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ON THE EFFECT OF COMPRESSIBILITY ON THE KELVIN-HELMHOLTZ INSTABILITY OF THE MAGNETOPAUSE

By

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Abstract

The effect of compressibility on the Kelvin-Helmholtz instability of the magnetopause is investigated, making use of the usual MHD equations. Unstable domains are obtained with respect to Mach number and the quantity the sound speed divided by the Alfvén speed, to the second power. It is found that as the ratio of the sound speed to the Alfvén speed becomes larger, the unstable domains tend to become wider, and overstability domains appear.

1. Introduction

The Kelvin-Helmholtz instability of the magnetopause has been considered as an origin of the ultra low frequency geomagnetic micropulsation, from ground magnetic observations and satellite ones (Atkinson and Watanabe [1966], Dungey and Southwood [1970], Samson et al. [1971]). The theoretical investigation on this instability of the magnetopause has been carried out by a number of researchers (Fejer [1964], Talwar [1964], Sen [1964, 1965], Lerche [1966], Southwood [1968], McKenzie [1970], Ong and Roderick [1972], Nagano and Inoue [1972]), but many problems remain unsolved.

According to Lerche's discussion [1966], it is concluded that as the waves with the shortest wavelength become the most unstable, the usual MHD assumption, under which the boundary layer is taken to be infinitesimal in thickness and the gyroradii of all particles are taken to be zero, is inapplicable to the stability of the magnetopause. However, taking into account the finite thickness of the boundary layer, Ong and Roderick [1972] showed that the effect of a finite layer stabilizes perturbations with a short wavelength. On the other hand, Nagano and Inoue [1972] took into account the finite length of ion Larmor radius and found that the effect of a finite radius tends to stabilize ones. Therefore, the MHD approach is considered still available for a short wavelength.

Sen [1964] studied the effect of compressibility on the Kelvin-Helmholtz instability and looked for the stability domains for the two limiting cases; the case of very small compressibility, that is, the Alfvén speed is much smaller than the sound speed, and the one of very large compressibility, that is, the Alfvén speed is much larger than the sound speed. He showed that an upper critical streaming speed appears for unstable domains. However, with regard to the magnetopause the Alfvén speed and the sound speed are the same order, and hence research for this case is required.

In this paper we discuss the effect of compressibility on the Kelvin-Helmholtz instability of the magnetopause, making use of the usual MHD equations. The unstable domains for some particular propagating modes are studied with respect to Mach number M and the quantity β the sound speed C_o divided by the Alfvén speed V_a, to the second power.

2. Dispersion relation

The usual MHD equations, assuming infinite conductivity and isotropic pressure, are used. We assume the magnetospheric plasma on one side of the interface (region I) and the solar wind plasma on the other side (region II). Both the directions of flow and magnetic field in the region I and II are assumed to be parallel to the interface. We perturb the equilibrium state of the interface by means of a small amplitude disturbance. The linearized equations for the first order perturbed physical quantities to vary as $\exp(i\boldsymbol{k}\cdot\boldsymbol{r}+\omega t)$ can be solved, and the dispersion relation can be obtained after some analyses, by using the boundary conditions.

According to Talwar's calculation [1964], the dispersion relation is written in the form

$$\rho_{o2}\{\Omega_2^2+(\boldsymbol{k}\cdot\boldsymbol{V}_{a2})^2\}\frac{[\{\Omega_2^2+(\boldsymbol{k}\cdot\boldsymbol{V}_{a2})^2\}(\Omega_2^2+k^2C_{o2}^2)+\Omega_2^2(\boldsymbol{k}\times\boldsymbol{V}_{a2})^2-\Omega_2^4]^{1/2}}{[\{\Omega_2^2+(\boldsymbol{k}\cdot\boldsymbol{V}_{a2})^2\}(\Omega_2^2+k^2C_{o2}^2)+\Omega_2^2(\boldsymbol{k}\times\boldsymbol{V}_{a2})^2]^{1/2}}$$
$$+\rho_{o1}\{\Omega_1^2+(\boldsymbol{k}\cdot\boldsymbol{V}_{a1})^2\}\frac{[\{\Omega_1^2+(\boldsymbol{k}\cdot\boldsymbol{V}_{a1})^2\}(\Omega_1^2+k^2C_{o1}^2)+\Omega_1^2(\boldsymbol{k}\times\boldsymbol{V}_{a1})^2-\Omega_1^4]^{1/2}}{[\{\Omega_1^2+(\boldsymbol{k}\cdot\boldsymbol{V}_{a1})^2\}(\Omega_1^2+k^2C_{o1}^2)+\Omega_1^2(\boldsymbol{k}\times\boldsymbol{V}_{a1})^2]^{1/2}}=0, \quad (1)$$

