Contributions, Geophysical Institute, Kyoto University, No. 14, 1974

# DISPERSION DUE TO THE RESIDUAL FLOW IN THE HYDRAULIC MODEL

By

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(Received August 31, 1974)

**Abstract**

The dispersion phenomena due to the tidal current are studied in a hydraulic model. A semidiurnal tide is provided for the model of Mikawa Bay, with horizontal and vertical scales of 1/2000 and 1/160, respectively. Effects of density stratification and wind are not considered. The diffusion of dyed water discharged continuously from a point source is investigated by a dye concentration analysis. Patterns of dye spreading are highly affected by the tidal residual flow. The one-dimensional diffusivity calculated from the distribution of the dye concentration is about $2.7\times10^6$ cm$^2$/sec in Mikawa Bay. It is in the same order as that of the dispersion coefficient due to the tidal residual flow.

## 1. Introduction

The distribution of a conservative material in an estuary is affected by both the advection and the diffusion. In a one-dimensional model, the mass transport by the fresh water discharge is considered to be the advective term, and that by the tidal current and other motions to be the diffusive term. With this method the magnitude of the diffusivity can be estimated, but the mechanism of the diffusion cannot be clarified.

In order to clarify the physical mechanism of the diffusion, the idea of the dispersion has been proposed. The basic principles of dispersion in shear flow were pointed out by Taylor [1954] in a study of dispersion in pipes. Taylor's analysis was applied to open channel flow by Elder [1959] and Fischer [1967]. Elder's result with some modifications was applied in estuaries by Bowden [1965] and Harleman [1966].

Fischer [1972] studied the Mersey Estuary and showed theoretically that the net vertical circulation does not induce a dispersion coefficient of the observed magnitude and transverse circulations are likely to be more important than vertical ones.

It is difficult, however, to measure the velocities at enough points to detect circulations in a real estuary. While, in a hydraulic model various measurements are relatively easy. So model experiments are useful to get the physical aspects of this problem.

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## 2. Prototype of Mikawa Bay

Mikawa Bay is encircled by the Chita and the Atsumi Peninsula and opens its mouth to the south-east part of Ise Bay as shown in Fig. 1. The tidal waves from the Pacific go through the Irako Channel and enter into Mikawa Bay.

The semidiurnal tide is dominant in Mikawa Bay. Spring tidal range is 1.3 m at Akabane facing to the Pacific and 1.9 m at Mikawa Harbour located at the bottom of Mikawa Bay. The latter is 50% greater than the former. The phase lag of the semidiurnal tide is 12° (24 min.) from the outer part of the bay to the inner.

According to the field observations, the tidal current is dominant in Mikawa Bay. The maximum velocity at the flood appears 3 hours before the high water, and that at the ebb 3 hours after the high water at Katahama.

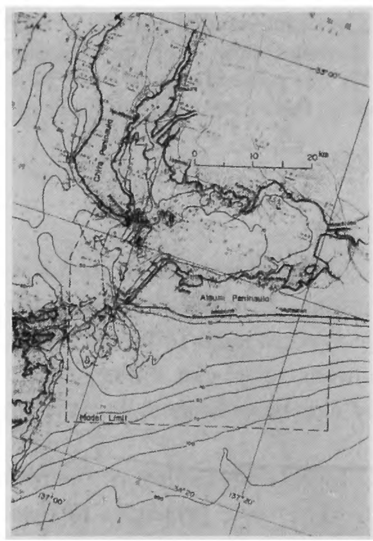


Fig. 1. Bathymetric chart of Mikawa Bay.

## 3. Model of Mikawa Bay

In the hydraulic model experiment for the tidal current, it is necessary for the following equations to be valid in order to hold a dynamical similitude between the prototype and the model;

$$t_r = x_r\, h_r^{-1/2} \tag{1}$$

and

$$n_r = x_r^{-1/2}\, h_r^{2/3}, \tag{2}$$

where $x$ is the horizontal length, $h$ the vertical length, $t$ the time, $n$ Manning's roughness coefficient, and the suffix $r$ denotes the ratio of the quantity in the prototype to that in the model.

