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PATTERN OF TIDAL DEFORMATION ON THE EARTH

By

Izuo OZAWA

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Abstract

The tidal deformation on the earth is classified into three types. The first type is formulated by a product of tidal numbers and the potential of the tide generating force like the tidal change of the gravity acceleration. The second type is formulated by a product of tidal numbers and the tidal displacement like the tidal tilting. The third type is formulated by a combination of the tidal potential, tidal numbers and the trigonometrical function of the latitude like the tidal strain.

The patterns of the latitudinal and the azimuthal distributions are calculated. And the possibility to obtain the ratio l/h is shown by means of the observations of these patterns of tidal changes.

1. Introduction

It is interesting to study what type of the tidal deformation is generated on the earth. It is usually said that the tidal deformation is the spheroidal type only. Some papers [Takeuchi (1950), Ozawa (1960, 1966)] have explained the individual tidal deformation in a few examples, but the systematic and general explanations have not been practised. In this paper, the general tidal changes on the earth are studied systematically. This study is useful to plan the observations of the earth tide over the world.

2. Classification of the earth tide

We may classify these tidal changes on the earth into three types as followings.

Tidal changes of the first type are formulated by the simple potential W_2 of the tide generating force and some tidal numbers, and are regardless of the latitude except W_2 itself. This type is originally founded on the vertical components of the displacement and so on. For example, the tidal change of the gravity acceleration Δg, the tidal change of the sea level ζ genrally belong to this type, and are formulated as follows,

$$\Delta g = \frac{2}{a}\left(1 + h - \frac{3}{2}k\right) W_2 , \tag{1}$$

$$\zeta = (1 + k - h)\frac{W_2}{g} , \tag{2}$$

where h and k are the Love's numbers, a, g and W_2 are the mean values of the earth's radius and the gravity acceleration on the surface of the earth, and the tidal potential of 2nd degree, respectively.

The vertical displacement u_r, the cubical dilatation Δ, the vertical extension e_{rr}, the diurnal components of the co-latitudinal and the prime vertical extensions $e_{\theta\theta}(1)$, $e_{\phi\phi}(1)$ on the earth tide also belong to the first type singularly as follows,

$$u_r = \frac{h}{g} W_2 , \tag{3}$$

$$\Delta = \left(a\frac{\partial h}{\partial r} + 4h - 6l\right) \frac{W_2}{ag} , \tag{4}$$

$$e_{rr} = \left(a\frac{\partial h}{\partial r} + 2h\right) \frac{W_2}{ag} , \tag{5}$$

$$e_{\theta\theta}(1) = (h - 4l) \frac{W_2}{ag} , \tag{6}$$

$$e_{\phi\phi}(1) = (h - 2l) \frac{W_2}{ag} , \tag{7}$$

where l, r, θ and ϕ are the Shida's number, and the radial, the co-latitudinal and the longitudinal vectors of the earth, respectively.

We know one more tidal change in this type. This is the tidal change of the rotational speed of the earth, δw, which is measured by the physical time scale. This change is proportional to the change of the moment of the inertia of the earth around the axis of the rotation, C, as follows [Iijima and Niimi (1971)],

$$\frac{\delta w}{w} = \frac{\delta C}{C} . \tag{8}$$

This speed change due to the change of the moment of the inertia is represented by $k' W_2$, and the tidal change of the rotational speed belongs to the first type. Of course, the true change of the speed of the rotation measured by the physical time is equal everywhere. But the change measured by the astronomical time is the function of the latitude and the longitude or the tidal displacement as shown in the following type.

