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"Trade Structure and Growth Effects of Taxation in a Two-Country World"

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Trade Structure and Growth Effects of Taxation in a Two-Country World*

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Abstract

This paper explores the long-run impacts of tax policy in a two-country model of endogenous growth with variable labor supply. We focus on international spillover effects of tax reforms under alternative trade structures. It is shown that if the instantaneous utility function of the representative family in each country is additively separable and if international capital mobility is absent, then a change in taxation in one country does not directly affect capital formation in the other country. Such a conclusion is fundamentally modified if international lending and borrowing are allowed. In the presence of financial capital mobility, a change in tax policy in one country directly diffuses to the growth performance of the other country, even though preference structures are assumed to be log-additive forms.

keywords: factor-income tax, consumption tax, equilibrium dynamics, two-country model, endogenous growth, variable labor supply

JEL classification: F43, O41

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1 Introduction

It has been well recognized that globalization of an economy significantly alters the effects of tax policy. First of all, in the presence of international trade of goods and services, the tax policy aiming at resource reallocation and income redistribution in the home country may have spillover effects on the economies of trade partners. Additionally, given development of world capital market, factor income taxation in general and capital income tax in particular directly affect investment decisions of foreign firms and households. Since a change in taxation in a country may give rise to relevant international spillover effects, the study on tax policy in a global setting should pay much attention not only to the trade patterns of goods and services but also to the degree of financial integration between the countries.

The central concern of this paper is to explore the relation between trade structure and the outcomes of tax policy in a global economy. We first examine a two-country world where each country engages in free trade of commodities in the absence of financial capital mobility. We then introduce international lending and borrowing into the base model and consider how financial integration affects impacts of fiscal action of an individual country. In the model without international capital mobility, each country produces a country-specific good and exchanges it with the other country's product. If the world capital market exists, in addition to free trade of commodities, the households in the home country freely lend to or borrow from the foreign households. We focus on fiscal interactions between the countries under these alternative settings of international trade.

More specifically, the basic setup of our study is a two-country model of endogenous growth with variable labor supply. We use a two-country version of the endogenous growth model with production externalities first presented Benhabib and Farmer (1994).[1] Our analytical basis is simple enough to treat endogenous growth of a two-country world in a highly tractable manner. The assumption of variable labor-leisure choice, however, provides us with a flexible framework for examining effects of various forms of taxation. We introduce factor income and consumption taxes into the baseline model and explore the effects of taxation in the world economy under alternative specifications of trade structure. In particular, we pay

[1] Behhabib and Farmer (1994) construct exogenous as well as endogenous growth models with external increasing returns. Our model is based on the endogenous growth version of their base model. Amano et al. (2009) explore tax incidence in the Behabib-Farmer model to consider the relation between fiscal outcomes and equilibrium indeterminacy.

our attention to international spillover effects of tax policy.

This paper presents two main findings. First, both domestic and international impacts of tax policy heavily depend on the trade structure. We show that if international lending and borrowing are impossible and if the instantaneous utility function of the representative household in each country takes a log-additive form, then the dynamic behavior of each country is independent of the other country's fiscal policy. Moreover, a change in taxation of one country does not affect the other country's growth performance. In contrast, if international lending and borrowing are allowed, a tax reform in one country directly affects the growth performance of the other country.

Our second finding is that the growth effects of taxation also depend on whether or not the balanced-growth path (BGP) of the world economy is unique. Our model of the world economy has a unique BGP if the degree of external increasing returns in each country is not strong enough. In this case, if there is no financial capital mobility, taxation in one country has a negative growth effect in that country. If financial capital mobility is possible, a change in a tax rate in one country yields complex global effects. The resulting effects on the growth performance of the world economy hinge critically upon the magnitudes of parameters involved in the model. If production external effects are sufficiently strong, then the global economy may have multiple balanced-growth equilibria. If this is the case, regardless of the presence of international financial market, tax policy generally yields qualitatively different effects depending on which BGP is realized. When the economy is on the BGP with a higher growth rate, the effects of tax policy are similar to those obtained in the case of unique BGP. By contrast, we obtain the opposite policy impacts, if the economy stays on the BGP with a lower growth rate. We examine the relation between the policy outcomes and the selection of a particular BGP in detail.

The issue of impacts of tax policy in global settings has attracted considerable attention in the literature. Early contributions such as Ihori (1991), Frenkel and Razin (1989 and 1991), Nielsen and Sørensen (1991) Ono and Shibata (1992), Bianconni and Turnovsky (1991) and Bianconi (1995) analyze two-country dynamic models with perfect financial capital mobility. Most of these studies discuss various effects of factor income taxes and government spending under the source-based principle of capital income taxation. In a similar vein, Lejour and Verbon (1998) examine a two-country growth model where capital is imperfectly mobile. All the contributions mentioned above employ exogenous (neoclassical) growth models where both countries produce homogenous goods. Therefore, in their analyses international lending and borrowing are equivalent to intertemporal trade of goods and services. Additionally, in their models reforms of fiscal policy yield level effects alone in the steady state equilibrium. By contrast, we assume that each country produces a country-specific good, so that our model with financial capital mobility treats intertemporal as well as intratemporal trades between the two countries. Moreover, since our model allows for endogenous growth, we can focus on the growth effects of various forms of taxation.

It is to be noted that Razin and Yuen (1996), Palomba (2007) and Iwamoto and Shibata (2008) explore the fiscal policy impacts in two-country endogenous growth models with capital mobility.[2] Those studies use two-period lived overlapping generations models with AK technologies and fixed labor supply. It is also assumed that both country produce homogeneous goods. Since our model allows intratemporal trade and endogenous labor supply, we can provide a more general analysis on the role of tax policy than the foregoing investigations that employ endogenous growth models.[3]

From the analytical view point, our modelling strategy is closely related to Turnovsky (1997, Chapter 7), Turnovsky (1999 and 2000) and Bianconi (1995). Turnovsky (1997) and Bianconi (2003) explore impacts of fiscal policy in two-country models where each country specializes in a country-specific product under perfect financial capital mobility. Hence, the trade structure of our model is essentially the same as these studies, but they use exogenous growth models so that growth effects of taxation is not discussed in their papers. On the other hand, Turnovsky (1999 and 2000) investigate various fiscal impacts by use of small-country models with endogenous growth and variable labor supply. Our analytical framework is a two, large-country version of the Turnovsky (1999 and 2000).

The reminder of the paper is organized as follows. The next section constructs the base model and examine the effects of tax policy in the absence of financial capital mobility. Section 3 introduces financial capital mobility into the base model to highlight how the policy effects

[2]Lejour and Verbon (1997) examine a two-country endogenous growth model with imperfect capital mobility. They use a representative-agent framework, but they assume that both countries produce homogeneous goods, so that commodity trade is ignored.

[3]Several authors such as Acemoglu and Ventura (2002), Bond et al. (2003) and Farmer and Lahiri (2005 and 2006) construct two (or multi) country, endogenous growth models that are more general than the AK growth model. Those studies, however, do not focus on fiscal policy.

are sensitive to the trade structure of the world economy. A brief concluding remark is given in Section 4.

2 The Model without Capital Mobility

2.1 Model Structure

There are two countries, country 1 and country 2. In each country there is a continuum of infinitely lived identical households with a unit mass. Each country has the same form of production function and the identical preference structure. In this paper we use the simplest representation of Armington's (1969) assumption: each country produces a country-specific, single good.[4] We assume that country 1 specializes in good x and country 2 specializes in good y. Each good can be either consumed or invested for physical capital accumulation. We also assume that the imported goods can be consumed, but they cannot be used for investment.[5] In the baseline model, households in each country cannot access to the international financial market, so that they can neither borrow from nor lend to the foreign households. In Section 4 we relax this restriction and incorporate financial capital mobility into our setting.

Production

The production technology of each country is described by

$$z_i = A k_i^a l_i^{1-a} \bar{k}_i^{\alpha-a} \bar{l}_i^{\beta-1+a}, \quad 0 < a < 1, \quad \alpha > a, \quad \beta > 1-a, \quad i = 1, 2, \tag{1}$$

where z_i, k_i and l_i respectively denote output, capital and labor input of country i. Here, $\bar{k}_i$ and $\bar{l}_i$ express external effects associated with the social levels of capital and labor in country i. The production function (1) means that the private technology under given levels of external effects satisfies constant returns but the social technology exhibits increasing returns with respect to the aggregate levels of capital and labor. We assume that those external effects are country specific so that there is no international spillover of production technologies. In addition, to make endogenous growth possible, we set $\alpha = 1$.[6] Hence, the social production

[4] See also Lloyd and Zhang (2006) for the Arminton's modelling.
[5] This setting is a simplified version of the two-sector model in which one sector produces country-specific trdable consumption goods and the other sector produces nontradable investment goods.
[6] Our formulation of production technology is first presented by Benhabib and Farmer (1994).

function derived by setting $\bar{k}_i = k_i$ and $\bar{l}_i = l_i$ is

$$z_i = Ak_i l_i^{\beta}, \quad i = 1, 2. \tag{2}$$

The commodity and factor markets in both countries are competitive. Firms maximize their instantaneous profits under given levels of external effects of production factors. Thus, letting r_i and w_i be the rate of return to capital and the real wage rate in country i, repectively, they are determined by $r_i = az_i/k_i$ and $w_i = (1-a)\, z_i/l_i$. Hence, the equilibrium levels of the rate of return to capital and the real wage are respectively written as

$$r_i = aAl_i^{\beta}, \quad i = 1, 2, \tag{3}$$

$$w_i = (1-a)\, Ak_i l_i^{\beta - 1}, \quad i = 1, 2. \tag{4}$$

Households

There is a representative household in each country. The households in country i consumes domestic as well as foreign goods and supply l_i units of labor in each moment. The objective functional of the representative household in country i is a discounted sum of utilities over an infinite horizon:

$$U_i = \int_0^{\infty} u\,(x_i, y_i, l_i)\, e^{-\rho t} dt, \quad \rho > 0; \quad i = 1, 2,$$

where x_i and y_i respectively denote consumption of x and y goods. By our assumption, y_1 is imported by country 1 and x_2 is imported by country 2. The instantaneous utility is assumed to be increasing in x_i and y_i, and decreasing in labor l_i. The standard concavity assumption is imposed on $u\,(.)$. We assume that the households in both countries have the same form of utility function and an identical time discount rate, ρ. In this paper we specify the instantaneous felicity function in the following manner:

$$u\,(x_i, y_i, l_i) = \theta \log x_i + (1-\theta) \log y_i - \frac{l_i^{1+\gamma}}{1+\gamma}, \quad 0 < \theta < 1, \quad \gamma > 0. \tag{5}$$

The flow budget constraint for the households in each country is given by

$$\dot{\omega}_i = (1 - \tau_i^r)\, r_i \omega_i + (1 - \tau_i^w)\, w_i l_i - m_i + T_i, \quad i = 1, 2, \tag{6}$$

where ω_i is the asset holding, m_i is real consumption expenditure and T_i denotes the real transfer from the domestic government. In addition, $\tau_i^r \in [0, 1)$ and $\tau_i^w \in [0, 1)$ denote the rates of capital and wage income taxes, respectively. For notational convenience, ω_i, w_i, T_i and m_i are expressed in terms of the good country i produces. Hence, if p denotes the price of good y in terms of good x, then the after-tax consumption spending in both countries are respectively determined by

$$m_1 = (1 + \tau_1^c)\,(x_1 + p y_1),$$

$$m_2 = (1 + \tau_2^c)\,(\frac{x_2}{p} + y_2).$$

In the above, $\tau_i^c \in [0, 1)$ represents the rate of consumption tax levied in country i.

