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REVIEW

ANALYSES OF MEASUREMENT TECHNIQUES OF ELECTRIC FIELDS AND CURRENTS IN THE ATMOSPHERE

By

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Abstract

Various sources of atmospheric electric fields and their characteristics are surveyed first. The measurement technique of electric fields with a DC type field mill is analyzed second, and then the antenna method for DC electric fields, currents and conductivity, and for AC electric fields is analyzed. Practical uses of these techniques are briefly described.

1. Introduction

There are two methods of measurement of atmospheric electric fields and currents described in this paper. In one of these we measure electric charges induced on the surface of a conductor exposed in the electric field, and this type of measurement apparatus is called a field mill. In the other method we measure potential differences between two conductors located separately in the atmosphere, and this is called an antenna method. Both types of measurement apparatuses were designed by a number of investigators in slightly different forms in the past and are described in the literature (see References). In this review we do not survey through these references but we analyze the principles of measurement techniques which we have used in our laboratory and which we think "best". "Best" means that a whole system of measurement including the sensor and the amplifier is simple, easy to handle, and stable for continuous measurements.

The analyzed principles of electric field measurements in this paper will be useful for the electric field measurement in the plasmas in the ionosphere, the magnetosphere and interplanetary space and also for electrostatic applications in engineering fields. The second method—the antenna method—can be applied to the measurements of the AC electric fields of electromagnetic waves in the frequency range of ELF and VLF.

2. Sources of electric fields and their characteristics

The atmosphere is fundamentally ionized by the galactic cosmic rays creating a conductivity profile exponentially increasing with height. Near the ground surface there are additional sources of ionization due to radioactive substances under the ground. In the region above approximately 50 km in the atmosphere there are ionizations due to solar Lyman α and hard X rays from the sun. In the stratosphere of the altitudes of 10–30 km where balloon measurements are made, there may be very simple production and loss processes of ions; the ionization source is simply the galactic cosmic rays, and the recombination is only process of ion loss. There is no effect of attachments with aerosols which play very important roles in the troposphere. However we must sometimes take into consideration the effects of carrying up through the tropopause from the troposphere of particulate material in the thunderstorm updraft, dust particles from volcanic eruption, or radioactive aerosols from nuclear explosions.

In the troposphere there are a number of electric generation mechanisms in which the thunderstorm cloud is considered the largest electric generator giving global geophysical effects. The thunderstorms occur mainly on the land areas centered at the equator and they have characteristic activity patterns in a day or in a season. The electric currents flowing out through the thunderstorm cloud are guided by the atmospheric conductivity profile and flow mainly upwards giving the upper conducting layer a high potential. The conductivity of this layer is high enough, so that the electric relaxation time is small enough to make the whole spherical layer the same potential. This potential is estimated to be about 300 kV. In other words the earth and the upper conducting layer make a kind of spherical shell condenser whose electrodes are given the potential of about 300 kV. The electricity stored in this condenser is discharged at the time constant of about 40 sec. The discharged current is measured as a vertical and downward current in the atmosphere, the magnitude being about 1×10^{-12} Amp/m^2. This current is considered constant with altitude in terms of continuity of electric current, and therefore the electric potential drops due to this current are different for each altitude and create an electric field profile exponentially decreasing with altitude.

In the ionosphere there may be a few current systems. In the middle and low latitudes, there is a S_q dynamo electric field in magnetically calm days, and besides there may be electric fields transferred through the magnetic fields from the magnetosphere on magnetically disturbed days. These ionospheric electric fields map into the stratosphere without severe attenuation. This can be easily expected by considering that the ionospheric height is about 100 km which is very small compared with the horizontal extent of the atmosphere.

Finally there exist the electric fields of both tropospheric and ionospheric origins in the atmosphere. The former gives the vertical electric field while the latter gives mainly the horizontal field. The variation pattern of the former is a type of UT diurnal variation, while that of the latter is a type of LT diurnal variation or irregular and time-limited variations due to magnetic disturbances. Therefore if three dimensional components of electric fields and currents can be measured in the stratosphere, each can be separated by different origins.

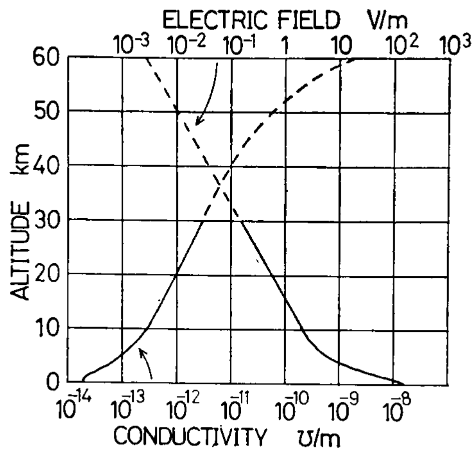

Fig. 1. Profiles of electric field and conductivity.

There are a number of artificial electric generating mechanisms near the ground surface. Space charges are produced from many kinds of burning processes. As the electric relaxation time of the air near the ground surface is about 10 minutes, these space charges are suspended in the atmosphere for a long time causing short period variations of electric field. The conductivity is decreased by many kinds of exaust gases or particulate material produced near the ground surface, deforming the electric field profile near the ground. The diurnal pattern of the electric field in the polluted area thus becomes very similar to that of the amount of pollution particles in the atmosphere.

When the thunderstorm cloud or rain cloud is actively working, the atmospheric electric field is greatly disturbed and shows characteristic forms of variations. The magnitude of such disturbances sometimes attains more than one hundred larger than the value in fair weather. Lightning causes many types of rapid field changes and also radiates the electromagnetic waves in wide frequency range.

Summarizing the characteristic features of atmospheric electric fields thus produced by a number of ways, (1) the electric field changes temporarily by many orders of magnitude larger than the normal value, and (2) the electric field decreases exponentially with altitude. The electric field and conductivity profiles are shown in Fig. 1. (3) The periods of variations range widely from DC up to some 100 MHz. The frequency spectrum of atmospheric electric fields and related electric component of electro-magnetic waves are shown in Fig. 2. (4) The equivalent inner resistance of the atmospheric electric source to be measured, which will be defined by Eq. (26), is extremely large and decreases exponentially with altitude. It is shown in Fig. 3.

In order to measure such electric fields with accuracy and stability, it is necessary to analyze the output impedance and the time response of the measuring apparatus as well as the dynamic frequency range of the phenomena to be measured.