The second type of the tidal change is formulated by the horizontal component of the tidal displacement itself and some tidal constants. The horizontal components u_θ, u_ϕ of the tidal displacement, the vertical deflection to the earth's axis d_θ, d_ϕ, the tidal tilting t_θ, t_ϕ, and the tidal rotational strain ω_θ, ω_ϕ, for the co-latitudinal θ and the longitutinal ϕ components, respectively, belong to this type as follows,

$$u_\theta = \frac{l}{g} \frac{\partial W_2}{\partial \theta} , \tag{9}$$

$$u_\phi = \frac{l}{g \sin\theta} \frac{\partial W_2}{\partial \phi} , \tag{10}$$

$$d_\theta = \frac{1+k-l}{al} u_\theta , \tag{11}$$

$$d_\phi = \frac{1+k-l}{al} u_\phi , \tag{12}$$

$$t_\theta = \frac{1+k-h}{al} u_\theta , \tag{13}$$

$$t_\phi = \frac{1+k-h}{al} u_\phi , \tag{14}$$

$$\omega_\theta = - \frac{l-h}{al} u_\phi , \tag{15}$$

$$\omega_\phi = \frac{l-h}{al} u_\theta . \tag{16}$$

Phases of these tidal changes are equal to that, ψ of the displacements on the vertical deflection for the earth's axis and for the ground, and are equal to $\pi/2-\psi$ on the rotational strain.

Particular changes in this type are the radial component of the rotational strain ω_r and the long period tide of $\omega_\theta(0)$, and these are always nil.

The third type of the tidal change is formulated by the combination of W_2, tidal numbers and trigonometrical functions of the latitude. The semi-diurnal components of the normal strains $e_{\theta\theta}(2)$, $e_{\phi\phi}(2)$ and of the longitudinal strain $e_{\theta\phi}(2)$, the diurnal component of the longitudinal strain $e_{\theta\phi}(1)$, and the long period's tide of the normal strain $e_{\theta\theta}(0)$, $e_{\phi\phi}(0)$ belong to this type. These are shown as follows,

$$e_{\theta\theta}(2) = \frac{h\sin^2\theta + 2l\cos2\theta}{\sin^2\theta}\frac{W_2}{ag}, \tag{17}$$

$$e_{\phi\phi}(2) = \frac{h\sin^2\theta - 2l(1+\sin^2\theta)}{\sin^2\theta}\frac{W_2}{ag}, \tag{18}$$

$$e_{\theta\phi}(2) = -\frac{4l\cos\theta}{\sin^2\theta}\tan2\phi\frac{W_2}{ag}, \tag{19}$$

$$e_{\theta\phi}(1) = \frac{4l\sin\theta}{\sin 2\theta}\tan\phi\frac{W_2}{ag}, \tag{20}$$

$$e_{\theta\theta}(0) = \frac{\left\{2l\cos2\theta + \frac{h}{3}(1-3\cos^2\theta)\right\}}{\frac{1}{3}(1-3\cos^2\theta)}\frac{W_2}{ag}, \tag{21}$$

$$e_{\phi\phi}(0) = \frac{\left\{2l\cos^2\theta + \frac{h}{3}(1-3\cos^2\theta)\right\}}{\frac{1}{3}(1-3\cos^2\theta)}\frac{W_2}{ag}, \tag{22}$$

$$e_{\theta\phi}(0) = 0\,. \tag{23}$$

The atmospheric tide (S_2' component, 12.00 hours' period) is given by Haurwitz (1956) shown as follows,

$$S_2' = b_2\,W_2^2 - b_4\,W_4^2\,, \tag{24}$$

where b_2 and b_4 are numerical constants, and W_2^2 and W_4^2 are semi-diurnal components of the potential which are expressed by solid harmonics of degree 2 and degree 4, respectively. And, S_2' is given also by Jaerisch (1907) as follows,

$$S_2' = f\sin\theta\cdot W_2^2\,. \tag{25}$$

S_2' of the Haurwitz's representation belongs to the first type, and that of the Jaerisch belongs to the third type.