When the flow in the model belongs to the turbulent regime the 4/3 power low, $K=\varepsilon^{1/3}L^{4/3}$ may hold, where $\varepsilon$ is the rate of energy dissipation. On the other hand, the ratio of the diffusivity $K_r$ is expressed by $K_r=x_r^2 t_r^{-1}$ from the dimensional analysis. Equating both diffusivities under the condition $L_r=x_r$, we get

$$U_r = \varepsilon_r^{1/3}\, x_r^{1/3}. \tag{3}$$

Assuming $\varepsilon_r=1$, the following equations are obtained from equations (1), (2) and (3),

$$h_r = t_r = x_r^{2/3}, \tag{4}$$

$$n_r = x_r^{-1/18} \tag{5}$$

As to the horizontal diffusivity, we get

$$K_r = x_r^{4/3}. \tag{6}$$

We use $x_r=2000$, $h_r=160$ and $t_r=160$, considering the capasity of the laboratory and the length of the tidal locus. The model is illustrated in Fig. 2, where the measuring points of the water level are shown by the mark $\otimes$ numbering from 1 to 12.

The tide and tidal current are well reproduced. Its details are discussed by Higuchi, et al. [1973]. The flow pattern at the flood in the model is shown in Fig. 3.

## 4. Residual flow

The tidal loci are shown in Fig. 4. In general, a float does not return to the

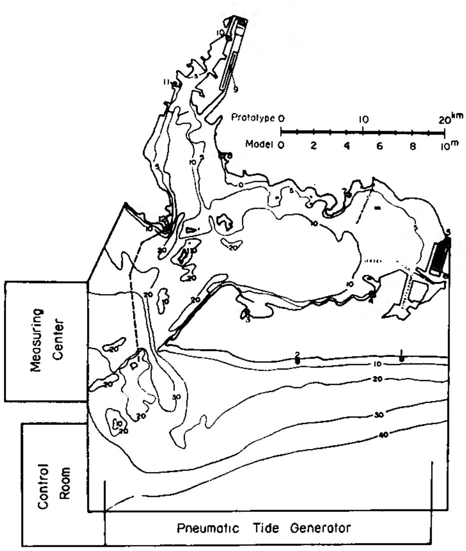


Fig. 2. Model of Mikawa Bay.

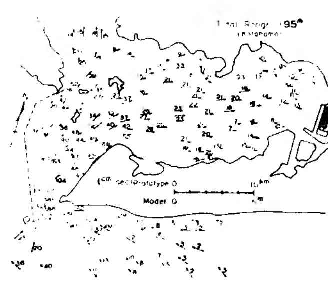

Fig. 3. Flow pattern at the flood (prototype).

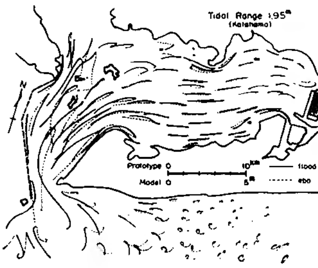


Fig. 4. Tidal loci (model).

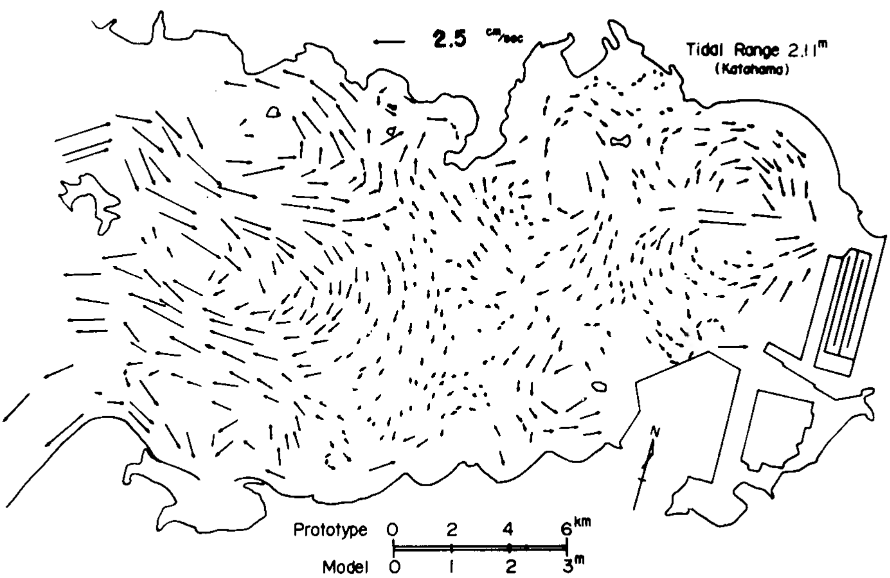


Fig. 5. Residual flow (model).

