Economics Bulletin

Volume 33, Issue 1

Dynamic voluntary advertising and vertical product quality

Yohei Tenryu
Graduate School of Economics, Kyoto University

Keita Kamei
Graduate School of Economics, Kyoto University

Abstract

We investigate the dynamic relationship between advertising and product quality under duopolistic competition. By using a simplified vertical product differentiation model with voluntary advertising, we show that the firm with larger market share has a larger advertising share and that there is a positive relationship between the difference in product quality and the number of customers in an industry.

We are grateful to Akihisa Shibata, Hiroaki Sasaki, and Katsuyuki Naito, as well as the editor and an anonymous referee, for their helpful comments. All errors are our own. Our research is financially supported by Grant-in-Aid for Specially Promoted Research (No. 23000001) and the G-COE program of Osaka University, ``Human Behavior and Socioeconomic Dynamics,'' as well as the Keio-Kyoto joint G-COE program, ``Raising Market Quality-Integrated Design of Market Infrastructure.''
Citation: Yohei Tenryu and Keita Kamei, (2013) "Dynamic voluntary advertising and vertical product quality", *Economics Bulletin*, Vol. 33 No. 1 pp. 564-574.
Contact: Yohei Tenryu - tenryu.yohei.25s@st.kyoto-u.ac.jp, Keita Kamei - keita.kamei@gmail.com.
Submitted: January 12, 2013. **Published:** March 04, 2013.

1 Introduction

Advertising is an important aspect of the behavior of firms, as reported by Huang et al. (2012). In an industry, even though firms compete with one another, they voluntarily advertise to persuade customers to buy their products over others (Friedman (1983); Martin (1993)).[1] This occurs because of the knowledge that there is a positive externality on voluntary advertising; that is, advertising benefits all other firms within an industry that produce the same industrial products. Advertising, therefore, can be interpreted as a public good (Roberts and Samuelson (1988); Piga (1998)). As the number of customers increases, all the firm within an industry increase their profits.

Voluntary advertising is frequently used by emerging industries and those that produce luxury goods. In emerging industries, competition over formats, such as that which occurred between the Blu-ray and HD DVD manufacturers, is common. Firms that produce products with a unique proprietary format use advertising to increase their market size. In industries that produce luxury goods, such as cigarettes, jewels, and brand-name goods, firms advertise in order to persuade new customers to buy their products.

Roberts and Samuelson (1988) and Piga (1998) investigate the relationship between advertising and production quality by using the model of product differentiation within an industry. Roberts and Samuelson (1988) empirically investigate the U.S. cigarette industry. They focused on the tar content in cigarettes and divided the industry into low- and high-tar markets; that is, high-tar cigarettes are produced by high-quality firms and low-tar ones are produced by low-quality firms. They found that, although both high- and low-quality firms advertised independently, advertising by high-quality firms benefits low-quality ones, and vice versa, and that the firms with the larger market share had the larger advertising share. Piga (1998) used the Hotelling (1929) location model and obtained results showing that market and advertising shares are positively correlated[2]. Furthermore, he showed that the industry size increases with the difference in firms' production efficiency.[3]

Although Piga (1998) provided a good explanation of the empirical results, a vertical differentiation model would be more suitable than one for horizontal differentiation for the analysis of emerging industries and those producing luxury goods. Therefore, adopting a simplified vertical product differentiation model, we investigate the dynamic relationship between advertising and production quality under duopolistic competition. The model leads us to confirm that the results obtained by Piga (1998) are robust; that is, we show that the firm with the larger share has the largest advertising share and that there is a positive relation between the difference in product quality[4] and the number of customers

1) Another type of advertising is implemented within an industry. Within the industry, firms produce physically almost identical goods and advertise to increase their shares. There is a considerable amount of literature dealing with this type of advertising. See Dockner et al. (2000) and Colombo and Lambertini (2003).

2) They study the dynamic relationship between advertising and pricing under duopolistic competition. Advertising has the main effect on the increase in the market size, and firms differ in production efficiency.