3. Patterns of amplitude and phase of the tidal change

The first type is given as combination of the tidal potential W_2 and the tidal constants, and so, the pattern of the change for the latitude is similar to W_2 itself. The pattern of the amplitude for the latitude in the declination of 0° and 28.44° are shown in Fig. 1. The diurnal component is always nil at the declination $\delta=0°$. The curve of $\sin^3$ in Fig. 1 shows the amplitude pattern of $\sin^3\theta$ for the colatitude θ like that of the atmospheric tide. The pattern of the first type is regardless of the ratio among the tidal numbers l, h and k. And so, the latitudinal pattern of this type is not useful to obtain the ratio of the tidal numbers. But, it is important to

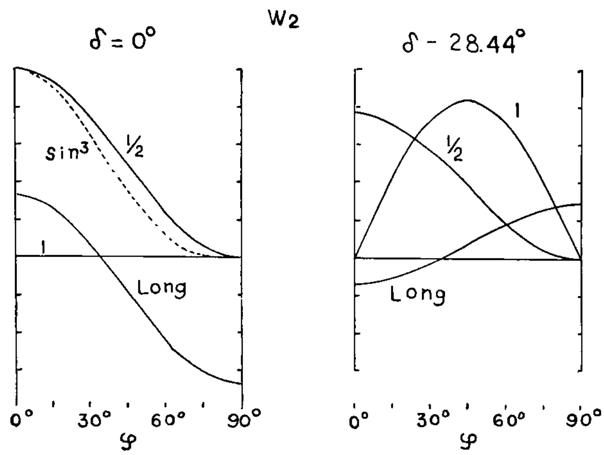

Fig. 1. The amplitude of W_2 versus the latitude φ at $\delta=0°$ and 28.44°.

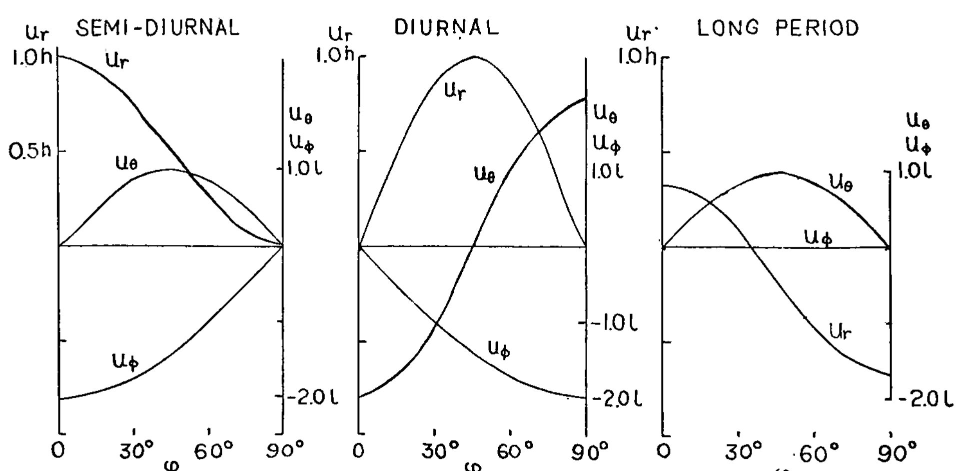

Fig. 2. The amplitude of the tidal displacement versus the latitude φ at $\delta=0°$ in the semi-diurnal and the long preiod's tides, and at $\delta=28.44°$ in the diurnal tide.

study the type of the global deformation on the earth tide.

Latitudinal patterns of the second type are represented by the components of the tidal displacement, u_θ or u_ϕ. Fig. 2 shows these components at $\delta=0°$ in the