The households' budget should satisfy the non-Ponzi-game scheme, so that it holds that

$$\lim_{t \to \infty} \omega_i\,(t) \exp\left(- \int_0^t (1 - \tau_i^r)\, r_i\,(s)\, ds\right) \geq 0, \quad i = 1, 2.$$

As a result, the following intertemporal budget constraint holds as well:

$$\begin{aligned} &\omega_i\,(0) + \int_0^\infty \exp\left(- \int_0^t (1 - \tau_i^r)\, r_i\,(s)\, ds\right) [(1 - \tau_i^w)\, w_i\,(t)\, l_i\,(t) + T_i\,(t)] dt \\ \geq &\int_0^\infty \exp\left(- \int_0^t (1 - \tau_i^r)\, r_i\,(s)\, ds\right) m_i\,(t)\, dt, \quad i = 1, 2, \end{aligned} \tag{7}$$

where $\omega_i\,(0)$ is the initial wealth holding of country i's households.

The Government

The government in each country neither lends to nor borrows from the domestic as well as foreign households. The government distributes back its total tax revenue to the domestic households. Therefore, the flow budget constraint for the government in country i is given by

$$T_1 = \tau_1^r r_1 \omega_1 + \tau_1^w w_1 l_1 + \tau_1^c (x_1 + p y_1), \tag{8}$$

$$T_2 = \tau_2^r r_2 \omega_2 + \tau_2^w w_2 l_2 + \tau_2^c (\frac{x_2}{p} + y_2). \tag{9}$$

Market Equilibrium Conditions

Since physical capital stocks are not traded, the market equilibrium conditions for the commodity markets are:

$$z_1 = x_1 + x_2 + \dot{k}_1, \tag{10}$$

$$z_2 = y_1 + y_2 + \dot{k}_2. \tag{11}$$

For simplicity, we assume that physical capital in each country does not depreciate.

Since we have assumed that international lending and borrowing are not allowed and that the government of each country runs a balanced budget, the only asset households own is the capital stock in each country. Thus the asset market equilibrium condition in each country is given by

$$\omega_i = k_i, \quad i = 1, 2. \tag{12}$$

Finally, in the absence of international financial market, the trade balance condition in the world market should hold in each moment of time:

$$p y_1 = x_2. \tag{13}$$

Perfect-Foresight Competitive Equilibrium

To sum up, the perfect-foresight competitive equilibrium (PFCE) of the world economy is defined in the following manner:

Definition: *The PFCE of the world economy holds if the following conditions are satisfied:*

(i) The firms maximize instantaneous profits under given levels of external effects, $\bar{k}_i$ *and* $\bar{l}_i$ $(i = 1, 2)$ *.*

(ii) Given rates of τ_i^c, τ_i^r *and* τ_i^w*, the households maximize their discounted sum of utilities under given sequences of prices,* $\{r_i(t), w_i(t), p(t)\}_{t=0}^{\infty}$*.*

(iii) The commodity and asset markets clear in each country and the trade balance condition (13) *holds at each moment of time.*

(iv) The government budget constraints (8) *and* (9) *are fulfilled at each moment of time.*

(v) External effects satisfy the consistency conditions such that $\bar{k}_i(t) = k_i(t)$ *and* $\bar{l}_i(t) =$ $l_i(t)$ for all $t \geq 0$.

2.2 Optimization Conditions

First, consider the following static problem for the representative household in country 1:

$$\max \ \theta \log x_1 + (1-\theta) \log y_1$$

subject to $(1+\tau_1^c)(x_1 + py_1) = m_1$. The resulting optimal choices of x_1 and y_1 are:

$$x_1 = \frac{\theta m_1}{1+\tau_1^c}, \quad y_1 = \frac{(1-\theta) m_1}{(1+\tau_1^c)p}. \tag{14}$$

Using (14) , we drive the instantaneous indirect subutility in such a way that

$$\hat{u}^1(m_1, p) = \log m_1 - (1-\theta)\log p + \theta \log \theta + (1-\theta)\log(1-\theta) - \log(1+\tau_1^c).$$

Similarly, we find that the static demand functions of the country 2's household are:

$$x_2 = \frac{\theta m_2 p}{1+\tau_2^c}, \quad y_2 = \frac{(1-\theta) m_2}{1+\tau_2^c}, \tag{15}$$

implying that the instantaneous indirect subutility of the representative households in country 2 is

$$\hat{u}^2(m_2, p) = \log m_2 + \theta \log p + \theta \log \theta + (1-\theta)\log(1-\theta) - \log(1+\tau_2^c).$$

To derive the optimization conditions for the representative household in each country, we set up the Hamiltonian function in which m_i is the control variable:

$$H_i = \hat{u}^i(m_i, p) - \frac{l_i^{1+\gamma}}{1+\gamma} + q_i\left[(1-\tau_i^r) r_i k_i + (1-\tau_i^w) w_i l_i - m_i + T_i\right], \quad i = 1, 2,$$

where q_i represents the shadow value of net asset evaluated in terms of utility. The necessary conditions for an optimum include the following:

$$1/m_i = q_i, \quad i = 1.2, \tag{16}$$

$$l_i^{\gamma} = q_i \left(1 - \tau_i^w\right) w_i, \quad i = 1, 2, \tag{17}$$

$$\dot{q}_i = q_i \left[\rho - \left(1 - \tau_i^r\right) r_i\right], \quad i = 1, 2, \tag{18}$$

$$\lim_{t \to \infty} q_i e^{-\rho t} k_i = 0, \quad i = 1, 2. \tag{19}$$

In the above (19) is the transversality condition.

Equations (4) and (17) give

$$l_i = \left[\left(1 - \tau_i^w\right)(1 - a) A k_i q_i\right]^{\frac{1}{\gamma + 1 - \beta}}, \quad i = 1, 2. \tag{20}$$

The above relations show that the labor supply in country i is positively (negatively) related to the implicit value of capital, $k_i q_i$, if $\gamma + 1 > \beta$ ($\gamma + 1 < \beta$). The aggregate output of country i is thus expressed as

$$z_i = A k_i \left[\left(1 - \tau_i^w\right)(1 - a) A k_i q_i\right]^{\frac{\beta}{\gamma + 1 - \beta}}, \quad i = 1, 2, \tag{21}$$

and the rate of return to capital is

$$r_i = a A \left[\left(1 - \tau_i^w\right)(1 - a) A k_i q_i\right]^{\frac{\beta}{\gamma + 1 - \beta}}, \quad i = 1, 2. \tag{22}$$

2.3 Dynamic System

From (13) , (14), (15) and (16), the equilibrium price of good y in terms of good x is written a

$$p = \frac{(1 - \theta)\left(1 + \tau_2^c\right) q_2}{\theta \left(1 + \tau_1^c\right) q_1}. \tag{23}$$

Namely, the relative price of good y in terms of good x is proportional to the relative shadow values of capital, q_2/q_1.

Let us denote $k_i q_i = v_i$. Here, v_i represents the utility value of capital in each country. Using this notation, we see that from (10), (11), (14), (15), (16) , (21) and (23), the capital accumulation equations of countries 1 and 2 are respectively rewritten as the following:

$$\frac{\dot{k}_i}{k_i} = A^{\frac{\gamma + 1}{\gamma + 1 - \beta}} \left[\left(1 - \tau_i^w\right)(1 - a) v_i\right]^{\frac{\beta}{\gamma + 1 - \beta}} - \frac{1}{v_{1i}} \frac{1}{1 + \tau_i^c}, \quad i = 1, 2. \tag{24}$$

Using (18) , (22) and (24), we obtain:

$$\dot{v}_i = [1 - a(1 - \tau_i^r)](Av_i)^{\frac{\gamma+1}{\gamma+1-\beta}}[(1 - \tau_i^w)(1 - a)]^{\frac{\beta}{\gamma+1-\beta}} + \rho v_i - \frac{1}{1 + \tau_i^c}, \quad i = 1, 2. \tag{25}$$

Consequently, the differential equations given by (25) constitute a complete dynamic system with respect to v_1 and v_2.

2.4 Balanced-Growth Equilibrium

The balanced growth of the world economy is established when v_1 and v_2 stay constant over time. Due to the assumption of log-additive utility functions, the dynamic behaviors of v_1 and v_2 are independent of each other and the steady-state condition in country i is given by

$$[1 - a(1 - \tau_i^r)](Av_i)^{\frac{\gamma+1}{\gamma+1-\beta}}[(1 - \tau_i^w)(1 - a)]^{\frac{\beta}{\gamma+1-\beta}} = \frac{1}{1 + \tau_i^c} - \rho v_i, \quad i = 1, 2. \tag{26}$$

It is easy to see that the steady-state value of v_i $(i = 1, 2)$ is uniquely given if $\gamma + 1 > \beta$. We also find that if $\gamma + 1 < \beta$, then either there is no steady state or there are dual steady states. Here, we assume that both countries have dual BGPs when $\gamma + 1 < \beta$. To sum up, we may state:

Proposition 1 *Suppose that the instantaneous utility function of the representative family in each country is additively separable between each commodity and labor. If $1 + \gamma > \beta$, the world economy has a unique BGP that satisfies global determinacy. If $1 + \gamma < \beta$, then there may exist four BGPs: one in which both countries grow at a lower rate is locally determinate, while the other three are locally indeterminate.*

Figure 1 depicts the phase diagram of (25) for the case of $1 + \gamma > \beta$ (so that the world economy is globally determinate). If this is the case, the world economy stays on the BGP and it has no transitional dynamics. In contrast, if $1+\gamma < \beta$, then the world economy involves four steady states. As Figure 2 shows, the steady state where both countries attain higher growth rates (lower values of v_1 and v_2) is a sink. The steady state where both countries attain lower growth rates (higher values of v_1 and v_2) is totally unstable and, hence, it exhibits local determinacy. The other two steady states are saddlepoints. These steady states also exhibit local indeterminacy, because the initial values of v_i $(= q_i k_i)$ are not specified in the perfect-foresight competitive equilibrium. Note that in the case of saddlepoint, the stable saddle path restrict the relation between v_1 and v_2 is restricted to the saddles path, the levels of v_1 and v_2 are totally indeterminate around the steady state with a higher growth rate that is a sink.

To explain how indeterminacy emerges on the BGP, following Bennett and Farmer (2000), we focus on the labor market equilibrium condition. Using (4) and $v_i = q_i k_i$, we can rewrite (17) as follows:

$$l_i^{\gamma}/q_i = (1-\tau_i^w)(1-a)\,Ak_i l_i^{\beta-1}. \tag{27}$$

Unlike Bennett and Farmer (2000), in our two-country model, we take the utility value of capital $q_i k_i$ rather than the marginal utility of consumption expenditure, $1/m_i$, as given since the capital value v_i is constant along the BGP. Using $(1-a)Ak_i l_i^{\beta-1} = w_i$ and (27) , we obtain

$$\log_i = \gamma \log l_i - \log q_i - \log(1-\tau_i^w), \tag{28}$$

which can be viewed as the Frisch labor supply curve in country i expressed by the utility value q_i. Notice that the elasticity of this labor supply curve in country i, evaluated on the BGP, is given by

$$\left(\left. \frac{d\log w_i}{d\log l_i} \right|_{l_i=\hat{l}_i} \right)^S = \gamma > 0. \tag{29}$$

Thus the Frisch labor supply curve has a positive slope.

