\\
\boldsymbol{B}&=\boldsymbol{B}_0+(b_x, b_y, b_z),\\
p&=p_0+\delta p,\\
\rho&=\rho_0,
\end{aligned}$$

where (u, v, w), (b_x, b_y, b_z) and δp denote the perturbations in velocity, magnetic field and pressure, respectively, and $\boldsymbol{U}_0$, $\boldsymbol{B}_0$, p_0 and ρ_0 are independent of x, y, z and t. We consider the case in which propagation is in the direction of magnetic field and look for a solution to the linearized equations, assuming all the perturbed quantities to vary as $f(y)\exp(ikz+\omega t)$.

According to Kalra's calculation [1967], the differential equation with respect to v is written as

$$[(\Omega_j^2+k^2V_{aj}^2)^2(D^2-k^2)-\nu_j^2k^2\Omega_j^2(D^2+2k^2)^2]v=0. \tag{7}$$

Here

$$\Omega_j = \omega + ikU_{oj},$$

$$V_{aj}^2 = \frac{B_{oj}^2}{4\pi\rho_{oj}},$$

$$D = \frac{d}{dy},$$

where $j=1$ and 2.

The solution to eq. (7) can be written as

$$v_1(y) = A_1 e^{l_1 y} + B_1 e^{m_1 y} \qquad y<0 \quad \text{(region I)}, \tag{8}$$

$$v_2(y) = A_2 e^{-l_2 y} + B_2 e^{-m_2 y} \qquad y>0 \quad \text{(region II)}, \tag{9}$$

where A_1, A_2, B_1 and B_2 are constants, and l_1, l_2, m_1 and m_2 are to be found from

$$l_j = k \tag{10}$$

$$m_j = \frac{(\Omega_j^2 + k^2 V_{aj}^2)}{\Omega_j \nu_j k}\left[1 - \frac{5\nu_j^2 k^4 \Omega_j^2}{2(\Omega_j^2 + k^2 V_{aj}^2)^2}\right]. \tag{11}$$

When we obtain eqs. (10) and (11), it is assumed that the FLR term is smaller than the inertia term, that is, $\nu k^2/\omega < 1$, and therefore only the first terms in $\nu k^2/\omega$ are retained. This holds also in the following derivations.

The boundary conditions at $y=0$ are (i) continuity of the normal component of velocity, that is, $[v/\Omega]=0$, (ii) continuity of the normal stress, $[\delta p + \Pi_{yy} + b_z B_0/4\pi]=0$, and (iii) continuity of the tangential stresses, $[\Pi_{zy}]=0$ and $[\Pi_{xy}]=0$ (the brackets [ ] denote the jump at the interface).

By using these boundary conditions, we obtain the following equations with respect to the constants A_1, A_2, B_1 and B_2

$$\Omega_2 A_1 + \Omega_2 B_1 = \Omega_1 A_2 + \Omega_1 B_2, \tag{12}$$

$$\frac{\rho_{01}(\Omega_1^2 + k^2 V_{a1}^2)}{\Omega_1 k} A_1 + 3\rho_{01}\nu_1 k B_1 = -\frac{\rho_{02}(\Omega_2^2 + k^2 V_{a2}^2)}{\Omega_2 k} A_2 - 3\rho_{02}\nu_2 k B_2, \tag{13}$$

$$\frac{\rho_{01}(\Omega_1^2 + k^2 V_{a1}^2)}{\Omega_1 k^2} B_1 = \frac{\rho_{02}(\Omega_2^2 + k^2 V_{a2}^2)}{\Omega_2 k^2} B_2, \tag{14}$$

$$\rho_{01}\nu_1 k A_1 + \frac{\rho_{01}(\Omega_1^2 + k^2 V_{a1}^2)}{\Omega_1 k} B_1 = -\rho_{02}\nu_2 k A_2 - \frac{\rho_{02}(\Omega_2^2 + k^2 V_{a2}^2)}{\Omega_2 k} B_2. \tag{15}$$

Eqs. (12), (13), (14) and (15) can be solved simultaneously for ω as a function of k by setting the determinant of the coefficients multiplying A_1, A_2, B_1 and B_2 equal to zero.

$$\begin{aligned}&2(\Omega_1^2+k^2V_{a1}^2)(\Omega_2^2+k^2V_{a2}^2)[\rho_{01}(\Omega_1^2+k^2V_{a1}^2)+\rho_{02}(\Omega_2^2+k^2V_{a2}^2)]\\&+k^2[\nu_2\Omega_2(\Omega_1^2+k^2V_{a1}^2)-\nu_1\Omega_1(\Omega_2^2+k^2V_{a2}^2)][\rho_{02}(\Omega_2^2+k^2V_{a2}^2)\\&-\rho_{01}(\Omega_1^2+k^2V_{a1}^2)]=0. \qquad (16)\end{aligned}$$

The dispersion relation which we now obtained is different from that derived by Kalra [1967].

