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OBSERVATION OF THE ROTATIONAL STRAIN, AND RELATION TO THAT OF EXTENSIONAL STRAINS IN THE EARTH TIDE

By

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Abstract

Observations by means of a rotationmeter were carried out at Otsu Observatory, and their analysed value for three months is given in this paper.

It has been proved that the gradient of the displacements with respect to their orthogonal directions can be obtained by use of the observations of the ground extensions in azimuth of more than three in the earth tide.

Then the results of the orthogonal gradient of the tidal horizontal displacement which is observed directly by means of a rotationmeter, which is calculated from the strain elements observed by extensometers, and which is calculated by the elastic theory in the earth tide are compared with each other.

There are good agreements between the amplitudes and the phase lags of the calculated values from the observations of the extensometers and that from the elastic theory. And there is an agreement between the amplitude of the observed values of M_2 wave by means of the rotationmeter and those obtained by the extensometers and the elastic theory in the earth tide.

1. Introduction

The crustal strains consist of not only the shear and dilatational strains but also the rotational strains. At the present time, many people acknowledge that the continents have been drifted rotationally like rigid bodies. And according to the analysis of trigonometrical surveys, such rotations have been found. These rotations will be made less in their analyses by the use of the surveys with optical waves, but the finite rotations will be found to the last extremity. These rotational strain should be as important as the fault in the mechanism of the earthquake.

According to the first approximation of the theory of the earth tide, there is no rotational strain with respect to the vertical. But, there is the shear strain, therefore the gradient of the tidal displacements with respect to the orthogonal azimuths, namely $\partial u_\theta / r \sin\theta \partial\phi$, $\partial u_\phi / r \partial\theta$ and so on are not always nil. These gradients can be observed directly by the use of rotationmeters.

The author has devised a rotationmeter which has high sensitivities of 10^{-8}/mm, and has carried out continuously observations of the gradient at Otsu Observatory since 1966. The analysis of the observations for three months is given in this paper. The result of the gradient is compared with that calculated from the observations with extensometers in some azimuths and that calculated from the elastic theory of the earth tide.

2. Fundamental theory

The rotational strain with respect to the vertical ω_r is given in the earth co-ordinate (r, θ, ϕ) as follows,

$$2\omega_r = \frac{1}{r^2 \sin\theta}\left\{\frac{\partial}{\partial\theta}(ru_\phi \sin\theta) - \frac{\partial}{\partial\phi}(ru_\theta)\right\}, \tag{1}$$

where r, θ and ϕ are the radial, the co-latitudinal and the longitudinal vectors, respectively, and u_ϕ and u_θ are the longitudinal and the co-latitudinal components of the displacement.

The components of the displacement in the earth tide are given approximately as

$$u_\theta = \frac{l}{g}\frac{\partial W_2}{\partial\theta},$$
$$u_\phi = \frac{l}{g\sin\theta}\frac{\partial W_2}{\partial\phi}, \tag{2}$$

where l, g and W_2 are Shida's number of the earth tide, the conventional numerical constant of the gravity acceleration at the earth's surface and the potential of the tide generating force which is shown as the solid harmonics of 2nd degrees, respectively.

From (1) and (2), we have as follows,

$$\left.\begin{aligned} \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial\phi} - u_\phi \cot\theta\right) &= \frac{1}{r}\frac{\partial u_\phi}{\partial\theta}, \\ \omega_r &= 0, \end{aligned}\right\} \tag{3}$$

where the left term of the first formula of (3) is the gradient of the co-latitudinal component of the tidal displacement respect to the prime vertical (eastward) direction, and the right term is that of the longitudinal component of the displacement respect to the co-latitude (southward). Namely, these gradients of the orthogonal components of the horizontal displacements respect to the rectangular azimuths are always equal.

And we have that the longitudinal strain $e_{\alpha,\alpha+\pi/2}$ is equal to twice of the gradient of the horizontal component of the tidal displacement respect to the orthogonal azimuth in a general coordinate. Where α and $\alpha+\pi/2$ are azimuths of the axes of the coordinate.