On the other hand, equation(4) presents the labor demand curve in such a way that

$$\log w_i = \log(1-a)\,A + \log k_i + (\beta-1)\log l_i, \tag{30}$$

Under a given level of k_i, the slope of the labor demand curve evaluated on the BGP is

$$\left(\left. \frac{d\log w_i}{d\log l_i} \right|_{l_i=\hat{l}_i} \right)^D = \beta - 1. \tag{31}$$

Accordingly, the difference between (29) and (31) is:

$$\left(\left. \frac{d\ln w_i}{d\ln l_i} \right|_{l_i=\hat{l}_i} \right)^S - \left(\left. \frac{d\ln w_i}{d\ln l_i} \right|_{l_i=\hat{l}_i} \right)^D = 1 + \gamma - \beta.$$

This implies that if $\gamma + 1 - \beta > 0$, the labor demand curve is less steeper than the Frisch labor supply curve : see Figure 3. In this case the labor supply and demand curves cross with 'normal' slopes. The opposite holds if $\gamma + 1 - \beta < 0$: see Figure 4 where the labor demand curve has a positive slope and it is steeper that the Frisch labor supply curve. Thus Proposition 1 states that indeterminacy of equilibria emerges if the labor supply and demand curves cross with wrong slopes.[7]

Note that from (18) and (23) the relative price p, which represents the terms of trade in our setting, evolves according to

$$\begin{aligned} \frac{\dot{p}}{p} &= (1-\tau_1^r)\, r_1 - (1-\tau_2^r)\, r_2 \\ &= (1-\tau_1^r)\, aA[(1-\tau_1^w)(1-a)\, Av_1]^{\frac{\beta}{1+\gamma-\beta}} \\ &\quad - (1-\tau_2^r)\, aA[(1-\tau_2^w)(1-a)\, Av_2]^{\frac{\beta}{1+\gamma-\beta}}. \end{aligned} \tag{32}$$

Since $-\dot{q}_i/q_i = \dot{k}_i/k_i = \dot{m}_i/m_i$ on the BGP, the above relation means that the steady-state change in the relative price is given by

$$\frac{\dot{p}}{p} = g_1^k - g_2^k, \tag{33}$$

where g_i^k denotes the growth rate of capital of country i in the steady state.

2.5 Growth Effects of Taxation

We are particularly concerned with the growth effects of taxation. Since the balanced-growth rate of capital in each country satisfies $g_i^k (= \dot{k}_i/k_i) = -\dot{q}_i/q_i = \dot{m}_i/m_i$, we obtain

$$\begin{aligned} g_i^k &= (1-\tau_i^r)\, r_i - \rho \\ &= aA\,(1-\tau_i^r)\,[(1-\tau_i^w)(1-a)\, Av_i]^{\frac{\beta}{1+\gamma-\beta}} - \rho, \quad i = 1, 2. \end{aligned}$$

[7] It has been intensively discussed that equilibrium indeterminacy of the Benhabib-Farmer model in small-open economy settings: see, for example, Weder (2000) and Meng and Velasco (2004). Our implication of indeterminacy conditions is similar to that obtained in the small-country models.

This relation gives

$$v_i = \frac{1}{A(1-a)(1-\tau_i^w)} \left[\frac{g_i^k + \rho}{aA(1-\tau_i^r)} \right]^{\frac{1+\gamma-\beta}{\beta}}, \quad i = 1, 2. \tag{34}$$

We see that the implicit value of capital, v_i, in the steady state is positively (negatively) related to the growth rate of capital, g_i^k, if $\gamma + 1 > \beta$ $(\gamma + 1 < \beta)$. Substituting (34) into (26) and arranging terms, we obtain the following:

$$\begin{aligned} &\frac{1 - a(1-\tau_i^r)}{(1-\tau_i^r) aA} (g_i^k + \rho) \\ &= (1-\tau_i^w)(1-a) \frac{1}{1+\tau_i^c} \left[\frac{g_i^k + \rho}{aA(1-\tau_i^r)} \right]^{\frac{\beta-(1+\gamma)}{\beta}} - \frac{\rho}{A}, \quad i = 1, 2. \end{aligned} \tag{35}$$

Figures 5-8 show the graphs of the left-hand and right-hand sides of (35). As shown in Figures 5 and 6, if $1 + \gamma > \beta$, then the graphs have a unique intersection. It is easy to see that a rise in each tax rate yields a downward shift of the locus of the right hand side and thus the balanced-growth rate of capital, g_i^k, will decline. Figures 7 and 8 depict the case of $\gamma + 1 < \beta$. In this case, the growth effect of a change in fiscal action depends on which BGP the economy stays. Since in Figure 7 a rise in τ_i^w or τ_i^c yields a downward shift of the locus of the right hand side, it reduces the balanced-growth rate if the economy is on the BGP with a higher growth rate. However, if the economy stays on the low-growth BGP, then a rise in every tax rate increases the balanced-growth rate.

To sum up, we have shown:

Proposition 2 *In the case of separable utility, if $1 + \gamma > \beta$, then we obtain: $dg_i^k/d\tau_i^r < 0$, $dg_i^k/d\tau_i^w < 0$, and $dg_i^k/d\tau_i^c < 0$. If $1 + \gamma < \beta$, then it holds that $dg_i^k/d\tau_i^w < 0$ and $dg_i^k/d\tau_i^c < 0$ if the economy stays on the BGP with a higher growth rate, while $dg_i^k/d\tau_i^w > 0$ and $dg_i^k/d\tau_i^c > 0$ on the BGP with a lower growth rate.*

The economic intuition for Proposition 2 is as follows.

(i) The case of $1 + \gamma > \beta$

Consider first the case where $1 + \gamma > \beta$. A rise in the after-tax price in the home country, $(1+\tau_1^c)p$, caused by an increase in τ_1^c, makes the consumption goods relatively more expensive than leisure.[8] This encourages households to raise demand for leisure and reduce labor supply. As a result, the level of employment declines when the Frisch labor supply curve is steeper than the labor demand curve. Since in our setting a lower employment depresses the rate of capital accumulation, which leads to a decline in the long-run growth rate in the home country. Graphically, this situation is depicted by a downward shift of the RHS of (35) in Figure 5.

If the government of the home country raises the tax rate on wage income, τ_1^w, then the after-tax real wage rate in the home country decreases. Since this lowers the opportunity costs of leisure, the substitution effect increases leisure and depresses labor supply. When $1+\gamma > \beta$ (so that the Frisch labor supply and demand curves cross with normal slopes), the reduction in labor supply is accompanied by a decrease in the equilibrium level of employment in the home country. Consequently, the long-run growth rate of the home country decreases, which is depicted by the downward shift of the RHS of (35) in Figure 5.

If the rate of tax on capital income in the home country, τ_1^r, rises, then the after-tax rate of return to capital decreases in the home country, which lowers the net return, $(1-\tau_1^r)r_1-\rho$. This causes the LHS of (35) to shift upward in Figure 6. Intuitively, the reduction in the after-tax rate of return raises the intertemporal price of future consumption, $1/[1+(1-\tau_1^r)r_1]$, so that future consumption becomes relatively expensive. As a consequence, the household increases current consumption and reduces saving. This impact depresses capital accumulation, which leads to a decline in the long-run growth rate in the home country. In addition, the lower after-tax rate of return to capital reduces the rate of change in the terms of trade, p, in (32). The resulting higher price of the good produced in the home country discourages the demand for that good and the growth rate of output of country 1 is depressed. This is shown by a downward shift of the RHS of (35) in Figure 6.

We should notice that, under the assumption in which the instantaneous utility function of the representative household is additively separable between consumption and labor supply, an increase in τ_i^c, τ_i^wor τ_i^r only affects the own country's capital value v_i and thus l_i. This arises because their impacts on the other country's capital value v_j are completely offset by the changes in the relative price p in (23).

(ii) The case of $1+\gamma < \beta$

[8] In what follows, we call country 1 (country 2) the home country (the foreign country).

Remember that in this case of $1 + \gamma < \beta$, the Frisch labor supply and labor demand curves cross with wrong slopes. Hence, when a rise in τ_1^c (or a rise in τ_1^w) decreases the household's labor supply, the resulting leftward shift of the Frisch labor supply curve rises the equilibrium level of employment l_1: see Figure 4. Moreover, an increase in τ_1^c (or τ_1^w) leads to the downward shift of the locus of RHS in (35) as illustrated in Figure 7. This changes the growth rate, g_1^k, in the respective steady states. In the low-growth steady state, g_1^k rises as a result of the increase in l_1, while g_1^k falls due to the reduction in l_1 on the BGP with a higher growth rate.

We see that the sign of $dg_i^k/d\tau_i^k$ is ambiguous on both BGPs. Since an increase in τ_1^r reduces the after-tax rate of return to capital in the home country, the LHS of (35) rotates counter-clock wise in Figure 8 under $1 + \gamma < \beta$. On the other hand, the decreased after-tax rate of return to capital reduces $\dot{p}/p$ in (32). The resulting higher future price of the good produced in the home country discourages the demand for that good, which makes the RHS of (35) to shift upward in Figure 8. As shown in the figure, whether g_1^k increases or decreases depends on the size of the relative movement of both loci.

In view of Propositions 1 and 2, we have found the following facts. First, since a change in any tax in one country does not affect the other countries growth performance, the divergence in growth rates between the two countries is absorbed by a change in $\dot{p}/p$. For example, a rise in τ_1^c lowers g_1^k, so that it depresses the rate of change in the terms of trade. Conversely, an increase in τ_2^c raises $\dot{p}/p$ on the BGP.

Second, it is worth emphasizing that the presence of indeterminacy does not alter the policy impacts in our model. As Proposition 2 states, we obtain a negative relation between taxation and long-term growth on the BGP with a higher growth rate that exhibits local indeterminacy. The unconventional policy effect, i.e. a rise in every tax increases the long-run growth rate, is established on the low-growth BGP that satisfies local determinacy. This result stems from the fact that the local indeterminacy emerges only when there are dual BGPs. This prevents us from obtaining a one-to-one correspondence between the comparative statics of policy change and the stability conditions in the balanced-growth equilibrium.

3 Financial Capital Mobility

Thus far we have assumed that households in each country neither borrow from nor lend to the foreign households. In this section we introduce international lending and borrowing into the base model examined in the previous section. We will show that this modification leads to substantial differences in the effects of taxation.[9]

3.1 Model Structure

The basic model structure in the presence of international lending and borrowing is the same as the model without capital mobility. Letting b_i be the stock of traded bonds (international IOUs) held by the households in country i. Here, b_1 is evaluated in terms of good x and b_2 is evaluated by good y. The constraints for the household's optimization problem in each country are now given by the following:

$$\dot{b}_i = (1 - \tau_i^r)(R_i b_i + r_i k_i) + (1 - \tau_i^w) w_i l_i - m_i - I_i + T_i, \quad i = 1, 2,$$

$$\dot{k}_i = I_i, \quad i = 1, 2,$$

where R_i is the real interest rate on b_i and I_i is investment on physical capital. The representative household in country i maximizes U_i subject to the above constraints and the initial holdings of physical capital, $k_i(0)$, and financial asset, $b_i(0)$.