When $\nu_1=\nu_2=0$ in eq. (16),

$$\rho_{01}(\Omega_1^2+k^2V_{a1}^2)+\rho_{02}(\Omega_2^2+k^2V_{a2}^2)=0. \qquad (17)$$

This equation is in accordance with the incompressible case of the dispersion relation that Lerche [1966], Sen [1964] and others obtained from the usual MHD equations.

3. Discussion of results

For simplification we take a particular case as follows: $B_{01}=B_{02}=B_0$ and $T_{01}=T_{02}=T_0$, where T denotes the temperature. Under the above conditions, the equilibrium state (eq. (6)) gives $\rho_{01}=\rho_{02}=\rho_0$, $V_{a1}=V_{a2}=V_a=B_0/\sqrt{4\pi\rho_0}$ and $\nu_1=\nu_2=\nu$. To simplify the dispersion relation still further, we set $U_{02}=-U_{01}=U_0$. Eq. (16) can then be written in the form

$$\begin{aligned}&\{[\omega^2-k^2(U_0^2-V_a^2)]^2+4k^2\omega^2U_0^2\}[\omega^2-k^2(U_0^2-V_a^2)]\\&\qquad+2\nu\omega k^4U_0^2[\omega^2+k^2(U_0^2-V_a^2)]=0. \qquad (18)\end{aligned}$$

When we put $\nu=0$, that is, zero Larmor radius, in eq. (18), we obtain

$$\omega^2-k^2(U_0^2-V_a^2)=0. \qquad (19)$$

This is the dispersion relation for the incompressible plasma which is derived in the usual MHD approximation. In this case, we can get the growth rate γ(for $U_0>V_a$)

$$\gamma=k(U_0^2-V_a^2)^{1/2}.$$

As shown in the above relation, the largest growth rate is attained when k tends to infinite value. This result is consistent with the discussion by Lerche.

In order to carry out the concrete computation for the values of growth rate γ as the function of wave number k (Here, only the real part γ of ω corresponding to zero imaginary part is selected. Certainly this is a significant solution to eq. (18).), we choose the reasonable quantities fitting the magnetopause as follows: $B_0=20\gamma$, N_0 (number density)$=50\ \mathrm{cm}^{-3}$ and $T_0=5\times10^{5}\,^\circ\mathrm{K}$. As the value of U_0, the three cases, that is, 75 km/sec, 100 km/sec and 200 km/sec are chosen.

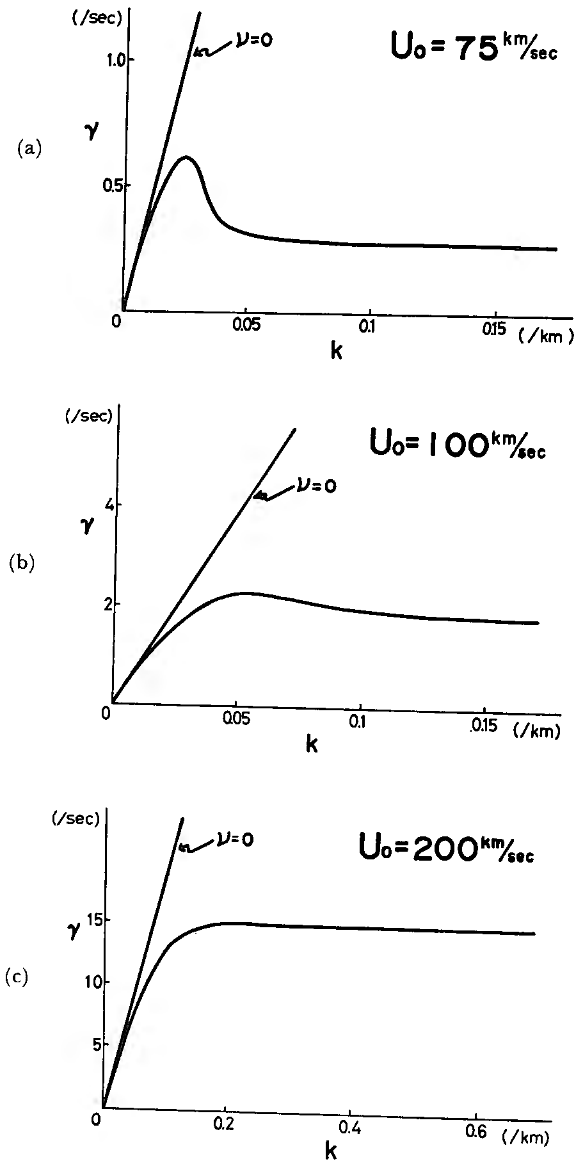

Fig. 2. The growth rate γ as a function of wave number k, taking $B_0=20\gamma$, $N_0=50\mathrm{cm}^{-3}$ and $T_0=5\times10^{5}\,^\circ K$. (a), (b) and (c) refer to $U_0=75$, 100 and $200\ km/sec$ respectively. The straight line corresponds to the case of zero Larmor radius.