3) In Piga (1998), the production efficiency depends on the marginal costs.

4) The quality can be regarded as the production efficiency in Piga (1998).

in the industry. Furthermore, we identify some different points in the results obtained by Piga (1998). First, whether or not high-quality firm obtains the larger market share depends on the maximum value of marginal willingness to pay and not on the marginal costs. Second, the increase in quality difference leads to an expansion of advertising by both types of firm. In Piga (1998), on the contrary, when the efficiency difference increases, the larger-share firm increases its advertising but the lower-share firm decreases it.

The structure of the remainder of the paper is as follows. Section 2 contains the basic setup. Section 3 is a derivation of the steady state and analysis of product quality difference; that is, how it affects the total advertising volume, the firm profit, and the total number of customers in the market is analyzed. Section 4 is the conclusion.

2 The Model

A high-quality firm, H and a low-quality firm, L exist in an economy. The high-quality firm produces high-quality goods, and the low-quality firm produces low-quality ones. The technology level of each firm is exogenously given by s_i for $i \in \{L, H\}$ and satisfies the relation, $s_H > s_L > 0$.

Consumers are uniformly distributed along the line with density, N, of consumers and have several preferences for goods, $\theta \in [0, \bar{\theta}]$. This parameter θ represents each consumer's marginal willingness to pay and $\bar{\theta}$ is the maximum value. According to preferences, each consumer is assumed to purchase one unit of goods from either a high-quality firm or a low-quality firm. We assume that the indirect utility function is $u = \theta s_i - p_i$, where p_i is good i's price. Hence, there is a threshold which characterizes the consumer who is indifferent between buying the high-quality goods and buying the low-quality goods.

$$\tilde{\theta} = \frac{p_H - p_L}{s_H - s_L}$$

Consumers who have the preferences, $\theta \in (0, \tilde{\theta})$ buy the low-quality goods, and consumers who have the preferences, $\theta \in (\tilde{\theta}, \bar{\theta})$ buy the high-quality goods. Therefore, there is no consumer who does not buy anything at all.

Using the indirect utility function, we can derive the demand functions,

$$Ny_L = N(\tilde{\theta} - 0) = N\left(\frac{p_H - p_L}{s_H - s_L}\right)(\geq 0), \tag{1}$$

$$Ny_H = N(\bar{\theta} - \tilde{\theta}) = N\left(\bar{\theta} - \frac{p_H - p_L}{s_H - s_L}\right)(\geq 0), \tag{2}$$

where y_L and y_H represent shares where consumers buy low-quality goods and high-quality goods, respectively.

The sum of the discounted present value of the profit for firm i, V_i, is

$$V_i = \int_0^\infty \pi_i(t)e^{-\rho t}dt = \int_0^\infty \left[N(t)y_i(t)(p_i(t) - c_i) - \mu A_i(t)^2\right] e^{-\rho t}dt, \tag{3}$$

where $\pi_i(t)$ is firm i's profit, c_i is the exogenous production cost, $A_i(t)$ is the investment in the advertisement, $\mu A_i(t)^2$ is the investment cost, μ is the exogenous positive parameter, and ρ is discount rate. For simplicity, we assume that the production cost is a linear function with respect to the quality, that is $c_i = s_i$. In the present model, the control variables are the price, $p_i(t)$ and the advertisement, $A_i(t)$, and the state variable is the number of consumers, $N(t)$.

The state variable evolves according to the following state equation,

$$\dot{N}(t) = \alpha(A_H(t) + A_L(t)) - \lambda N(t), \tag{4}$$

where $\alpha > 0$ is the exogenous advertising efficiency parameter and the initial stock $N(0)(> 0)$ is given. $\lambda > 0$ is the depreciation rate, which implies that, if firms do not advertise, consumers lose their interest in the industry goods. This state equation is as in Piga (1998). It implies that advertising is cooperative; the advertising of a firm also benefits the other firm. In other words, the advertising has public good characteristics.