The Hamiltonian function of the optimization problem for the households in country i is set as

$$\begin{aligned} H_i &= \hat{u}^i(m_i, p) - \frac{l_i^{1+\gamma}}{1+\gamma} + q_i \left[(1 - \tau_i^r)(R_i b_i + r_i k_i) + (1 - \tau_i^w) w_i l_i - m_i - I_i + T_i\right] + \lambda_i I_i \\ i &= 1, 2. \end{aligned}$$

The necessary conditions for an optimum include the following:

$$1/m_i = q_i = \lambda_i, \quad i = 1, 2, \tag{36}$$

[9] Hu and Mino (2009) study the effect of international financial integration in a two-county, exogenous growth model with production externalities. Hu and Mino (2013) investigate the same issue in the context of two-country Heckscher-Ohlin model with social constant returns. Our analytical framework used below is a endogenous-growth counterpart of Hu and Mino (2009).

$$l_i^{\gamma} = q_i \left(1 - \tau_i^w\right) w_i, \qquad i = 1, 2, \tag{37}$$

$$\dot{q}_i = q_i \left[\rho - \left(1 - \tau_i^r\right) R_i\right], \quad i = 1, 2, \tag{38}$$

$$\dot{\lambda}_i = \lambda_i \left[\rho - \left(1 - \tau_i^r\right) r_i\right], \qquad i = 1, 2, \tag{39}$$

together with the tansversality conditions

$$\lim_{t \to \infty} e^{-\rho t} q_i k_i = 0; \qquad \lim_{t \to \infty} e^{-\rho t} \lambda_i b_i = 0, \qquad i = 1, 2,$$

and the non-Ponzi-game scheme:

$$\lim_{t \to \infty} \exp\left(-\int_0^t R_i(s)\, ds\right) b_i(t) \geq 0, \qquad i = 1, 2.$$

From (36), (38) and (39), it holds that

$$R_i = r_i, \qquad i = 1,\ 2. \tag{40}$$

The financial integration means that the real rate or return to holding bonds is the same in both countries, so that it always holds that

$$R_1 = R_2 + \frac{\dot{p}}{p},$$

which leads to

$$\frac{\dot{p}}{p} = r_1 - r_2, \tag{41}$$

Namely, the terms of trade between goods x and y varies according to the discrepancy between the real rates of return to capital in both countries.

The equilibrium condition in the bond market is

$$b_1 + p b_2 = 0. \tag{42}$$

Note that the homogeneity of private technologies gives $z_i = r_i k_i + w_i l_i$ $(i = 1, 2)$. Hence, from the flow budget constraints for the households and the government in each country, the dynamic equations of b_1 and b_2 are:

$$\dot{b}_1 = r_1 b_1 + x_2 - p y_1, \tag{43}$$

$$\dot{b}_2 = \left(r_1 - \frac{\dot{p}}{p}\right) b_2 + y_1 - \frac{x_2}{p}. \tag{44}$$

Equations (43) and (44) respectively describe the current accounts of country 1 and 2.[10]

3.2 Dynamic System with Capital Mobility

Conditions (22) and (41) yield:

$$\begin{aligned} \frac{\dot{p}}{p} &= r_1 - r_2 \\ &= aA\left[\left(1-\tau_1^w\right)(1-a)Av_1\right]^{\frac{\beta}{\gamma+1-\beta}} - aA\left[\left(1-\tau_2^w\right)(1-a)Av_2\right]^{\frac{\beta}{\gamma+1-\beta}}. \end{aligned} \tag{45}$$

The capital stock in each country evolves in the following manner:

$$\frac{\dot{k}_1}{k_1} = A^{\frac{\gamma+1}{\gamma+1-\beta}}\left[\left(1-\tau_1^w\right)(1-a)v_1\right]^{\frac{\beta}{\gamma+1-\beta}} - \frac{\theta\left(1/v_1\right)}{1+\tau_1^c} - \frac{\theta\left(1/v_1\right)\left(pq_1/q_2\right)}{1+\tau_2^c},$$

$$\frac{\dot{k}_2}{k_2} = A^{\frac{\gamma+1}{\gamma+1-\beta}}\left[\left(1-\tau_2^w\right)(1-a)v_2\right]^{\frac{\beta}{\gamma+1-\beta}} - \frac{(1-\theta)(1/v_2)}{1+\tau_2^c} - \frac{(1-\theta)\left(1/v_1\right)\left(q_2/pq_1\right)}{1+\tau_1^c}.$$

From (22), (38) and (40) the implicit price of capital in each country follows:

$$\dot{q}_i/q_i = \rho - \left(1-\tau_i^r\right)aA\left[\left(1-\tau_i^w\right)(1-a)Av_i\right]^{\frac{\beta}{\gamma+1-\beta}}, \qquad i=1,2.$$

Now define: $v_i \equiv q_i k_i$ $(i=1,2)$ and $h \equiv pq_1/q_2$. We also denote

$$r_i\left(v_i\right) \equiv aA\left[\left(1-\tau_i^w\right)(1-a)Av_i\right]^{\frac{\beta}{\gamma+1-\beta}}, \quad i=1,2. \tag{46}$$

Then the dynamic equations of p, k_i and q_i displayed above can be summarized as the

[10] Using conditions (43)and (44), we obtain $\dot{b}_1 + p\dot{b}_2 = r_1\left(b_1 + pb_2\right) - \dot{p}b_2$. Thus (42) gives $\dot{b}_1 + p\dot{b}_2 + \dot{p}b_2 = 0$, which is consistent with the bond-market equilibrium condition given by (42).

following complete dynamic system with respect to v_1, v_2 and h:

$$\dot{v}_1 = v_1 \left[\rho + \left(\frac{1}{a} - (1 - \tau_1^r) \right) r_1 (v_1) \right] - \frac{\theta h}{1 + \tau_2^c} - \frac{\theta}{1 + \tau_1^c}, \tag{47}$$

$$\dot{v}_2 = v_2 \left[\rho + \left(\frac{1}{a} - (1 - \tau_2^r) \right) r_2 (v_2) \right] - \frac{(1-\theta)(1/h)}{1 + \tau_1^c} - \frac{1-\theta}{1 + \tau_2^c}, \tag{48}$$

$$\frac{\dot{h}}{h} = \frac{\dot{p}}{p} + \frac{\dot{q}_1}{q_1} - \frac{\dot{q}_2}{q_2} = \tau_1^r r_1 (v_1) - \tau_2^r r_2 (v_2) . \tag{49}$$

In our model, the initial capital stock holdings of both countries, $k_1 (0)$ and $k_2 (0)$, are historically given. However, in the absence of financial frictions in the world bond market, the initial asset positions, $b_1 (0)$ and $b_2 (0)$, may be adjusted instantaneously, so that the initial level of relative price that satisfies the equilibrium condition of the bond market, $b_1 (0) + p (0) b_2 (0) = 0$, is not predetermined. Therefore, all the state variables, $v_1 (t)$, $v_2 (t)$ and $h (t)$, are forward-looking variables, implying that the local determinacy of the equilibrium path near the balanced-growth equilibrium requires that the linearly approximated system of (47), (48) and (49) have three unstable roots. We explore equilibrium determinacy in the next subsection.

3.3 Balanced-Growth Equilibrium

In the balanced-growth equilibrium v_i and h stay constant over time. When $\dot{v}_1 = \dot{v}_2 = \dot{h} = 0$ in (47), (48) and (49), the following conditions hold:

$$\rho + \left[\frac{1}{a} - (1 - \tau_1^r) \right] r_1 (v_1) - \frac{\theta / v_1}{1 + \tau_1^c} - \frac{\theta h / v_1}{1 + \tau_2^c} = 0, \tag{50}$$

$$\rho + \left[\frac{1}{a} - (1 - \tau_2^r) \right] r_2 (v_2) - \frac{(1-\theta)/v_2}{1 + \tau_2^c} - \frac{(1-\theta) (1/v_2 h)}{1 + \tau_1^c} = 0, \tag{51}$$

$$\tau_2^r r_2 (v_2) = \tau_1^r r_1 (v_1) . \tag{52}$$

From (46) condition (52) is rewritten as

$$\tau_2^r \left[(1 - \tau_2^w) (1 - a) A v_2 \right]^{\frac{\beta}{\gamma + 1 - \beta}} = \tau_1^r \left[(1 - \tau_1^w) (1 - a) A v_1 \right]^{\frac{\beta}{\gamma + 1 - \beta}} .$$

This equation yields the following relation between v_1 and v_2:

$$v_2 = \Phi v_1, \tag{53}$$

where

$$\Phi \equiv \left(\frac{\tau_1^r}{\tau_2^r}\right)^{\frac{1+\gamma-\beta}{\beta}} \frac{1-\tau_1^w}{1-\tau_2^w}.$$

Equation (53) shows that in the steady state the value of capital (in terms of utility) of country 2 relative to that of country 1 depends on the relative magnitudes of factor income tax rates in both countries. Such a direct link between the values of capitals held in both countries stems from free mobility of financial asset between the two countries. For example, if $\tau_1^w = \tau_2^w$, then it follows from (53) that

$$\begin{aligned} \text{sign } (v_1 - v_2) &= \text{sign } (\tau_2^r - \tau_1^r) \text{ if } 1+\gamma > \beta, \\ \text{sign } (v_1 - v_2) &= \text{sign } (\tau_1^r - \tau_2^r) \text{ if } 1+\gamma < \beta. \end{aligned}$$

That is, if the rate of tax on wage income is the same in both countries and if labor externalities are small enough to satisfy $1+\gamma > \beta$, then a higher capital income tax in country 1 leads to a lower relative value of capital in country 1. This intuitively plausible result, however, fails to hold, if labor externalities are sufficiently large to satisfy $\beta > 1+\gamma$. Similarly, if $\tau_1^r = \tau_2^r$, then

$$\text{sign } (v_1 - v_2) = \text{sign } (\tau_1^w - \tau_2^w),$$

The balanced-growth rate of capital in country i is given by $g_i^k = (1-\tau_i^r)\, r_i(v_i) - \rho$ and, hence, (52) yields

$$g_1^k - g_2^k = r_1(v_1) - r_2(v_2). \tag{54}$$

It is to be noted that from (41) the relative price on the BGP varies in the following manner:

$$\frac{\dot{p}}{p} = (1-\tau_1^r)\, r_i(v_i) - (1-\tau_2^r)\, r_2(v_2). \tag{55}$$

Again, (52) means that (55) becomes $\dot{p}/p = r_1(v_1) - v_2(v_2)$, so that $\dot{p}/p = g_1^k - g_2^k$. Since this condition is the same as (33), the growth rate and price change differentials on the BGP between the home and foreign countries are characterized in the same manner as in the model without financial integration. However, the presence of financial capital mobility yields the key difference in policy effects that are not observed in the absence of international lending and borrowing: growth performance of each country depends not only on her own tax policy but also on the tax policy of the foreign country. Focusing on this point, we explore growth effects of taxation in the following subsection.