Figs. 2(a), 2(b) and 2(c) refer to the computed results for $U_0=75$, 100 and 200 km/sec respectively. For comparison, we also note the well-known result for the case $\nu=0$. As shown in Fig. 2, the effect of FLR tends to stabilize the perturbations with larger wave number (i.e. shorter wavelength). As Lerche pointed out, it can be said that the effect of finite Larmor radius of particles should be taken into account. Ong and Roderick [1972] obtained the critical wave number k_c such that the growth rate is zero for $k>k_c$ in the case of the shear layer of finite thickness, but in our case no such critical wave number exists. For $U_0=75$ km/sec, the peak corresponding to the maximum growth rate is remarkable (Fig. 2(a)). On the contrary, as U_0 increases, the peak becomes more inconspicuous though the growth rate has larger values as a whole. This may be due to the fact that the assumption of incompressibility in the present analysis becomes questionable in the larger flow speed.

The assumption, $\nu k^2/\omega<1$, is not valid when $k>2\pi/R_{Li}$. Therefore, it is desirable to analyze the stability problem of the magnetospheric boundary, relaxing the above approximation and considering the plasma to be compressible.

Acknowledgement

The authors would like to thank Professor H. Maeda and Dr. T. Ogawa for critical reading of the manuscript and helpful discussions.

References

Anderson, K. A., J. H. Binsack and D. H. Fairfield, 1968; Hydromagnetic disturbances of 3- to 15-minute period on the magnetopause and their relation to bow shock spikes, J. Geophys. Res., **73,** 2371–2386.

Aubry, M. P., M. G. Kivelson and C. T. Russell, 1971; Motion and structure of the magnetopause, J. Geophys. Res., **76,** 1673–1696.

Dungey, J. W. and D. J. Southwood, 1970; Ultra low frequency waves in the magnetosphere, Space Sci. Rev., **10,** 672–688.

Ershkovich, A. I. and A. A. Nusinov, 1972; Geomagnetic tail oscillations, Cosmic Electrodynamics, **2,** 471–480.

Fejer, J. A., 1964; Hydromagnetic stability at a fluid velocity discontinuity between compressible fluids, Phys. Fluids, **7,** 499–503.

Heppner, J. P., M. Sugiura, T. L. Skillman, B. G. Ledley and M. Campbell, 1967; OGO-A magnetic field observations, J. Geophys. Res., **72,** 5417–5471.

Kalra, G. L., 1967; Effect of finite ion Larmor radius on the stability of superposed fluids, Can. J. Phys., **15,** 1579–1585.

Kaufmann, R. L. and A. Konradi, 1969; Explorer 12 magnetopuase observations: large-scale nonuniform motion, J. Geophys. Res., **74,** 3609–3627.

Konradi, A. and R. L. Kaufmann, 1965; Envidence for rapid motion of the outer boudary of the magnetosphere, J. Geophys. Res., **70,** 1627–1637.

Lerche, I., 1966; Validity of the hydromagnetic approach in discussing instability of the magnetospheric boundary, J. Geophys. Res., **71,** 2365–2371.

McKenzie, J. F., 1970; Hydromagnetic wave interaction with the magnetopause and the bow shock, Planet. Space Sci., **18,** 1–23.

Nayyar, N. K. and S. K. Trehan, 1970; Effect of gyro-viscosity on Rayleigh-Taylor instability of a plasma, J. Plasma Phys., **4,** 563–571.

Ong, R. S. B. and N. Roderick, 1972; On the Kelvin-Helmholtz instability of the earth's magnetopause, Planet. Space Sci., **20,** 1–10.

Rosenbluth, M. N., N. A. Krall and N. Rostoker, 1962; Finite Larmor radius stabilization of "weakly" unstable confined plasmas, Nucl. Fusion, Suppl. Part **1,** 143–150.

Sen, A. K., 1963; Stability of hydromagnetic Kelvin-Helmholtz discontinuity, Phys. Fluids, **6,** 1154–1163.

Sen, A. K., 1964; Effect of compressibility of Kelvin-Helmholtz instability in a plasma, Phys. Fluids, **7,** 1293–1298.

Sen, A. K., 1965; Stability of the magnetospheric boundary, Planet. Space Sci., **13,** 131–141.

Singh, S. and H. Hans, 1966; Magnetic viscosity and the stability of superposed fluids in a magnetic field, Nucl. Fusion, **6,** 6–8.

Southwood, D. J., 1968; The hydromagnetic stability of the magnetospheric boundary, Planet. Space Sci., **16,** 587–605.

Talwar, S. P., 1964; Hydromagnetic stability of the magnetospheric boundary, J. Geophys. Res., **69,** 2707–2713.

Thompson, W. B., 1961; The dynamics of high temperature plasma, in *Reports on Progress in Physics,* The Physical Society, London, **24,** 363–424.

Thompson, W. B., 1962; *An Introduction to Plasma Physics,* Pergamon Press, 224 pp.