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<td>Author(s)</td>
<td>Ashida, Yasumasa</td>
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Kyoto University
Study on Propulsive Characteristics of Magnetic Sail and Magneto Plasma Sail by Plasma Particle Simulations

Yasumasa ASHIDA
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Abstract

Magnetic Sail is a spacecraft propulsion system that generates an artificial magnetosphere to block solar wind particles and uses the imparted momentum to accelerate a spacecraft. The derivative of magnetic sail, Magneto Plasma Sail inflate the artificial magnetosphere by a plasma injection from the spacecraft. The momentum caught by the inflated magnetosphere is transfer to the spacecraft as a larger thrust. However, the propulsive characteristics of magnetic sail and magneto plasma sail has not been quantified even thought the many previous works were carried out. This is because the large computational efforts is required to simulate the momentum transfer process including the plasma kinetics and the scaling law of the propulsive characteristics cannot be obtained under the one simulation model.

In the present study, the plasma flow around Magnetic Sail and Magneto Plasma Sail is simulated from the ion inertial scale, where ion Larmor motion is comparable with the artificial magnetosphere size, to the electron inertial scale, where electron Larmor radius and Debye length are comparable with the magnetosphere size. The steady-state model of plasma flow around magnetic sail including ion kinetic effects had to be developed by using Flux-Tube model to enable us to drastically shorten the simulation time required to reveal the propulsive characteristics with various parameters. Using the high performance computing techniques and the recently developed peta-scale supercomputer, the highly parallelized simulation code based on Full Particle-in-Cell (PIC) model was also developed to take the electron kinetic effects into consideration.

Three type of simulation: three-dimensional Flux-Tube model, two-dimensional Full-PIC model and three-dimensional Full-PIC model are hence performed to reveal scaling law of the propulsive characteristics of Magnetic Sail and Magneto Plasma Sail. The thrust of magnetic sail is approximately proportional to the magnetic moment of onboard superconductive coil on the electron inertial scale even though the thrust is approximately proportional to the 2/3 power of the magnetic moment on the ion inertial scale. The thrust changes from the electron inertial scale to the ion inertial scale nonlinearly. The empirical formula of thrust was obtained by the combination of Flux-Tube model and Full-PIC model. It was also revealed that these propulsive characteristics are caused by the finite Larmor motion and charge separation between ion and electron.

The solar wind which is a natural phenomenon changes its parameters such as number density and velocity every moment. The propulsive characteristics about solar wind variation were also examined. The thrusts on the electron inertial scale and the ion inertial scale are proportional
to the 1.15 power of number density and the 0.67 power of number density, respectively. This propulsive characteristics cause the difference in the flexibility of the interplanetary flight missions by magnetic sail since the average plasma density is inversely proportional to square of the sun-spacecraft distance. It was also revealed that the magnetic sail on the ion inertial scale, which thrust is inversely proportional to the 1.3 power of the sun-spacecraft distance is more suitable for the deep space flight than the magnetic sail on the electron inertial scale, which thrust is inversely proportional to the 2.3 power of the sun-spacecraft distance.

The increase in thrust of Magneto Plasma Sail utilizing plasma injection is demonstrated by two- and three-dimensional simulations. It becomes clear that the increase in thrust is dependent on the Larmor radius of injection plasma and the plasma energy at injection point. By simulations with various plasma parameters and one-dimensional theoretical analysis, the parameters which maximize a thrust were examined. The maximum increase (thrust of MPS / thrust of magnetic sail) is 97 but the thrust gain (thrust of MPS / (thrust of magnetic sail + thrust of plasma jet)) is only 0.4 by same condition. On the contrary, the maximum thrust gain 5.2 is obtained and the relation of trade-off between the increase in thrust and the thrust gain is revealed.

Moreover, the propulsive characteristics other than thrust: thrust-mass ratio, thrust-power ratio and specific impulse, were also examined. The thrust-mass ratio is approximately constant on the electron inertial scale and gradually decreases on the ion inertial scale as the magnetic moment of the onboard coil becomes large. The thrust-power ratio is mainly depends on the plasma injection velocity and, when the thrust gain is large the thrust-power ratio is also large. However, it was revealed that the high thrust-mass ratio, the high thrust-power ratio and specific impulse cannot be achieved simultaneously. Under the limitation of the present technology and the propulsive characteristics, the feasible demonstration missions of Magnetic Sail and Magneto Plasma Sail are also proposed. The optimized trajectory obtained by Genetic Algorithm shows that large acceleration is obtained even the small-scale Magnetic Sail by the flight via an inner planet. The artificial halo orbit at Lagrange points are also proposed as the candidate of the magnetic sail mission.

Concluding the present thesis, the scaling law of the propulsive characteristics of Magnetic Sail and Magneto Plasma Sail are successfully revealed, with a few suggestions for realization of future interplanetary missions.
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Nomenclature

\( \alpha \)    attack angle

\( B \)    magnetic field

\( \beta \)    IMF direction

\( \beta_{jet} \)    kinetic beta of plasma injection

\( B_{IMF} \)    magnetic flux density of Interplanetary Magnetic Field

\( B_{jet} \)    magnetic flux density of plasma injection

\( B_{MP} \)    magnetopause magnetic flux density

\( \gamma \)    steering angle

\( \gamma_{jet} \)    generalized momentum of plasma injection

\( c \)    light speed

\( \Delta x \)    grid spacing

\( \Delta t_{field} \)    time step for momentum equation

\( \partial t \)    time step

\( E_{MP} \)    Magnetopause electric field

\( E \)    electric field

\( \epsilon_0 \)    permittivity

\( E_p \)    electric power

\( F_{mag} \)    thrust of magnetic sail
$F_{\text{MPS}}$ thrust of MPS

$I_{\text{coil}}$ coil current

$I_{\text{SP}}$ specific impulse

$\mathbf{J}$ current density

$\mathbf{J}_{\text{MP}}$ magnetopause current density

$k_B$ Boltzmann constant

$\lambda_D$ Debye length

$L_{\text{MHD}}$ Theoretical magnetosphere size based on MHD approximation

$\dot{m}$ mass flow rate

$\mu_0$ magnetic permeability

$M$ magnetic moment

$m_e$ electron mass

$m_i$ ion mass

$N_{\text{SW}}$ solar wind density

$n_e$ electron number density

$n_i$ ion number density

$\omega_p$ plasma frequency

$p$ static pressure

$p_e$ electron static pressure

$p_i$ ion static pressure

$Q$ charge density

$q_e$ electron charge

$q_i$ ion charge

$\rho$ mass density
$R$  sun-spacecraft distance

$r_{eL}$  electron Larmor radius

$R_{coil}$  coil radius

$r_{iL}$  ion Larmor radius

$r_{jet}$  Larmor radius of plasma injection

$T_e$  solar wind temperature

$V_{SW}$  solar wind velocity

$V_A$  Alfven velocity

$v_e$  electron velocity

$v_i$  ion velocity

$V_s$  speed of sound

$V_{th}$  thermal velocity
Chapter 1

General Introduction

1.1 Introduction

Beginning in the 20th century, the invention of rocket enables human being to extend their activity onto the space. In recent years, the satellites launched with the rocket are used for the communication, earth observation and etc. These technologies become indispensable to the present-day life. Many satellites aiming at a scientific probe are also launched in order to reveal the history of the universe and solar system. There are two kinds of these satellites, earth orbiter and space probe. Hubble Space Telescope of NASA performing a optical observation and SUZAKU of JAXA using X-ray for the observation are earth orbiters. The space probe leaves the earth and explores other planets. Planetary exploration requires the prolonged flight to the target planet (Fig. 1.1 [1]) and improvement in the propulsion system of launch capability with a rocket and satellite loading, etc. are needed.

In the search for an inner solar system, Mariner 2 in 1962, Mariner 4 in 1964 and Mariner 10 in 1973 of NASA approached Venus, Mars and Mercury, respectively. Although NOZOMI of JAXA was launched towards Mars in 1998, it has not resulted in the injection to a Mars orbit. In comet exploration, Giotto of ESA reached the comet Halley in 1998, and Deep Space1 of NASA has arrived at the comet Borrelly. HAYABUSA launched in 2003 [2] landed at asteroid Itokawa, and succeeded in the sample return. Although launched AKATUKI towards Venus in 2010, an orbital injection goes wrong and it is waiting for the following opportunity. BepiColombo of Japan-Europe joint development is due to launch towards Mercury in 2015. The ion engine is carried in DeepSpace1, HAYABUSA and BepiColombo among these space probes, and prolonged missions were made possible.

In the search for an outer solar system (Fig. 1.2 [1]), on the contrary, the United States took the lead. Pioneer 10, Pioneer 11 and Voyager 1 observed Jupiter and Saturn. Voyager 2 approached Neptune and Uranus for the first time. Also just now, these satellites are continuing the flight for 30 years or more aiming at the exterior of the solar system. Detailed exploration was carried
out not only to a flyby but also to the circumference orbit by Galileo of Jupiter, and Cassini of Saturn. Furthermore, NewHorizons was launched in 2006 towards Pluto where exploration is not yet performed, and the arrival in 2015 is planned. In these explorations of the deep space, in order to enlarge the acceleration of spacecraft by launch with a rocket, the weight of a spacecraft is restrained. Moreover, since power generation by a solar cell becomes difficult, the radioisotope battery is carried.

In order to realize more efficient deep space explorations, in addition to the chemical propulsion of a rocket etc., and the electric propulsion used by the search for an inner solar system, the propulsion system called the sail propulsion which generates an efficient thrust by using the environment of an interplanetary space is proposed. The solar sail using the sunlight is proved by IKAROS of JAXA. Moreover, electric sail, Magnetic Sail, and Magneto Plasma Sail are proposed using the solar wind which is the high-speed plasma flow which blows off from the sun.
1.2 Propulsion System Utilizing Solar Wind

1.2.1 Solar Wind and Magnetosphere

Interplanetary space is filled with low density and essentially collisionless plasma. This plasma called as the solar wind is emitted to the interplanetary space with the solar activity. The solar wind consists of mainly proton and electron. Each proton or electron in the solar wind obeys the equation of motion with the external electric and magnetic force. The density and temperature of the solar wind are typically $5 \times 10^6 \text{ m}^{-3}$ and 10 eV, respectively. The solar wind velocity at 1 AU (Earth orbit) is approximately between 300 km/s and 900 km/s. A magnetic field, so called Interplanetary Magnetic Field (IMF), accompanies the solar wind.

When the planet which had a magnetic field like the earth exists in the interplanetary space which this solar wind fills, a magnetosphere (magnetic cavity) is formed by the interaction of solar wind plasma and a magnetic field (Fig. 1.3 [1]). In the front of a magnetosphere, Bow shock is formed, where the supersonic plasma flow of the solar wind is slowed down and the shock wave occurs. The inside and the exterior of a magnetosphere are separated in the domain called magnetopause (magnetosphere boundary). The incoming solar wind into the magnetosphere induces the magnetopause current. In addition, the plasma which trespassed upon the inside of a magnetosphere from the domain called a cusp and a magnetic tail forms the equatorial ring current which flows into an opposite direction with the magnetopause current. The magnetic field structure of a magnetosphere is decided by such complicated current system as illustrated in Fig. 1.4 [3]. Moreover, the IMF also participates in the structure of this magnetosphere, and causes dynamic change of the magnetosphere called magnetic reconnection.

Figure 1.3: Interaction between the solar wind and the magnetic field of the Earth. The magnetosphere is formed around the Earth [1].
CHAPTER 1. GENERAL INTRODUCTION

1.2.2 Magnetic Sail

Magnetic Sail [4, 5, 6] is a spacecraft propulsion system that aims at quick interplanetary flight in deep space explorations. Zubrin and Andrews first provided the concept of magnetic sail in 1991 [4]. As shown in Fig. 1.5, they conceptually designed a spacecraft with a large hoop superconductive coil, which could produce an artificial magnetic cavity (magnetosphere) to reflect the solar wind particles approaching the coil. Due to this interaction, the solar wind flow will lose its momentum, and a corresponding repulsive force would exert on the coil to accelerate the magnetic sail spacecraft in the anti-sun direction. Magnetic Sail is expected to provide an efficient orbital transfer in interplanetary space since it directly converts the momentum of the solar wind into thrust by using a superconductive coil without consuming fuels. Zubrin et al. [4] theoretically estimated that a 20 N-class magnetic sail is possible by making a 100-km diameter magnetosphere. It is impossible by current technologies to launch and expand such huge structure in outer space. Therefore, it is difficult to bring Magnetic Sail to practical use.

1.2.3 Magneto Plasma Sail

Due to an unrealistically large coil structure (several kilo meters in diameter) necessary for a magnetic sail, the magnetic sail did not gain much interest. In 2000, the concept of Magnetic Sail attained renewed interest when the idea to make a large magnetosphere and a corresponding large thrust by employing a compact coil (several meters in diameter) with a plasma jet was proposed by Winglee et al. [7, 8] instead of deploying a large-scale coil by Zubrin. Winglee’s concept is illustrated in Fig. 1.6. This concept is called Mini-Magnetospheric Plasma Propulsion (M2P2) or Magneto Plasma Sail (MPS). Magneto Plasma Sail forms a strong magnetic field using not only a superconducting coil but also plasma jets from the spacecraft as shown in Fig. 1.6. The small artificial magnetosphere by on-board coil currents is inflated to the larger magnetosphere.
1.2. PROPULSION SYSTEM UTILIZING SOLAR WIND

by injected plasma jets from the spacecraft. This method of creating the magnetic field is called as the magnetic inflation. The magnetic inflation needs some propellant, but can eliminate the use of large diameter coil. Hence, Magneto Plasma Sail is more realistic propulsion system than Magnetic Sail.

Fig. 1.7 shows the MPS demonstration spacecraft model proposed by JAXA. A coil 4 m in diameter is carried. The weight of the coil is restricted to 200 kg and 100 mN-class thrust generation is required for the mission feasibility. Table 1.1 represents the comparison of Magneto Plasma Sail with Ion Engine and Solar Sail. Magneto Plasma Sail aims at coexistence of the high thrust level and high efficiency.