To examine the existence of the balanced-growth equilibrium, it is useful to rewrite (50) as

$$h = \frac{1+\tau_2^c}{\theta}\left[\rho v_1 + \left(\frac{1}{a} - (1-\tau_1^r)\right) r_1(v_1)\, v_1 - \frac{\theta}{1+\tau_1^c}\right].$$

Similarly, equation (51) is rewritten as

$$h = \frac{1-\theta}{1+\tau_1^c}\left[\rho v_2 + \left(\frac{1}{a} - (1-\tau_2^r)\right) r_2(v_2)\, v_2 - \frac{1-\theta}{1+\tau_2^c}\right]^{-1}.$$

In view of (46) and (53), the above equations are respectively expressed in the following manner:

$$\begin{aligned} h &= \frac{1+\tau_2^c}{\theta}\left[\left(\frac{1}{a} - 1 + \tau_1^r\right) a\left[(1-\tau_1^w)(1-a)\right]^{\frac{\beta}{1+\gamma-\beta}} (Av_1)^{\frac{1+\gamma}{1+\gamma\gamma-\beta}} + \rho v_1 - \frac{\theta}{1+\tau_1^c}\right] \\ &\equiv F(v_1), \end{aligned} \tag{56}$$

$$\begin{aligned} h &= \frac{1-\theta}{1+\tau_1^c}\left[\left(\frac{1}{a} - 1 + \tau_2^r\right) a\left[(1-\tau_2^w)(1-a)\right]^{\frac{\beta}{1+\gamma-\beta}} (A\Phi v_1)^{\frac{1+\gamma}{1+\gamma-\beta}} + \rho\Phi v_1 - \frac{1-\theta}{1+\tau_2^c}\right]^{-1} \\ &\equiv G(v_1), \end{aligned} \tag{57}$$

Equations (56) and (57) jointly determine the steady-state values of $v_1\ (= q_1 k_1)$ and $h\ (= pq_1/q_2)$. The steady-state level of $v_2\ (= q_2 k_2)$ is then given by (53).

It is easy to confirm that if $1+\gamma > \beta$, then $F(v_1)$ monotonically increases with v_1, while $G(v_1)$ monotonically decreases with v_1. Thus there is a unique set of steady-state levels of v_1, v_2 and h: see Figures 9 and 10. If $1+\gamma < \beta$, then we find:

$$\lim_{v_1\to 0} F(v_1) = +\infty, \quad \lim_{v_1\to+\infty} F_1(v_1) = +\infty, \quad \lim_{v_1\to 0} G(v_1) = 0, \quad \lim_{v_1\to+\infty} G(v_1) = 0.$$

It is also seen that $F(v_1)$ has a unique minimum and $G(v_1)$ has a unique maximum. Therefore, $F(v_1)$ is a U-shaped function, while $G(v_1)$ is an inverse U-shaped function. Therefore, if it exists, there are dual balanced-growth equilibria: see Figure 11.

As for the local determinacy of the balanced-growth equilibrium, we need to check the local behavior of $(47), (48)$ and (49) around the steady state. The coefficient matrix of the linearized dynamic system is given by

$$J = \begin{bmatrix} \rho + \left(\frac{1}{a} - (1-\tau_1^r)\right)[r_1'(v_1)v_1 + r_1(v_1)] & 0 & -\frac{\theta}{1+\tau_2^c} \\ 0 & \rho + \left(\frac{1}{a} - (1-\tau_2^r)\right)[r_2'(v_2)v_2 + r(v_2)] & \frac{1-\theta}{1+\tau_1^c}\left(\frac{1}{h^2}\right) \\ h\tau_1^r r_1'(v_1) & -h\tau_2^r r_2'(v_2) & 0 \end{bmatrix}. \tag{58}$$

In the above v_1, v_2 and h denote their steady-state values. Appendix A demonstrates that if $1+\gamma > \beta$, all of the characteristic roots of J have positive real parts, so that the unique balanced-growth path of the world economy holds local determinacy. In the case of $1+\gamma < \beta$, the matrix J has at least one stable root and, hence, local indeterminacy always holds, regardless whether or not there is a unique BGP. The following proposition summarizes the characterization of the balanced-growth equilibrium of the world economy:

Proposition 3 *Suppose that international lending and borrowing are allowed. Then if $1+\gamma > \beta$, the world economy has a unique BGP that satisfies local determinacy. If $1+\gamma < \beta$, then the world economy may have dual BGPs, both of which are locally indeterminate.*[11]

3.4 Global Impacts of Taxation

We now investigate impacts of tax policy in the world economy with financial integration. The steady-state conditions displayed above have already suggested that the presence of international lending and borrowing strengthens the spillover effects of tax policy even though we have assumed simple log-additive preferences.

To examine the growth effects of taxation, it is helpful to use (56) and (57). The steady-state level of v_1 is determined by condition $F(v_1) = G(v_1)$. Once the steady-state value of v_1 is given, the corresponding level of v_2 is determined by (53). Thus we may inspect impacts

[11] Remember that in the world economy without financial integration, there may exist four BGPs. The BGP of the world economy with financial capital mobility involves two BGPs at most.

of taxation by observing how changes in tax rates shift the graphs of $F(v_1)$ and $G(v_1)$.

(i) *The case of* $1+\gamma>\beta$

Figures 9 and 10 depict the graphs of $h=F(v_1)$ and $h=G(v_1)$ under $1+\gamma>\beta$. As figures show, in this case there is a unique balanced-growth equilibrium for the feasible region of $h>0$.

First, suppose that country 1 (the home country) raises the consumption tax rate, τ_1^c. Unlike the case without financial capital mobility, this change directly diffuses to the steady-state condition for country 2 (the foreign country), because (57) involves τ_1^c. It is easy to confirm that an increase in τ_1^c shifts the graphs of $h=F(v_1)$ and $h=G(v_1)$ upward and downward, respectively. As a result, the steady-state level of v_1 decreases, while the impact on h is ambiguous. Here, we should note that the balanced-growth rate of capital in each country is respectively given by

$$\begin{aligned} g_1^k &= (1-\tau_1^r)\, r_1(v_1)-\rho=(1-\tau_1^r)\, aA\left[(1-\tau_1^w)(1-a)\, Av_1\right]^{\frac{\beta}{1+\gamma-\beta}}-\rho, \\ g_2^k &= (1-\tau_2^r)\, r_2(\Phi v_1)-\rho=(1-\tau_2^r)\, aA\left[(1-\tau_2^w)(1-a)\, A\Phi v_1\right]^{\frac{\beta}{1+\gamma-\beta}}-\rho. \end{aligned}$$

When deriving the second equation shown above, we use (53). Since a higher τ_1^c lowers the steady-state values of v_1 and v_2 under $1+\gamma>\beta$, the relations between g_i^k $(i=1,2)$ and v_1 given above reveal that a higher τ_1^c depresses the balanced-growth rates of both countries. In the similar manner, a rise in τ_2^c also lowers the growth rates of both countries on the BGP of the world economy.

Intuitively, the initial impact of a rise in τ_1^c on the home country is basically the same as in the model without financial capital mobility: a higher τ_1^c makes consumption goods more expensive than leisure, which encourages the households to increase demand for leisure and reduce labor supply. Since the Frisch labor supply curve is steeper than the labor demand curve under $1+\gamma>\beta$, the equilibrium level of employment falls. This depresses the rate of return to capital, so that the long-run growth rate of the home country declines. Such a conclusion is the same as the model without financial capital mobility. The pivotal difference is that, in the presence of financial capital mobility, the long-run growth rate of the foreign country also falls as a result of an increase in the consumption tax rate in the home country. This is because an increase in the relative price of good x caused by a higher τ_1^c expands import of good y by the home country. To meat such an increase in demand, a higher amount of output of the foreign country used for export and, therefore, capital accumulation in the foreign country falls. Such a negative impact yields a lower growth of the foreign economy. Consequently, a rise in the consumption tax in a country has a global impact.

Next, consider the effects of a change in the wage income tax. A higher τ_1^w reduces the after-tax real wage rate, and thereby the supply of labor decreases. Although v_1 remains unchanged, the level of employment falls due to the decrease in the labor supply under $1+\gamma > \beta$. This reduction depresses output, $z_1 = Ak_1 l_1^{\beta}$, which in turn makes the graph of $h = F(v_1)$ given by (56) shift downward (see Figure 10). As Figure 10 demonstrates, a rise in τ_1^w increases v_1. From (20) and (22) an increase in v_1 $(= k_1 q_1)$ enhances labor supply and thus r_1 rises, but these increases fall short of their original levels. Hence, the new balanced-growth rate of the home country is lower than that in the original one.

On the other hand, the output of the foreign country is also smaller than in the original one, which implies lower employment l_2, so does $r_2 = aAl_2^{\beta}$. As a result, capital accumulation of the foreign country is also discouraged, so that its long-run growth rate declines.

Finally, assume that the home country raises the rate of capital income tax, τ_1^r. When τ_1^r increases, the after-tax rate of return to capital in the home country, $(1-\tau_1^r)r_1$, falls, which leads to an increase in $\dot{q}_1/q_1$ due to (38) and (40). Given $v_1 = q_1 k_1$ (i.e., given r_1), this impact is described by an upward shift of the graph of $h = F(v_1)$ in Figure 9. The lower pre-tax rate of return to capital $r_1(v_1)$ discourages the household's saving in the home country. This depresses the demand for investment goods $\dot{k}_1$. Although q_1 rises, Figure 9 implies that v_1 falls. Recalling that $\dot{k}_1/k_1 = -\dot{q}_1/q_1 = (1-\tau_1^r)r_1(v_1) - \rho$ and that $r_1(v_1)$ increases with v_1, the new balanced-growth rate of the home country is lower than that in the original one.

It is worth emphasizing that when $1+\gamma > \beta$, a higher rate of capital income tax in the home country yields a higher growth of the foreign economy. To see this, note that from (55) the reduction in $r_1(v_1)$ caused by a rise in τ_1^r lowers $\dot{p}/p$. This means that in the future the foreign goods (good y) less expensive relative to the home goods (good x). Hence, , the foreign country will enhance her export, which raises the output of the foreign country. Therefore, the capital accumulation of the foreign country is accelerated and thus the long-run growth rate rises.

(ii) The case of $1+\gamma<\beta$

Figure 11 depicts the case of $1+\gamma<\beta$. Again, there generally exist dual BGPs. We can demonstrate that comparative statics results on the BGP with a lower higher level of v_1 are the same as those in Proposition 3. It is also seen that on the BGP with a higher v_1, most of the results are reversed.

As previously shown, an increase in τ_1^c (or τ_1^w) shifts the Frisch labor supply curve in Figure 4 to the left, which leads to an increase in the level of employment in the case of $1+\gamma<\beta$. The increase in τ_1^c also reduces the consumption demand for domestic output, x_1, as well as that for foreign output, y_1 due to higher prices. As a result, the graph of curve $h=F(v_2)$ shifts upward, while the graph of $h=G(v_1)$ in Figure 11 shifts downward. Figure 11 shows that in the low-growth balanced-growth equilibrium, $v_1=q_1k_1$ falls as a result of a rise in τ_1^c (or τ_1^w). Equation (20) means that the level of employment rises, which accelerates capital accumulation and the rate of growth. This mechanism is exactly the same as that in the low-growth balanced-growth equilibrium without financial integration under $1+\gamma<\beta$ shown by Figure 7.

In contrast, as shown in Figure 11, a higher τ_1^c raises v_1 on the high-growth BGP with a lower v_1. From (20) we find that the level of employment decreases, which reduces the long-run growth rate. This mechanism is precisely the same as that on the high-growth BGP without international lending and borrowing under $1+\gamma<\beta$: see, again, Figure 7.

As for the growth effects of capital-income taxation, if the world economy stays on the BGP where both coutries attain higher growth rates (lower levels of v_1 and v_2), the the growth effects of a change in τ_i^r are the same as the case of $1+\gamma>\beta$: a higher τ_i^r $(i=1,2)$ raises v_1 and v_2, so that the after-tax rate of return to capital of each country is depressed. Hence, a rise in the rate of capital income tax in one country reduces the balanced-growth rates of both countries. On the other hand, a higher τ_i^r decreases the steady-state levels of v_1 and v_2 on the BGP with lower growth rates (higher v_1 and v_2). Since a smaller v_i increases the before rate of return to capital under $1+\gamma<\beta$, a rise in τ_i^r on the after-tax rate of return is ambiguous. Thus the growth effects of a rise in τ_i^r may be negative or positive depending on the parameter magnitude involved in the model.