initial location after one tidal cycle; that is, the tidal locus does not close. This residue is called "the tidal residue", which suggests the existence of the residual flow (constant flow). The residual flow pattern in the model is shown in Fig. 5. The velocity of the residual flow is several percent of the tidal current as being clear in comparing Fig. 3 with Fig. 5. Due to the lack of the data of the prototype, we cannot compare the residual flow in the model with that in the prototype.

The study of a residual flow has not been well developed. The residual flow is thought to be formed by the effect of the configration on the tidal current and the river discharge as suggested by Yamada, et al. [1971].

## 5. The diffusion experiment with a continuous point source

The dyed water, which is colored by fluorescence soda to a concentration of

75 ppm, is discharged continuously at a rate of 0.69 $cm^3/sec$ from the point A in Fig. 6. The spring tide is provided.

At Stns. 7, 11, 21 and 27 we sampled the water column from the surface to the bottom, with a glass tube of 6 mm in inner diameter, at the times of high water and of low water of every fifth cycle, in order to study the time change of dye concentration. At all 27 stations, simultaneous samplings were carried out at the times of flood, high water, ebb and low water, at 80th and 100th cycles after the beginning of discharge, in order to study the horizontal distribution of dye concentration.

The time change of dye concentration is shown in Fig. 7. The abscissa shows the time in tidal cycle, the ordinate the ratio of the dye concentration in per mill (‰) to that of the discharged dyed water. In Fig. 8 the horizontal distribution of dye concentration at 90th tidal cycle is shown, which is obtained by averaging the values at 80th cycle and 100th cycle.

Comparing Fig. 5 with Fig. 8, the distribution of dye concentration seems highly dependant on the tidal residual circulation.

## a. One-dimensional model

We introduce a one-dimensional co-ordinate from the point A to the mouth of the bay as shown in Fig. 9.

Figure 10 shows the averaged dye concentration at sections a, b, c, d, e, and f as shown in Fig. 9. The one-dimensional equation of diffusion is as follows,

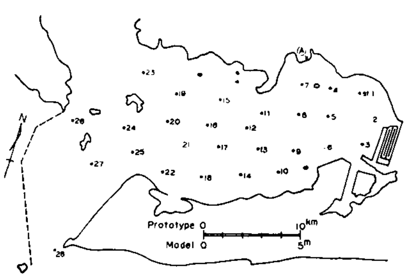


Fig. 6. Stations of discharging and sampling.

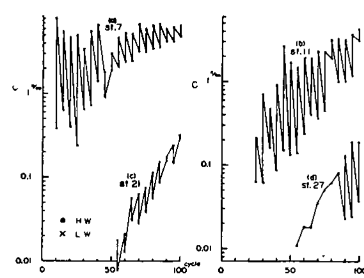


Fig. 7. Time change of dye concentration.

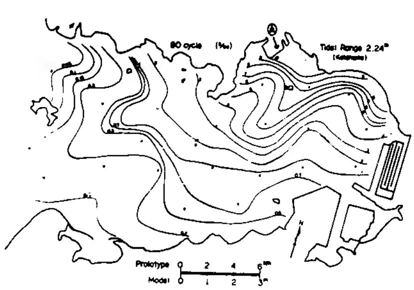


Fig. 8. Horizontal distribution of dye concentration.

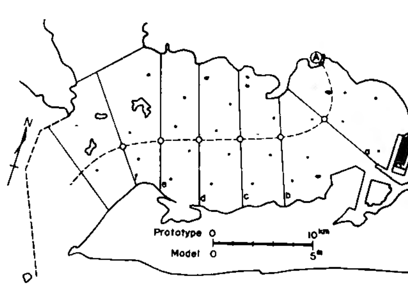


Fig. 9. One-dimensional co-ordinate and the sections used for averaging.