It is noteworthy that the state equation and firms' profit functions are linear with respect to the state variable. In addition, as discussed below, the control variables are independent from the state variable, and open-loop strategies do not depend on the initial state. This is called the linear state game, which has the property that the open-loop equilibrium is Markov perfect.[5)] Therefore, in the analysis below, we use the Hamiltonian function to solve this duopolistic game.

3 Duopoly Equilibrium

Although advertising is cooperative, firms compete in prices. In what follows, we solve each firm's problem, derive the steady-state duopoly equilibrium, and then examine comparative statics.

3.1 The low-quality firm's problem

The low-quality firm's decision follows from maximizing (3) subject to (1) and (4), given the initial stock $N(0)$ and high-quality firm's strategies. The current value Hamiltonian of the low-quality firm is as follows.

$$\mathcal{H}_L = N(t)\left[\frac{p_H(t) - p_L(t)}{s_H - s_L}\right](p_L(t) - s_L) - \mu A_L(t)^2 + \phi_L(t)\left[\alpha(A_H(t) + A_L(t)) - \lambda N(t)\right],$$

5) See Dockner, et al. (2000), section 7.3.

where ϕ_L represents the co-state variable associated with (4). The first-order conditions are the following:

$$p_L(t) = \frac{p_H(t) + s_L}{2}, \tag{5}$$

$$\phi_L(t) = \frac{2\mu}{\alpha} A_L(t), \tag{6}$$

$$\dot{\phi}_L(t) = (\lambda + \rho)\phi_L(t) - \left[\frac{p_H - p_L}{s_H - s_L}\right](p_L(t) - s_L), \tag{7}$$

$$0 = \lim_{t\to\infty} \phi_L(t) N(t) e^{-\rho t}. \tag{8}$$

3.2 The high-quality firm's problem

The high-quality firm's decision follows from maximizing (3) subject to (2) and (4), given the initial stock $N(0)$ and low-quality firm's strategies. The current value Hamiltonian of the high-quality firm is

$$\mathcal{H}_H = N(t)\left[\bar{\theta} - \frac{p_H(t) - p_L(t)}{s_H - s_L}\right](p_H(t) - s_H) - \mu A_H(t)^2 + \phi_H(t)\left[\alpha(A_H(t) + A_L(t)) - \lambda N(t)\right].$$

where ϕ_H represents the co-state variable associated with (4). The first-order conditions are the following:

$$p_H(t) = \frac{\bar{\theta}(s_H - s_L) + s_H + p_L(t)}{2}, \tag{9}$$

$$\phi_H(t) = \frac{2\mu}{\alpha} A_H(t), \tag{10}$$

$$\dot{\phi}_H(t) = (\lambda + \rho)\phi_H(t) - \left[\bar{\theta} - \frac{p_H - p_L}{s_H - s_L}\right](p_H(t) - s_H), \tag{11}$$

$$0 = \lim_{t\to\infty} \phi_H(t) N(t) e^{-\rho t}. \tag{12}$$

To analyze the steady state, we impose the following assumption.

Assumption 1. *The initial co-state variables are assumed to be*

$$\phi_L(0) = \frac{(\bar{\theta} + 1)^2 (s_H - s_L)}{9(\lambda + \rho)}, \quad \phi_H(0) = \frac{(2\bar{\theta} - 1)^2 (s_H - s_L)}{9(\lambda + \rho)}.$$

This assumption guarantees that the transversality conditions for both firms hold.[6)]

6) See Appendix A.

3.3 Optimal Value and Steady-State Value

From (5) and (9), we obtain the equilibrium prices,

$$p_H^* = \frac{2\bar{\theta}(s_H - s_L) + 2s_H + s_L}{3} \tag{13}$$

$$p_L^* = \frac{\bar{\theta}(s_H - s_L) + s_H + 2s_L}{3} \tag{14}$$

These prices do not depend on time; i.e., in equilibrium these values are constant over time. Furthermore, each firm's price depends not only on its own technology but also on that of the opponent.