Let the direction cosines of the axis of the azimuth α for the co-latitude and the longitude be l_2 and m_2, respectively, and those of the $\alpha+\pi/2$ be l_3 and m_3, we have the relation among the $e_{\alpha,\alpha+\pi/2}$, $e_{\theta\theta}$, $e_{\phi\phi}$ and $e_{\phi\theta}$ as follow,

$$e_{\alpha,\alpha+\pi/2} = 2e_{\theta\theta}l_2l_3 + 2e_{\phi\phi}m_2m_3 + e_{\theta\phi}(l_2m_3 + l_3m_2) \tag{4}$$

where $e_{\theta\theta}$, $e_{\phi\phi}$ and $e_{\theta\phi}$ are extensional strain on the co-latitude, that on the latitude and the longitudinal strain on the co-latitude and the longitude, respectively. $e_{\theta\theta}$, $e_{\phi\phi}$ and $e_{\theta\phi}$ are calculated from the observed extensions e_i in the directions more than three azimuths by use of the following relation

$$e_i = e_{\theta\theta} l_i^2 + e_{\phi\phi} m_i^2 + e_{\theta\phi} l_i m_i \tag{5}$$

The components of the M_2 and O_1 waves of the potential of the tide generating force W_2 at Otsu ($\pi/2-\theta=34°59'$) are as follows,

$$W_2(M_2) = 1.5971\times 10^{-8} ag\cos 2H,$$
$$W_2(O_1) = 0.9281\times 10^{-8} ag\cos H, \tag{6}$$

where a and H are the distance from the earth's center to the observatory and the hour angle of the component wave, respectively.

From formulae (2), (3) and (6), we have the M_2 and O_1 waves' components of the gradient of the tidal displacements u_θ or u_ϕ with respect to the prime vertical or the co-latitude, respectively are given as follows

$$M_2 := -4.375\times 10^{-8} l \sin 2H,$$
$$O_1 := 2.594\times 10^{-8} l \sin H. \tag{7}$$

Put the coordinate in the azimuth of 38°, we have the longitudinal strain $e_{38°,38°+\pi/2}$ and the gradient of the $(38°+\pi/2)$ component of the tidal displacement with respect to the azimuth 38°, $\partial u_{38+\pi/2}/\partial u_{38}$ from formulae (2), (3), (4) and (6) or (7) as follows,

$$e_{38°,38°+\pi/2}(M_2) = 9.9394\times 10^{-8} l\cos(2H-351.78°),$$
$$e_{38°,38°+\pi/2}(O_1) = 3.1186\times 10^{-8} l\cos(H-157.78°),$$
$$\frac{\partial u(38°+\pi/2)}{\partial x(38°)}(M_2) = 4.9697\times 10^{-8} l\cos(2H-351.78°),$$
$$\frac{\partial u(38°+\pi/2)}{\partial x(38°)}(O_1) = 1.5593\times 10^{-8} l\cos(H-157.78°). \tag{I}$$

Principle of the rotationmeter (I. Ozawa, [1970]): In Fig. 1, the two fixed point $B(x, o)$ and $C(o, y)$ on the ground are displaced to $B'(x, \Delta y)$ and $C'(\Delta x, y)$, hence the rotation and the longitudinal strain are given as,

$$\text{rotation:} \qquad \frac{1}{2}\left(\frac{\Delta y}{x} - \frac{\Delta x}{y}\right),$$

$$\text{longitudinal strain:} \qquad \left(\frac{\Delta y}{x} + \frac{\Delta x}{y}\right).$$

If the xy plane is the horizontal plane on the earth's surface and $x=y$, $\Delta x = \Delta y$ in the earth tide.

The mechanism of our rotationmeter is shown in Fig. 2 as following. It consists

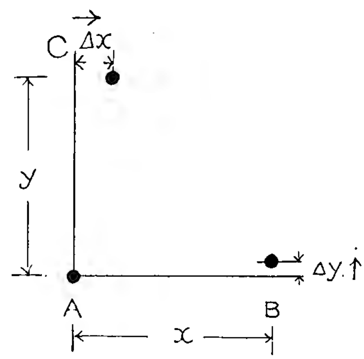

Fig. 1. The principle of the longitudinal and rotational strains.

of two canti-levers which are fixed with A and B on the ground. Two canti-levers are put in parallel, the distance between these canti-levers is very little. The change of the distance at their free ends is observed.

3. Observations

The observations have been carried out with a rotationmeter at Otsu which is at the north latitude of 34°59.6′ and the east longitude of 135°51.5′ since 1966. The rotationmeter (I. Ozawa, 1970) consists of two canti-lever whose lengths are six meters, and are put in the azimuth of S 38° W-. The usual sensitivity is about $2\times10^{8-}/$mm.