1.2.4 Scale of the Artificial Magnetosphere

The magnetospheric size $L_{MHD}$ shown in Fig. 1.5 is derived from the pressure equilibrium at the magnetopause (the boundary of the magnetosphere, where the magnetopause current is induced by the solar wind plasma) [9]. Based on MHD approximation, the solar wind dynamic pressure balances the magnetic pressure of the magnetic field generated by a magnetic sail spacecraft as Eq. (1.2.1). In the case of 3D dipole magnetic field, Eq. (1.2.1) is set to Eq. (1.2.2). By solving Eq. (1.2.2) about $L_{MHD}$, the magnetospheric size is obtained as Eq. (1.2.3) with the MHD approximation. In same manner, by solving Eq. (1.2.4) for 2D dipole magnetic field, the magnetosphere size is obtained as Eq. (1.2.5).

\[ \frac{1}{2} N_{SW} m_i V^2_{SW} = \frac{B^2}{2\mu_0} \]  

(1.2.1)
CHAPTER 1. GENERAL INTRODUCTION

Figure 1.6: Schematic illustration of Magnetic Sail and MPS. By a plasma injection, the dipole magnetic field of the magnetic sail is expanded.

Figure 1.7: MPS spacecraft model proposed by JAXA. A 200 kg superconductive coil and 100 mN-class thrust generation is required for missions.
### 1.2. PROPULSION SYSTEM UTILIZING SOLAR WIND

Table 1.1: Comparison of Magneto Plasma Sail with Ion Engine and Solar Sail [10, 11]

<table>
<thead>
<tr>
<th>Spec</th>
<th>Magneto Plasma Sail (Jupiter Mission)</th>
<th>Ion Engine</th>
<th>Solar Sail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propellant required</td>
<td>Propellant required (H, Xe etc.)</td>
<td>Propellant required (Ar, Xe etc.)</td>
<td>No propellant required</td>
</tr>
<tr>
<td>Isp&gt;3000 s</td>
<td>Isp=3000-4500 s</td>
<td>Isp=∞</td>
<td></td>
</tr>
<tr>
<td>Thrust=1 N@4 kW</td>
<td>Thrust=several mN-200 mN</td>
<td>Thrust=10 mN (14 m×14 m)</td>
<td></td>
</tr>
<tr>
<td>Proportional to</td>
<td>Proportional to</td>
<td>Proportional to</td>
<td></td>
</tr>
<tr>
<td>1/(Sun distance)²/³</td>
<td>1/(Sun distance)²</td>
<td>1/(Sun distance)²</td>
<td></td>
</tr>
<tr>
<td>Efficiency=250 mN/kW</td>
<td>Efficiency=20-30 mN/kW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life=TBD</td>
<td>Life=8000-18000 h</td>
<td>Life=to be verified</td>
<td></td>
</tr>
<tr>
<td>Operation</td>
<td>Extension mechanism not required</td>
<td>Extension mechanism not required</td>
<td>Extension mechanism required</td>
</tr>
<tr>
<td></td>
<td>Thrust vector control by attitude control</td>
<td>Thrust vector control in all direction</td>
<td>Thrust vector control by attitude control</td>
</tr>
</tbody>
</table>

\[
\frac{1}{2}N_{SW}m_iV_{SW}^2 = \frac{1}{2\mu_0} \left( \frac{\mu_0 M}{4\pi L_{MHD}^2} \right)^2
\]  

(1.2.2)

\[
L_{MHD} = \left( \frac{\mu_0 M^2}{16\pi^2 N_{SW}m_iV_{SW}^2} \right)^{\frac{1}{2}}
\]  

(1.2.3)

\[
\frac{1}{2}N_{SW}m_iV_{SW}^2 = \frac{1}{2\mu_0} \left( \frac{\mu_0 M}{2\pi L_{MHD}^2} \right)^2
\]  

(1.2.4)

\[
L_{MHD} = \left( \frac{\mu_0 M^2}{4\pi^2 N_{SW}m_iV_{SW}^2} \right)^{\frac{1}{4}}
\]  

(1.2.5)

When the typical parameters of the solar wind, \(N_{SW}=5 \times 10^6 \text{ m}^{-3}\), \(V_{SW}=500 \text{ km/s}\) are assumed, the magnetic flux density at the magnetopause is estimated approximately as \(B_{MP}=40 \text{ nT}\). Then the ion Larmor radius at the magnetopause is calculated as \(r_{iL} \sim 100 \text{ km}\) and the electron Larmor radius is calculated as \(r_{eL} \sim 50 \text{ m}\). If \(L_{MHD}\) and the ion Larmor radius \(r_{iL}\) satisfy the condition, \(L_{MHD}>>r_{iL}\), the finite Larmor effect can be neglected and the plasma flow can be treated as a single fluid on the MHD scale. On the contrary, if \(L_{MHD}\) and \(r_{iL}\) are comparable and \(L_{MHD}>>r_{eL}\) at the magnetopause, the ion’s finite Larmor radius effect should be considered and electrons can be assumed as a fluid on the ion inertial scale. When \(L_{MHD}\) is smaller than \(r_{iL}\), both ion’s and electron’s finite Larmor radius effect should be considered. We call this scale length of the magnetosphere \((L_{MHD}<r_{iL} \text{ and } L_{MHD}>r_{eL})\) as the electron inertial scale. In addition to the electron kinetic effects, in small magnetosphere, the charge separation also should be considered.
Table 1.2: Typical parameters of the solar wind

<table>
<thead>
<tr>
<th>Solar wind velocity $V_{SW}$ [m/s]</th>
<th>Solar wind density $N_{SW}$ [m$^{-3}$]</th>
<th>Plasma temperature $T_e$ [eV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^5$</td>
<td>$5 \times 10^6$</td>
<td>10</td>
</tr>
</tbody>
</table>

![Diagram of solar wind parameters](image)

Figure 1.8: Scale of the artificial magnetosphere and schematic illustration of trajectories of ion and electron around magnetopause. On the electron inertial scale, the kinetics of both ion and electron become significant.

since Debye length $\lambda_D \sim 10$ m becomes comparable with the magnetosphere size on the electron inertial scale. Full kinetic simulation such as Full-PIC simulation should be performed to reveal the thrust characteristics of magnetic sail and MPS on the electron inertial scale.

To generate large magnetosphere, the large superconductive coil in several km in radius is required. On the present superconductive technology and rocket technology, such a large superconductive coil can not be available in space. The several meters coil in radius is realistic. By the small radius coil, only small artificial magnetosphere on the electron inertial scale can be obtained.

In Fig. 1.8, the simulation scale of the magnetic sail is summarized. On the MHD scale, the interaction between the magnetic field and plasma is started by a thin boundary layer. On the ion inertial scale and the electron inertial scale, the ion penetrates into the magnetic field and the broad boundary layer is expected.

### 1.3 Overview of Previous Studies

The studies of Magnetic Sail and MPS are performed by the laboratory experiments, numerical simulations and analytical approaches. After several studies indicated a possibility that Magneto Plasma Sail drastically shorten a exploration mission time, many studies about this propulsion
system have been conducted in Washington University, NASA, ISAS/JAXA, and other research institutes.

In the experimental approach, by using the scale down model of Magnetic Sail, the formation of the artificial magnetosphere is confirmed [12, 13]. Using the pendulum stand, the thrust generation of Magnetic Sail is directly measured [14, 15]. The magnetic inflation is also performed to demonstrate the concept of MPS. As a result, the larger magnetosphere is obtained and the increase in thrust is expected although no one successively measures the thrust of scale down MPS directly. However, the experimental parameters are restricted by the size of experimental equipment and the actual phenomena are not necessarily reflected in experiment. In particular, the collisionless plasma cannot be generated in an experiment and direct collision of plasma diffuses plasma. The scale of the artificial magnetosphere is restricted on the MHD scale.

In addition, the characteristics of the superconductive onboard spacecraft are investigated in laboratory, and the design techniques for space is now established [16, 17].

The analytical approaches are also performed to predict the thrust characteristics of magnetic sail and MPS. In MHD scale, the Newtonian model [18] of the earth magnetosphere is also useful for the magnetic sail. Cattell et al. [19] in 2005 described that 3-ton superconductive coil is required to generate the thrust of 700 mN by the theoretical approach based on the conservation of magnetic flux and MPS is non-competitive with the existing propulsion system based on the thrust-mass ratio. On the other hand, the MPS demonstrator spacecraft model as shown in Fig. 1.7 is proposed by JAXA/NEC. The spacecraft is assumed to equip the 200 kg superconductive coil and it is required to generate 1 mN class thrust force without the magnetic expansion and 100 mN class thrust force with the magnetic expansion. The orbital change technique by control of such a small level thrust is also examined [20, 21]. Although the mission analysis of large-scale magnetic sails (ion inertial scale ~ MHD scale, which are unrealizable with present technology) has already been conducted [22], what the kind of the mission for which a small-scale magnetic sail is suitable has not been determined. In addition, as a new way to improve thrust generation capability by MPS, Plasma Magnet concept using the rotating magnetic field was proposed by Slough [23] and the ground experiment was performed. The use of dipole plasma equilibrium (equatorial ring-current concept) rather than the frozen-in concept using a high Alfvén Mach number plasma flow is also proposed for the MPS concept [24]. However, it is still left to the numerical analysis by full kinetic simulations whether these new concepts successively improve the thrust of MPS or not. As the new concept of utilizing the solar wind, the use of both magnetic field and electric field are also proposed, but only rough analysis was performed [25].

Thus, the numerical simulation is required to understand the phenomena around MPS and to obtain the thrust characteristics. Following Winglee et al., Saha et al. [26] and Khazanov et al. [27] summarized issues that should be clarified about MPS concept. That is,

1) How is the momentum transferred from solar wind to the spacecraft? What is the interaction of the coil current and the current induced in the magnetosphere?
2) Can the magnetosphere really be produced using plasma injection?

3) What is the magnitude of the total thrust force generated on the spacecraft? Is the thrust competitive with the thrust generated by the existing propulsion system?

4) What numerical approach should be used to model a MPS?

The most important study is to reveal the thrust production process of Magneto Plasma Sail. The physics in the above two processes has to be precisely clarified to evaluate the performance of MPS. In order to use MPS for the actual missions, it is important to clarify 3) issue.

These processes of Magneto Plasma Sail have to be analyzed considering the scale of plasma-magnetic field interaction. In case that the ion Larmor radius is much smaller than the characteristic length of plasma, physics in plasma - magnetic field interaction can be considered based on the magnetohydrodynamics (MHD) and the effect of finite ion Larmor radius (i.e. ion kinetic effect) can be neglected. The magnetohydrodynamics is the fluid description of plasma, and the plasma flow is strongly coupled with the magnetic field. However, when the ion Larmor radius becomes comparable to the characteristic length, the ion kinetic effect cannot be neglected. Physical model has to be carefully selected according to the interaction scale. In the case that the ion Larmor radius of the injected plasma is much less than the scale length of the spacecraft (several meters), the magnetic field inflation can be studied based on the magnetohydrodynamics. Meanwhile, the ion Larmor radius of the solar wind is about km. First, Winglee et al. performed the study based on the MHD model and showed high performance of Magneto Plasma Sail. However, some researchers [27, 28, 29] indicated that a study taking into account the ion kinetic effect was needed. Therefore, Magneto Plasma Sail has been studied by both the magnetohydrodynamic and the ion kinetic approaches. It has been shown clearly that full kinetic simulation is required to model MPS already [30]. However, no one has been able to demonstrate other issues by using the full kinetic simulation. This is mainly because that the full kinetic simulation such as Full-PIC (Particle-In-Cell) requires huge computational resource. The realistic conditions such as the feasible spacecraft model and Interplanetary Magnetic Field (IMF) should be also assumed to model MPS.

Not only studies about Magneto Plasma Sail but also studies about Magnetic Sail are summarized because the knowledge obtained by the studies about Magnetic Sail is useful for studying Magneto Plasma Sail.

Previous studies are briefly overviewed and remaining problems are described below.

1.3.1 Studies on Magnetic Sail

Following Zubrin’s estimation, several numerical simulations were performed in order to accurately evaluate the thrust of Magnetic Sail.

On the MHD scale, Nishida et al. [31, 32] revealed the thrust generation mechanism of a magnetic sail by performing ideal MHD (Magnetohydrodynamics) simulations in two-dimension and three-dimension. The attitude stability of the magnetic sail are also investigated. However,
Hall effects are neglected in these simulation and the artificial resistivity model are required to simulate the magnetic reconnection.

On the ion inertial scale, Fujita [33] carried out particle simulations called Hybrid-PIC [34] to evaluate the thrust affected by ion kinetic effects in various magnetosphere sizes. Eqs. (1.3.1) and (1.3.2) are the thrust model of magnetic sail obtained by Fujita. Although it is unreliable in the range of $L_{MHD} < r_{iL}$ since Hybrid-PIC simulation neglects electron kinetics, it is often used in the examination of a thrust. More reliable thrust model is required for the mission analysis.

$$F = C_d \times \frac{1}{2} m_i N_{SW} V_{SW}^2 \times \pi L_{MHD}^2$$  \hspace{1cm} (1.3.1)

$$C_d = \begin{cases} 
2.9 \times \exp \left[ -0.35 \times \left( \frac{r_{iL}}{L_{MHD}} \right)^2 \right] & \text{for } r_{iL} < L_{MHD} \\
2.4 / \left( \frac{r_{iL}}{L_{MHD}} \right) \times \exp \left[ -0.17 / \left( \frac{r_{iL}}{L_{MHD}} \right)^2 \right] & \text{for } r_{iL} > L_{MHD} 
\end{cases}$$  \hspace{1cm} (1.3.2)

Kajimura et al. [35] performed similar Hybrid-PIC simulations to reproduce experiment results. These researches successfully revealed the thrust characteristics of a magnetic sail, and currently, the simulation techniques are applied to evaluate Magneto Plasma Sail, which inflates the magnetosphere of magnetic sail with a plasma jet from a spacecraft to obtain a large thrust level. However, to calculate the time evolutions of electromagnetic field and plasma flow around a magnetic sail, Hybrid-PIC simulation spends much time and take large amount of memory. These methods are hence not suitable for surveying a lot of design parameters of a magnetic sail such as coil size, coil current, parameters of the solar wind plasma, etc.