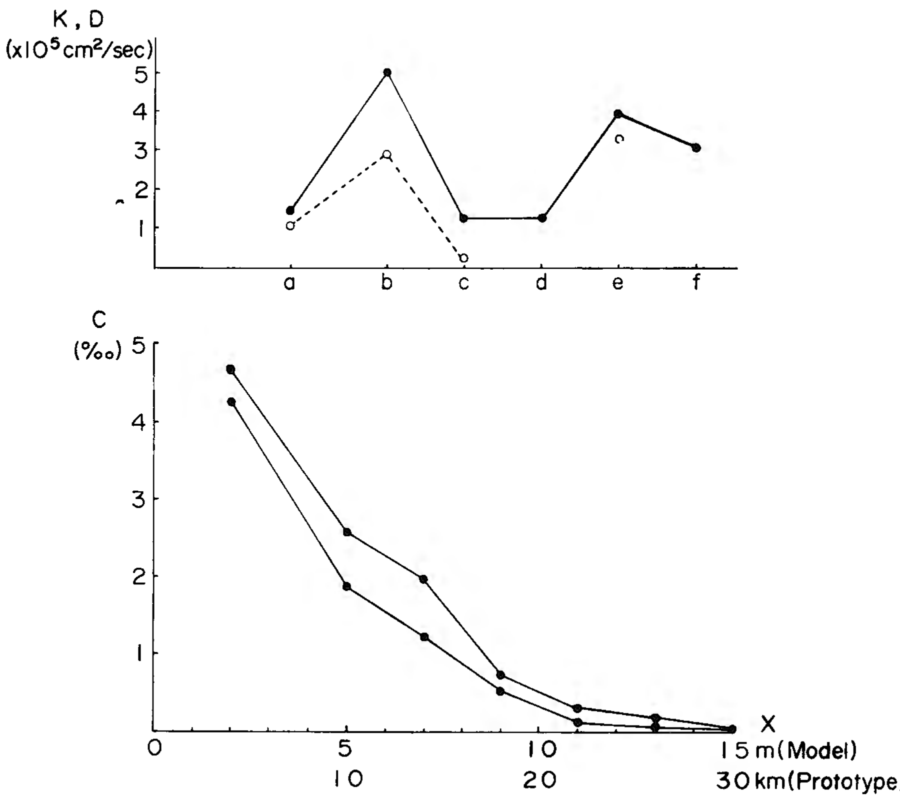


Fig. 10. Upper: diffusivity (real line) and dispersion coefficient (dotted line).
Lower: sectional mean concentration of dye.

$$A\frac{\partial C}{\partial t}+Q\frac{\partial C}{\partial x}=\frac{\partial}{\partial x}\left(A\cdot K\cdot\frac{\partial C}{\partial x}\right), \tag{7}$$

where $A$ is the cross-sectional area, $C$ the dye concentration, $Q$ the net transport of water through the section and $K$ the diffusion coefficient. The diffusion coefficient is expressed in the form of finite-difference as

$$K=\frac{A\cdot\Delta x\cdot\frac{\Delta C}{\Delta t}+Q\cdot\Delta C}{A\cdot\frac{\Delta C}{\Delta x}}. \tag{8}$$

The one-dimensional diffusion coefficient is shown in the upper portion of Fig. 10. By averaging, we obtain the value of $2.7\times10^5$ cm$^2$/sec as the one-dimensional diffusivity in Mikawa Bay.

**b. Two-dimensional model**

In the one-dimensional model, the dye flux by the residual flow and the tidal

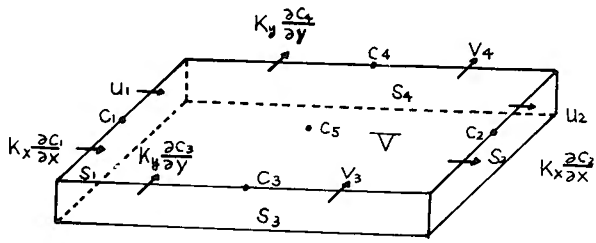


Fig. 11. Box model.

current were considered to be the diffusive term, of which cross-sectional mass transport averages are zero. Here we introduce boxes in the bay as shown in Fig. 11 and consider the dye flux by the residual flow to be the advective term and that by the tidal current to be the diffusive term.