Photo 1 shows examples of the records of the rotationmeter of the former and new types. The former type records the change of the distance between its two canti-levers, but the new type's (I. Ozawa [1970]) records are separated into two curves, which show the change of the distances between the free ends of the levers and the center point on the ground, *O* in the Fig. 2.

The levers of this rotationmeter are put in the azimuth, and the change of the distance in that of S 52° E—between the free ends of both canti-levers. So, this instrument records the change of $\partial u_{S52°E}/\partial x_{S38°W}$.

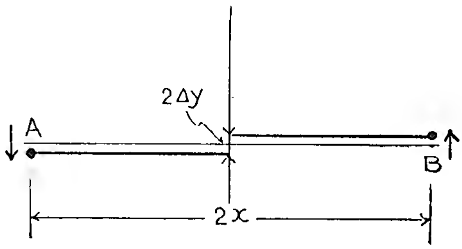

Fig. 2. The principle of the rotationmeter.

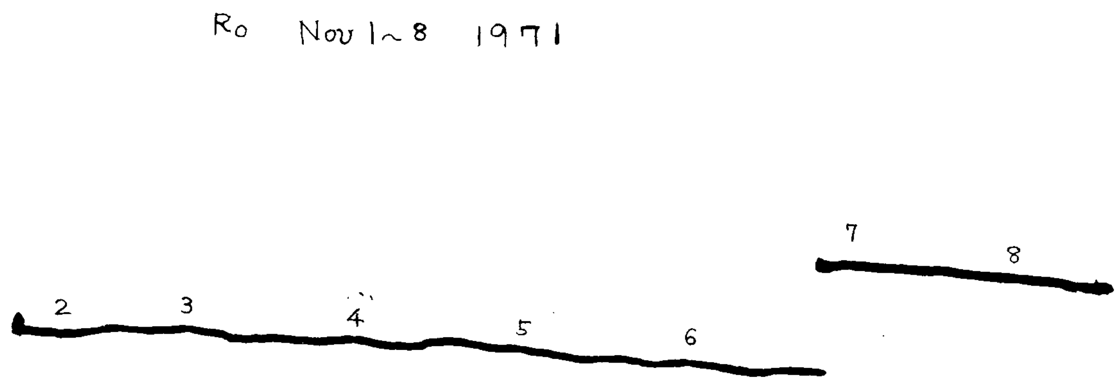

Photo 1 (a). The record for a week observed with the former type rotationmeter.

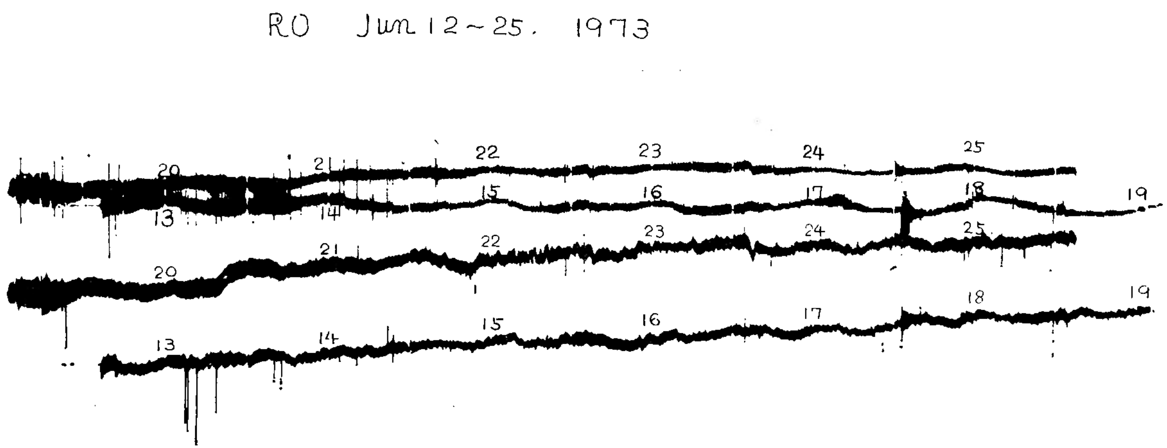

Photo 1 (b). The record for two weeks observed with the new type rotationmeter.

The change of the distance is transformed into the tilting of the axces of the horizontal pendulum, and its rotation of the pendulum is amplified with the optical lever on the photographic papers.