The following proposition summarizes our findings:

Proposition 4 *Suppose that $1+\gamma > \beta$. Then the BGP of the world economy is uniquely given and the following cross-country and own-country growth effects of tax reforms hold:*

$$\begin{aligned} dg_i^k/d\tau_j^c &< 0, \quad dg_i^k/d\tau_j^w < 0, \quad dg_i^k/d\tau_j^r > 0, \quad i,j=1,2,\ i \neq j, \\ dg_i^k/d\tau_i^c &< 0, \quad dg_i^k/d\tau_i^w < 0, \quad dg_i^k/d\tau_i^r < 0, \quad i=1,2. \end{aligned}$$

Proposition 5 *Suppose that $1+\gamma < \beta$ and that the world economy has dual BGPs. Then on the BGP with higher growth rates of both countries, the following growth effects of taxation hold:*

$$\begin{aligned} dg_i^k/d\tau_i^c &< 0, \quad dg_i^k/d\tau_i^w < 0, \quad dg_i^k/d\tau_j^c < 0, \quad dg_i^k/d\tau_j^w < 0, \\ dg_i^k/d\tau_i^r, \quad dg_i^k/d\tau_j^r &< 0 \quad i,j=1,2. \end{aligned}$$

If the world economy stays on the BGP with lower growth rates of both countries, then the growth effects of taxation are given by

$$\begin{aligned} dg_i^k/d\tau_i^c &> 0, \quad dg_i^k/d\tau_i^w > 0, \quad dg_i^k/d\tau_j^c > 0, \quad dg_i^k/d\tau_j^w > 0, \\ dg_i^k d\tau_i^r &\gtrless 0, \quad dg_i^k/d\tau_j^r \lessgtr 0, \quad i,j=1,2. \end{aligned}$$

4 Conclusion

The central message of this paper is that the growth effects of tax reforms in the global economy heavily depends on the trade structure. By use of a two-country model of endogenous growth with variable labor supply, we have examined the growth effects of taxation under alternative specifications of trade structures. We have shown that the presence of financial capital mobility plays a significant role as to how a change in tax policy in one country affects the other country's growth performance. Our study demonstrates that when inspecting growth effect of fiscal actions in an open-economy setting, we should carefully consider what kind of specification of trade structure can capture the reality well.

Our discussion has relied on specific functional forms of preferences and technologies. To obtain results that are useful policy recommendations, we should use more general modelling than that employed in this paper. As we have seen, even in our simple setting analytical examination of policy impacts are rather complex. This fact suggests that we should explore numerical consideration to generalize our discussion. Such a generalization is an urgent task in our future study.

Appendix A

This appendix confirms the determinacy conditions given by Proposition 3. First, note that from (46) we obtain

$$r'(v_i) = \frac{\beta}{1+\gamma-\beta}\frac{r(v_i)}{v_i}, \quad i = 1, 2.$$

Taking this relation into account, we see that the coefficient matrix J in (58) is expressed as

$$J = \begin{bmatrix} \Gamma_1(v_1) & 0 & -\frac{\theta}{1+\tau_2^c} \\ 0 & \Gamma(v_2) & \frac{1-\theta}{1+\tau_1^c}\left(\frac{1}{h^2}\right) \\ h\tau_1^r r_1'(v_1) & -h\tau_2^r r_2'(v_2) & 0 \end{bmatrix},$$

where v_1, v_2 and h take their steady-state values and

$$\begin{aligned} \Gamma_i(v_i) &\equiv \rho + \frac{1+\gamma}{1+\gamma-\beta}\left[\frac{1}{a} - (1-\tau_i^r)\right] r_i(v_i), \quad i = 1, 2, \\ r_i'(v_i) &= \frac{\beta}{\gamma+1-\beta} aA^{\frac{1+\gamma}{1+\gamma-\beta}} \left[(1-\tau_i^w)(1-a)v_i\right]^{\frac{2\beta-\gamma-1}{1+\gamma-\beta}}, \quad i = 1, 2. \end{aligned}$$

We can confirm that if $1+\gamma > \beta$, then

$$r_i'(v_i) > 0, \quad \Gamma_i(v_i) > 0, \quad i = 1, 2.$$

If $1+\gamma < \beta$, then

$$\begin{aligned} r_i'(v_i) &< 0, \quad i = 1, 2, \\ \text{sign}\, \Gamma_i(v_i) &= \text{sign}\left\{\rho + \frac{1+\gamma}{\gamma+1-\beta}\alpha A^{\frac{1+\gamma}{1+\gamma-\beta}}\left[(1-\tau_i^w)(1-a)\right]^{\frac{\beta}{\gamma+1-\beta}} v_1^{\frac{2\beta-1-\gamma}{1+\gamma-\beta}}\right\}, \quad i = 1, 2. \end{aligned}$$

The characteristic equation of J is written as

$$\phi(\lambda) = \lambda^3 - (\text{Tr } J)\,\lambda^2 + a_2\lambda - \det J = 0,$$

where λ denotes the characteristic root of J and

$$\begin{aligned}
\text{Tr } J &= \Gamma_1 + \Gamma_2, \\
\det J &= \frac{\theta h}{1+\tau_2^c}\Gamma_2 \tau_1^r r_1 + \frac{1-\theta}{1+\tau_1^c}\left(\frac{1}{h}\right)\tau_2^r r_2 \Gamma_1, \\
a_2 &= \Gamma_1\Gamma_2 + \frac{\theta}{1+\tau_1^r} r_1' + \frac{1-\theta}{1+\tau_1^c}\left(\frac{1}{h}\right)\tau_2^r r_2'.
\end{aligned}$$

According to the Routh–Hurwitz criterion, the number of the roots of $\phi(\lambda)$ with positive real parts equals the number of changes in signs of the following sequence:

$$\left\{1,\ -\text{Tr } J,\ a_2 - \frac{\det J}{\text{Tr } J},\quad -\det J\right\}. \tag{59}$$

Note that

$$a_2 - \frac{\det J}{\text{Tr } J} = \frac{1}{\Gamma_1+\Gamma_2}\left[\Gamma_1\Gamma_2(\Gamma_1+\Gamma_2) + \frac{\theta h}{1+\tau_2^c}\tau_1^r r_1' \Gamma_1 + \frac{1-\theta}{1+\tau_1^c}\left(\frac{1}{h}\right)\tau_2^r r_2' \Gamma_2\right].$$

First, suppose that $1+\gamma > \beta$. In this case it holds that $\text{Tr } J > 0$, $\det J > 0$ and $a_2 - \left(\frac{\det J}{\text{Tr } J}\right) > 0$. Hence, the sequence (59) changes signs three times, which means that all of the characteristic roots of J have positive real parts. Since v_1, v_2 and h are unpredetermined variables, this means that the balanced growth path is locally determinate in the case of $1+\gamma > \beta$. On the other hand, if $1+\gamma < \beta$ and if $\Gamma_i' < 0$, then we see that $\text{Tr } J < 0$ and $\det J > 0$. This shows that J has two roots with negative real parts, so that the BGP is locally indeterminate. In addition, if $\Gamma_1 > 0$ and $\Gamma_2 > 0$, then $\text{Tr } J > 0$ and $\det J < 0$. Therefore, J has one negative, real root. Again, there is a continuum of converging paths around the balanced growth equilibrium. As a result, regardless of the number of the BGP, equilibrium indeterminacy holds under $1+\gamma < \beta$.

Appendix B

In Propositions 4 and 5, the magnitudes of own-country and cross-country growth effects of the labor income and consumption taxes are given by the following:

$$\frac{dg_1^k}{d\tau_1^w} = \frac{1}{|D|}\frac{\theta(1-\theta)}{(1-\tau_1^w)(1+\tau_1^c)v_1 h}\left[\frac{1}{(1+\tau_1^c)h}+\frac{1}{1+\tau_2^c}\right]\frac{\beta}{1+\gamma-\beta}(1-\tau_1^r)r_1,$$

$$\frac{dg_1^k}{d\tau_2^w} = \frac{1}{|D|}\frac{\theta(1-\theta)}{(1-\tau_2^w)(1+\tau_2^c)v_2}\left[\frac{1}{(1+\tau_1^c)h}+\frac{1}{1+\tau_2^c}\right]\frac{\beta}{1+\gamma-\beta}(1-\tau_1^r)r_1,$$

$$\frac{dg_1^k}{d\tau_1^c} = \frac{1}{|D|}\frac{\theta(1-\theta)}{(1+\tau_1^c)^2 v_1 h}\left[\frac{1}{(1+\tau_1^c)h}+\frac{1}{1+\tau_2^c}\right]\frac{\beta}{1+\gamma-\beta}(1-\tau_1^r)r_1,$$

$$\frac{dg_1^k}{d\tau_2^c} = \frac{1}{|D|}\frac{\theta(1-\theta)}{(1+\tau_2^c)^2 v_2}\left[\frac{1}{(1+\tau_1^c)h}+\frac{1}{1+\tau_2^c}\right]\frac{\beta}{\gamma+1-\beta}(1-\tau_1^r)r_1,$$

$$\frac{dg_2^k}{d\tau_1^w} = \frac{1}{|D|}\frac{\theta(1-\theta)}{(1-\tau_1^w)(1+\tau_1^c)v_1 h}\left[\frac{1}{(1+\tau_1^c)h}+\frac{1}{1+\tau_2^c}\right]\frac{\beta}{1+\gamma-\beta}(1-\tau_2^r)r_2,$$

$$\frac{dg_2^k}{d\tau_2^w} = \frac{1}{|D|}\frac{\theta(1-\theta)}{(1-\tau_1^w)(1+\tau_1^c)v_2}\left[\frac{1}{(1+\tau_1^c)h}+\frac{1}{1+\tau_2^c}\right]\frac{\beta}{1+\gamma-\beta}(1-\tau_2^r)r_2,$$

$$\frac{dg_2^k}{d\tau_1^c} = \frac{1}{|D|}\frac{\theta(1-\theta)}{(1+\tau_1^c)^2 v_1 h}\left[\frac{1}{(1+\tau_1^c)h}+\frac{1}{1+\tau_2^c}\right]\frac{\beta}{1+\gamma-\beta}(1-\tau_2^r)r_2,$$

$$\frac{dg_2^k}{d\tau_2^c} = \frac{1}{|D|}\frac{\theta(1-\theta)}{(1+\tau_2^c)^2 v_2}\left[\frac{1}{(1+\tau_1^c)h}+\frac{1}{1+\tau_2^c}\right]\frac{\beta}{\gamma+1-\beta}(1-\tau_2^r)r_2,$$

where $|D|$ denotes the determinant of coefficient matrix of the linealized system consisting of (50), (51) and (53). When $(1-\tau_1^r)r_1 > (<)(1-\tau_2^r)r_2$, i.e., $r_1(v_1) > (<)r_2(\Phi v_1)$ on the BGP due to (52), then $dg_1^k/d\tau_1^w > (<)dg_2^k/d\tau_1^w$, $dg_1^k/d\tau_2^w > (<)dg_2^k/d\tau_2^w$, $dg_1^k/d\tau_1^c > (<)dg_2^k/d\tau_1^c$, $dg_1^k/d\tau_2^c > (<)dg_2^k/d\tau_2^c$. It is seen that from (20) and (22) , if $r_1 > r_2$, then $l_1 > l_2$. Given a higher r_1 (i.e. a higher R in (40)), the magnitude of the tax effect on l_1 is larger than that on l_2. Therefore, the growth effect of the tax change in the home country is larger than in the foreign country.