where

$$\Omega_j=\omega+i\boldsymbol{k}\cdot\boldsymbol{U}_{oj}, \quad \boldsymbol{V}_{aj}=\frac{\boldsymbol{B}_{oj}}{\sqrt{4\pi\rho_{oj}}}, \quad j=1 \text{ or } 2.$$

Here ρ_o, $\boldsymbol{U}_o$, $\boldsymbol{B}_o$ and C_o are respectively the mass density, the flow velocity, the magnetic field and the sound speed that represent the equilibrium values on either side of the boundary layer, and the subscripts 1 and 2 denote the values in the region I and II respectively.

Let us consider a particular case as follows; $\rho_{o1}=\rho_{o2}=\rho_o$, $\boldsymbol{U}_{o2}=-\boldsymbol{U}_{o1}=\boldsymbol{U}_o$, $\boldsymbol{B}_{o1}=\boldsymbol{B}_{o2}=\boldsymbol{B}_o$, $\boldsymbol{V}_{a1}=\boldsymbol{V}_{a2}=\boldsymbol{V}_a$ and $C_{o1}=C_{o2}=C_o$. We now take ϕ to be the angle between $\boldsymbol{k}$ and $\boldsymbol{B}_o$, and Ψ between $\boldsymbol{k}$ and $\boldsymbol{U}_o$. If we define non-dimensional parameters

$$X=\frac{\omega L_o}{\sqrt{V_a^2+C_o^2}}, \quad k^*=kL_o, \quad M=\frac{U_o}{\sqrt{V_a^2+C_o^2}} \quad \text{and} \quad \beta=\frac{C_o^2}{V_a^2},$$

where L_o represents the characteristic length, eq.(1) becomes

$$[X^2-k^{*2}A-2ik^*XM_\Psi][X^2-k^{*2}B-2ik^*XM_\Psi]^{1/2}[(X^2-k^{*2}M_\Psi^2)^2-4k^{*2}X^2M_\Psi^2$$

$$+k^{*2}(X^2-k^{*2}B)+2ik^*XM_\Psi\{2(X^2-k^{*2}M_\Psi^2)+1\}]^{1/2}+[X^2-k^{*2}A+2ik^*XM_\Psi]$$
$$\times[X^2-k^{*2}B+2ik^*XM_\Psi]^{1/2}[(X^2-k^{*2}M_\Psi^2)^2-4k^{*2}X^2M_\Psi^2+k^{*2}(X^2-k^{*2}B)$$
$$-2ik^*XM_\Psi\{2(X^2-k^{*2}M_\Psi^2)+1\}]^{1/2}=0. \qquad (2)$$

Here

$$A=M_\Psi^2-\frac{1}{1+\beta}\cos^2\phi,$$

$$B=M_\Psi^2-\frac{\beta}{(1+\beta)^2}\cos^2\phi,$$

where

$$M_\Psi=M\cos\Psi.$$

We rationalize eq.(2) and obtain an eighth degree integral expression in X as follows;

$$X^8+C_1X^6+C_2X^4+C_3X^2+C_4=0, \qquad (3)$$

where

$$C_1=2(2M_\Psi^2+1),$$
$$C_2=6M_\Psi^4+2M_\Psi^2+\frac{1+3\beta}{(1+\beta)^3}\cos^2\phi\cdot\{2(1+\beta)-\cos^2\phi\},$$
$$C_3=2\Big[2M_\Psi^6-M_\Psi^4+\frac{\cos^2\phi}{(1+\beta)^3}\{2(1-\beta^2)-(1+3\beta)\cos^2\phi\}M_\Psi^2$$
$$+\frac{\beta\cos^4\phi}{(1+\beta)^4}(2+3\beta-\cos^2\phi)\Big],$$
$$C_4=\Big[M_\Psi^2-\frac{\cos^2\phi}{1+\beta}\Big]\Big[M_\Psi^6-\Big(2-\frac{\cos\phi^2}{1+\beta}\Big)M_\Psi^4+\frac{2\beta\cos^2\phi}{(1+\beta)^3}(2+2\beta-\cos^2\phi)M_\Psi^2$$
$$-\frac{2\beta^2\cos^4\phi}{(1+\beta)^4}\Big].$$

As eq.(3) is rationalized, we must check the consistency between eq.(2) and (3).