On the electron inertial scale, Akita et al. [36] and Moritaka et al. [37] performed Full-PIC simulation with the artificial mass ration between ion and electron, $m_i/m_e \sim 25$ in two-dimension. However, the thrust level of the magnetic sail can be obtained by the artificial parameter since the scaling law is not revealed yet. Hence, the simulation by realistic plasma parameter is required for the thrust characteristics on the electron inertial scale.

It is also noted that these simulations did not include Interplanetary Magnetic Field (IMF), which may significantly affect the above obtained thrust gain. In the Earth’s magnetosphere, IMF plays a very important role since the magnetic reconnection distorts the structure of the magnetosphere and transfers the momentum of plasma through the magneto tail. Only Nishida et al. [38] reflected the influence of IMF in thrust by performing 2D ideal-MHD simulation. However, it is difficult to reproduce the phenomena of a magnetic reconnection correctly by the ideal-MHD simulation including artificial viscosity and numerical diffusion.

In the field of the space plasma physics, the interaction of the solar wind with magnetized asteroids and lunar magnetic anomaly is also examined [39, 40, 41, 42]. These studies reproduced the observation by spacecraft by Hall-MHD or Hybrid-PIC simulation.
1.3.2 Studies on Magnetic Inflation

Several simulations of magnetic inflation have been previously performed. An analytical study based on the ideal MHD equations was also conducted by Parks et al. [43] and showed that the magnetic field strength decreased as $r^{-1}$. Hybrid-PIC simulations with the ion kinetic effects were performed to examine the expansion of the magnetosphere without the solar wind flow under different initial condition [28, 44, 45]. As a result, these studies revealed that the magnetic field could be expanded depending on ion Larmor radius and Alfvén speed of the injected plasma even if the ion kinetic effect is considered. By using two-dimensional Full-PIC simulation including the electron kinetics, Moritaka et al. [46] also demonstrated the magnetic inflation without the solar wind plasma by assuming the artificial mass ratio ($m_i/m_e \sim 25$).

However, the thrust of MPS was never evaluated by Full-PIC simulation despite of the demonstrative analysis by MHD and Hybrid-PIC simulation. In addition, the interaction between the solar wind and the inflated magnetic field is still unknown when the electron kinetics is taken into the consideration.

1.3.3 Studies on Magneto Plasma Sail

Early investigations [7] suggested that the propulsion system that is called as Magneto Plasma Sail (MPS) was a promising space propulsion system, which might drastically shorten the trip time of deep space missions. These investigations were based on the assumptions that a significant thrust force proportional to the area of a magnetosphere could be produced with a compact coil that can be equipped onboard a spacecraft. To understand MPS performance, many studies were conducted using Magnetohydrodynamics (MHD) [47, 48] and Hybrid-PIC techniques [27, 29, 49]. From these simulations, it was revealed that the thrust by MPS, $F_{MPS}$, is larger than that of magnetic sail, $F_{mag}$, (i.e., $F_{MPS}/F_{mag}>1$), supporting Winglee’s concept. The thrust by MPS were, however, limited as $F_{MPS}/F_{mag} < 10$.

A detailed explanation for the limited thrust was proposed by Nishida et al. [48], who performed ideal-MHD simulation to obtain the maximum $F_{MPS}/F_{mag}$ of about 2 for a low kinetic beta plasma jet; in contrast, for high kinetic beta plasma jets, $F_{MPS}/F_{mag}$ became approximately zero even though the magnetospheric size was intensively inflated. A much better thrust performance was obtained in the case of ion inertial scale magnetospheres, in which $F_{MPS}/F_{mag}$ up to 8 was obtained by Hybrid-PIC simulation with a low kinetic beta plasma jet. These simulations showed that the idea of MPS was rather promising on the ion inertial scale magnetospheres, but $F_{MPS}/F_{mag}$ was not as large as expected in the early investigations.

In fact, to make the MPS system effective, $F_{MPS}$ should be larger than a net thrust force, $F_{mag} + F_{jet}$, in which the contribution from plasma jet ($F_{jet}$) is also incorporated. Summarizing the results obtained by the MHD and Hybrid-PIC simulations, we found that the effective thrust gain $F_{MPS}/(F_{mag} + F_{jet})$ never exceeded unity (i.e., $F_{MPS}/(F_{mag} + F_{jet})=1$), and this means
Table 1.3: Summary of previous studies by numerical simulation

<table>
<thead>
<tr>
<th>Target</th>
<th>Dimension</th>
<th>Simulation model</th>
<th>Ion kinetics</th>
<th>Electron kinetics</th>
<th>IMF</th>
<th>Isp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nishida et al., 2006 Magnetic sail</td>
<td>2D</td>
<td>MHD</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Nishida et al., 2007 MPS</td>
<td>3D</td>
<td>MHD</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Nishida et al., 2012 Magnetic sail</td>
<td>2D</td>
<td>MHD</td>
<td>N/A</td>
<td>N/A</td>
<td>Considered</td>
<td>N/A</td>
</tr>
<tr>
<td>Fujita, 2009 Magnetic sail</td>
<td>3D</td>
<td>Hybrid-PIC</td>
<td>Considered</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Khazanov et al., 2005 Magnetic sail</td>
<td>3D</td>
<td>Hybrid-PIC</td>
<td>Considered</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Winske et al., 2005 Magnetic inflation</td>
<td>3D</td>
<td>Hybrid-PIC</td>
<td>Considered</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Kajimura et al., 2010 Magnetic inflation</td>
<td>3D</td>
<td>Hybrid-PIC</td>
<td>Considered</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Moritaka et al., 2010 Magnetic inflation</td>
<td>2D</td>
<td>Full-PIC</td>
<td>Considered</td>
<td>$m_i/m_e \sim 25$</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Moritaka et al., 2010 Magnetic sail</td>
<td>2D</td>
<td>Full-PIC</td>
<td>Considered</td>
<td>$m_i/m_e \sim 25$</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Ashida et al., 2011 Magnetic sail</td>
<td>3D</td>
<td>Flux-Tube</td>
<td>Considered</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Ashida et al., 2012 MPS</td>
<td>2D</td>
<td>Full-PIC</td>
<td>Considered</td>
<td>Considered</td>
<td>Considered</td>
<td>Considered</td>
</tr>
<tr>
<td>Ashida et al., 2013 MPS</td>
<td>3D</td>
<td>Full-PIC</td>
<td>Considered</td>
<td>Considered</td>
<td>N/A</td>
<td>Considered</td>
</tr>
</tbody>
</table>

that no effective configuration of MPS was found in the previous studies.

On the electron inertial scale, the simulation of MPS itself is not performed yet because of large requirement of the computational resource. The propulsive characteristics such as the specific impulse, the thrust-mass ratio and the thrust-power ratio are also not revealed on the scale.

Finally, Table 1.3 represents the summary of previous studies.

### 1.4 Objectives

It is expected that the size of the artificial magnetosphere is valid from several 100 m to several 100 km according to the mission requirement. However, the thrust characteristics of MPS are not understood in multi-scale and only the thrust on the limited condition is obtained. Parametric study and thrust characteristic model is insufficient to design missions. In this study, by the develop-
CHAPTER 1. GENERAL INTRODUCTION

ment of the steady-state model of magnetosphere and HPC technique, the parameter dependence of Magnetic Sail and Magneto Plasma Sail thrust is revealed. The thrust performance, such as the specific impulse, thrust-mass ratio and thrust-power ratio is also examined by assuming realistic spacecraft model. The comparison with the existing space propulsion system is represented to reveal the feasibility and practicality of magnetic sail and Magneto Plasma Sail.

1.5 Outline of This Thesis

The contents consist of eight chapters as follows.

Background of the present study and summaries of previous studies are described in the chapter 1.

In the chapter 2, the numerical models used in the present study are described and some numerical tests are given to validate our numerical code. In this study, three computational techniques are prepared according to the simulation scale. The classic fluid-analysis method (MHD model), the steady-state model (Flux-Tube model) and full kinetic model (Full-PIC model) in accordance with the ultimate cause of plasma physics are used properly appropriately. The MHD model aims to calculate the thrust exerted on a magnetic sail spacecraft without taking plasma kinetics into consideration. The objective of the Flux-Tube model is to construct a simplified numerical model for magnetic sail with short computational time and reduced memory requirement in order to survey the effects of various design parameters. The Flux-Tube model neglects wave propagation (electromagnetic wave, acoustic wave, etc.) and it solves only a steady state of a plasma flow including the ion’s finite Larmor effect. Therefore the Flux-Tube model is expected to quickly estimate a thrust level with low capacity of the computational memory. The full kinetic simulation is also developed by using HPC techniques to perform the large-scale simulations on the peta-scale supercomputers.

In the chapter 3, the simulation of Magnetic Sail is performed on the ion inertial scale. The newly developed Flux-Tube model is used. By neglecting the plasma waves in short time period, only steady state of plasma flow is considered. The fast thrust estimation enable us to perform the simulations with various parameters. The thrust characteristics of magnetic sail with various magnetosphere sizes are analyzed and the thrust formula is obtained. In addition, the comparison between Flux-Tube model and MHD model, that is, ion kinetic model and fluid model is performed to examine the effects of ion kinetics on the thrust generation of the Magnetic Sail.

In the chapter 4, Full-PIC simulation is performed on the ion inertial scale to the electron inertial scale. To reduce the calculation effort, the artificial plasma condition is adopted and structure of magnetosphere is simulated. Change of the magnetosphere structure by the existence of the influence of electron kinetics is considered. The influence of the interplanetary magnetic field on the generation of magnetosphere is also examined.

In the chapter 5, the simulation of magnetic sail is performed on the electron inertial scale.
1.5. OUTLINE OF THIS THESIS

The Full-PIC simulation with realistic plasma parameters is performed in both two dimensions and three dimensions by using HPC techniques. The thrust characteristics are obtained also on this scale. The parametric study is performed about magnetic sail design parameters and solar wind plasma parameters. Discussion using the theoretical model is also proposed. In addition, comparison between two-dimensional magnetosphere and three-dimensional magnetosphere are performed.

In the chapter 6, the simulation of MPS is performed on the electron inertial scale using large-scale Full-PIC simulation. We use the realistic plasma parameters to assume the spacecraft model feasible in the present technology. The increase in thrust and the thrust gain are examined by various plasma injection parameters and Magneto Plasma Sail design parameters. The parametric dependency in thrust increase is revealed and design of MPS is optimized.

In the chapter 7, the thrust characteristics of Magnetic Sail and Magneto Plasma Sail such as the thrust level, the specific impulse and thrust-mass ratio are summarized and the empiric formula of thrust generation is proposed. In addition, using the empiric formula, realistic mission is analyzed by using Genetic Algorithm.

Finally, the conclusion of the present study is presented in the chapter 8.
Chapter 2

Numerical Models

2.1 Introduction

Simulation scale of Magnetic Sail and Magneto Plasma Sail is varied form MHD scale (magnetosphere size $L_{MHD}>>r_{iL}$) to the electron inertial scale ($L_{MHD}<r_{iL}$ and $L_{MHD}>r_{eL}$) as mentioned in Chapter 1. The thrust generation of Magnetic sail and Magneto Plasma Sail have to be analyzed considering the scale length of plasma - magnetic field interaction, properly. Depending on the each simulation scale and the assumption, hence, MHD model (single fluid model), Flux-Tube model (particle ion and fluid electron model), Full-PIC model (particle ion and particle electron model) are required to reveal the thrust characteristics of Magnetic Sail and Magneto Plasma Sail.

We first introduce three simulation models used in this study and the validity of simulation code is checked by simple condition.

2.2 Assumptions

In the case of a magnetic sail in interplanetary space, an ion mean free path is approximately $1.5 \times 10^8$ km. It is much longer than the typical length of a magnetosphere we consider, $L_{MHD}=100$ m $\sim 1000$ km. Direct collisions between particles hence can be neglected in the magnetic sail simulations. According to the magnetosphere size, the way through which the magnetic field and plasma particles interact changes. When the finite Larmor radius of plasma is shorter than magnetosphere size, the fluid approximation is valid. Otherwise, the plasma should be treated as particles. If the magnetosphere size excels enough from the Debye length, electric charge neutrality can be assumed. In considering only change of frequency lower than the plasma frequency, a displacement current is neglected and the characteristic velocity is set to Alfven velocity. Finally, we summarized assumptions in Table 2.1.
### Table 2.1: Assumptions on three simulation models

<table>
<thead>
<tr>
<th></th>
<th>MHD model</th>
<th>Flux-Tube model</th>
<th>Full-PIC model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetosphere size</td>
<td>$L_{MHD} &gt;&gt; r_{iL}$</td>
<td>$L_{MHD} \sim r_{iL}$</td>
<td>$L_{MHD} &lt; r_{iL}$</td>
</tr>
<tr>
<td>Ion kinetics</td>
<td>N/A (fluid)</td>
<td>Considered</td>
<td>Considered</td>
</tr>
<tr>
<td>Electron kinetics</td>
<td>N/A (fluid)</td>
<td>N/A (fluid)</td>
<td>Considered</td>
</tr>
<tr>
<td>Charge separation</td>
<td>N/A (quasi neutral)</td>
<td>N/A (quasi neutral)</td>
<td>Considered</td>
</tr>
<tr>
<td>Characteristic velocity</td>
<td>Alfvén velocity</td>
<td>Alfvén velocity</td>
<td>Light speed</td>
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<tr>
<td>Characteristic length</td>
<td>Non</td>
<td>Ion inertial length</td>
<td>Debye length</td>
</tr>
</tbody>
</table>

#### 2.3 Magnetohydrodynamic Model

The numerical model of Ideal Magnetohydrodynamic (MHD) model [50] aims to calculate the thrust exerted on a magnetic sail spacecraft without taking plasma kinetics into consideration. Treating plasma as a fluid, the very large magnetosphere $L_{MHD} \gg r_{iL}$ is calculated. Such a large magnetosphere is not realistic for the spacecraft thrust system. We use the ideal MHD model as the candidate for comparison. Moreover, since the physical phenomena are simplified in ideal MHD model, it is useful also for an understanding of a phenomena around Magnetic Sail.