From (7), (11), and Assumption 1, the co-state variables in equilibrium are constant,

$$\phi_L^* = \phi_L(0) = \frac{(\bar{\theta}+1)^2(s_H - s_L)}{9(\lambda+\rho)}, \quad \phi_H^* = \phi_H(0) = \frac{(2\bar{\theta}-1)^2(s_H - s_L)}{9(\lambda+\rho)}.$$

This immediately leads to equilibrium advertising,

$$A_L^* = \frac{\alpha(\bar{\theta}+1)^2(s_H - s_L)}{18\mu(\lambda+\rho)}, \tag{15}$$

$$A_H^* = \frac{\alpha(2\bar{\theta}-1)^2(s_H - s_L)}{18\mu(\lambda+\rho)}. \tag{16}$$

Advertising by each firm is constant over time. Therefore, it is clear that the dynamical system is described by only (4).

The steady-state stock of the number of consumers is obtained by setting $\dot{N} = 0$:

$$N^* = \frac{\alpha^2(5\bar{\theta}^2 - 2\bar{\theta} + 2)(s_H - s_L)}{18\lambda\mu(\lambda+\rho)}. \tag{17}$$

It is easy to check that the term $5\bar{\theta}^2 - 2\bar{\theta} + 2$ in the numerator is always positive.[7] This implies that the consumer density in the steady state is always positive.

Next, we consider the stability of the system by solving (4). Substituting (15) and (16) into (4), we obtain

$$N(t) = \frac{\alpha^2(5\bar{\theta}^2 - 2\bar{\theta} + 2)(s_H - s_L)}{18\lambda\mu(\lambda+\rho)} + \left[N(0) - \frac{\alpha^2(5\bar{\theta}^2 - 2\bar{\theta} + 2)(s_H - s_L)}{18\lambda\mu(\lambda+\rho)}\right] e^{-\lambda t} \tag{18}$$

This implies that, for any initial stock $N(0)$, N converges to the steady-state N^* due to the positivity of λ. In addition, when the initial stock $N(0)$ is smaller than the steady-state value N^*, N is monotonically increasing until achieving the steady state value. On the other hand, when the initial stock $N(0)$ is larger than N^*, N is monotonically decreasing until achieving the steady state value.

7) Translating the term $5\bar{\theta}^2 - 2\bar{\theta} + 2$ into the standard form, we get $5\left(\bar{\theta} - \frac{1}{5}\right)^2 + \frac{9}{5} > 0$ for all $\bar{\theta}$.

Finally, we compare the profits between both firms. Both firms' lifetime values are

$$V_L = \frac{N(0)(\bar{\theta}+1)^2(s_H - s_L)}{9(\lambda+\rho)} + \frac{\alpha^2(\bar{\theta}+1)^2(3\bar{\theta}^2 - 2\bar{\theta}+1)(s_H - s_L)^2}{108\rho\mu\lambda(\lambda+\rho)^2}, \tag{19}$$

$$V_H = \frac{N(0)(2\bar{\theta}-1)^2(s_H - s_L)}{9(\lambda+\rho)} + \frac{\alpha^2(2\bar{\theta}-1)^2(2\bar{\theta}^2+1)(s_H - s_L)^2}{108\rho\mu\lambda(\lambda+\rho)^2}. \tag{20}$$

For (19), the first term in the RHS is positive. Since the term $3\bar{\theta}^2 - 2\bar{\theta} + 1$ is always positive,[8] the second term is also positive. (20) is always positive. Therefore, we obtain the following proposition.