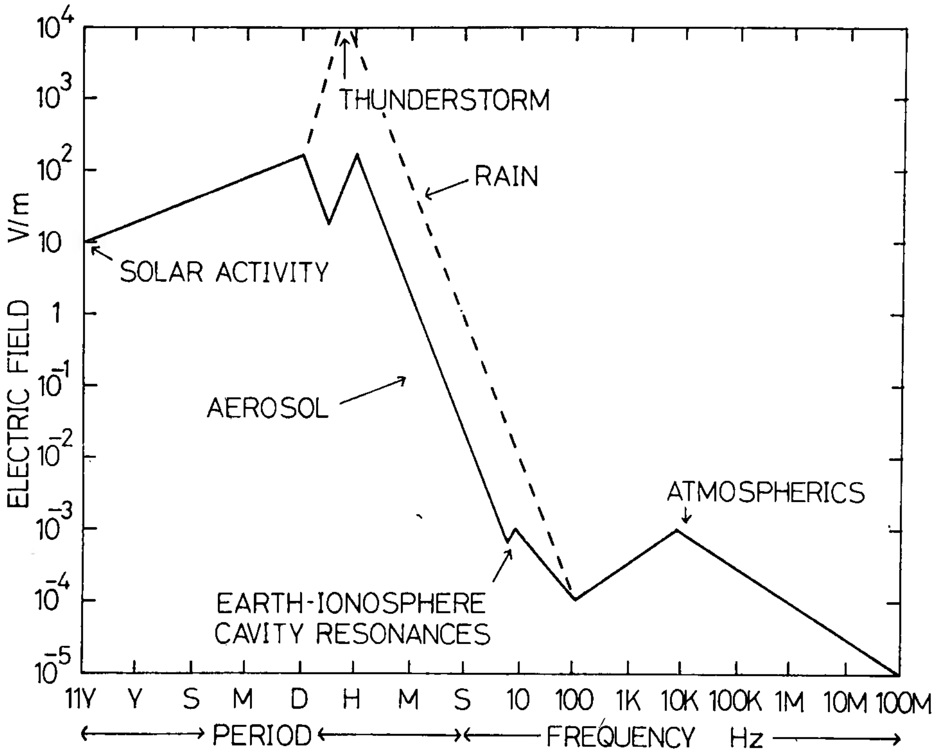

Fig. 2. Frequency spectrum of electric field.

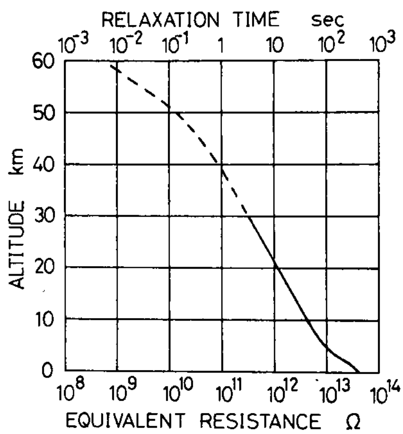

Fig. 3. Relaxation time and equivalent resistance for the antenna of 10 pF.

3. Field mill

There are two types of field mills, AC and DC types. In the field mill of the AC type, the output signal is AC and an AC amplifier is used. The sign of the electric field is discriminated by a phase detecting circuit. On the other hand in the field mill of the DC type, a DC output current is amplified with a high impedance DC amplifier. Both field mills of AC and DC types have favourable and unfavourable points, but the latter is much easier to handle than the former. In this paper the field mill of the DC type will be analyzed.

The field mill is sometimes called a mechanical collector or a rotating collector. The electric field measuring system including a field mill and an amplifier is called a field meter.

3. 1. The working principle of a DC type field mill

Two symmetric conducting sectors are insulated and rotate in a horizontal plane around an axis perpendicular to the sectors in the shielding cover which has two windows of the same sectorial shape. The rotating sectors are exposed to the external electric field successively after being shielded. The moving vane is grounded at a moment when it is exposed to the electric field, then it rotates by 90° and is connected

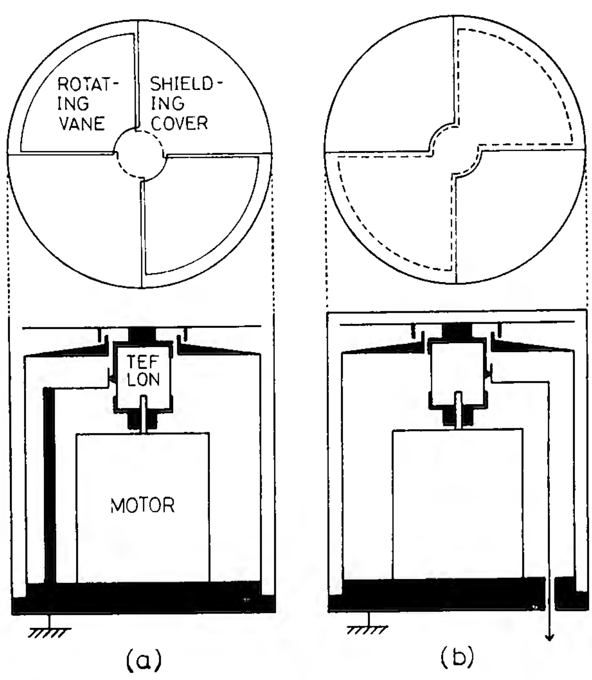

Fig. 4. Top and side views of the field mill.
(a) The rotating vane is grounded.
(b) The rotating vane is connected to the amplifier.

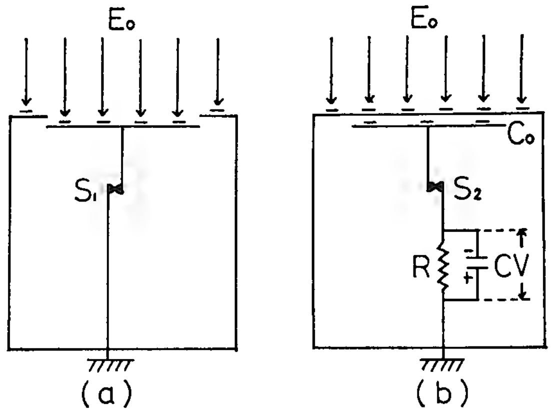

Fig. 5. Schematic diagram of the field mill. (a) and (b) correspond to those in Fig. 4.

to the amplifier when it is completely shielded. Figure 4 shows the top and side views of the field mill, and Fig. 5 is a schematic diagram which shows the working principle of the field mill.

Let the total area of the rotating sectors be S. When the rotating sectors are entirely exposed to the external electric field, the charge Q_0 induced on the sector surface is expressed by

$$Q_0=\varepsilon_0 E_0 S, \tag{1}$$

where ε_0 is the atmospheric permitivity which is practically the same as the permitivity of free space. The unit of MKSA system is used. The electric field E_0 on the moving sectors is generally different from the electric field E on the plane area without the field mill, that is

$$E=kE_0. \tag{2}$$

k is a constant which depends on the configuration of the place where the field mill is set and the geometrical shape of the field mill. The value of k is determined by a plane reduction (refer to 3. 5.).

After disconnecting from the ground, the moving sectors carry the charge Q_0 into the shielding cover. It is then connected to the input of the amplifier. The equivalent circuit is shown in Fig. 6, where V_s is an equivalent source potential, S_1 is an equivalent switch for grounding, S_2 is an equivalent switch for connecting the sectors to the amplifier. C_0 is the electrostatic capacitance of the rotating sectors, and R and C are the input resistance and capacitance of the amplifier including the coaxial cable guiding a signal from the field mill to the amplifier. S_1 is closed and S_2 is open when the rotating sectors are entirely exposed to the external electric field. C_0 is charged at this instance. S_1 is open and S_2 is closed when the rotating sectors are entirely shielded from the external field. The input pontential of the amplifier V_0 at this instance is

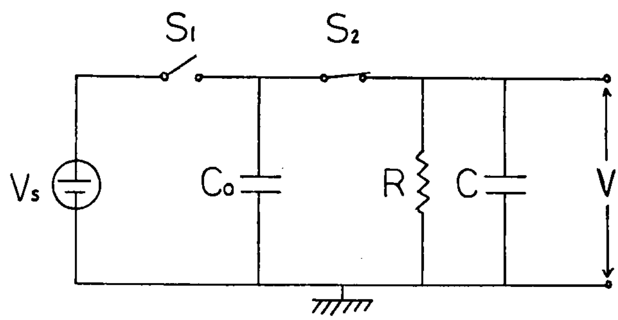

Fig. 6. Equivalent circuit of the field mill.

given by

$$V_0 = \frac{Q_0}{C_0 + C} = \frac{\varepsilon_0 E_0 S}{C_0 + C}. \tag{3}$$

At the next instance S_2 is open and the rotating sectors are disconnected from the amplifier, and then the input potential of the amplifier is decreased as

$$V_1 = \frac{\varepsilon_0 E_0 S}{C_0 + C} e^{-t_1/\tau}, \tag{4}$$

where

$$\tau = RC, \tag{5}$$

and t_1 is the time lapsed after the first charging C.