**2.3.1 Basic Equations**

The full set of the plasma fluid equation consists of two species, ion and electron, are described as,

$$\nabla \cdot \mathbf{E} = \frac{(n_i q_i + n_e q_e)}{\epsilon_0}$$ \hspace{1cm} (2.3.1)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ \hspace{1cm} (2.3.2)

$$\nabla \cdot \mathbf{B} = 0$$ \hspace{1cm} (2.3.3)

$$\nabla \times \mathbf{B} = \mu_0 (n_i q_i \mathbf{v}_i + n_e q_e \mathbf{v}_e) + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$ \hspace{1cm} (2.3.4)

$$m_j n_j \left[ \frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \cdot \nabla) \mathbf{v}_j \right] = q_j n_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \nabla p_j \hspace{0.5cm} j = i, e$$ \hspace{1cm} (2.3.5)

$$\frac{\partial n_j}{\partial t} = \nabla \cdot (n_j \mathbf{v}_j) = 0 \hspace{0.5cm} j = i, e$$ \hspace{1cm} (2.3.6)

$$p_j = C (m_j n_j)^\gamma \hspace{0.5cm} j = i, e$$ \hspace{1cm} (2.3.7)

The one-fluid MHD equations are derived from the two-fluid equations Eqs. (2.3.5), (2.3.6) and
2.3. MAGNETOHYDRODYNAMIC MODEL

At this point, the information on the temperature difference between electrons and ions is annihilated by assuming temperature equilibration. Consequently, the number of momentum equations is reduced by one. This permits the definition of the following one-fluid variables:

\[ \rho \equiv n_i m_i + n_e m_e \quad (2.3.8) \]

\[ Q \equiv n_i q_i + n_e q_e \quad (2.3.9) \]

\[ \rho v \equiv n_i m_i v_i + n_e m_e v_e \quad (2.3.10) \]

\[ J \equiv q_i n_i v_i + q_e n_e v_e \quad (2.3.11) \]

\[ p \equiv p_i + p_e = (n_i + n_e) k_B T \quad (2.3.12) \]

Multiplying the equations by the mass and adding them gives mass conservation equation;

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad (2.3.13) \]

In a similar way, the momentum conservation equation is given by adding the Eqs. (2.3.5) and assuming the quasi charge-neutrality \( n_i \approx n_e; \)

\[ \frac{\partial}{\partial t} (\rho v) + \nabla \cdot (\rho vv) + \nabla p - J \times B = 0 \quad (2.3.14) \]

Here, the relation \( m_e/m_i \ll 1 \) is used to transform the equation. Multiplying Eqs. (2.3.5) by the charge over mass quotients and adding them results in the generalized Ohm’s law. The usual Ohm’s law is obtained by neglecting small terms of the generalized Ohm’s law (the time derivative term and so on);

\[ E + v \times B - \frac{1}{n e} J \times B = 0 \quad (2.3.15) \]

The final term in Eq. (2.3.15) is identified as the Hall term. Physically, the Hall term decouples ion and electron motion on the ion inertial scale. In ideal MHD approximation, the Hall term is neglected.

The energy conservation equation is described as

\[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 + \frac{p}{\gamma - 1} \right) + \nabla \cdot \left\{ \left( \frac{1}{2} \rho v^2 + \frac{p}{\gamma - 1} + p \right) v \right\} - J \cdot E = 0 \quad (2.3.16) \]
, where \( \gamma = \frac{5}{3} \) is the specific heat ratio.

To obtain the self-consistent MHD equations, the Maxwell’s equations are needed. The Maxwell’s equations are as follows;

\[
\nabla \times E = -\frac{\partial B}{\partial t} \\
\nabla \times B = \mu_0 J \\
\n\nabla \cdot B = 0
\]

In the magnetohydrodynamics, the Poisson’s law is not considered as an initial condition on the Ampere’s law. The displacement current term of the Ampere’s law is negligible. In these set of equation is called as ideal MHD equations, in which the heat flux, the viscosity and Hall effect are neglected. In addition, ideal MHD equation is valid in the condition that the magnetic Reynolds number is sufficient large;

\[
R_m \equiv \frac{\mu_0 L_c v_c}{\eta} \gg 1
\]

where \( L_c \) and \( v_c \) are the characteristic length and velocity, respectively.

The ideal MHD equations express conservation of the main macroscopic quantities of a plasma, viz. mass, momentum, energy, and magnetic flux. The conservation form of the ideal MHD equations are

\[
\frac{\partial U}{\partial t} + \nabla \cdot F = 0
\]

\[
U = \begin{bmatrix} \rho \\ \rho v \\ B \\ e \end{bmatrix}
\]

\[
F = \begin{bmatrix} \rho v \\ \rho vv + \left( p + \frac{B^2}{2\mu_0} \right) I - \frac{BB}{\mu_0} \\ vB - Bv \\ e + p + \frac{B^2}{2\mu_0} v - \frac{(v \cdot B)B}{\mu_0} \end{bmatrix}
\]

where the energy density is
2.3. MAGNETOHYDRODYNAMIC MODEL

\[ e = \frac{1}{2} \rho v^2 + \frac{p}{\gamma - 1} + \frac{B^2}{2\mu_0} \]  
\[(2.3.24)\]

The ideal MHD equations are non-dimensionalized using the characteristic length, the characteristic density and the characteristic magnetic flux density. The characteristic velocity, time, pressure and current density are defined using as below.

\[ v_c = \frac{B_c}{\sqrt{\mu_0 \rho_c}} \]  
\[(2.3.25)\]

\[ t_c = \frac{L_c}{v_c} \]  
\[(2.3.26)\]

\[ p_c = \rho_c v_c^2 \]  
\[(2.3.27)\]

\[ J_c = \frac{B_c}{\mu_0 L_c} \]  
\[(2.3.28)\]

The non-dimensional ideal MHD equations are written in the conservation form as follows;

\[ \frac{\partial U}{\partial t} + \nabla \cdot F = 0 \]  
\[(2.3.29)\]

\[ U = \begin{bmatrix} \rho \\ \rho v \\ B \\ e \end{bmatrix} \]  
\[(2.3.30)\]

\[ F = \begin{bmatrix} \rho v \\ \rho v + \left( p + \frac{B^2}{2} \right) I - B B \\ vB - Bv \\ e + p + \frac{B^2}{2} - (v \cdot B) B \end{bmatrix} \]  
\[(2.3.31)\]

where the energy density is

\[ e = \frac{1}{2} \rho v^2 + \frac{p}{\gamma - 1} + \frac{B^2}{2} \]  
\[(2.3.32)\]
2.3.2 Algorithm of the Ideal Magnetohydrodynamic Code

One dimensional form of ideal MHD equations in conservation form is given as

\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0 \tag{2.3.33}
\]

The difference form which fills the conservation law of the equation is given as

\[
U_{i}^{n+1} = U_{i}^{n} - \frac{\Delta t}{\Delta x} \left( \tilde{F}_{i+\frac{1}{2}} - \tilde{F}_{i-\frac{1}{2}} \right) \tag{2.3.34}
\]

The magnetic field and plasma density \((U)\) is defined in grid point as shown in Fig. 2.1.

\(\tilde{F}\) represents the numerical flux on the cell surface. The original Lax-Fridrich scheme defines the numerical flux as

\[
\tilde{F}_{i+\frac{1}{2}} = \frac{1}{2} \left( F_{i} + F_{i+1} \right) - \frac{1}{2} \frac{\Delta x}{\Delta t} \left( U_{i+1} - U_{i} \right) \tag{2.3.35}
\]

This scheme includes the strong numerical viscosity and the result obtained by this scheme is very diffusive. Hence, this scheme is not suitable for the actual use. Total Variation Diminishing (TVD) Lax-Fridrich scheme is used in fact. The TVD Lax-Fridrich reduces the numerical viscosity. Time evolution is solved by the Predictor-Corrector Method. Predictor is given as

\[
U_{i}^{n+\frac{1}{2}} = U_{i} - (F_{i+1}^{n} - F_{i-1}^{n}) \frac{\Delta t}{4\Delta x} \tag{2.3.36}
\]
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Using the predictor, final value, corrector, is calculated. High order accuracy is achieved by MUSCL interpolation and TVD solution is achieved by introducing MINMOD limiter:

\[ U^R_{i+\frac{1}{2}} = U_{i+1}^{n+\frac{1}{2}} - \frac{1}{4} \left[ (1 - \kappa) \tilde{\Delta}_{i+\frac{3}{2}} + (1 + \kappa) \tilde{\Delta}_{i+\frac{1}{2}} \right] \]  
(2.3.37)

\[ U^L_{i+\frac{1}{2}} = U_{i}^{n+\frac{1}{2}} + \frac{1}{4} \left[ (1 - \kappa) \tilde{\Delta}_{i-\frac{1}{2}} + (1 + \kappa) \tilde{\Delta}_{i+\frac{1}{2}} \right] \]  
(2.3.38)

\[ \tilde{\Delta}_{i+\frac{1}{2}} = \text{minmod} \left( \Delta_{i+\frac{1}{2}}, \omega \Delta_{i-\frac{1}{2}} \right) \]  
(2.3.39)

\[ \tilde{\Delta}_{i+\frac{1}{2}} = \text{minmod} \left( \Delta_{i+\frac{1}{2}}, \omega \Delta_{i+\frac{3}{2}} \right) \]  
(2.3.40)

\[ \text{minmod} \left( x, y \right) = \text{sgn} \left( x \right) \cdot \max \left\{ 0, \min \left[ |x|, y \cdot \text{sgn} \left( x \right) \right] \right\} \]  
(2.3.41)

\[ \Delta_{i+\frac{1}{2}} = U_{i+1}^{n+\frac{1}{2}} - U_{i}^{n+\frac{1}{2}} \]  
(2.3.42)

\( U^R \) and \( U^L \) represents the quantity at right side and left side of cell surface, respectively. \( \kappa = -1, \omega = 1 \) give the upwind second-order accurate scheme. The numerical flux on the cell surface \( \tilde{F} \) is calculated by using \( U^R \) and \( U^L \) as

\[ \tilde{F}_{i+\frac{1}{2}} = \frac{1}{2} \left[ F \left( U^R_{i+\frac{1}{2}} \right) + F \left( U^L_{i+\frac{1}{2}} \right) \right] - \frac{1}{2} r \left( U^R_{i+\frac{1}{2}} - U^L_{i+\frac{1}{2}} \right) \]  
(2.3.43)

\( r \) is the maximum eigenvalue of Jacobian matrix \( \partial F / \partial U \). \( r \) is given by

\[ r = |v_x| + c_f \]  
(2.3.44)

\[ c_f^2 = \frac{1}{2} \left\{ (a^*)^2 + \sqrt{(a^*)^4 - 4a^2b_x^2} \right\} \]  
(2.3.45)

\[ b_x = \frac{B_x}{\sqrt{\rho}} \]  
(2.3.46)

\[ (a^*)^2 = \frac{\gamma p + B^2}{\rho} \]  
(2.3.47)

\[ a^2 = \frac{\gamma p}{\rho} \]  
(2.3.48)

In three dimensions, the same procedure is repeated for y-direction and z-direction, respectively.

The above TVD Lax-Fridrich scheme include error of \( \nabla \cdot B \). In order to suppress the error,
the source term

\[
S_o = \begin{bmatrix}
0 \\
-(\nabla \cdot B) B \\
-(\nabla \cdot B) V \\
-(\nabla \cdot B) B \cdot V
\end{bmatrix}
\]  

is added to the basic equations:

\[
\frac{\partial U}{\partial t} + \nabla \cdot F = S_o
\]

The source term is evaluated by second-order accurate central-difference scheme.

### 2.3.3 Numerical Tests

The one-dimensional shock tube problem is performed, and this problem is proposed by Brio and Wu and solved by many other authors as a numerical test [51]. In this problem, the initial state are given by

\[
(\rho, v_x, v_y, v_z, B_y, B_z, p) = \begin{cases}
(1.0, 0.0, 0.0, 0.0, 0.0, 1.0, 1.0) & \text{for } x<400 \\
(0.125, 0.0, 0.0, 0.0, 0.0, 1.0, 1.0) & \text{for } x>400
\end{cases}
\]

with \(B_x = 0.75\) and \(\gamma = 2\). Note that the hydrodynamic data is the same as the Sod's Riemann problem. This problem is solved using the one-dimensional form of ideal MHD model, Eq. (2.3.33), and the Predictor-Corrector scheme for the time integration with \(\Delta t = 0.2\). The computational domain is set to be \([0:800]\) with 800 grid points (\(\Delta x = 1.0\)). Fig. 2.2 shows the computational setup of this problem.

The solution after 400 time steps is shown in Fig. 2.3. A fast rarefaction wave (FR) and a slow compound wave (SM) propagate from the origin to the left. A contact discontinuity (C), a slow shock (SS) and a fast rarefaction wave (FR) propagate to the right. The exact solution is obtained by fine grid (8000 grid, \(\Delta x = 0.1\)) simulation. The numerical solution by the TVD Lax-Friedrich scheme is in good agreement with the results by Brio and Wu, and many other authors.