The analyses of the record of the rotationmeter for three months whose epoch is November 1st, 1970 is shown as follows,

$$\left.\begin{array}{ll} \text{for } M_2\text{-wave} & 0.25\times 10^{-8}\cos(2t-22°), \\ \text{and for } O_1\text{-wave} & 1.27\times 10^{-8}\cos(t-236°). \end{array}\right\} \qquad \text{(II)}$$

4. Considerations

As mentioned in Chapter 2, the rotational strain around the vertical ω_r in the earth tide is nil. Therefore, we are able to calculate the gradients of the tidal displacements with respect to the orthogonal directions by using the observations of extensional strains in the azimuths of more than three.

The present author (I. Ozawa, [1966]) has obtained the strain elements of the earth tide as follows,

	for M_2-wave	for O_1-wave
$e_{\theta\theta}$	$0.612\times 10^{-8}\cos(2t-25.7°)$	$0.558\times 10^{-8}\cos(t-23.6°)$,
$e_{\phi\phi}$	$1.109\times 10^{-8}\cos(2t-359.0°)$	$0.698\times 10^{-8}\cos(t-5.0°)$,
$e_{\theta\phi}$	$1.080\times 10^{-8}\cos(2t-172.7°)$	$0.972\times 10^{-8}\cos(t-202.0°)$.

From these results and the formulae (3) and (6), we have

$$\frac{1}{r}\frac{\partial u_\phi}{\partial\theta}=\begin{cases} M_2: & 0.540\times 10^{-8}\cos(2t-172.7°), \\ O_1: & 0.486\times 10^{-8}\cos(t-202.0°), \end{cases}$$

and

$$\frac{\partial u_{S52°E}}{\partial u_{N38°E}}=\begin{cases} M_2: & 0.233\times 10^{-8}\cos(2t-329.1°), \\ O_1: & 0.181\times 10^{-8}\cos(t-220.7°). \end{cases} \qquad \text{(III)}$$

According to the comparisons of the results (II) which is the direct observed values by means of the rotationmeter, (I) which is the pure theoretical value and (III) which is the indirect observed value by means of the extensometers, the amplitudes and the phase lags of (I) and (III) have a good agreement each other in the case of $l=0.047$ or 0.116 for M_2 and O_1, respectively.

Although the ω_r component of the rotation is free from the load tide, if the strain elements which produce the result (III) were compensated with the load tide, the agreements between (I) and (III) would be better. The results (I) and (II) have a agreement in the amplitude of M_2-wave, and their phase lags have the relation of an approximation. The author wants to have more sensible and longer analyses of the observations by means of these instruments, and he wants to consider also the effects of the load tide in more precise theory.

5. Observation of the rotation of the rotation

We also have an interesting to observations of a rotation of a rotation of the tidal displacement U. The (rot rot) of the displacement U with respect to the vertical, z, is shown as

$$(rot\ rot\ \boldsymbol{U})_z=\frac{1}{2}\left(\frac{\partial\omega_y}{\partial x}-\frac{\partial\omega_x}{\partial y}\right)$$
$$=\frac{1}{4}\left\{\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\right\}$$
$$=\frac{1}{4}\left\{\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\left(\frac{\partial^2 w}{\partial x_2}+\frac{\partial^2 w}{\partial y_2}\right)\right\}$$
$$=\frac{1}{4}\frac{\partial\Delta}{\partial z}, \tag{8}$$

where

$$\Delta=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial w}+\frac{\partial w}{\partial z} \quad \text{and} \quad w=\frac{h}{g}W_2,$$

because $\partial^2w/\partial x_2+\partial^2w/\partial y_2+\partial^2w/\partial z_2=0$ in the earth tide,
From these relations and formula (1), we have,

$$\frac{\partial\Delta}{\partial z}=(3h+5ah'+a^2h''-6al'-6l)\frac{W_2}{a^2g} \tag{9}$$

where h, h', h'', and l' is Love's number of the earth tide, the 1st derivative with respect to r, the 2nd derivative of h, and 1st derivative of l with respect to r, respectively.

For the constants in the formula (9), we have these numerical values as $h=0.556$, $l=0.073$, $ah'=-1.405$, and $al'=-0.630$ (I. Ozawa, [1966]). But we have not ever obtained the numerical value of a^2h'' from the observations. Thus we are not able to calculate the numerical value of $(rot\ rot\ \boldsymbol{U})_z$. This value can be obtained by observations of the tidal dilatations at two different depths in the crust.

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