The rotating sectors are again exposed to the external field and gains the induced charge, and then go to be connected to the input circuit of the amplifier. The total electric charges on the capacitance C at this time are given by

$$\varepsilon_0 E_0 S + C V_1 = \varepsilon_0 E_0 S + \frac{C \varepsilon_0 E_0 S}{C_0 + C} e^{-T/\tau},$$

where T is the period half a rotation of the motor used. When S_2 is open, the charge on C is given by $C/(C_0+C)$ times this value. Thus the input potential of the amplifier after the second charging is given by

$$V_2 = \frac{\varepsilon_0 E_0 S}{C_0 + C} \left(1 + \frac{C}{C_0 + C} e^{-T/\tau}\right) e^{-t_2/\tau}. \tag{6}$$

After the nth charging the potential is given by

$$V_n = \frac{\varepsilon_0 E_0 S}{C_0 + C} \left(\sum_{n=1}^{n} r^{n-1}\right) e^{-t_n/\tau}, \tag{7}$$

where

$$r \equiv \frac{C}{C_o+C} e^{-T/\tau}. \tag{8}$$

As the relation $|r|<1$ is always held, the series in Eq. (7) is converged as

$$\sum_{n=1}^{n} r^{n-1} = \frac{1-r^n}{1-r}. \tag{9}$$

As the relation $e^{-T/\tau} \simeq 1$ is held when we choose the values of C and R so as to be

$$\tau \gg T, \tag{10}$$

then

$$V_\infty = \frac{\varepsilon_0 E_0 S}{C_o} e^{-t_\infty/\tau}, \tag{11}$$

when $n \to \infty$. Since the exponential term in Eq. (11) also becomes to unity during the period T, the input voltage to the amplifier is finally given by

$$V = \frac{\varepsilon_0 E_0 S}{C_o}. \tag{12}$$

From Eq. (12) E_0 can be obtained by measuring V. The sensitivity of the field mill is therefore proportional to the total area of the rotating sectors and is inversely proportional to the capacitance C_0.

3. 2. The equivalent impedance of the field mill

The equivalent impedance of the field mill Z is defined by the ratio of the open circuit voltage V to the short circuit current I. Therefore Z is expressed by

$$Z = \frac{V}{I} = \frac{Q_0/C_0}{Q_0/T} = \frac{T}{C_o}. \tag{13}$$

From Eq. (13) Z is given by the ratio of the period T to the capacitance C_0. This impedance is the most important factor when an amplifier is designed for the field mill.

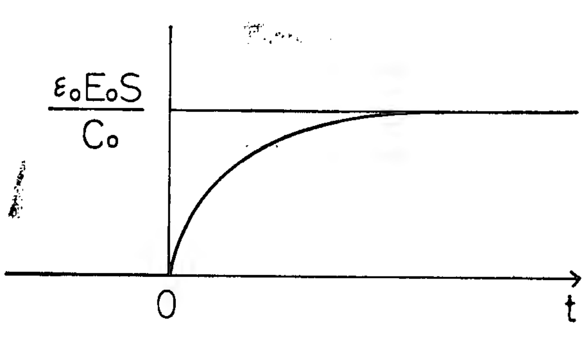

Fig. 7. Response curve of the field mill.

3. 3. The time constant of the field mill

When the external field is suddenly applied from 0 to E_o by the step function, the output of the field mill follows exponentially as shown in Fig. 7. Taking the condition (10) into consideration and from Eqs. (7), (8), (9) and (11), the time constant τ_{FM} of this field mill is calculated in the following.

$$\frac{\varepsilon_0 E_o S}{C_o}(1-e^{-t/\tau_{FM}})=\frac{\varepsilon_0 E_o S}{C_o+C}\cdot\frac{1-r^n}{1-r}. \tag{14}$$

As $r\simeq C/(C_o+C)$, Eq. (14) becomes

$$e^{-t/\tau_{FM}}=r^n. \tag{15}$$

The charging number n_0 for $t=\tau_{FM}$ is given by

$$n_0=-\frac{1}{\ln r}=\frac{1}{\ln\left(\frac{C_o+C}{C}\right)}. \tag{16}$$

The time constant of the field mill is then given by

$$n_0 T=\frac{T}{\ln\left(\frac{C_o+C}{C}\right)}. \tag{17}$$

3. 4. An example of the actual field mill

A schematic model of an actual field mill is already given in Fig. 4. Since this field mill is an all weather type, rain water does not enter the field mill. The rotating sectors are insulated with teflon and are connected to the motor axis by a piece of the teflon cylinder. Since the teflon is located on the motor, it is always warmed and insulation is kept high. The field mill has run continuously for several years with the use of a synchronous motor of 1800 rpm. The following is a list of the equivalent circuit constants of the field mill.

$C_o=30$ pF

$T=1/60$ sec$=0.0167$ sec

$S=5.6\times10^{-3}$ m^2

$C=670$ pF (when coaxial cable of 10 m length is used)

$R=5\times10^9\ \Omega$

$\tau=RC=3.35$ sec

$V=\frac{\varepsilon_0 E_o S}{C_o}=0.165$ V (when $E_o=100$ V/m)

$$I=\frac{\varepsilon_0 E_0 S}{T}=2.97\times 10^{-10}\text{ Amp}$$

$$Z=\frac{T}{C_o}=5.6\times 10^{8}\ \Omega$$

$$\tau_{FM}=\frac{T}{\ln\left(\frac{C_o+C}{C}\right)}=0.381\text{ sec}$$

It is clear from the above values that the field mill is a minor current source with a high impedance, so that the amplifier must have a character of an impedance changer.

3. 5. Plane reduction and calibration of the field mill

In order to get E_0 from measurement of V using Eq. (12), it is necessary to know the effective area of the rotating sectors S and their total capacitance C_0. In order to get the true field E from E_0 in Eq. (2), it is necessary to know the coefficient of plane reduction k. The absolute measurement of the electric field for the plane reduction must be done in the center of the plane area of over about 100 m^2 which is not too distant from the field mill (within about 100 m).

Two horizontal parallel conducting wires of length about 5 m are set at the heights of 1 and 2 m. Small electric collectors using a radioactive substance are connected to the middle of the wires. The electric potential difference between the two wires is measured with an electrostatic voltmeter. Reading of the voltmeter indication must be done every one to two minutes during one to two hours. The electric field on the plane surface thus obtained is compared with the recording of the field mill. From this comparison the coefficient of plane reduction will be given. Although k is constant unless the setting condition of the field mill changes, it is desirable to make a plane reduction approximately once a year. An actual example is given in Ogawa and Tanaka [1970].