### 2.3.4 Simulation Model

Three-dimensional computational domain of MHD model and calculation flow are shown in Figs. 2.4 and 2.5, respectively. The uniform spacing grid is used. In typical case, \(512 \times 512 \times 512\) grid is included. The domain decomposition is adopted as shown in Fig. 2.4. The computational domain
2.3. MAGNETOHYDRODYNAMIC MODEL

Figure 2.2: The computational setup for the one-dimensional shock tube problem along $x$-axis.

Figure 2.3: One-dimensional shock tube test problem. The numerical solution of TVD Lax-Friedrich scheme and exact solution are plotted: a) mass density and b) magnetic flux density along $x$-axis.
is divided into small sub domain and parallelly calculated by many processors. The numerical flux at the boundary of small sub domain is exchange by Message Passing Interface (MPI) [52]. The solar wind is assumed to flow in a positive direction of the $z$-axis. The initial condition of the MHD model simulation is set as

\[
\begin{cases}
\rho_{\text{initial}} = \frac{B^2}{V_{Alf}^2} & \rho_{\text{initial}} > 0.01\rho_{SW} \\
\rho_{\text{initial}} = 0.01\rho_{SW} & \rho_{\text{initial}} \leq 0.01\rho_{SW}
\end{cases}
\]  

(2.3.52)

\[
p_{\text{initial}} = 0.1p_0
\]

(2.3.53)

\[
V_{\text{initial}} = 0
\]

(2.3.54)

\[
B_{\text{initial}} = B_{\text{dipole}}
\]

(2.3.55)

in order to prevent the very fast Alfvén velocity at magnetic dipole center from ruining the calculation.

In the inner boundary, the physical value of the flow is constant at the initial value. In inflow boundary ($z=-N/2$), the variables are fixed by the solar wind parameters. In outflow boundary, the value of one inner sides are extrapolated.

From the Courant condition, the time step size is defined as

\[
\Delta t = \min \left( \frac{\Delta x}{|v_x| + C_{fx}^j}, \frac{\Delta x}{|v_y| + C_{fy}^j}, \frac{\Delta x}{|v_z| + C_{fz}^j} \right)
\]

(2.3.56)

\[
C_{fj} = \frac{1}{2} \left( \frac{\gamma p + B^2}{\rho} \right) + \sqrt{\left( \frac{\gamma p + B^2}{\rho} \right)^2 - 4 \left( \frac{\gamma p}{\rho} \right)^2 \left( \frac{B_j^2}{\rho} \right)^2} \quad j = x, y, z
\]

(2.3.57)

### 2.4 Flux-Tube Model

On the ion inertial scale, where the ion kinetics has dominant effect, the thrust of magnetic sail and Magneto Plasma Sail has been simulated by Hybrid-PIC simulation. However, the quick estimation is required to survey the various parameters. Hence, we choose the Flux-Tube model. The objective of the Flux-Tube model is to construct a simplified numerical model for magnetic sail with short computational time and reduced memory requirement in order to survey the effects of various design parameters. The Flux-Tube model neglects wave propagation (electromagnetic wave, acoustic wave, etc.) and it solves only a steady state of a plasma flow including the ion’s finite Larmor effect. Therefore the Flux-Tube model is expected to quickly estimate a thrust level with low capacity of the computational memory. However, the Flux-Tube models developed for hall thrusters [53] and ion engine grid [54, 55] do not include the effect of induced magnetic field.
2.4. FLUX-TUBE MODEL

Figure 2.4: Computational domain and boundary condition used in MHD model. The domain is divided to small sub domain for parallelized computation.

Figure 2.5: Calculation flow of ideal MHD model.
In the case of magnetic sail, the induced magnetic field has to be considered.

Therefore we constructed a new Flux-Tube model including the induced magnetic field. Then we performed numerical simulations for a magnetic sail with a large magnetosphere (>100 km) and compared thrust and magnetosphere characteristics with the results by earlier studies to confirm the accuracy of the new method.

The numerical model of Flux-Tube model aims to calculate the thrust exerted on a magnetic sail spacecraft taking the finite Larmor radius effect into consideration. The magnetic sail thrust required for the future mission is from sub N to several N and the Flux-Tube model evaluates the thrust of N-class magnetic sail in the steady state. When the typical parameters of the solar wind are assumed, the magnetic flux density at the magnetopause is estimated approximately as 40 nT. Then the ion Larmor radius at the magnetopause is calculated as \( r_{iL} \sim 100 \) km and the electron Larmor radius is calculated as \( r_{eL} \sim 50 \) m. The magnetosphere size of the N-class magnetosphere is corresponding to \( L_{MHD} \sim r_{iL} \) and \( L_{MHD} \gg r_{eL} \). When \( L_{MHD} \) and \( r_{iL} \) are comparable at the magnetopause, ion kinetic effects, such as the finite Larmor radius effect, should be considered. On the contrary, when \( L_{MHD} \) is much larger than \( r_{eL} \), electron motion can be assumed as a fluid. In this scale length, where we call as the ion inertial scale, Hybrid-PIC model can be used for the thrust calculation. However, the large computational resource is required for three-dimensional simulation of magnetic sail and Magneto Plasma Sail. The various plasma waves, which is not affected on the thrust generation, is also included in the Hybrid-PIC simulation and the fluctuation of thrust is occurred. Here, the average thrust in the steady state is more suitable for the mission design than the short period thrust fluctuation (\( \sim \) plasma frequency or Alfvén velocity). Hence, we developed the Flux-Tube model, which treat only steady state of plasma flow. It is expected that the required computational resource becomes smaller than that of Hybrid-PIC model.

In the case of a magnetic sail in interplanetary space, an ion’s mean free path \( (\delta \sim 1.5 \times 10^8 \) km) is much longer than the typical length of a magnetosphere \( (L_{MHD} \sim 100 \) km). Hence, direct collisions between plasma particles can be neglected. Also, the wave-particle interaction, through which the solar wind plasma particles change their momentum even though \( \delta \gg L \), is neglected in the model by considering only the steady state field, that is, electrostatic field and magneto static field. Then solar wind plasma is treated as a completely collision less flow and the Flux-Tube model could not treat the shock wave correctly. As for the charge separation, if the magnetospheric size \( L_{MHD} \) is sufficiently larger than the Debye length \( (L_{MHD} \gg \lambda_D) \), then charge neutrality can be assumed by neglecting the charge separation. When \( L_{MHD} \) is several 100 km and the steady state of plasma flow is assumed, it is larger than the Debye length, which is approximately \( \lambda_D \sim 10 \) m in the solar wind. The steady state is also achieved over a much longer time than the plasma oscillation and the neutrality assumption is valid over this time scale.

In addition, only ions are treated as particles and electrons are treated as a fluid. As for the thermal velocity, the ion temperature in the solar wind is approximately 1 eV and the thermal velocity, approximately 10 km/s, is smaller than the solar wind bulk velocity \( V_{SW} \), 400 km/s. So
the ion’s temperature of the solar wind can be neglected. On the other hand, the electron thermal velocity, approximately 2000 km/s, is larger than the solar wind bulk velocity and the thermal motion of electrons is expected to affect the formation of a magnetosphere: hence the electron temperature is considered in the basic equations below. Then, it is expected the Flux-Tube model we proposed should be adaptable to the magnetic sail in the ion inertial scale since the obvious shock is formed when the magnetospheric size \( L_{MHD} \) satisfies the condition, \( L_{MHD} \gg r_iL \). The magnetic sail in the ion inertial scale generates \( N \)-class thrust and the thrust should be evaluated for a future orbital transfer system in interplanetary space. Here, it is thought that 20%~30% error of thrust evaluation is permissible since the uncertainties such as solar wind variation and attitude of spacecraft is expected in the mission.

2.4.1 Basic Equations

Ions are treated as particles and ion trajectories are calculated under the assumption of a non-time varying electromagnetic field. The equations for ion motion are

\[
\frac{dr_i}{dt} = v_i \quad (2.4.1)
\]

and

\[
m_i \frac{dv_i}{dt} = e(E + v_i \times B) \quad (2.4.2)
\]

On the other hand, electrons are treated as a fluid. The equation of the electron motion is given by

\[
en_e (E + u_e \times B) + k_B T_e \nabla n_e = 0 \quad (2.4.3)
\]

assuming massless fluid. The second term in Eq. (2.4.3) represents the pressure of the electron fluid and the electron temperature \( T_e \) is considered here. In the magnetosphere, the electron temperature differs between the parallel direction to the magnetic field and the perpendicular direction to the magnetic field. However, it is expected that the massless electron has less effects for the thrust and the isotropic electron temperature is assumed for simplicity in the model.

Furthermore, assuming quasi-neutrality

\[
n_e = n_i \quad (2.4.4)
\]

and combining this equation with Ampere’s law

\[
\nabla \times B_p = \mu_0 J_{total} = en_i \mu_0 (u_i - u_e) \quad (2.4.5)
\]

, an explicit expression for electric field given by


\[ E = \left( \frac{1}{e n_i \mu_0} \nabla \times B_p - u_i \right) \times B - \frac{1}{en_i} k_B T_e \nabla n_i \tag{2.46} \]

is obtained by eliminating \( n_e \) and \( u_e \). Here, \( B_p \) represents the magnetic flux density induced by plasma current. That is, total magnetic flux density is represented as \( B = B_p + B_0 \). \( B_0 \) is the initial magnetic field generated by coil current. Finally, substituting Eq. (2.4.6) into Faraday law

\[ \partial B / \partial t = -\nabla \times E \tag{2.4.7} \]

is written in only one variable \( B \) since \( B_p \) is replaced as \( B_p = B - B_0 \). By the time integration of Eq. (2.4.8) until a steady state \( (\partial B / \partial t = 0) \), the magnetic field in the next iteration step is obtained. The value of the magnetic field, ion density and electric field is defined at grid point as same as the MHD simulation as show in Fig. 2.1.

### 2.4.2 Algorithm of the Flux-Tube Code

The calculation consists of two different parts:

1. the ion trajectory calculation.
2. the static field calculation.

The ion trajectory calculation tracks particles in a non-time varying electromagnetic field by assuming a steady state instead of solving the time evolution of the electromagnetic field. The Buneman-Boris method is used for the trajectory calculation. From particle trajectories, a density distribution and a current distribution are obtained. Then a self-consistent electromagnetic field is obtained by solving the Maxwell’s equations. The electromagnetic field is again assumed to be in a steady state. Thus the particle trajectories and the electromagnetic field are repeatedly solved until both variables are converged to obtain a true steady state of the plasma flow and the electromagnetic field.

The detail of the calculation sequence is illustrated in Fig. 2.6. At the beginning of each calculation, the initial magnetic field configuration \( B_0 \) is given as an input parameter. In the main loop in Fig. 2.6, the ion trajectory calculation is performed. When a non-time varying electromagnetic field is assumed, ion trajectories are traced by integrating Eq. (2.4.1) and Eq. (2.4.2) using the Buneman-Boris method to conserve the momentum of particles [56]. Then the trajectory of each ion is calculated from the in-flow boundary to out-flow boundary. The electromagnetic field at each particle’s position is linearly interpolated by the grid point value. From the ion trajectories, the ion density and the current density are calculated to satisfy the conservation of mass. In this
method, which is usually called as a “Flux-Tube method”, the ion density of $k_{th}$ grid is calculated as

$$n^k = \sum_l \frac{IT^k_l}{eV_{grid}}$$  \hspace{1cm} (2.4.9)

by counting the number of ions passing through the grid and their residence time $T^k_l$ in the $k_{th}$ grid. Here, $I$ is the ion current produced by respective ion trajectories. The current density is calculated as the product of the ion density and average velocity of ions.

In the sub-loop in Fig. 2.6, the electromagnetic field for the next ion trajectory calculation is renewed by Eq. (2.4.8) with the ion density and the current density obtained in the ion trajectory calculation. It is desirable to calculate the static electromagnetic field directly from

$$\nabla \times \mathbf{E} = 0$$  \hspace{1cm} (2.4.10)

However, the partial differential equation (Eq. (2.4.10)) is unstable. To prevent the unstable nature of the equation, the induction equation (Eq. (2.4.7)) is employed and the magnetic field $\mathbf{B}$ is time integrated by the fourth-order Runge-Kutta method until $\partial \mathbf{B} / \partial t = 0$, hence $\nabla \times \mathbf{E} = 0$, is satisfied. Here, the time step used for the integration $\Delta t_{field}$ is a different variable from the variable used for the ion trajectory calculation $\Delta t_{particle}$. The magneto static field obtained by the integration is substituted to Eq. (2.4.8) and the electrostatic field is also obtained. However, Eq. (2.4.8) diverges if an area where the ion density is zero exists, since Eq. (2.4.8) contains the division by the ion density. Hence, to prevent the divergence, a lower limit has to be set to the ion density.

These calculations are repeated until the electromagnetic field in the sub-loop and the thrust in the main loop coincide within a tolerable error. Here, the thrust is calculated by two methods. In the first method, the electromagnetic force between the induced current by a plasma flow and the coil current onboard spacecraft is calculated, and the electromagnetic force is designated as $\mathbf{F}_{mag}$:

$$\mathbf{F}_{mag} \equiv \oint \left( \sum \sum \frac{\mu_0 J S dl \times r}{4\pi r^3} \right) \times I_{coil} dr$$  \hspace{1cm} (2.4.11)

The summations represents the discrete integral in the three-dimensional calculation space and and the contour integral represents the integration along the coil current path. The second method uses a change in the solar wind momentum passing over a control volume to derive a thrust, $\mathbf{F}_{flow}$:

$$\mathbf{F}_{flow} \equiv -\oint \left( m_i n_i u_i u_i + \frac{B \cdot B}{2\mu_0} I - \frac{BB}{\mu_0} \right) \cdot \mathbf{n} dS$$  \hspace{1cm} (2.4.12)

The surface integral represents the integration on the arbitrary surface including the onboard coil.
Table 2.2: Parameters used in one-dimensional test simulation of Flux-Tube model

<table>
<thead>
<tr>
<th>Solar wind parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity ( V_{SW} )</td>
<td>( 4 \times 10^5 \text{m/s} )</td>
</tr>
<tr>
<td>Density ( N_{SW} )</td>
<td>( 5 \times 10^6 \text{m}^{-3} )</td>
</tr>
<tr>
<td>Electron temperature ( T_e )</td>
<td>( 10^5 \text{K} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Computational region</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid spacing ( \Delta x )</td>
<td>10 km</td>
</tr>
<tr>
<td>Grid number</td>
<td>128</td>
</tr>
</tbody>
</table>

In the first method, the magnetic field near a coil is calculated based on the Biot-Savart law, and the amount of thrust is estimated by integrating the electromagnetic force along a coil current as in Eq. (2.4.11). In the second method, the thrust is calculated by an arbitrary surface integral of the momentum as in Eq. (2.4.12). Note that \( \mathbf{u}_i \), \( \mathbf{u}_i \), \( \mathbf{B} \), and \( \mathbf{I} \) are tensor. The terms in Eq. (2.4.12) represent the momentum exchange by ion motions, the magnetic pressure and the Maxwell stresses, respectively. The thrust calculated by the second method is only correct when the calculation is converged.