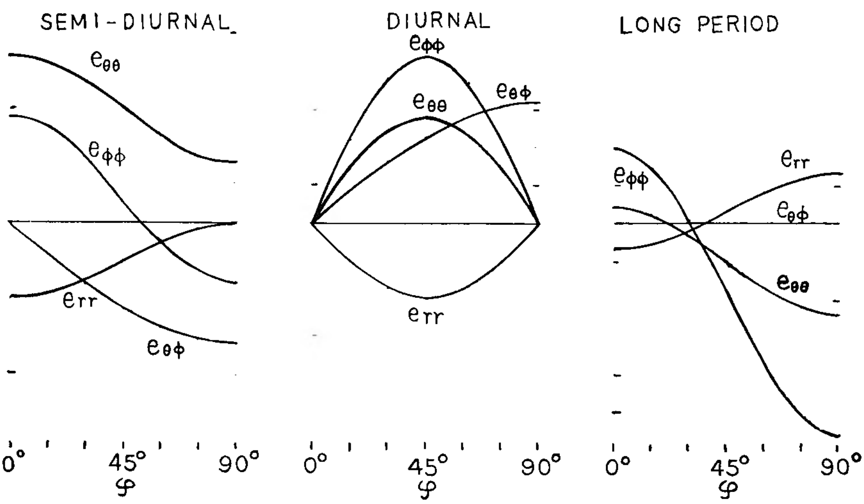

Fig. 3. The components of the tidal strains versus the latitude φ at $h=0.60$ and $l=0.08$. These patterns are at $\delta=0°$ in the semi-diurnal and the long period's tides, and at $\delta=28.44°$ in the diurnal tide.

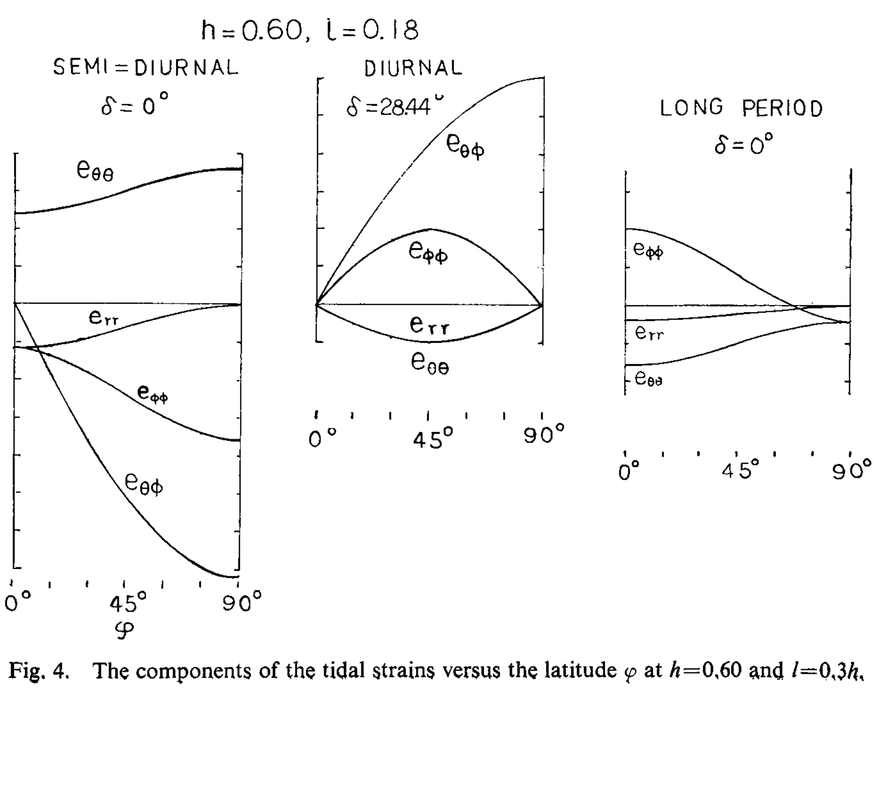

Fig. 4. The components of the tidal strains versus the latitude φ at $h=0.60$ and $l=0.3h$.

semi-diurnal and in the long period's tides, and at $\delta=28.44°$ in the diurnal tide. These patterns are also regardless of the ratio of the tidal numbers.

Fig. 3 shows the latitudinal patterns of the tidal strain components at $h=0.60$, $l=0.08$, and $a\frac{\partial h}{\partial r}=-1.40$. The patterns of the semi-diurnal and the long period's tide are at $\delta=0°$, and that of the diurnal tide is at $\delta=28.44°$.

Fig. 4 shows the latitudinal pattern of the components of the tidal strain at $h=0.60$, $l=0.18$ ($l=0.3\ h$) and $a\frac{\partial h}{\partial r}=-1.32$ in which are type of the deformation is the spheroidal and the cubical dilatation is nil. These patterns of the semi-diurnal and the long period's tide are at $\delta=0°$, that of the diurnal tide is at $\delta=28.44°$.