When the atmospheric electric field is observed continuously for a long time, it is necessary to make calibration in order to check the sensitivity of whole system of the field mill and the amplifier. To do this a metal plate previously prepared is attached at the fixed distance parallel to the rotating sectors of the field mill. A fixed potential is applied to the plate making an artificial electric field. A deflection in the recording is checked to see whether it has been kept constant or not since the last calibration. Applying several different voltages, linearity of the field mill can be examined. It is desirable to make calibration at least once a week.

3. 6. Errors of the field mill

An error is caused by the contact potential between the different kinds of metals like aluminum or brass which are used for assembling the field mill. When the contact plane is clear, the error due to this effect is negligible. During the long term measurement when the field mill is exposed to rain and pollution, the contact plane rusts.

The contact potential in these circumstances will be very large and the resulting error will become up to the equivalent field of 10 V/m.

When it rains, a rain charge will come into the measurement through the rotating sectors. There is a relation between the rain current and the electric field. Applying Ogawa [1961]'s measurement, the observed rain current is 4×10^{-3} esu/cm^2 sec$=1.3\times10^{-8}$ Amp/m^2. The effective rain current into the field mill is 7.3×10^{-11} Amp. The electric field at that time is 1,000 V/m which makes the equivalent current of $\varepsilon_0 E_0 S/T=3.0\times10^{-9}$ Amp. The error due to the rain current amounts to about 4%. The sign of the rain charge is opposite to the electric field but the rain current has the effect of intensifying the electric field in the amplifier.

When it rains, raindrops splash on the rotating sector surface and a charge separation occurs. In this case the positive current will enter the amplifier, resulting in an effective reduction of the electric field.

3. 7. Examples of the amplifier for the field mill

As described in 3. 4. the field mill is a minor current source with high impedance, so the amplifier should be of high input impedance, or an impedance changer. An

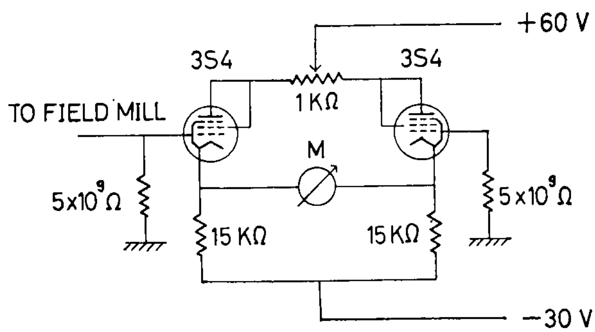

Fig. 8. High impedance DC amplifier for the field mill.

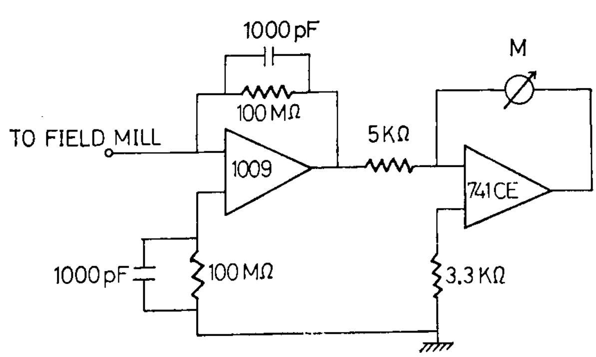

Fig. 9. Amplifier for the field mill using operational amplifiers.

example of a simple and stable amplifier for the field mill is given in Fig. 8. Two miniature vacuum tubes 3S4's are used as triodes, and the anode voltage is adjusted to about 1.0 V and the bias voltage to about 6 V. The recorder in Fig. 8 is a recording ammeter of ± 1 mA full scale with the inner resistance of 7 kΩ. This amplifier is very durable and is good for several years of continuous recording.

It became difficult to get 3S4. Instead the operational amplifier is useful for the field mill amplifier. Fig. 9 is an example of this type. In this circuit $\tau = RC = 0.1$ sec satisfies the condition (10). For the use of the operational amplifier refer to the Handbook of Operational Amplifier Application (Burr—Brown).

4. The antenna method

4. 1. Analysis of a parallel plate antenna

In the antenna method we measure a potential difference between two conductors in the electric field. When it is applied near the ground surface, one side of the conductors may be the ground. For the convenience of analysis, let a pair of parallel plates be insulated, and placed separately in the direction of electric field in the air of conductivity σ and of electric field E_0. In the following analysis the plates are assumed to be so large that the edge effect is neglected. The potential applied between the plates is

$$V_0 = E_0 h, \tag{18}$$

where h is called an effective height, and in this case it equals to the true separation d, i.e.,

$$h = d. \tag{19}$$

Between the two plates is connected a parallel RC circuit which is an input of the amplifier. The charges induced on the antenna capacitance C_0 and the amplifier input capacitance C after enough time elapsed since the connection, are shown in Fig. 10.

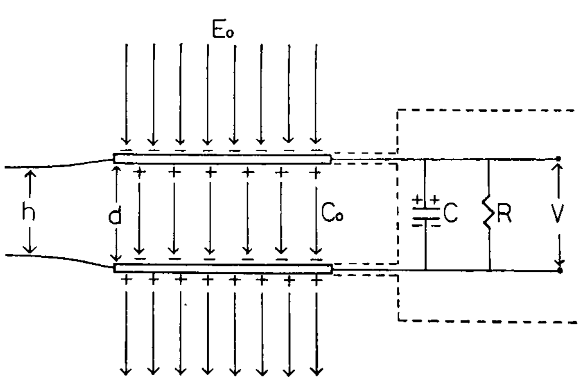

Fig. 10. Schematic diagram of the parallel plate antenna.

The relation of the antenna effective height h to the true separation d is given by

$$h<d. \tag{20}$$

In order to minimize the difference between h and d, and the electric field deformation due to adding of C, it is intuitively recognized in Fig. 10 that C is required to be much smaller than the antenna capacitance C_o. Then

$$C_o \gg C. \tag{21}$$

This condition will finally be derived from the following analysis. Therefore the antenna effective height is assumed to be equal to the antenna true separation. Let the charge on C be $Q_{t=0^-}$, then the charge on C_0, $Q'_{t=0^-}$, is given by

$$Q'_{t=0^-}=\frac{C_0}{C}Q_{t=0^-}. \tag{22}$$

Now let the external electric field change suddenly from E_0 to E by step function. The charge on C at this instance ($t=0^+$) $Q_{t=0^+}$ is given by

$$Q_{t=0^+}=Q_{t=0^-}+\frac{C}{C_0+C}\varepsilon_0(E-E_0)S, \tag{23}$$

where S is the area of the plates. The charge changes with time depending on (1) the conduction current σE flowing into the plates from the external atmosphere, (2) the conduction current between the antenna plates $\sigma q/\varepsilon_0$ (q is the charge density on the inner surfaces of the antenna), and (3) the current through the resistance, R, V/R, where V is the amplifier input voltage. In the above consideration an effect of displacement current is neglected. Thus the time change of the charge on C is expressed by the differential equation

$$\frac{dQ}{dt}=\frac{C}{C_0+C}\left(\sigma ES-\frac{\sigma C_0 Q}{\varepsilon_0 C}-\frac{Q}{CR}\right). \tag{24}$$