2.4.3 Numerical Tests

First, in order to validate the Flux-Tube model we proposed, one-dimensional simulation of a magnetosphere boundary was performed. An ion and an electron’s trajectory in the magnetopause are schematically shown in Fig. 2.7. By the particle drift motion, the magnetopause current \( \mathbf{J}_{MP} \) is induced and the magnetic field is confined inside the magnetosphere. This magnetosphere structure has a big influence on the process through which the magnetic sail generates the thrust since the particles lose their momentum at the magnetopause. The magnetic field around the one-dimensional magnetopause is analytically calculated using

\[
\frac{x}{\sqrt{2r_{iL}}} = \ln \left( \tan \left( \frac{\Phi}{2} \right) \right) + 2 \cos \Phi \tag{2.4.13}
\]

and

\[
B = \sqrt{2B_{MP}} \sin \Phi \tag{2.4.14}
\]

[9]. Here, values of \( B_{MP} = 40 \text{ nT} \) and \( r_{iL} = 100 \text{ km} \) were obtained assuming the typical parameters listed in Table 2.3 and \( \Phi \) varies from 0 to \( \pi/4 \).

This one-dimensional magnetopause structure is numerically simulated by the Flux-Tube method we proposed. The one-dimensional computational region of 1280 km size is divided into 128 grids. The original magnetic field is defined to be inversely proportional to the cube of the distance as

\[
B_{y0} = 4.0 \times 10^{-9} \left( \frac{1.6 \times 10^5 - x}{1.6 \times 10^5} \right)^{-3} \tag{2.4.15}
\]
2.4. FLUX-TUBE MODEL

Figure 2.6: Calculation flow of Flux-Tube code
CHAPTER 2. NUMERICAL MODELS

In addition, the solar wind parameters are listed in Table 2.2. The time step $\Delta t_{\text{particle}}$ for the ion trajectory calculation in the main loop is set to a value sufficiently shorter than the ion Larmor cycle. The time step $\Delta t_{\text{field}}$ for the time integration of Eq. (10) in a sub-loop is set to satisfy

$$\frac{\Delta x}{\Delta t_{\text{field}}} > V_A$$

(2.4.16)

The simulation result and the theoretical result are shown in Fig. 2.8. The bold line, the dotted line and the dashed line represent the theoretical result, the numerical result by the Flux-Tube model and the original magnetic field defined by Eq. (2.4.15), respectively. The magnetic field distribution around the magnetopause ($x<800$ km) from the Flux-Tube model agrees well with that of the theory. In addition, the Flux-Tube model provides the magnetic field distribution inside magnetosphere ($x>800$ km). Thus, it can be said that the Flux-Tube model correctly simulates the magnetic field around the magnetic sail.

In addition, the effect of the grid spacing $\Delta x$ on the steady state solution for the magnetosphere was investigated. The 1280 km one-dimensional computational area was divided to 128 grids, 64 grids and 32 grids. The grid spacing is 10 km, 20 km and 40 km, respectively, as listed in Table 2.3. The one-dimensional magnetopauses calculated for each condition are shown in Fig. 2.9. Three results are in agreement with the magnetospheric size defined as the point where the magnitude of the magnetic field changes from $B/B_0>1$ to $B/B_0<1$. However, as the grid spacing becomes larger, the gradient of the magnetic field around the boundary decreases. This is because the magnetopause current density $J_{MP}$ that causes the magnetopause magnetic field distribution
Figure 2.8: One-dimensional magnetic flux density of magnetopause along the $x$-axis calculated by Flux-Tube model (dashed line), original magnetic flux (dotted line) and theory (bold line).

Table 2.3: Parameters used in one-dimensional test simulation of Flux-Tube model

<table>
<thead>
<tr>
<th>Grid spacing, km</th>
<th>Grid number</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>128</td>
</tr>
<tr>
<td>20</td>
<td>64</td>
</tr>
<tr>
<td>40</td>
<td>32</td>
</tr>
</tbody>
</table>

has the magnitude error since an error arises between the true value of magnetic field and the interpolated value from grids as the grid spacing becomes larger. When the gradient of the magnetic field around the boundary is decreased, the ions can penetrate further into the magnetosphere and some ions, which should be reflected in the true magnetosphere, are not reflected by the magnetic field when the error is included. Hence, the thrust for the case with the lager grid spacing may become slightly smaller. However, it should fit within the tolerable error assumed for the Flux-Tube model, that is, 20% to 30% error.

### 2.5 Full Particle-in-Cell Model

The demonstrator spacecraft has small diameter superconductive coil and only weak magnetic field; hence, small magnetosphere can be generated. When the typical parameters of the solar wind, the ion Larmor radius and the electron Larmor radius at the magnetopause are calculated as $r_{iL} \sim 100$ km and $r_{eL} \sim 50$ m, respectively. When the magnetosphere size $L_{MHD}$ is smaller than $r_{iL}$, both ion’s and electron’s finite Larmor radius effect should be considered. In addition to the finite Larmor radius effect, when the Debye length $\lambda_D$ is competitive with the magnetosphere
size ($L_{MHD} \sim \lambda_D$), the charge separation should be taken into consideration. We call this scale length of the magnetosphere ($L_{MHD} < r_{iL}$ and $L_{MHD} > r_{eL}$) as the electron inertial scale. Full-PIC simulation is performed to reveal the thrust characteristics of magnetic sail and MPS on the electron inertial scale.

Previous studies elucidated the thrust characteristics of magnetic sails that have a large magnetosphere (>100 km) using Hybrid-PIC model and MHD model. These simulation techniques are currently being used to evaluate MPS, which inflates the magnetosphere of the original magnetic sail using a plasma jet from the spacecraft to obtain a higher thrust level [26, 27, 29, 48]. However, the thrust of a magnetic sail has not been evaluated for a small-scale magnetosphere (<100 km), which can be expected in a demonstration spacecraft. The difficulty is that the kinetic effects of electrons must be considered in such a small-scale magnetosphere, and neither MHD simulation nor Hybrid-PIC simulation is suitable for doing so [30]. Full-PIC simulation, which treats both ions and electrons as particles, is needed in order to take into account the particle nature of plasma, including finite Larmor radius effects and charge separation. The interplanetary magnetic field can also be taken into consideration without any artificial assumption of model.

The solar wind mainly consists of ions (protons) and electrons. Typical parameters of the solar wind at an Earth orbit are listed in Table 1.2. In the case of a magnetic sail in the interplanetary space, an ion mean free path is approximately $1.5 \times 10^8$ km. It is much longer than the typical length of a magnetosphere we consider by Full-PIC model, $L_{MHD} = 100$ m~<~10 km. Hence, direct collisions between particles can be neglected in the magnetic sail simulations. Here, the magnetospheric size $L_{MHD}$ is derived from the pressure equilibrium at the magnetopause (the boundary of the
magnetosphere, where the magnetopause current is induced by the solar wind plasma). Based on MHD approximation, the solar wind dynamic pressure balances the magnetic pressure of the 2D dipole magnetic field generated by a magnetic sail spacecraft as Eq. (1.2.5) and the magnetic pressure of the 3D dipole magnetic field generated by a magnetic sail spacecraft as Eq. (1.2.3), respectively.

### 2.5.1 Basic Equations

Full-PIC simulation treats both ions and electrons as particles in order to consider the finite Larmor radius effects. In addition, the charge separation and the effects of the electric field at the magnetopause are considered.

Full-PIC simulation solves the equation of motion,

\[
\frac{d\mathbf{r}_j}{dt} = \mathbf{v}_j, j = i, e
\]  

and

\[
m_j \frac{d\mathbf{v}_j}{dt} = q_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}), j = i, e
\]

and traces the precise motion of each ion and electron using the Buneman-Boris method [56].

From the particle trajectories, a density distribution and a current distribution are obtained according to the PIC weighting method. Maxwell equations,

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]  

and

\[
\frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \mu_0 \mathbf{J}
\]

are solved by using the Finite-difference time-domain (FDTD) method to obtain a self-consistent electromagnetic field.

The electric field is corrected electrostatically by Poisson’s equation;

\[
\nabla \cdot \mathbf{E} = \frac{e(n_i - n_e)}{\varepsilon_0}
\]  

in order to conserve the energy.

### 2.5.2 Algorithm of the Full Particle-in-Cell Code

Maxwell equations are solved by using Finite Differential Time domain (FDTD) method. The magnetic field and the electric field are defined at the surface center and the edge center, respectively, as shown in Fig. 2.10. As shown in Fig. 2.11, the magnetic field is updated a half-step every since the Buneman-Boris method to track particles requires the electromagnetic field \(t=T+1/2\).
The positions of particles are also updated every half time step using the velocity at $T+1/2$. The electromagnetic field at the particle position is linearly interpolated from the value at the definition point of the electromagnetic field.

From the particle trajectories, a density distribution and a current distribution are obtained according to the PIC weighting method. The density distribution is defined at volume center of Fig. 2.10 and the current density is defined at the same point with the electric field.

Poisson’s equation is used for the correction of the electrostatic field in order to achieve the energy conservation law. From the original electric field calculated by FDTD method, the correction of the static electric field can be written as

$$E_{correct} = E_{FDTD} + E_{error}$$  \hspace{1cm} (2.5.6)

The Poisson’s equation is rewritten using $E_{err}$,

$$\nabla \cdot E_{err} = \frac{e(n_i - n_e)}{\epsilon_0} - \nabla \cdot E_{FDTD}$$  \hspace{1cm} (2.5.7)

By using the relation

$$E_{err} = -\nabla \phi_{err}$$  \hspace{1cm} (2.5.8)

$$\nabla^2 \phi_{err} = -\frac{e(n_i - n_e)}{\epsilon_0} + \nabla \cdot E_{FDTD}$$  \hspace{1cm} (2.5.9)

is obtained. This equation is solved by using FFT method and $E_{err}$ is eliminated from the
Figure 2.11: Calculation flow of Full-PIC code.
The open boundary condition is adopted in the simulations of magneto plasma sail. We use Perfectly Matched Layer (PML) conditions [57]. The PML boundary condition introduce the artificial resistance and magnetic resistivity into the Maxwell equations;

$$\frac{\partial B}{\partial t} = -\nabla \times E + \sigma_m B$$  \hspace{1cm} (2.5.10)

and

$$\frac{1}{c^2} \frac{\partial E}{\partial t} = \nabla \times B - \mu_0 J + \sigma E$$  \hspace{1cm} (2.5.11)

The magnetic resistivity is defined as

$$\sigma_m = \frac{\sigma}{\epsilon_0}$$  \hspace{1cm} (2.5.12)

to prevent electromagnetic wave from reflecting from the boundary. The PML boundary cannot use in a dispersible medium like plasma. Therefore, the boundary for the particle, where eliminate particle from calculation is installed before the PML boundary for the electromagnetic field. Fig. 2.12 shows the definition of sigma along the one-dimensional axis. The sigma becomes large in non-physical region.

### 2.5.3 Parallelization and Improvement in Calculation Speed

Full-PIC model requires large computational resource in order to trace large amount of particles. In the typical three-dimension simulation, 40 billion particles are included in the computational domain. Hence Computational load becomes huge in order to calculate the trajectory of huge numbers of plasma particles. That has prevented from simulating the plasma flow around small-scale magnetic sail and obtaining the thrust characteristics. However, large 3D full-PIC simulations have recently become possible owing to the improvement on the parallel computing techniques.
Table 2.4: Results of weak and strong scaling test by 3D Full-PIC code from 4 CPUs to 1024 CPUs

<table>
<thead>
<tr>
<th></th>
<th>Base condition</th>
<th>1024 CPUs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak scaling</td>
<td>40 s @ 4 CPUs</td>
<td>41 s</td>
</tr>
<tr>
<td></td>
<td>1.03/1 (ideal)</td>
<td>~97 %</td>
</tr>
<tr>
<td>Strong scaling</td>
<td>662 s @ 4 CPUs</td>
<td>41 s</td>
</tr>
<tr>
<td></td>
<td>16.1/16 (ideal)</td>
<td>~101 %</td>
</tr>
</tbody>
</table>

Our simulation code is also parallelized by using both OpenMP and MPI. As shown in Fig. 2.13a, the computational domain is divided to small sub region along z-axis. Each sub region is calculated by one process. The electromagnetic field and particle stranding the boundary is communicated by MPI. The ghost cell is adopted in order to transfer the boundary value as shown in Fig. 2.14. The particle involved in the sub region is divided smaller groups as shown in Fig. 2.13b. The each particle group is calculated by different thread. Thus, two kinds of parallelization techniques are combined.