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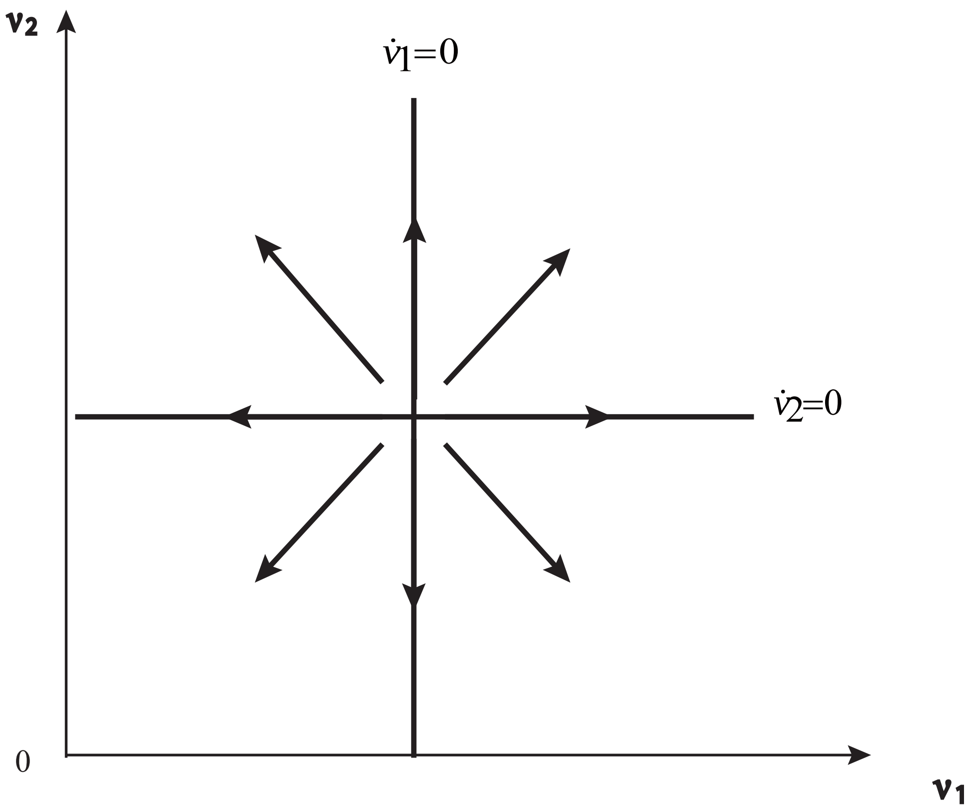