The current flowing into the plates I is given by

$$I=\sigma\iint_s E_n dS=\frac{\sigma}{\varepsilon_0}C_0 V, \tag{25}$$

and the equivalent antenna resistance V/I is given by

$$r\equiv\frac{V}{I}=\frac{\varepsilon_0}{C_0\sigma}. \tag{26}$$

Using Eq. (26), the capacitance of the parallel plate antenna

$$C_0=\frac{\varepsilon_0 S}{d}, \tag{27}$$

and the relation $Q=CV$, Eq. (24) can be transformed to the relation of electric potential. The first term of the right of Eq. (24) can be rewritten $\sigma ES=(C_0\sigma/\varepsilon_0)(\varepsilon_0 S/d)\times(Ed/C_0)=Ed/r$. The second term is $-\sigma C_0 Q/\varepsilon_0 C=-V/r$. The third term is $-Q/CR=-V/R$. Then Eq. (24) becomes

$$\frac{dV}{dt}+\frac{1}{C_0+C}\,\frac{(r+R)}{rR}V-\frac{1}{(C_0+C)}\,\frac{Ed}{r}=0. \tag{28}$$

The general solution to Eq. (28) is

$$V=e^{-\int\{1/(C_0+C)\}\{(r+R)/rR\}dt}\left\{\int e^{\int\{(r+R)/(C_0+C)\}dt}\frac{1}{(C_0+C)}\,\frac{Ed}{r}dt+\text{Const.}\right\}. \tag{29}$$

The integrations on the right of Eq. (29) are carried on, then

$$V=\frac{R}{r+R}Ed+\text{Const.}\,e^{-\{(r+R)/rR(C_0+C)\}t}. \tag{30}$$

From the initial condition Const. in Eq. (30) is at $t=0^+$

$$\text{Const.}=V_{t=0^+}-\frac{R}{r+R}Ed=\frac{Q_{t=0^+}}{C}-\frac{R}{r+R}Ed. \tag{31}$$

Substituting Eq. (23) into Eq. (31)

$$\text{Const.}=\frac{Q_{t=0^-}}{C}+\frac{\varepsilon_0 S}{C_0+C}(E-E_0)-\frac{R}{r+R}Ed. \tag{32}$$

The first term on the right of Eq. (32) is obtained from Eq. (30) when $t\to\infty$, and then Eq. (32) becomes

$$\text{Const.}=\frac{R}{r+R}E_0 d+\frac{C_0 d}{C_0+C}(E-E_0)-\frac{R}{r+R}Ed. \tag{33}$$

Equation (33) is rewritten

$$\text{Const.}=\left(\frac{C_0}{C_0+C}-\frac{R}{r+R}\right)(E-E_0)d. \tag{34}$$

Substituting Eq. (34) into Eq. (30),

$$V=\frac{R}{r+R}Ed+\left(\frac{C_0}{C_0+C}-\frac{R}{r+R}\right)(E-E_0)de^{-t/\tau}, \tag{35}$$

where

$$\tau=\frac{rR(C_0+C)}{r+R}. \tag{36}$$

The first term on the right of Eq. (35) gives the sensitivity, and the second term is a transient term giving the time response of this antenna system. The time constant of the antenna system is given by Eq. (36).

For a practical use of this antenna method the second term on the right of Eq. (35) is designed to be neglected, so that

$$V(t)=\frac{R}{r+R}E(t)d. \tag{37}$$

Equation (37) indicates that the time varying electric field can be measured without any time delay or time advance. In this case

$$\frac{C_0}{C_0+C}-\frac{R}{r+R}=0. \tag{38}$$

From Eq. (38)

$$C_0r=CR. \tag{39}$$

This is called by Kasemir and Ruhnke (1958) the condition of phase matching. In Eq. (37) if

$$r\ll R, \tag{40}$$

then

$$E=\frac{V}{d}. \tag{41}$$

The electric field is obtained simply and directly from the measurement of V. Equation (38) can be combined with the Inequality (40) to yield

$$C_0\gg C. \tag{21}$$

This was obtained at the beginning of this analysis. From the conditions (21) and (40) it becomes apparent that the antenna of relatively large capacitance with the small capacitance and large resistance at the amplifier input must be used for the measurement of the electric field. Making the antenna capacitance larger is an opposite condition for making the antenna separation d larger. If we let the second term on the right of Eq. (35) be close to zero, then there is no need to consider the antenna time constant.

Although Eq. (35) is derived from the analysis of a pair of parallel plates, the antenna capacitance C_0 is the only coefficient relating to the antenna form. Therefore the result of this analysis can be applied to any form of antenna. Even if the antenna form is complex and the antenna capacitance cannot be estimated, the antenna effective height can be close to the antenna true separation when only conditions (21) and (40) are satisfied, and thus the electric field can be measured from Eq. (41).

4.2. The equivalent circuit of the antenna

Let a charged spherical conductor be placed in the atmosphere of conductivity σ, then the initial charge Q_0 is dissipated by conduction current in the direction of

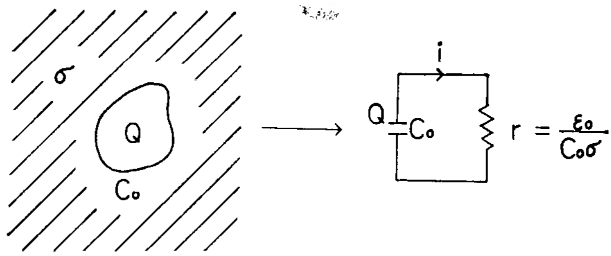

Fig. 11. Equivalent circuit of a charged conductor.

electric field on the surface of the conductor. The charge on the conductor at any time is given by

$$Q=Q_0e^{-t/(\varepsilon_0/\sigma)}. \tag{42}$$

On the other hand when the initial charge Q_0 on a capacitance C_0 is discharged through a resistance r, the charge on the capacitance C_0 at any time is given by

$$Q=Q_0e^{-t/C_0r}. \tag{43}$$

Equations (42) and (43) have the same form. From this comparison a charged spherical conductor may be equivalent to the circuit in Fig. 11. Let Eq. (42) be equal to Eq. (43),

$$\frac{\varepsilon_0}{\sigma}=C_0r. \tag{44}$$

The lefthand term is called atmospheric relaxation time, and the righthand term is the time constant of the equivalent circuit. In Eq. (43) there is no need to be spherical for the conductor, any form of conductor in the atmosphere can be transferred to the equivalent circuit in Fig. 11.

It can be proved that r on the right of Eq. (44) is obtained by integrating the resistance over the conducting sphere from its radius a to infinity. Let a distance from the center of the sphere be ρ, then

$$r=\int_a^\infty \frac{d\rho}{4\pi\rho^2\sigma}=\frac{1}{4\pi a\sigma}. \tag{45}$$

Considering that the capacitance of the spherical conductor is given by

$$C_0=4\pi\varepsilon_0 a, \tag{46}$$

Eq. (44) is derived from Eq. (45).