These patterns of the components of the tidal strain except $e_{\theta\phi}$ change in accordance with the ratio of the tidal numbres, l/h. The latitudes in which the strain components $e_{\theta\theta}$ and $e_{\phi\phi}$ of the semi-diurnal and the long period's tides are nil change with the ratio l/h. If we have the latitudinal or azimuthal pattern of the tidal strain, we can obtain the ratio l/h.

In order to compare, Fig. 5 shows the relation of the strain components versus the latitude φ in the tortional type's deformation in which the radial component of the displacement and the cubical dilatation are nil.

Fig. 6 and 8 show these azimuthal patterns of the amplitude and the phase of the semi-diurnal and the long period's components of the tidal extensions versus latitude φ at $h=0.60$, $l=0.08$, $a\frac{\partial h}{\partial r}=-1.40$, and $\delta=0°$, respectively. Fig. 7 shows that

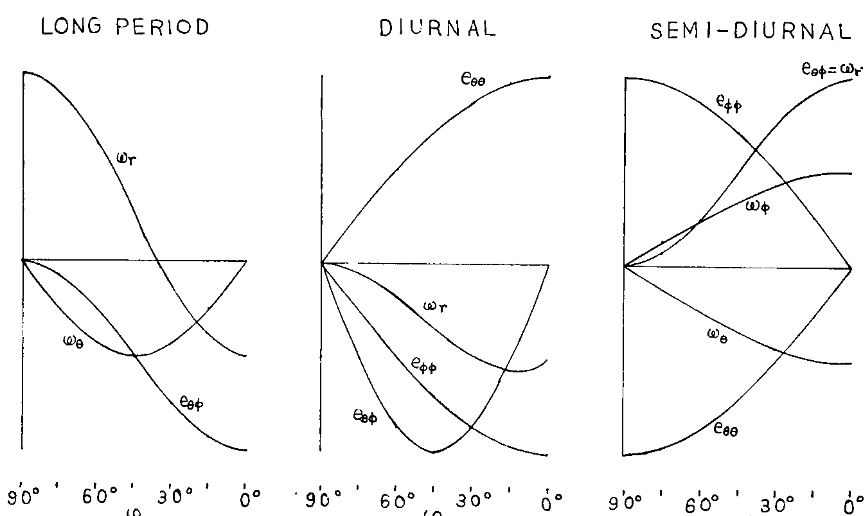

Fig. 5. The components of the strain of the torsional type deformation versus the latitude φ. The radial displacement and the cubical dilatation are nil.

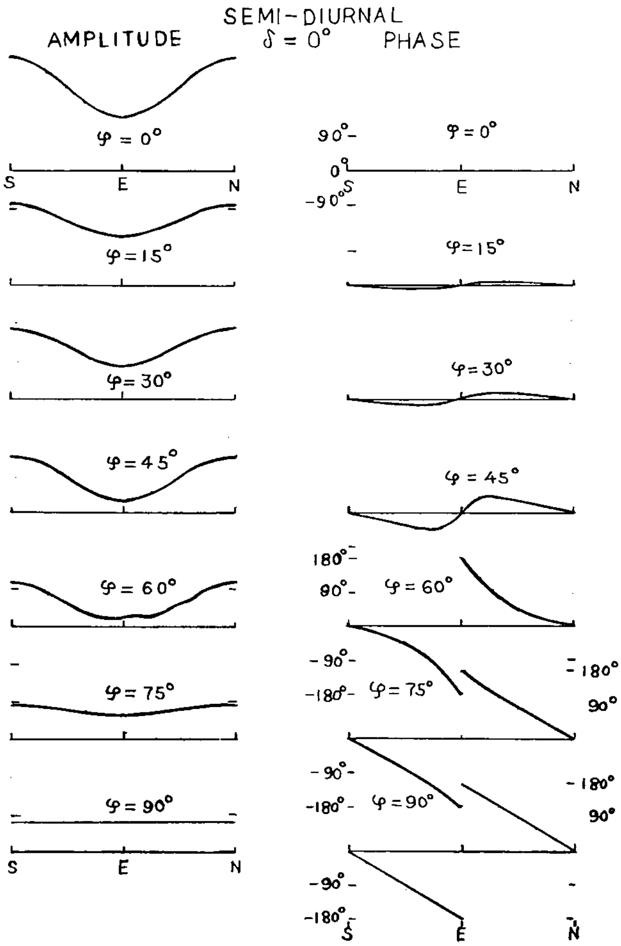

Fig. 6. The azimuthal pattern of the semi-diurnal component of the tidal extension versus the latitude φ, at h=0.60 and l=0.08.

of the diurnal component of the tidal extension at h=0.60, l=0.08, $a\frac{\partial h}{\partial r}=-1.40$, and δ=28.44°.