Proposition 1. *If $\bar{\theta} < 2$, the lifetime value and the advertising share of the low-quality firm is larger than those of the high-quality firm . If $\bar{\theta} > 2$, the lifetime value and the advertising share of the low-quality firm is smaller than those of the high-quality firm. If $\bar{\theta} = 2$, both firms have the same lifetime values and advertising shares.*

Proof. See Appendix B. □

This proposition illustrates that the high-quality firm is not always dominant. In the developed markets or countries, people have preferences for higher-quality goods; thus, the high-quality firm can obtain a larger market share and earn more profit than the low-quality firm. Conversely, in the less developed markets or countries people prefer less variety; thus, the low-quality firm can obtain a larger market share and earn more profit.[9]

3.4 Comparative Statics

We now examine how the optimal values change when each firm's technology changes. Firstly, the increases in the quality, s_i for $i \in \{H, L\}$ yield

$$\frac{\partial A_L^*}{\partial s_H} = \frac{\alpha(\bar{\theta}+1)^2}{18\mu(\lambda+\rho)} > 0, \quad \frac{\partial A_H^*}{\partial s_H} = \frac{\alpha(2\bar{\theta}-1)^2}{18\mu(\lambda+\rho)} > 0, \tag{21}$$

$$\frac{\partial A_L^*}{\partial s_L} = -\frac{\alpha(\bar{\theta}+1)^2}{18\mu(\lambda+\rho)} < 0, \quad \frac{\partial A_H^*}{\partial s_L} = -\frac{\alpha(2\bar{\theta}-1)^2}{18\mu(\lambda+\rho)} < 0. \tag{22}$$

This result implies that a change in quality difference has the same effect on the behavior of both the low-quality firm and the high-quality firm. In other words, the increase in s_H always leads to an expansion of advertising by both types of firm. On the other hand, the increase in s_L always leads to a reduction in advertising by both types of firm.

Differentiating (17) with respect to s_i for $i \in \{L, H\}$ yields

$$\frac{\partial N^*}{\partial s_H} = \frac{\alpha^2(5\bar{\theta}^2 - 2\bar{\theta}+2)}{18\lambda\mu(\lambda+\rho)} > 0, \quad \frac{\partial N^*}{\partial s_L} = -\frac{\alpha^2(5\bar{\theta}^2 - 2\bar{\theta}+2)}{18\lambda\mu(\lambda+\rho)} < 0. \tag{23}$$

8) In the same way, we can obtain the standard form, $9\left(\bar{\theta} - \frac{1}{3}\right)^2 + 2 > 0$ for all $\bar{\theta}$.
9) We can easily verify that $y_H^* \gtreqless y_L^*$ if and only if $\bar{\theta} \gtreqless 2$.

This implies that if the difference in firms' technology becomes larger; i.e., if the technology level of the high-quality firm increases, both firms invest in more advertisement and, as a result, can get more consumers. This is illustrated in Figure 1.

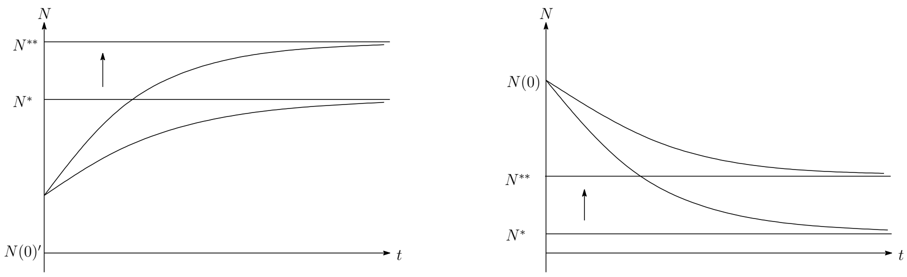

Figure 1: The effects of increase in s_H

In such a situation, we obtain the following proposition.

Proposition 2. *The lifetime values of both firms are increasing functions of the technology of the high-quality firm and decreasing functions of that of the low-quality firm.*

Proof. See Appendix C. □

The proposition implies that both types of firm prefer larger differences of technology between the high-quality firm and the low-quality firm. When the difference becomes smaller, since the monopoly power of the industry becomes small, both firms' incentives to advertise disappear. As a result, the number of consumers decreases, and neither firms increases its earnings. Therefore, the low-quality firm has no incentive to produce higher-quality goods.