4.3. Analysis of a transient phenomenon of the antenna equivalent circuit (Refer to textbooks in electric circuits)

When two conductors of electrostatic capacitance $2C_0$ are placed separately by a

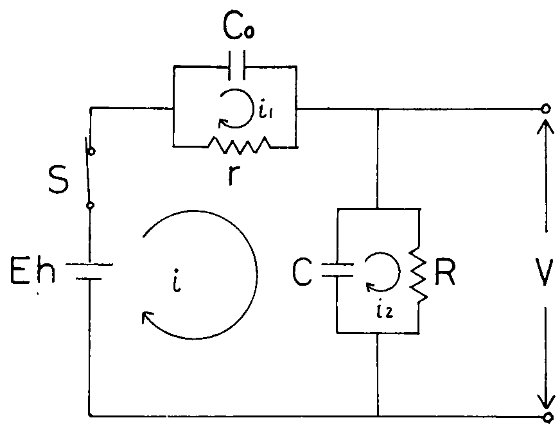

Fig. 12. Antenna equivalent circuit for analysis of a transient phenomenon.

distance d in the direction of electric field E and both conductors are connected through a parallel CR circuit. The equivalent circuit is given in Fig. 12, where Eh is the equivalent electric potential source to be measured. In order to analyze a transient phenomenon in this circuit, a switch S is inserted in the circuit. When S is open, no charge is accumulated on C_0 and C. Let us estimate a potential drop V across the resistance R after S is closed. Let charge on C_0 and C be q_1 and q_2 and currents in each element circuit be i, i_1 and i_2 which flow in the direction of the arrows respectively, then apply the Kirchhoff's law,

$$r(i-i_1)+R(i-i_2)-Eh=0, \tag{47}$$

$$r(i_1-i)+\frac{q_1}{C_0}=0, \tag{48}$$

$$R(i_2-i)+\frac{q_2}{C}=0, \tag{49}$$

$$\left.\begin{aligned}\frac{dq_1}{dt}&=i_1,\\ \frac{dq_2}{dt}&=i_2.\end{aligned}\right\} \tag{50}$$

Substituting Eq. (50) into Eqs. (47), (48), and (49),

$$r\left(i-\frac{dq_1}{dt}\right)+R\left(i-\frac{dq_2}{dt}\right)-Eh=0, \tag{51}$$

$$r\left(\frac{dq_1}{dt}-i\right)+\frac{q_1}{C_0}=0, \tag{52}$$

$$R\left(\frac{dq_2}{dt}-i\right)+\frac{q_2}{C_0}=0. \tag{53}$$

Transient terms are assumed to be the following when $Eh=0$.

$$\left.\begin{aligned} i&=Ie^{pt},\\ q_1&=Q_1e^{pt},\\ q_2&=Q_2e^{pt}.\end{aligned}\right\} \tag{54}$$

Substituting Eq. (54) into Eqs. (51), (52) and (53), then from Eqs. (52) and (53),

$$Q_1=\frac{rI}{rp+\dfrac{1}{C_0}} \tag{55}$$

and

$$Q_2=\frac{RI}{Rp+\dfrac{1}{C}} \tag{56}$$

are obtained. Substituting Eqs. (55) and (56) into Eq. (51) and solving in terms of p, then

$$p=-\frac{r+R}{rR(C_0+C)}\,. \tag{57}$$

Substitution of Eqs. (55) and (56) into Eq. (54) yields the transient terms.

$$\left.\begin{aligned} i_t&=Ie^{pt},\\ q_{1t}&=\frac{rI}{rp+\dfrac{1}{C_0}}e^{pt},\\ q_{2t}&=\frac{RI}{Rp+\dfrac{1}{C}}e^{pt}.\end{aligned}\right\} \tag{58}$$

Next the stational terms are expressed as

$$\left.\begin{aligned} i_s&=\frac{Eh}{r+R},\\ q_{1s}&=C_0ri_s=\frac{C_0r}{r+R}Eh,\\ q_{2s}&=CRi_s=\frac{CR}{r+R}Eh.\end{aligned}\right\} \tag{59}$$

From Eqs. (58) and (59), general solutions are given by

$$i=\frac{1}{r+R}Eh+Ie^{pt},$$

$$\left.\begin{aligned} q_1&=\frac{C_0 r}{r+R}Eh+\frac{r}{rp+\dfrac{1}{C_0}}Ie^{pt},\\ q_2&=\frac{CR}{r+R}Eh+\frac{R}{Rp+\dfrac{1}{C}}Ie^{pt}. \end{aligned}\right\} \tag{60}$$

The integration constant I in Eq. (60) can be determined from the initial conditions. When the switch S is closed, C_0 and C are charged at $t=0^+$. The total amount of charges on the righthand electrode of C_0 and the upper electrode of C is not changed before and after the switch S is closed. Since the charge quantity is zero at $t=0^-$, it is also zero at $t=0^+$. Therefore

$$q_{1t=0^+}=q_{2t=0^+}, \tag{61}$$

and

$$\frac{q_{1t=0^+}}{C_0}+\frac{q_{2t=0^+}}{C}=Eh. \tag{62}$$

Combination of Eqs. (61) and (62) yields

$$q_{1t=0^+}=q_{2t=0^+}=\frac{C_0C}{C_0+C}Eh. \tag{63}$$

Substituting Eq. (63) into the third equation in Eq. (60), then the integration constant I can be estimated at $t=0^+$.

$$I=\frac{CRp+1}{R}\left(\frac{C_0}{C_0+C}-\frac{R}{r+R}\right)Eh. \tag{64}$$

Substitution of Eq. (64) into the third equation in Eq. (60) yields

$$V=\frac{q_2}{C}=\frac{R}{r+R}Eh+\left(\frac{C_0}{C_0+C}-\frac{R}{r+R}\right)Ehe^{pt}. \tag{65}$$

Since from Eq. (57)

$$-\frac{1}{p}=\frac{rR(C_0+C)}{r+R}, \tag{66}$$

Eq. (65) is equal to Eq. (35), the result of analysis of a pair of parallel plates. When $C_0r=CR$, the second term on the right of Eq. (65) is equal to zero, and the circuit in Fig. 12 works as a voltage divider independent of frequency of the source.

4.4 Measurement of atmospheric current with the antenna

The inner resistance of a voltmeter is always larger than that of the electric source to be measured, and the inner resistance of a current meter is always smaller than that of the electric source to be measured. Therefore when the measurement of electric current in the atmosphere with the antenna is intended, a reverse inequality to (40) must be used, i.e.,

$$r \gg R. \tag{67}$$

In order to make the transient term in Eq. (35) small, the next relation is needed.

$$C_0 \ll C. \tag{68}$$

When Inequalities (67) and (68) hold, considering $r=\varepsilon_0/C_0\sigma$,

$$V \simeq \frac{R}{r} Ed = ARi, \tag{69}$$

where A is the antenna effective area given by

$$A = \frac{C_0 d}{\varepsilon_0}. \tag{70}$$

In Eq. (69) the input voltage V to the amplifier is proportional to the current density i.

When both conditions of electric field measurement (40) and of current measurement (67) are not satisfied, $r \simeq R$, then the stational term in Eqs. (35) or (65) becomes

$$V = \frac{R}{r+R} Eh = \frac{C_0 R\sigma h}{\varepsilon_0 + C_0 R\sigma} E = \frac{C_0 Rh}{\varepsilon_0 + C_0 R\sigma} i. \tag{71}$$

In this case the effects of electric field and conductivity or of electric current and conductivity will come into the measurement of $V(t)$; the measurement of pure electric field or current becomes impossible.