These azimuthal patterns of the tidal extension change in accordance with the ratio l/h. These azimuthal pattrens of these extensions are not simple. For example, these curves of their amplitudes of these semi-diurnal extensions versus the azimuth on the polar coordinat of two dimensions are ellipse at the low latitude, ovaloid at the middle latitude, elliptic gear at the high latitude, and a circle on the pole at h=0.60 and l=0.08, respectively.

Azimuthal patterns of these tidal displacements on the semi-diurnal component in the various latitude are shown in Figs. 9 and 10.

In the semi-diurnal component, curves of their amplitude pattrens versus the

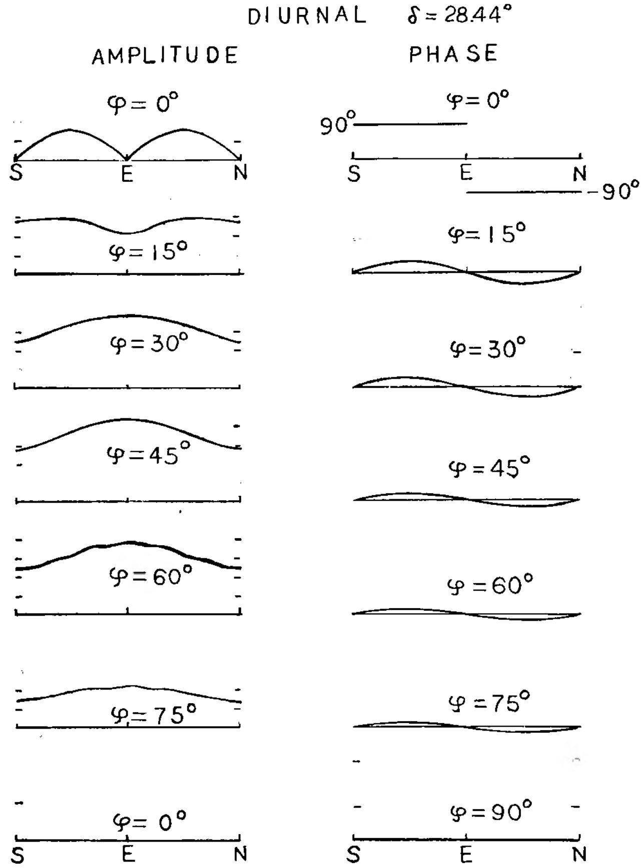

Fig. 7. The azimuthal pattern of the diurnal component of the tidal extension versus the latitude, at h=0.60 and l=0.08.

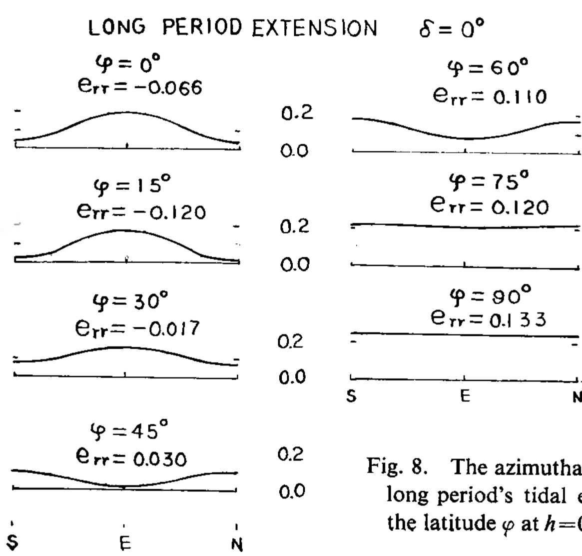

Fig. 8. The azimuthal pattern of the long period's tidal extension versus the latitude φ at h=0.60 and l=0.08.