Fig. 2.15 and Table 2.4 shows the scalability of the parallelization. In weak scaling test, the particle number per thread is constant and ideally execution time is constant. In strong scaling test, total particle number is constant. Hence, as the CPU number increase, the particle number handled by one CPU becomes smaller. Fig. 2.15 shows the results of 2D simulation. The left vertical axis and the right vertical axis represent execution time and the acceleration rate to 4 CPUs, respectively. The improvement in calculation speed proportional to the CPU numbers is achieved. In three-dimensional our Full-PIC simulation code, both the strong scaling ($5 \times 10^8$ particles in total) and the weak scaling ($5 \times 10^5$ particles / CPU) are checked. The scalability of 101 % and 97 % are achieved in strong scaling and weak scaling, respectively, as shown in Table 2.4.

In addition to the parallelization, in order to improve the calculation speed, the sorting and load balancing are implemented. The calculation speed of Full-PIC simulation depends on the memory access speed interpolating the electric magnetic field and current density. By sorting the portion of the plasma particle, CPU can access the field data continuously and cash-miss ratio becomes smaller. In the simulation code of this study, the list structure as shown in Fig. 2.16 are used for the particle data handling. The list structure refers the memory address using pointer and can arbitrarily switch the calculation sequence. In addition, the plasma injected from spacecraft is easily add to last of the list structure. Sorting by list structure requires only $O(N)$ calculation compared with the bubble sort $O(N^2)$ and merge sort $O(N \log N)$. By complementing the sorting, the calculation speed is improved by 30 % as shown in Fig. 2.17. The horizontal axis and the vertical axis represent the simulation step and calculation time.

When the particle numbers contained in the each region differ, the expectation occurs and computational resource is consumed vainly. In the simulation code, to balance the particle number per thread, the thread number per process are dynamically changes as schematically illustrated in Fig. 2.18. In initial state, the particles are scattered uniformly. As the simulation step goes on, the particle count becomes uneven. The region where particle number is high is calculated by the
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Figure 2.13: Hybrid parallelization: a) domain is divided to small region by domain decomposition and b) particles are divided to small groups by particle parallelization.

Figure 2.14: Ghost cell used for transferring of boundary data by domain decomposition.
2.5. FULL PARTICLE-IN-CELL MODEL

Figure 2.15: Scalability of 2D Full-PIC code from 4 CPUs to 32 CPUs.

Figure 2.16: List structure for particle data handling. The list structures for threads are prepared and accessed in parallel.
many CPUs to finish the one cycle of calculation approximately same time.

### 2.5.4 Numerical Tests

In order to validate the simulation code, plasma wave propagations are tested. The distribution plasma oscillation without external magnetic field is given by

\[
\omega^2 = c^2 k^2 + \omega_p^2
\]

[58]. Here, \( \omega \) is frequency, \( k \) is wave number and \( \omega_p \) is plasma frequency. As shown in Fig. 2.19, the distribution corresponding to the plasma oscillation is correctly obtained by Full-PIC simulation. Here, plasma density and temperature corresponding to the solar wind is assumed. The contour represent the magnitude of electric field and the temporal-spatial FFT is performed. The blue line and green line represents the tangential line at \( k = 0 \) and \( k = \pm \infty \).

When external magnetic field parallel to the wave propagation is induced, R-mode wave:

\[
\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 - (\omega_c/\omega)}
\]

and L-mode wave:

\[
\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 - (\omega_c/\omega)}
\]

is obtained as Fig. 2.20. Especially, the R-mode wave propagate in \( \omega < \omega_c \) is called as Whistler
2.5. **FULL PARTICLE-IN-CELL MODEL**

![Figure 2.18: Schematic illustration of load balancing.](image)

![Figure 2.19: Wave propagation without external magnetic field.](image)
CHAPTER 2. NUMERICAL MODELS

Figure 2.20: Wave propagation with external magnetic field parallel to wave.

\[ \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_{UH}^2} \]

is obtained as Fig. 2.21. Here, \( \omega_{UH}^2 = \omega_p^2 + \omega_e^2 \). Thus, our simulation code can deal with the electromagnetic wave correctly.

Energy conservation law in the simulation is also evaluated by the two-stream instability. The instability can be observed by an energetic particle stream when ions and electrons have different drift velocity. The energy from the particles can lead to plasma wave excitation. The simulation distinguished between the velocity of ion and electron. The simulation result is shown in Fig. 2.22. The horizontal axis and vertical axis represents position of particles and velocity. Initial velocities of ion and electron are set as 0 m/s and \( 1.5 \times 10^{-7} \) m/s and the initial velocity fluctuation of electron is added. Ion and electron exchange energy through an electromagnetic field, and the wave.

Figure 2.21: Wave propagation with external magnetic field perpendicular to wave.
instability grows. However, as shown in Fig. 2.23, the total energy and kinetic energy of plasma are constant.

2.6 Summary

The simulation scale of Magnetic Sail and Magneto Plasma Sail varies from MHD scale to the electron inertial scale and various simulation methods valid in each scale are required. We developed three simulation codes adopting ideal-MHD model, Flux-Tube model and Full-PIC model. Ideal-MHD model performs the single fluid approximation and not suitable for the simulation on ion inertial scale and electron inertial scale. The Flux-Tube model, especially, is newly proposed by this study in order to achieve the fast and exact thrust estimation on the ion inertial scale. The validity of newly developed Flux-Tube code is confirmed by the comparison of one-dimensional simulation of magnetopause structure with theoretical analysis results. As a result, it was revealed that that two structures are well in agreement. Full-PIC model, which requires large computational resource, is developed by using the parallelization techniques. The Full-PIC model has been required for the plasma simulation not only Magneto Plasma Sail but also other electric propulsion system to revel the plasma physics affected by plasma kinetics. However, the large computational cost prevents the Full-PIC simulation from performing. As a result, the high scalability and high efficiency simulation is achieved. The validity of the three simulation codes is also described by performing the test simulations: wave propagation, energy conservation.

According to the simulation models represented in this chapter, the thrust characteristics of Magnetic Sail and Magneto Plasma Sail are revealed in below chapters.
Figure 2.23: Energy conservation of Full-PIC simulation along time evolution of two-stream instability. Total energy in space is constant.
Chapter 3

Simulation of Magnetic Sail on Ion Inertial Scale

3.1 Introduction

On the ion inertial scale magnetosphere, the ion Larmor motion affects the thrust generation of magnetic sail and hence the ion kinetics should be considered. However, the simulation method such as Hybrid-PIC method, which takes the ion kinetics into the consideration, requires large computational resource. Therefore, the parametric study for the thrust characteristics of ion inertial scale magnetic sail spend much simulation time and sufficient thrust characteristics for the mission design and spacecraft design is not acquired.

As a computational method, hence, we chose a Flux-Tube model in which ions are treated as flux-tubes as we newly developed in Chapter 2. The Flux-Tube model neglects wave propagation (electromagnetic wave, acoustic wave, etc.) and it solves only a steady state of a plasma flow including the ion’s finite Larmor effect. Therefore the Flux-Tube model is expected to quickly estimate a thrust level with low capacity of the computational memory. First, we performed numerical simulations for a magnetic sail with a large magnetosphere and compared thrust and magnetosphere characteristics with the results by MHD simulation and earlier studies to confirm the accuracy of the new method. In addition, we reveal the thrust characteristics of magnetic sail with various magnetosphere sizes by parametric survey using Flux-Tube model.

3.2 Computational Settings

The initial magnetic field configuration according to the coordinate system as shown in Fig. 3.1 is given as an input parameter. The calculation domain around magnetic sail is divided into small cubic elements in order to apply the finite difference method for the electromagnetic field calculation. The solar wind is assumed to flow in a positive direction of the z-axis. So ions are
arranged at equal intervals on the in-flow boundary with uniform velocity in a positive direction of the $z$-axis by neglecting thermal motions. The spacecraft with a large hoop coil is fixed to the origin of the computational domain. In the simulation, the superconductive characteristics of the coil are neglected, and the coil is represented by a single current loop. As Figs. 3.1 a) and b) indicate, two cases, according to the angle between the coil axis and the solar wind velocity, are simulated. In the parallel case, the magnetic moment of the coil is parallel to the $z$-axis and, in the perpendicular case, the magnetic moment of the coil is directed to the $y$-axis.

According to the proposed analysis model (Chapter 2), plasma simulation of a magnetic sail was performed. The assumed parameters and conditions are listed in Table 3.1, and typical parameters of the solar wind at the Earth orbit are used. Under this condition, the size of a magnetosphere calculated by Eq. (1.2.3) is $L_{MHD} \sim 500$ km. At the magnetopause, the magnetic flux density is approximately 40 nT, and hence the ion Larmor radius is $r_{iL} \sim 100$ km. The size of calculation domain was $4800 \times 4800 \times 4800$ km and it was divided to $65 \times 65 \times 65$ by equally spaced orthogonal grids. The grid width is 75 km. From the in-flow boundary at $z=-2400$ km, 16 particles per cell with a total of 65536 particles were arranged at regular intervals. The thrust becomes approximately constant with more than 16 particles per cell. Then the grid and particle number is set as the necessary and sufficient condition for the fast thrust estimation. The time step to solve the momentum equations was set to $\Delta t_{\text{particle}} = 10^{-2}$ s which is sufficient to solve the ion trajectories correctly. Ions pass through a grid over several time steps and this time step is sufficiently shorter than the ion Larmor cycle. To solve Eq. (2.4.8) stably, the time step is limited by Courant condition as $\Delta x/\Delta t_{\text{field}} > V_A$. Here, by assuming a very large coil radius (250 km), the magnetic flux density on the coil becomes 50 $\mu$T and the Alfven velocity $V_A$ is approximately $5 \times 10^5$ m/s. Therefore, when $\Delta x$ is 75 km, it is necessary for $\Delta t_{\text{field}}$ to satisfy the condition of
3.3 Simulation Results and Discussion

### Validation of Flux-Tube Model via Magnetosphere Simulation

In Chapter 2, the one-dimensional magnetosphere structures on ion inertial scale were simulated by using new developed Flux-Tube model and the simulation result was compared by the theoretical analysis. It is confirmed that the Flux-Tube model can deal with the formation of the artificial magnetosphere. Then, we start to simulate the three-dimensional magnetosphere by Flux-Tube model to reveal the thrust characteristics of magnetic sail on the ion inertial scale.

Figures 3.2 shows the plasma density distribution. In the figure, a) and b) represent the parallel case and the perpendicular case, respectively. In Fig. 3.2, c) and d) represent the simulation result by ideal MHD model in same condition with Flux-Tube model.

A wake region, where the plasma density is low, is formed in the lee region of the coil (z>0 km) as shown in Fig. 3.2 a) since ions cannot penetrate the magnetic field generated by the coil. A similar wake region is also found in Fig. 3.2 b). The density distribution in the parallel case (Fig. 3.2a) is symmetric around the z-axis, and in the perpendicular case (Fig. 3.2b), the density distribution is symmetric about the xz-plane and asymmetric about the yz-plane. This is because ions are subjected to the same directional Lorentz force in the x>0 km and x<0 km region as shown in Fig. 3.3. As shown in Fig. 3.2c, the density distribution obtained by ideal MHD simulation in parallel case is symmetric around the z-axis as well as the result of Flux-Tube model. However, the density distribution of ideal MHD model in perpendicular case (Fig. 3.2d) considerably differs from that of Flux-Tube model (Fig. 3.2b). The density distribution is symmetric about the xz-plane and the yz-plane. This is because the finite Larmor effects and Hall effects are neglected in ideal MHD model. On the ion inertial scale, the ion kinetics should be taken into consideration to simulate the structure of magnetosphere correctly.

Some ion trajectories that approach the region near the coil are shown in Fig. 3.4. In the parallel case (Fig. 3.4a), trajectories are symmetric to the z-axis. Ions that go through the edge of the

### Table 3.1: Parameters used in the simulation for \( L_{MHD} \sim 500 \) km

<table>
<thead>
<tr>
<th>Solar wind parameters</th>
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<tbody>
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<td>( 4 \times 10^5 ) m/s</td>
</tr>
<tr>
<td>Density ( N_{SW} )</td>
<td>( 5 \times 10^6 ) m(^{-3})</td>
</tr>
<tr>
<td>Electron temperature ( T_e )</td>
<td>( 10^5 ) K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spacecraft parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil radius ( R_{coil} )</td>
<td>( 2.5 \times 10^5 ) m</td>
</tr>
<tr>
<td>Coil current ( I_{coil} )</td>
<td>200 kA turn</td>
</tr>
<tr>
<td>Magnetic moment ( M )</td>
<td>( 3.9 \times 10^{16} ) Wb · m</td>
</tr>
</tbody>
</table>

\( \Delta t_{field} < 1.5 \times 10^{-4} \) s. In practice, \( \Delta t_{field} \) was set as \( 1.0 \times 10^{-4} \) s.
CHAPTER 3. SIMULATION OF MAGNETIC SAIL ON ION INERTIAL SCALE

(a) Flux-Tube model, parallel case.  (b) Flux-Tube model, perpendicular case.

(c) MHD model, parallel case.  (d) MHD model, perpendicular case.

Figure 3.2: Calculated plasma density for $L_{MHD} = 500$ km magnetosphere ($R_{coil} = 250$ km, $I_{coil} = 200$ kATurn, $M = 3.9 \times 10^{16}$ Wbm) in whole computational domain by Flux-Tube model and ideal MHD model.
wake area, as trajectory $\beta$ and $\gamma$, make a turn in large Larmor radius and lose their momentum to contribute to thrust production. Also, ions, which reach nearer the coil following trajectory $\alpha$, are trapped by a strong magnetic field during their Larmor motion and finally reflected to contribute to thrust production. By comparison, ions passing through the edge of the computational domain as trajectory $\delta$ do not contribute to thrust production since these ions are not affected by the magnetic field and travel in a straight line. In the perpendicular case (Fig. 3.4b), ion trajectories are symmetric about the $xz$-plane and asymmetric about the $yz$-plane the same as the density distribution. As shown in Fig. 3.4b, ions that go through the trajectory as $\alpha 2$ are trapped in the magnetosphere and make a current loop similar to the equatorial ring current in the Earth’s magnetosphere (see also Fig. 1.4). Ions traveling along trajectory $\alpha 1$ are also trapped and descend to the coil center along the magnetic field line. Other ions are scattered by the magnetic field, resulting in a thrust.