When the condition of phase matching (39) is not satisfied, the phenomenon of overshoot or overdamping appears in the measurement of time varying electric field or current. For such an effect of mismatching, refer to Ruhnke (1961).

4.5. Measurement of electric conductivity

If two kinds of parallel circuits of CR, one of which satisfies the conditions (21) and (41), and the other satisfies the conditions (67) and (68), are connected alternatively to the same antenna, the electric field and current can be measured alternatively. From these measurements the conductivity can be estimated assuming the quasistatic state.

$$\sigma = \frac{i}{E}. \tag{72}$$

There is another method of estimating σ from the antenna time constant. Before the measurement of electric field, the input is shorted, then the output of the amplifier rises exponentially. In this case considering $r=\varepsilon_0/C_0\sigma$, σ is obtained from the antenna constant τ given in Eq. (36);

$$\sigma=\frac{\varepsilon_0}{C_0}\left(\frac{C_0+C}{\tau}-\frac{1}{R}\right). \tag{73}$$

τ is directly read from the exponential rise curve of record and is substituted into Eq. (73). If the conditions of electric field measurement (21) and (40) are satisfied, then Eq. (73) can be simplified.

$$\sigma=\frac{\varepsilon_0}{\tau}. \tag{74}$$

4.6. An example of the antenna for stratospheric measurement

In Fig. 13 is given an antenna system for the measurements of three dimensional components of electric fields, currents, and conductivity in the stratosphere. Four s eel wires 10 m in length and 2 mm in diameter are used as the antennas for the horizontal components of field and current. A wire is hung from each end point of the four insulating rods stretched in the rectangular directions in a horizontal plane. The horizontal components are measured from the potential differences between pairs of counter antennas. This horizontal antenna system is hung by the same kind of wire which is used as an antenna for the vertical component. The vertical field is obtained

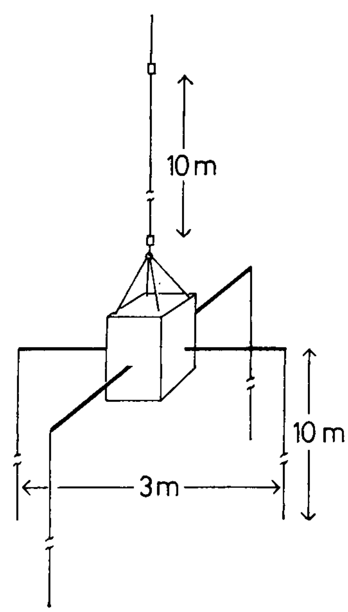

Fig. 13. Antenna system for measurement of three dimensional electric fields, currents, and conductivity in the stratosphere.

by measuring a potential difference between this wire and one of the wires for the horizontal components. If two sets of C and R are used alternatively by switching, then the electric fields and currents can be measured alternatively. Conductivity is also measured in the way described in 4.5. The results of the actual measurements with this antenna system of electric fields, currents, and conductivity in the stratosphere will be published in the near future.

4.7. Measurement of air-earth current

A conducting sphere is used as an antenna for the measurement of air-earth current when it is lifted to a certain height from the ground. The shape of the antenna is not necessarily spherical but any form of conductor can be used if its electrostatic capacitance can be estimated. The operational amplifier of ultra low bias current can be used as an inverting type amplifier for the measurement of air-earth current. The equivalent circuit is shown in Fig. 14.

Since the relaxation time of the air near the ground surface is about 500 sec, if the antenna of capacitance of 10 pF is used, the equivalent antenna resistance becomes $r \simeq 5 \times 10^{13} \Omega$. This impedance can be directly used as an imput impedance of the inverting type amplifier. If $R=10^{11}\Omega$ and $C=5{,}000$ pF are used in the feedback circuit, then the phase matching condition (39) is satisfied and the conditions of electric current measurement (67) and (68) are also satisfied When the antenna effective height (h) is 2 m, for example, then the output voltage of the operational amplifier is calculated by

$$V=\frac{R}{r}Eh. \tag{75}$$

A calculation by Eq. (75) with an assumed electric field of 100 V/m gives the potential of 0.4 Volt. Eq. (75) is transformed to Eq. (69) and V is proportional to the current

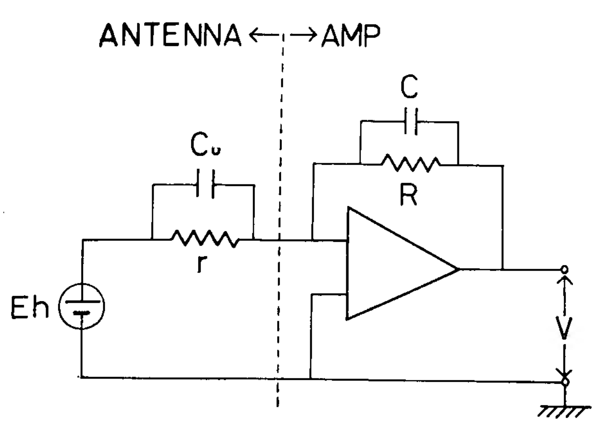

Fig. 14. Equivalent circuit for measurement of air-earth current.

density i. The effective area of this antenna is $A = C_0 h / \varepsilon_0 = 2.26\ \mathrm{m}^2$. The current density i must be $i = V/AR = 1.77 \times 10^{-12}\ \mathrm{Amp/m^2}$.

4.8. Measurements of AC (ELF and VLF) electric fields

The antenna method can be applied not only for DC electric field but also for AC electric field component of electromagnetic waves in ELF and VLF. The antenna of the capacitance of 10 pF is used in the following analysis. It is seen in Fig. 3 that the equivalent antenna resistance decreases with altitude from the ground surface. It has the value of about $10^{11}\Omega$ at 40 km height. On the other hand the impedance of the antenna capacitance C_0 is for the frequency of 10 Hz, $|1/j\omega C_0| = 1.6 \times 10^9 \Omega$. Therefore for AC electric fields in the altitude range below about 40 km,

$$r \gg \left| \frac{1}{j\omega C_0} \right|, \tag{76}$$

where $\omega = 2\pi f$ and f is the frequency. From the relation of (76) r can be neglected and the equivalent circuit is shown in Fig 15. In this case the potential difference across the resistance R is given by

$$V = \frac{Z}{Z_0 + Z} Eh, \tag{77}$$

where

$$Z_0 = -j \frac{1}{C_0 \omega}, \tag{78}$$

and

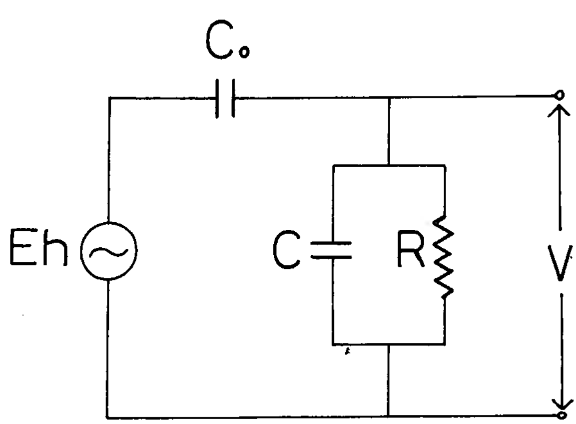

Fig. 15. Equival nt circuit for measurement of AC electric field.