The dimensions of a box are 4 km along the majer($x$) axis of the tidal ellipse and 1.5 km along the minor($y$) axis. In Fig. 11, $C$ is the dye concentration, $S$ the side area, $V$ the volume of the box, $K_x$ and $K_y$ the diffusivity in $x$ and $y$ direction, respectively. The two-dimensional equation of diffusion is as follows,

$$\frac{\partial C}{\partial t} = -\frac{\partial}{\partial x}(uC) - \frac{\partial}{\partial y}(vC) + \frac{\partial}{\partial x}\left(K_x \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial C}{\partial y}\right). \tag{9}$$

It is expressed in the form of finite-difference as,

$$V\frac{\Delta C_5}{\Delta t} = S_1u_1C_1 - S_2u_2C_2 + S_3v_3C_3 - S_4v_4C_4 + S_1K_x\frac{\Delta C_1}{\Delta x}$$

$$- S_2K_x\frac{\Delta C_2}{\Delta x} + S_3K_y\frac{\Delta C_3}{\Delta y} - S_4K_y\frac{\Delta C_4}{\Delta y}. \tag{10}$$

The unknown factors are $K_x$ and $K_y$ for each box. Their values can be get, if we assume that they, respectively, are the same as those of the neighbouring box. This assumption may be allowable, because the difference of the tidal currents is small in the neighbouring two boxes and so the differences of $K_x$ and $K_y$ in the neighbouring boxes may not be large.

Figure 12 shows the advective and diffusive dye fluxes. In this area the mean water depth is 13 m and the maximum velocity of tidal current is about 20 cm/sec. We obtain $K_x=4.7\times10^4$ cm²/sec and $K_y=1.7\times10^4$ cm²/sec in the central part of the bay. These values are smaller by one order than the one-dimensional diffusivity.

**c. Dispersion model**

According to Fischer's model [1972], we divide the current velocity into 4 parts,

$$u(x, y, t) = u_0(x) + u_1(x, t) + u_s(x, y) + u'(x, y, t), \tag{11}$$

where $u_0$ is the sectional mean velocity of the dyed water, that is $u_0=\langle\bar{u}\rangle$. Here, the overbar shows the cross-sectional average and the angle brackets an average over

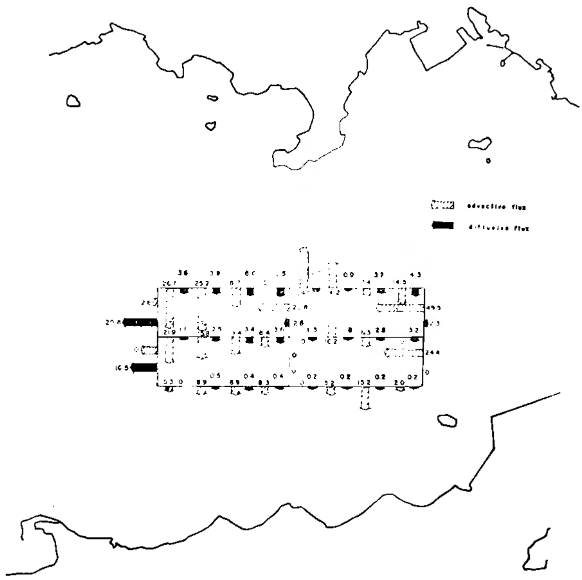

Fig. 12. Dye fluxes in the box model.

one tidal cycle. $u_1=\bar{u}-u_0$, which shows the mean velocity of the tidal current in the cross-section. $u_s$ means the velocity of the residual flow, and $\bar{u}_s=0$. $u'$ is the deviation of the velocity, and $\bar{u}'=<u'>=0$.