Figure 1: The case of $1+\gamma>\beta$

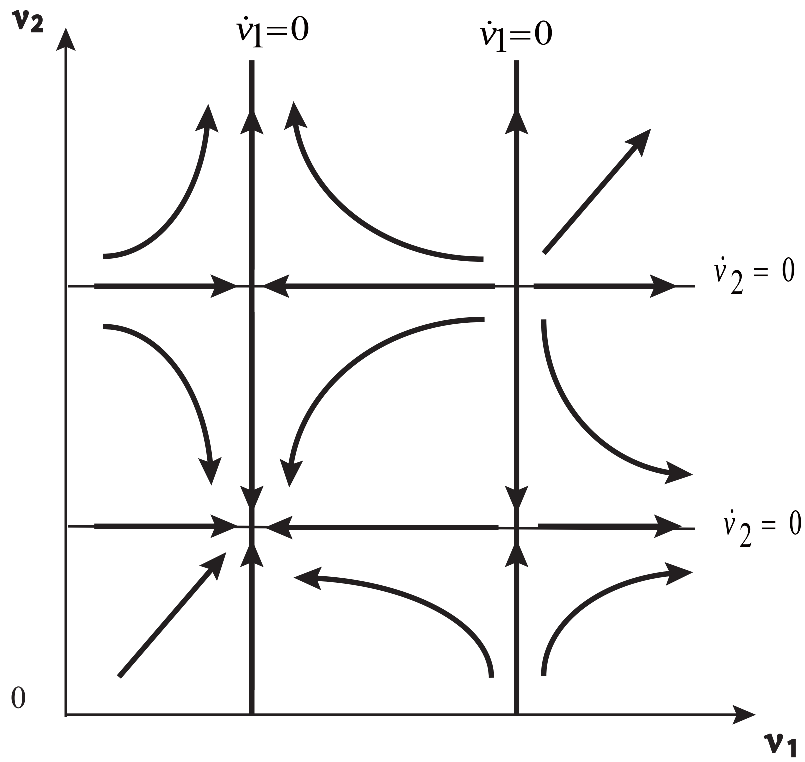

Figure 2: The case of $1+\gamma<\beta$

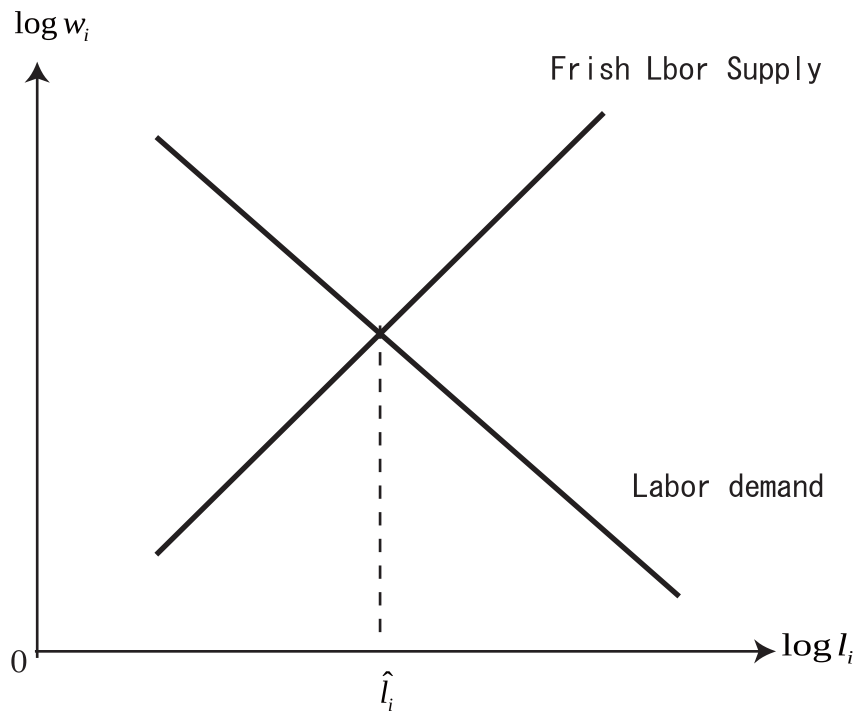

Figure 3: : The case of $1+\gamma>\beta$

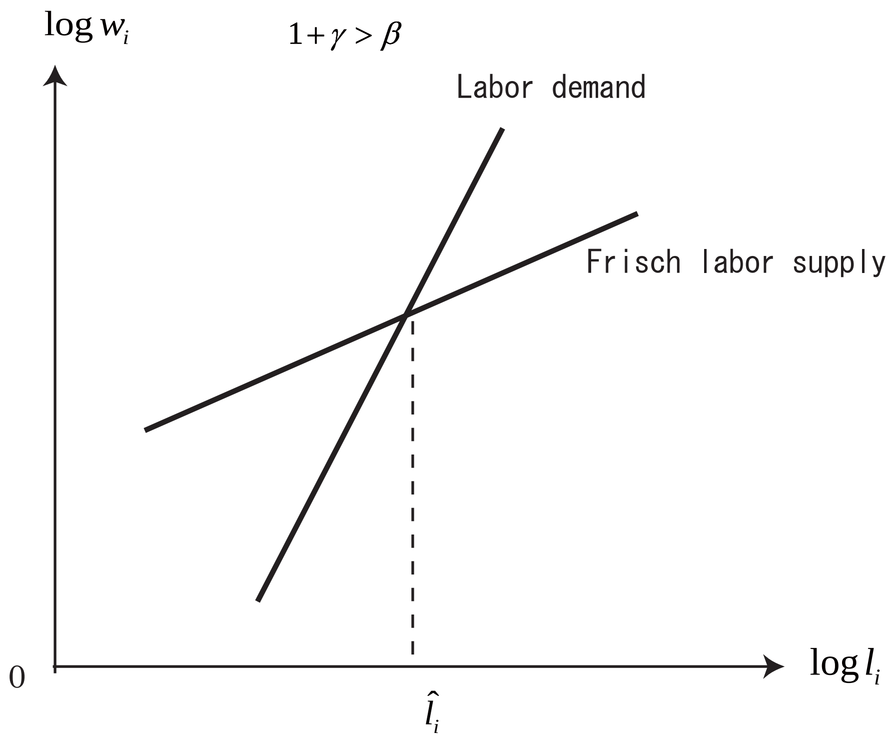

Figure 4: The case of $1+\gamma<\beta$

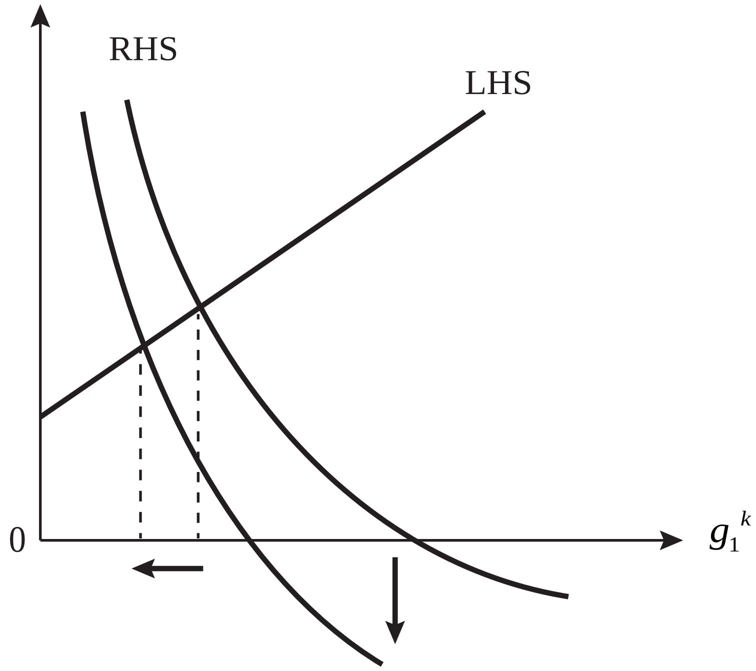

Figure 5: Effects of changes in τ_1^c and τ_1^w under $1+\gamma>\beta$

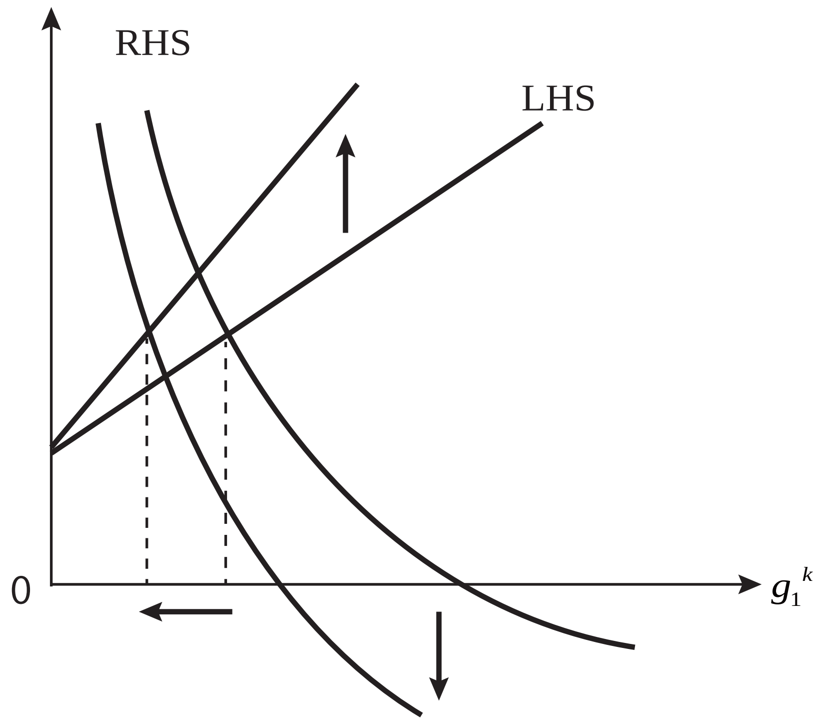

Figure 6: Effect of a change in τ_1^r under $1+\gamma>\beta$

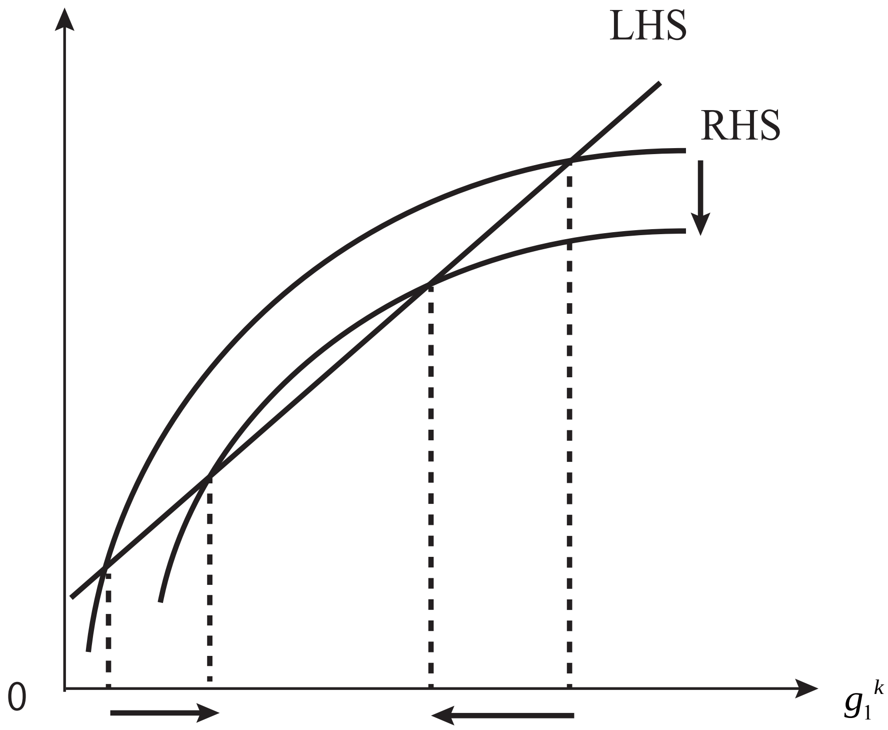

Figure 7: Effects of changes in τ_1^c and τ_1^w under $1+\gamma < \beta$

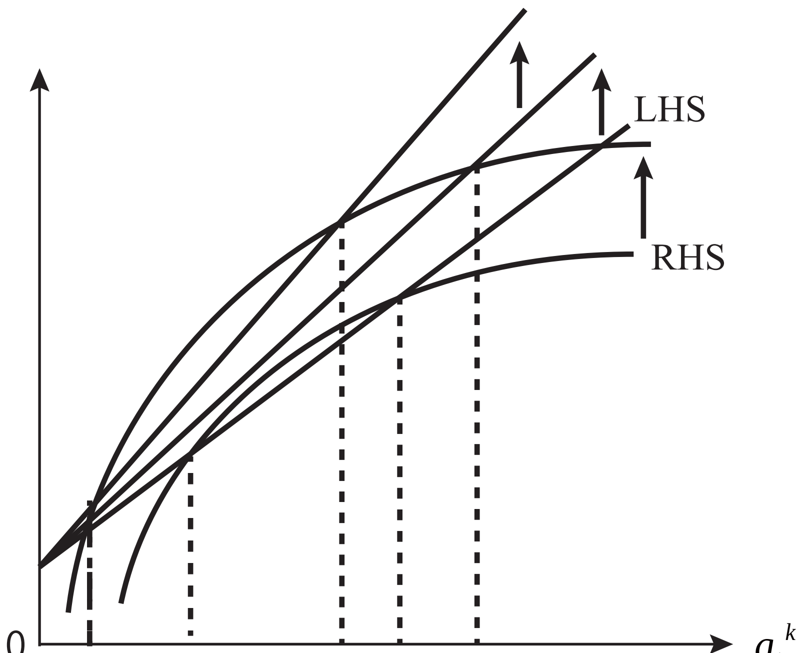

Figure 8: Effect of a change in τ_1^r under $1+\gamma < \beta$

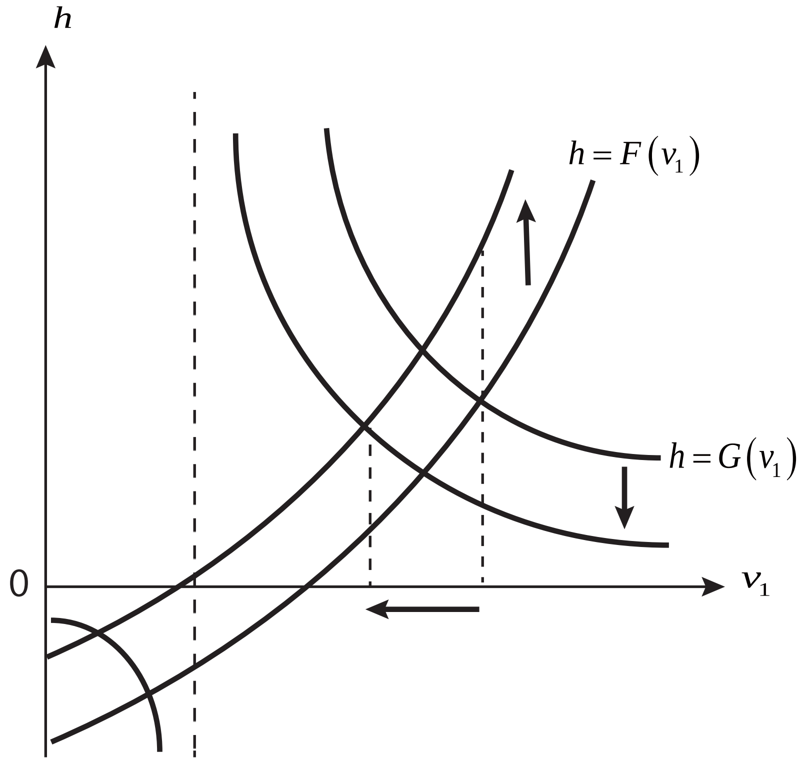

Figure 9: Effects of changes in τ_1^c and τ_1^rchange in τ_1^r under $1+\gamma > \beta$

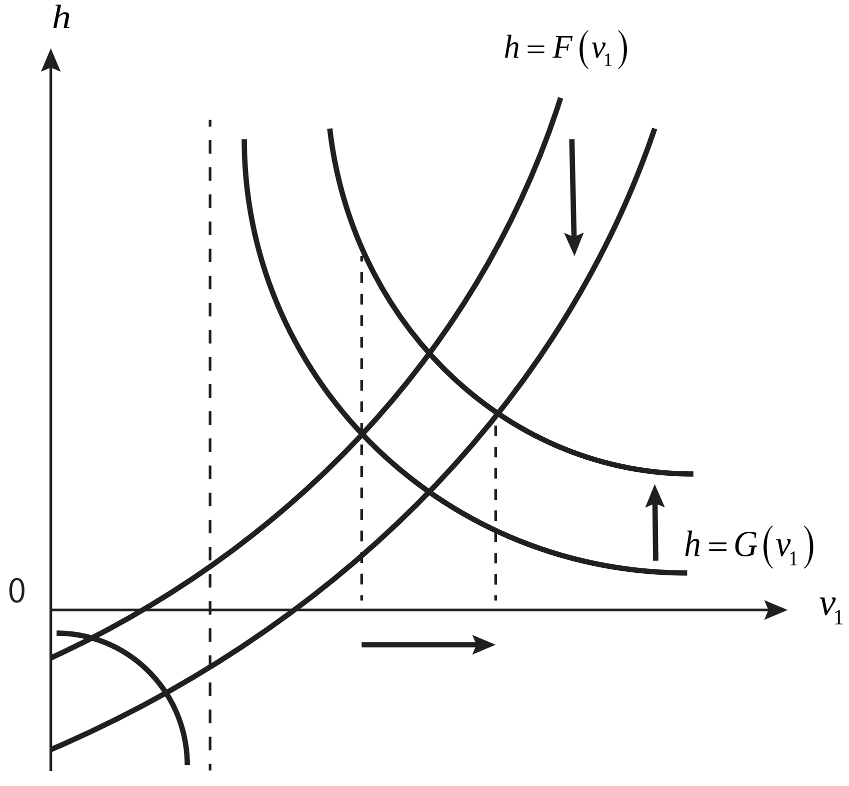

Figure 10: Effect of a change in τ_1^w under $1+\gamma > \beta$

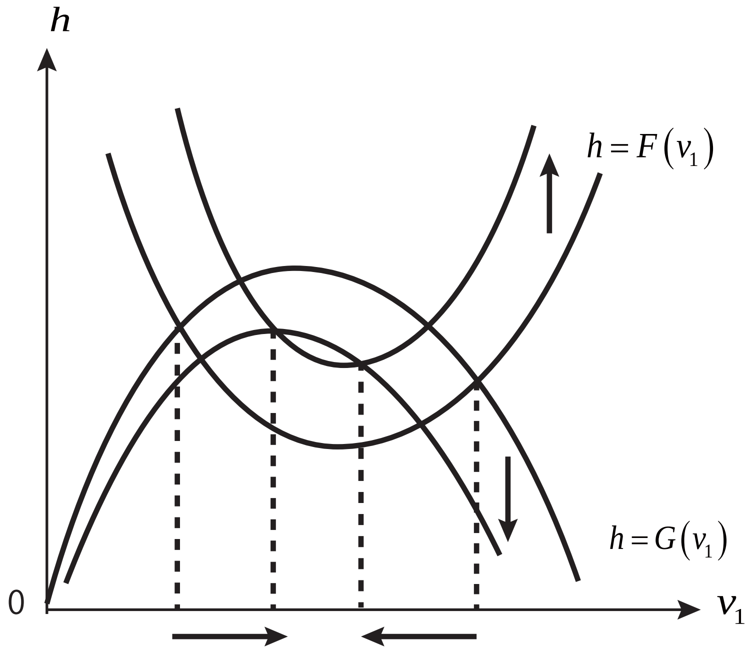

Figure 11: Effect of a change in τ_1^c under $1+\gamma<\beta$