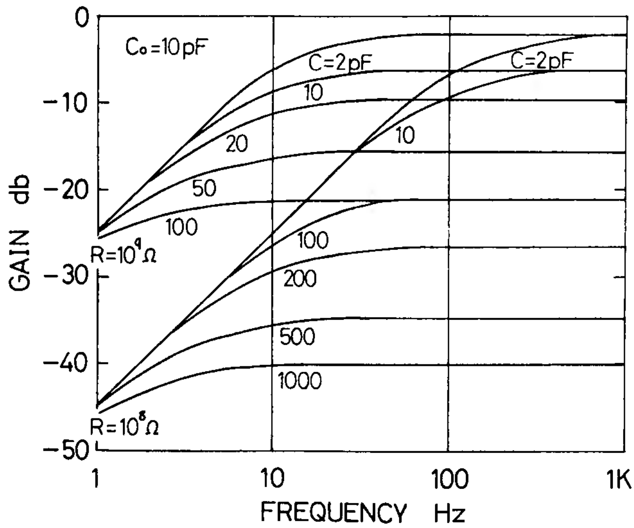

Fig. 16. Frequency response of the ball antenna ($C_0=10$ pF).

$$Z=\frac{R}{1+j\omega CR}. \tag{79}$$

Substitution of Eqs. (78) and (79) into Eq (77) yields

$$V=\frac{\omega C_0 R}{\sqrt{\omega^2R^2(C_0+C)^2+1}}Eh. \tag{80}$$

If the resistance larger than $10^{10}\Omega$ is used for R, then

$$\omega^2R^2(C_0+C)^2\gg 1. \tag{81}$$

If the antenna is designed so as to be

$$C_0\gg C, \tag{82}$$

then

$$V\simeq Eh. \tag{83}$$

In this case the antenna output is independent of the frequency of signals and no attenuation is achieved. Even when $R=10^9$ or $10^8\Omega$ is used, the precise electric field component can be calculated from Eq. (80). The results of calculations for several different input capacitances are shown in Fig. 16.

For the antenna which satisfies the condition (82) a spherical or any shape of cavity antenna is used, in which the preamplifier directly connected to the inside of the cavity is used. Thus the value of C can be minimized. This type of antenna is called the ball antenna. Sometimes it is called the spherical antenna, or the capacity antenna in literature. For practical use of the ball antenna refer to Ogawa et al. [1966a, b].

4.9. Plane reduction and calibration of the antenna

The antenna is sometimes used on the roof of a building. In this case the antenna effective height must be estimated including the effects of the building as well as of the supporting post of the antenna. The plane reduction must be done in the same way described in 3.5. When the antenna is used for the air-earth current or AC electric field component in ELF and VLF, the DC electric field must be measured with the same antenna. A small radioactive collector is attached to the neutral point of the antenna and the electric potential is measured by a static voltmeter, while the absolute measurement of the electric field must be made simultaneously in the plane area near the antenna.

The antenna calibration is made by applying an artificial electric field to the antenna. For the measurements of AC electric fields the signals of fixed frequency and of fixed amplitude are radiated from the radiator antenna which is placed a certain distance apart from the ball antenna.

Details are described in Ogawa and Tanaka [1971] and Clayton et al. [1973].

5. Concluding remarks

The atmospheric electric field is the most fundamental element in the atmospheric electricity. It will be said that the study of atmospheric electricity begins with a measurement of the electric field and ends in the same. Anyone who first knows the electric field of more than 100 V/m in the atmosphere is surprised to hear this and wonders why such a field exists. It seems to be easy to measure such a large voltage, but in actual attempts he will soon feel the difficulty not only in the measurement but also in the interpretation of the data obtained. This is because the atmospheric electric field contains too much data about the atmosphere; the eiectric field at any point is affected by the space from that point to infinity. This is a specifie character of the electric field. The atmospheric electric field contains data about thunderstorm clouds, rain clouds, snow clouds, fog, haze, mist and many other weather processes, aerosol generation and transportation, and many other pollution processes near the ground surface. On the other hand the atmospheric electric field is mixed with the ionospheric and magnetospheric electric fields and may be mixed even with the electric field of interplanetary space. The electric field in plasma-filled space is a measure of the dynamics of its constituents. It is one proof that the mechanisms of aurorae and magnetic substorms are studied very actively by measuring the electric fields in the stratosphere at high latitudes.

A clear demonstration of atmospheric disturbance due to radioactive fallout from nuclear explosions since 1951 is given by seeing the secular variation of atmospheric electric field. In Fig. 17 is shown the correlation of the electric field observed at Kakioka Magnetic Observatory to the sunspot numbers since 1930. The relatively regular positive correlation between both is largely disturbed after 1951, since artificial

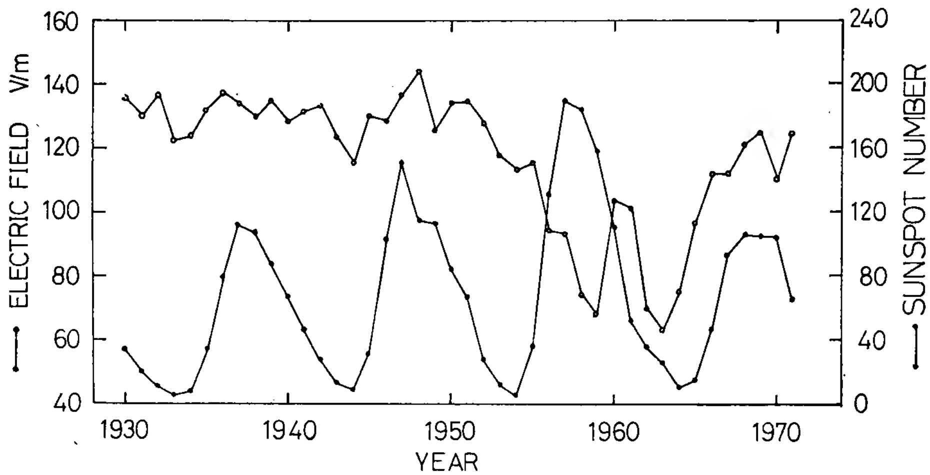

Fig. 17. Correlation between electric field and sunspot numbers. Electric field is disturbed by nuclear explosions in the atmosphere since 1951.

nuclear explosions were frequently carried out in the atmosphere. It is clear from Fig. 17 that even at the present time the electric field has not recovered to the natural pattern.

That the electric field contains too much such data sometimes brings important truths, but sometimes only complexity and irritation. Nevertheless it can be said that the measurement of the electric field is still fundamental to the study of atmospheric electricity.

In this review no distinct definition of the electric field is made and little attention is paid to the sign of the electric field. The term potential gradient is very often used in the strict sense and is more popular in atmospheric electricity. However the term electric field is more understandable for people in fields other than atmospheric electricity. This is why the term electric field is used throughout instead of potential gradient.

This review is not very inclusive, especially in the references. A supplementary review will be prepared in the future.

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