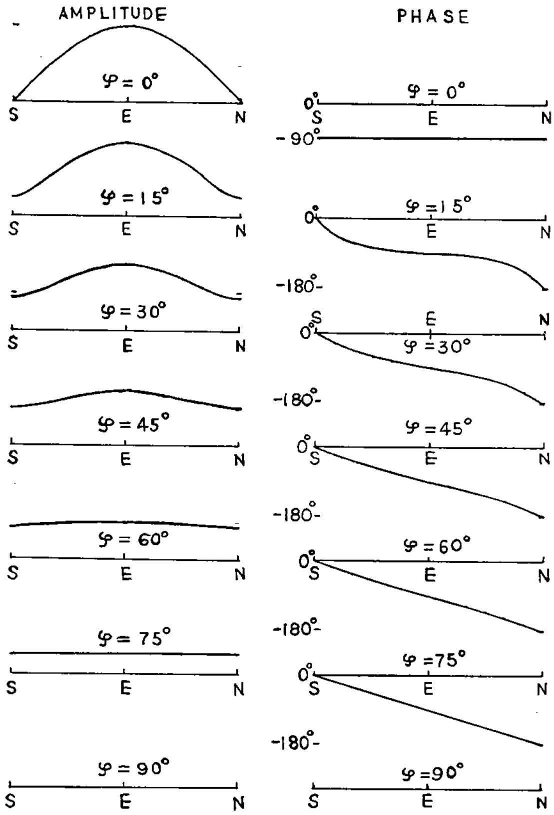

Fig. 9. The azimuthal pattern of the semi-diurnal component of the tidal displacement versus the latitude φ.

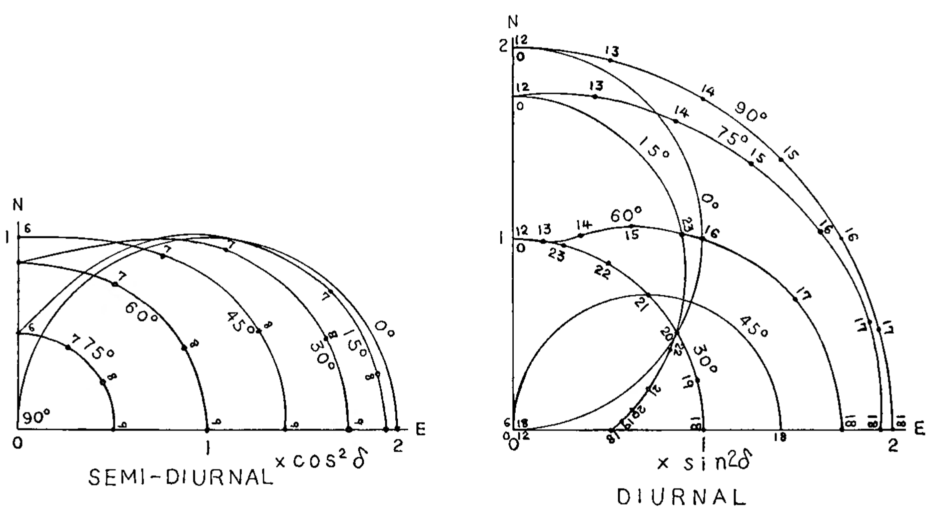

Fig. 10. The azimuthal pattern of the semi-diurnal and the diurnal components of the tidal displacement versus the latitude.

azimuth on the polar coordinate are a point at the pole, ellipse in the high latitude which are higher than 45°, ovaloid at the low latitude lower than 45°, and two circles contacted each other in the meridional direction at the equator, respectivly.

In the diurnal component, curves of their amplitude patterns versus the azimuth on the polar coordinate are a circle at the pole, two circles contacted in the prime vertical direction at the equator, two circles contacted in the meridional direction at the latitude of 45°, and ovaloid at the other latitude, respectively.

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