4 Conclusion

We investigated the dynamic relationship between advertising and production quality under oligopolistic competition. Using a simplified version of the vertical product differentiation model with voluntary advertising, we confirm that the results obtained by Piga (1998) are robust; that is, we showed that the firm with a larger market share has a larger advertising share and that there is a positive relationship between the difference in product quality and the number of customers. In addition, we found two differences from the results of Piga (1998). First, the shares of high-quality and low-quality firms are determined by the maximum value of marginal willingness to pay and not the marginal costs. Second, a change in quality difference has the same effect on the advertising behavior of both low-quality and high-quality firms.

References

[1] Colombo, L., Lambertini, L. (2003) "Dynamic advertising under vertical product differentiation" *Journal of Optimization Theory and Applications* **119**, 261-280.

[2] Dockner, E., Jorgensen S., Long, N.V., Sorger, G. (2000) *Differential Games in Economics and Management Science*, Cambridge University Press: Cambridge.

[3] Friedman, J.W. (1983) "Advertising and Oligopolistic Equilibrium" *The Bell Journal of Economics* **14**, 464-473.

[4] Hotelling, H.H. (1929) "Stability in Competition" *Economic Journal* **39**, 41-57.

[5] Huang, J., Leng, M., Liang, L. (2012) "Recent Developments in Dynamic Advertising Research" *European Journal of Operations Research* **220**, 591-609.

[6] Martin, S. (1993) *Advanced Industrial Economics*, Blackwell Publishers: Oxford.

[7] Piga, C. (1998) "A Dynamic Model of Advertising and Product Differentiation" *Review of Industrial Organization* **13**, 509-522.

[8] Roberts, J., Samuelson, L. (1988) "An Empirical Analysis of Dynamics, Nonprice Competition in an Oligopolistic Industry" *RAND Journal of Economics* **19**, 200-220.

Appendix A. Transversality Conditions

Using (7), (11), (13), and (14), we solve differential equations for the co-state variables.

$$\phi_L(t) = \frac{x_L}{\lambda+\rho} + \left[\phi_L(0) - \frac{x_L}{\lambda+\rho}\right] e^{(\lambda+\rho)t} \tag{24}$$

$$\phi_H(t) = \frac{x_H}{\lambda+\rho} + \left[\phi_H(0) - \frac{x_H}{\lambda+\rho}\right] e^{(\lambda+\rho)t} \tag{25}$$

where

$$x_L = \frac{(\bar{\theta}+1)^2(s_H - s_L)}{9}, \text{ and } x_H = \frac{(2\bar{\theta}-1)^2(s_H - s_L)}{9}$$

We substitute these equations into (4),

$$\dot{N} = \frac{\alpha^2}{2\mu}\left(\frac{x_L + x_H}{\lambda+\rho} + \left[\phi_L(0) + \phi_H(0) - \left(\frac{x_L + x_H}{\lambda+\rho}\right)\right] e^{(\lambda+\rho)t}\right) - \lambda N(t).$$

Then, we solve this differential equation,

$$N(t) = \left[N(0) - \frac{x_N}{\lambda} - \frac{\phi_N(0) - x_N}{2\lambda+\rho}\right] e^{-\lambda t} + \frac{x_N}{\lambda} + \frac{\phi_N(0) - x_N}{2\lambda+\rho} e^{(\lambda+\rho)t}. \tag{26}$$

where

$$x_N = \frac{\alpha^2}{2\mu}\left(\frac{x_L + x_H}{\lambda+\rho}\right), \text{ and } \phi_N(0) = \frac{\alpha^2}{2\mu}(\phi_H(0) + \phi_L(0)).$$