3. Discussion of results

First, when $\phi=90°$, that is, the wave vector $\boldsymbol{k}$ is perpendicular to the magnetic field $\boldsymbol{B}_o$, eq.(3) is solved into factors and a physically nontrivial equation is obtained in the form

$$X^4+2k^{*2}(M_\Psi^2+1)X^2+k^{*4}M_\Psi^2(M_\Psi^2-2)=0. \qquad (4)$$

In this case the instability condition is $0<M\cos\Psi<\sqrt{2}$.

Second, when $\phi=0°$, that is, the wave vector $\boldsymbol{k}$ is parallel to the magnetic field $\boldsymbol{B}_o$, the following equation is obtained from eq.(3) in the same way as mentioned above.

$$X^4+2k^{*2}\left(M_\Psi^2+\frac{\beta}{1+\beta}\right)X^2+k^{*4}\left\{M_\Psi^4-\frac{2\beta}{1+\beta}M_\Psi^2+\frac{2\beta^2}{(1+\beta)^3}\right\}=0. \tag{5}$$

In this case the instability condition is

$$\left[\frac{\beta}{\beta+1}\left\{1-\left(\frac{\beta-1}{\beta+1}\right)^{1/2}\right\}\right]^{1/2}<M\cos\Psi<\left[\frac{\beta}{\beta+1}\left\{1+\left(\frac{\beta-1}{\beta+1}\right)^{1/2}\right\}\right]^{1/2}.$$

In the case of the angle ϕ other than $0°$ and $90°$, eq.(3) is too complicated to be solved analytically, and so let us calculate it with the computer and check the consistency between eq.(2) and (3).

Figs. 1(a), (b), (c) and (d) refer to the diagrams of the curves $M\cos\Psi$ as functions of β, for $\phi=0°$, $30°$, $60°$ and $90°$ respectively. When $\phi=0°$, we can see that no unstable domain appears for $\beta<1$ and an instability domain does for $\beta>1$. On the contrary, when $\phi=30°$ and $60°$, unstable domains appear for $\beta<1$ and overstability