The dye concentration is expressed in the same way as the velocity.

$$C(x,y,t) = C_0(x) + C_1(x,t) + C_s(x,y) + C'(x,y,t)\,. \tag{12}$$

The dye flux through a cross-section is expressed as,

$$M = \frac{1}{T}\int^T\int_A (uC)\,dAdT\,. \tag{13}$$

If we assume $<u_1C_1>\approx 0$, the equation (13) is rewriten as follows,

$$M = C_0Q + A_0\,(\overline{u_sC_s} + <\overline{u'C'}>)\,, \tag{14}$$

where $Q$ is the rate of flow of the discharged water, and $A_0$ an averaged cross-sectional area.

The dye flux by the residual flow is shown in Fig. 13. According to Fischer [1972] the dispersion coefficient is as follows,

$$D_1 = \frac{1}{dC_0/dx}\cdot\overline{u_sC_s}\,. \tag{15}$$

The values are shown in Fig. 10 by marks ○ connected with the dotted line. From this figure, it could be concluded that the one-dimensional diffusivity consists chiefly of the dispersion coefficient due to the residual flow.

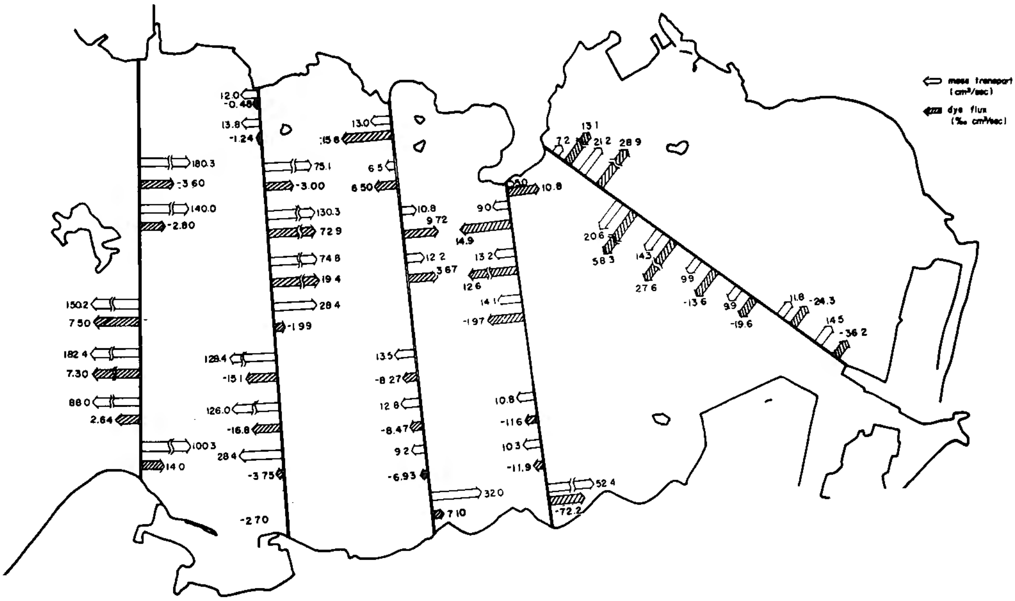


Fig. 13. Mass transport and dye flux by the residual flow.

## 6. Conclusion

In this paper, we have investigated the effect of the tidal residual flow on the dispersion in a shallow tidal bay by means of a hydraulic model experiment of Mikawa Bay.

As a result, the following are revealed,

1) The tide and tidal currents are well reproduced.
2) The distribution of dye concentration is highly affected by the tidal residual flow.
3) One-dimensional diffusivity in Mikawa Bay is $2.7 \times 10^5$ cm²/sec.
4) Two-dimensional diffusivities in the central part of the bay are $K_x = 4.7 \times 10^4$ cm²/sec and $K_y = 1.7 \times 10^4$ cm²/sec.
5) The dispersion due to the tidal residual flow plays a more important role in the distribution of the material in a shallow tidal bay as Mikawa Bay than the diffusion due to the tidal current itself.


### Acknowledgements

The author expresses his sincere thanks to Prof. H. Higuchi of Ehime Univ. and Prof. H. Kunishi of Kyoto Univ. for their encouragements and valuable suggestions and to Mr. M. Kashiwai and Mr. Y. Kitagawa for their assistances in the experiment.

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