We now check that the transversality conditions hold. From (8),

$$\begin{aligned}
&\lim_{t\to\infty} \phi_L(t) N(t) e^{-\rho t} \qquad (27)\\
&= \lim_{t\to\infty}\Bigg(M\frac{x_L}{\lambda+\rho}e^{-(\lambda+\rho)t} + M\left(\phi_L(0) - \frac{x_L}{\lambda+\rho}\right) + \frac{x_L x_N}{\lambda(\lambda+\rho)}e^{-\rho t}\\
&+ \frac{x_N}{\lambda}\left(\phi_L(0) - \frac{x_L}{\lambda+\rho}\right)e^{\lambda t} + \frac{x_L}{\lambda+\rho}\left(\frac{\phi_N(0) - x_N}{2\lambda+\rho}\right)e^{\lambda t}\\
&+ \left(\phi_N(0) - \frac{x_N}{2\lambda+\rho}\right)\left(\frac{\phi_N(0) - x_N}{2\lambda+\rho}\right)e^{(2\lambda+\rho)t}\Bigg)
\end{aligned}$$

where

$$M = N(0) - \frac{x_N}{\lambda} - \frac{\phi_N(0) - x_N}{2\lambda+\rho}$$

To hold the transversality condition requires that

$$\phi_L(0) = \frac{x_L}{\lambda+\rho}, \text{ and } \phi_H(0) = \frac{x_H}{\lambda+\rho}. \tag{28}$$

Condition (28) is also required for the transversality condition of the high-quality firm to hold.

Appendix B. Proof of Proposition 1

Subtracting (20) from (19) yields

$$V_L - V_H = (2-\bar{\theta})\left[\frac{N(0)(s_H - s_L)\bar{\theta}}{3(\lambda+\rho)} + \frac{\alpha^2(s_H - s_L)\bar{\theta}(5\bar{\theta}^2 - 2\bar{\theta} + 2)}{108\rho\mu\lambda(\lambda+\rho)^2}\right], \tag{29}$$

and subtracing (16) from (15) yields

$$A_L^* - A_H^* = \frac{\alpha\bar{\theta}(s_H - s_L)}{6\mu(\lambda+\rho)}(2-\bar{\theta}). \tag{30}$$

Since the terms in the square brackets in (29) and the coefficient of $(2-\bar{\theta})$ in (30) are positive, the signs of (29) and (30) depend on the term $(2-\bar{\theta})$. Therefore, if $\bar{\theta} \lesseqgtr 2$, $V_L \gtreqless V_H$ and $A_L^* \gtreqless A_H^*$.

Appendix C. Proof of Proposition 2

We can easily check the signs by differentiating both firms' profit functions with respect to s_i for $i \in \{L, H\}$.

$$\begin{aligned}
\frac{\partial V_L}{\partial s_H} &= \frac{N(0)(\bar{\theta}+1)^2}{9(\lambda+\rho)} + \frac{\alpha^2(\bar{\theta}+1)^2(3\bar{\theta}^2 - 2\bar{\theta} + 1)(s_H - s_L)}{54\rho\mu\lambda(\lambda+\rho)^2} > 0, \\
\frac{\partial V_L}{\partial s_L} &= -\left[\frac{N(0)(\bar{\theta}+1)^2}{9(\lambda+\rho)} + \frac{\alpha^2(\bar{\theta}+1)^2(3\bar{\theta}^2 - 2\bar{\theta} + 1)(s_H - s_L)}{54\rho\mu\lambda(\lambda+\rho)^2}\right] < 0, \\
\frac{\partial V_H}{\partial s_H} &= \frac{N(0)(2\bar{\theta}-1)^2}{9(\lambda+\rho)} + \frac{\alpha^2(2\bar{\theta}-1)^2(2\bar{\theta}^2 + 1)(s_H - s_L)}{54\rho\mu\lambda(\lambda+\rho)^2} > 0, \\
\frac{\partial V_H}{\partial s_L} &= -\left[\frac{N(0)(2\bar{\theta}-1)^2}{9(\lambda+\rho)} + \frac{\alpha^2(2\bar{\theta}-1)^2(2\bar{\theta}^2 + 1)(s_H - s_L)}{54\rho\mu\lambda(\lambda+\rho)^2}\right] < 0.
\end{aligned}$$