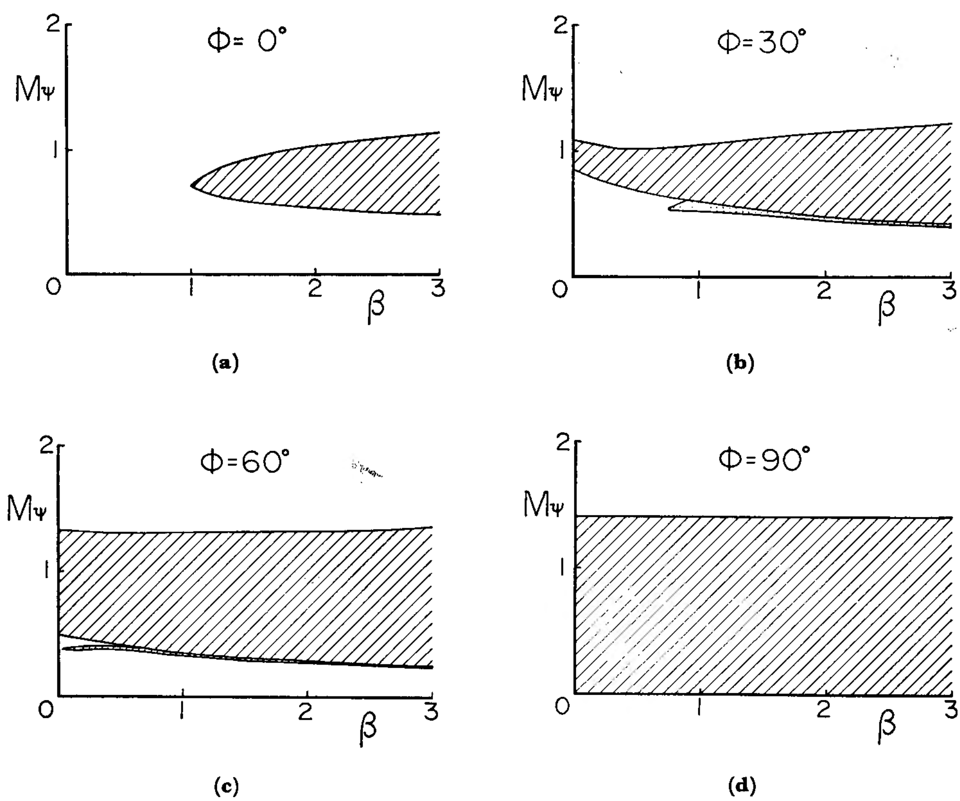

Fig. 1. The curves M_Ψ as functions of β, where $M_\Psi=M\cos\Psi$. (a), (b), (c) and (d) refer to $\phi=0°$, $30°$, $60°$ and $90°$ respectively. The shaded regions are the instability domains and the dotted regions are the overstability domains.

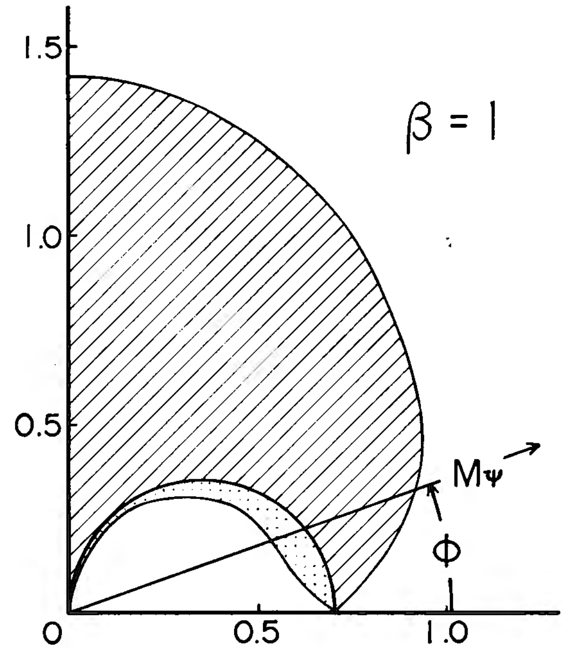

Fig. 2. The curves for the polar plots of M_Ψ as functions of ϕ in the case of $\beta=1$, where $M_\Psi=M\cos\Psi$. The shaded region is the instability domain and the dotted region is the overstability domain.

domains unexpectedly make their appearance. In general, as the angle ϕ becomes larger, the unstable domains tend to become wider, and also as β becomes larger, the unstable domains do. However, when $\phi=90°$, an instability domain is settled independent of β.

Fig. 2 refers to the diagram for the polar plots of $M\cos\Psi$ as functions of ϕ in the case of $\beta=1$. As ϕ varies, the upper limits of the unstable domains vary by about twice, while the lower limits do large.

As a matter of course, as the angle Ψ between $\boldsymbol{k}$ and $\boldsymbol{U}_0$ becomes larger, the unstable domains do wider and wider. The border lines of the instability domains are found to coincide with ones from $C_4=0$ in eq.(3). The border lines of the overstability domains are yet unsolved analytically, and so it is desirable to solve them in the future.

In order to obtain the values of β on the magnetopause, we take typical values as follows; a particle density $N_0=10\ \text{cm}^{-3}$, a temperature $T_0=8\times10^5\ °\text{K}$ and a magnetic field intensity $B_0=18\gamma$ corresponding to the front of it, and $N_0=5\ \text{cm}^{-3}$, $T_0=5\times10^5\ °\text{K}$ and $B_0=8\gamma$ corresponding to the tail. These values give $\beta\approx0.8$ for the front and $\beta\approx1.3$ for the tail. Because unstable domains become wider as β does larger, it is said that the tail becomes unstable more easily.

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