In addition to the Larmor motion of each ion, ions and electrons make a drift motion by a magnetic field and an electric field at the boundary of a magnetosphere (i.e. magnetopause) and the motion induces a magnetopause current. By the magnetopause current, the magnetic field inside magnetosphere is strengthened and the magnetic field outside magnetosphere is weakened from the initial magnetic field. In the parallel case as shown in Fig. 3.5a, the magnetopause current makes a counterclockwise loop around the $z$-axis when observed from the sun direction; in this case, the coil current makes a clockwise current loop around the $z$-axis. In the perpendicular case (Fig. 3.5b), two current loops opposite to the coil current loop are formed at the magnetopause.
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Figure 3.4: Close-up view of plasma distribution and typical ion trajectories for $L_{MHD}=500$ km magnetosphere ($R_{coil}=250$ km, $I_{coil}=200$ kATurn, $M=3.9 \times 10^{16}$ Wbm) by Flux-Tube model; a) parallel case and b) perpendicular case.

around both the south pole ($y>0$ km) and the north pole ($y<0$ km) of the magnetic sail.

The coil current and the plasma current loop which is opposite to the coil current repulsively interact and produce the thrust of the magnetic sail toward the anti-sun direction in both parallel and perpendicular cases. The thrust in the parallel case is calculated to be 1500 N and the thrust in the perpendicular case is calculated to be 1640 N. Thus, the thrust slightly changes according to the coil axis direction. When these thrusts are evaluated in terms of a drag coefficient defined by Eq. (3.3.1), these become $C_d = 2.9$ and 3.1, respectively.

$$C_d = \frac{F}{0.5m_i N_{SW} V_{SW}^2 (\pi L_{MHD}^2)}$$  \hspace{1cm} (3.3.1)

The convergence of the calculation is checked by the magnetic field and the thrust. First, Fig. 3.6 represents the residual of the magnetic field in the sub-loop shown in Fig. 2.6, which is obtained from the integration of Eq. (2.4.8). The horizontal axis in the graph shows the number of occurrence in the sub-loop and the vertical axis shows the residual log ($B_{now} - B_{prev}$) at a certain fixed grid point ($x$, $y$, $z$)=(0 km, 0 km, 1200 km). Here, $B_{MP}$ is 40 nT. It is found that the residual becomes 1/1000 or less of the initial value by 5000-step time integration, and the calculation is converged. Thus, at each sub-loop step, the magnetic field, which satisfies $\partial B/\partial t = 0$, was obtained by an iterating electromagnetic field calculation.

Next, the time history of the thrust calculation in the main loop is shown in Fig. 3.7. The horizontal axis and the vertical axis are the numbers of iteration steps in the main loop and the thrust, respectively. The solid line represents the thrust calculated from the change of solar wind momentum, $F_{flow}$ (Eq. (2.4.12)), and the dashed line represents the thrust calculated by
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Figure 3.5: Calculated current flow and plasma density distribution for $L_{MHD}=500$ km magnetosphere ($R_{\text{coil}}=250$ km, $I_{\text{coil}}=20$ kATurn, $M=3.9 \times 10^{16}$ Wbm) by the Flux-Tube model; a) parallel case and b) perpendicular case.

Figure 3.6: Time history of residual log $\left(\frac{B_{noue}-B_{\text{prev}}}{B_{MP}}\right)$ in the sub loop (Fig. 7.7) for $L_{MHD}=500$ km magnetosphere ($R_{\text{coil}}=250$ km, $I_{\text{coil}}=20$ kATurn, $M=3.9 \times 10^{16}$ Wbm) in the parallel case.
electromagnetic force, $F_{\text{mag}}$ (Eq. (2.4.11)). The thrust calculation is performed every five iteration steps because the calculation of the thrust itself takes much time. As a result, thrusts calculated by two different methods are converged to the same value at 50 iterations. After that, the force is almost constant, and a good convergence rate is obtained for the Flux-Tube model.

In order to check how the assumption of the steady state in the Flux-Tube model affects the thrust evaluation, we compared the result in the parallel case with an ideal MHD (Magnetohydrodynamics) simulation’s result and a Hybrid-PIC simulation’s result using the same parameters listed in Table 3.1. The MHD simulation treats the solar wind plasma as a single fluid by neglecting the finite Larmor radius effect of plasma particles. The TVD Lax-Friedrich Scheme and a spherical-coordinate system with axial symmetry are used for the calculation. The outer boundary is set at the position of 1000 km in radius and the inner boundary is set at the position of 400 km in radius. The Hybrid-PIC simulation treats ions as individual particles and electrons as a fluid as same as the Flux-Tube model. By solving the time evolution of the electromagnetic field, the Hybrid-PIC model can treat waves such as acoustic waves that are neglected in the Flux-Tube model. In summary, Table 3.2 represents the phenomena and the conditions assumed in each simulation.

First, the thrust and the magnetospheric size calculated by each method are listed in Table 3.3. Thrusts calculated by the MHD simulation and the Hybrid-PIC simulation are obtained as 1600 N and 1560 N, respectively. This is slightly larger than the thrust calculated by the Flux-Tube model, 1500 N. However, the difference of three forces is within 10%. This means that the Flux-Tube model is a good model for estimating the thrust of a magnetic sail.
3.3 SIMULATION RESULTS AND DISCUSSION

Table 3.2: Phenomena and calculation condition in each simulation

<table>
<thead>
<tr>
<th>Phenomena</th>
<th>Flux-Tube</th>
<th>Hybrid-PIC</th>
<th>ideal-MHD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite Larmor radius effect</td>
<td>Considered</td>
<td>Considered</td>
<td>N/A</td>
</tr>
<tr>
<td>Interaction between magnetic field and plasma</td>
<td>Considered</td>
<td>Considered</td>
<td>Considered</td>
</tr>
<tr>
<td>Acoustic wave (low frequency wave)</td>
<td>N/A</td>
<td>Considered</td>
<td>Considered</td>
</tr>
<tr>
<td>Shock</td>
<td>N/A</td>
<td>Considered</td>
<td>Considered</td>
</tr>
<tr>
<td>Electromagnetic wave</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Charge separation</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 3.3: Thrust calculated by each method ($M=3.9 \times 10^{16}$ Wbm)

<table>
<thead>
<tr>
<th>Method</th>
<th>Thrust F [N]</th>
<th>Magnetosphere size L [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux-Tube</td>
<td>1500</td>
<td>728</td>
</tr>
<tr>
<td>Hybrid-PIC</td>
<td>1560</td>
<td>728</td>
</tr>
<tr>
<td>ideal MHD</td>
<td>1600</td>
<td>942</td>
</tr>
</tbody>
</table>

In Fig. 3.8, the magnitude of the magnetic flux density along the $x$-axis is plotted. In simulation results, the magnetospheric size is defined as the point where the magnitude of the magnetic field changes from $B/B_0>1$ to $B/B_0<1$ and slightly differs from the magnetosphere size calculated by Eq. (1.2.3), 500 km. The blue line, the green line, the purple line and the red line represent the MHD result, the Flux-Tube result, the Hybrid-PIC result and the initial magnetic field, respectively. The structure of the magnetic field inside magnetosphere calculated by the Flux-Tube model agrees well with that calculated by the Hybrid-PIC, and the magnetosphere size is almost the same. On the other hand, the structure of the magnetic field outside magnetosphere is slightly different. This is because the ion temperature is neglected and the ion static pressure is not considered in the Flux-Tube model. The diamagnetic current:

$$J_D = (k_B T_i + k_B T_e) \frac{B \times \nabla n}{B^2}$$

(3.3.2)

becomes smaller by neglecting the ion temperature. As a result, the magnetic field calculated by the Flux-Tube model extends to a further position than the Hybrid-PIC result as seen in Fig. 3.8. The difference between the MHD simulation and the Hybrid-PIC simulation is large: the magnetospheric size calculated by the MHD simulation is approximately 1.2 times larger than that of the Hybrid-PIC simulation. Such a large difference is caused whether the finite Larmor radius effect is included or not. When the finite Larmor effect is considered as in the Hybrid-PIC and the Flux-Tube model, particles can penetrate the small magnetic field around the magnetopause and the boundary layer has a finite width, approximately corresponding to ion’s Larmor radius. In
CHAPTER 3. SIMULATION OF MAGNETIC SAIL ON ION INERTIAL SCALE

Figure 3.8: Calculated magnitude of the magnetic flux density along the $x$-axis for $L_{MHD}=500$ km magnetosphere ($R_{coil}=250$ km, $I_{coil}=20$ kATurn, $M=3.9 \times 10^{16}$ Wbm) in the parallel case.

In contrast, the plasma fluid and the magnetic field are sharply divided in the MHD simulation, which neglects the finite Larmor effect and the boundary layer width becomes zero, since the magnetic field cannot penetrate into the solar wind plasma. Therefore the magnetic flux density by the MHD simulation rapidly becomes small outside the magnetosphere. In summary, the magnetospheric size calculated by the Flux-Tube and the Hybrid-PIC models and the magnetosphere size of the MHD are different due to the disparity of the width of the boundary layer.

In Fig. 3.9, the ion density distributions in the $xz$-plane calculated by each method are shown. The MHD simulation forms the clear bow shock where the plasma flow is compressed and the density becomes higher in front of the magnetosphere as Fig. 3.9b. In contrast, in the Hybrid-PIC result, the shock formation is ambiguous, Fig. 3.9c, and no shock is formed using the Flux-Tube model, Fig. 3.9a.

In addition, to observe the details, the density distribution of ions along the $z$-axis is shown in Fig. 3.10. The solid line, the dash line and the dotted line represent MHD, Flux-Tube and Hybrid-PIC results, respectively. There are areas where the ion density increases to 4 and 5 times of the solar wind density outside the magnetosphere ($L_{MHD}=500$ km) in the MHD and Hybrid-PIC simulation results. The ion density in the MHD result increases from $z=-800$ km to -400 km and the ion density in the Hybrid-PIC simulation moderately increases from $z=-1000$ km to 0 km. These are expected to correspond to the shock region where a supersonic plasma flow changes into a sub-sonic flow. On the other hand, in the Flux-Tube model, the ion density does not increase outside the magnetosphere since no shock is formed. Instead, the ion density slightly increases from $z=-200$ km to 0 km as shown in Fig. 3.10 since ions are trapped as a result of the Larmor
3.3. SIMULATION RESULTS AND DISCUSSION

(a) Flux-Tube model.

(b) MHD model.

(c) Hybrid-PIC model.

Figure 3.9: Calculated plasma density distributions for $L_{MHD}=500$ km magnetosphere ($R_{coil}=250$ km, $I_{coil}=20$ kATurn, $M=3.9 \times 10^{16}$ Wbm) in the $xz$-plane in the parallel case.
CHAPTER 3. SIMULATION OF MAGNETIC SAIL ON ION INERTIAL SCALE

Figure 3.10: Calculated plasma density distributions along the $z$-axis for $L_{\text{MHD}}=500$ km magnetosphere ($R_{\text{coil}}=250$ km, $I_{\text{coil}}=20$ kATurn, $M=3.9 \times 10^{16}$ Wbm) in the parallel case. (3D Flux-Tube model)

motion in the magnetic cusp. These differences are also attributed to the fact that the finite Larmor effect of ions is neglected in the MHD simulation. If the finite Larmor effect is considered as in the Hybrid-PIC simulation, the interaction between the acoustic wave and the solar wind plasma becomes smaller on the ion inertial scale, and the bow shock in front of the magnetosphere should become too weak to affect the formation of the magnetosphere. Hence, the Hybrid-PIC simulation, in which the finite Larmor effect correctly is included, represents phenomena on the ion inertial scale better than the MHD simulation that contains the strong bow shock. As for the Flux-Tube model, the finite Larmor effect of ions is considered and the bow shock is not formed since the steady state is assumed and the unsteady phenomena including waves do not appear. Then the magnetic field and the ion density distribution similar to that found with the Hybrid-PIC simulation are formed and the thrust is well estimated by the Flux-Tube model. It is also noted that the form of the magnetosphere is more heavily affected by the finite Larmor effect rather than the unsteady phenomena such as waves within the magnetospheric size we consider.

Finally, the computational cost of the Flux-Tube model is compared with the cost of the MHD simulation and the Hybrid-PIC simulation. The computer used for the comparison has an Intel® Xeon E5504 processor and the simulation codes using the methods are not parallelized. The Flux-Tube model requires 200 MB memory and half a day to reach a steady state. This is below the 1/10 cost if it is compared with the cost by the Hybrid-PIC simulation (2 GB memory capacity and four days to calculate the thrust for the same conditions). The MHD simulation required less memory and approximately same calculation time with the Flux-Tube model. However, the
finite Larmor effect is not included in the MHD simulation and the perpendicular case cannot be simulated because of axial symmetric coordinate system. Hence, the Flux-Tube model enables us to calculate the thrust of magnetic sail in a steady state in a fast and efficient way.

### 3.3.2 Thrust of Magnetic Sail

The Flux-Tube model enabled fast and accurate thrust evaluation of a magnetic sail by assuming only a steady state as described in above section. Then simulations with various magnetospheric sizes $L_{MHD}$ were performed to estimate the thrust characteristics of a magnetic sail when the coil axis was parallel (parallel case) or perpendicular (perpendicular case) to the solar wind. Parameters associated with the solar wind are the same as in Table 3.1 and the magnetospheric size is varied as $L_{MHD}=15$ km, 80 km, 150 km, 350 km, 500 km, 650 km and 800 km. The detailed parameters are listed in Table 3.4.

Results are shown in Fig. 3.11. The thrust of a magnetic sail becomes larger as the magnetosphere becomes larger as shown in Fig. 3.11 in both the parallel case and the perpendicular case. This is because more particles are scattered by a larger size magnetosphere. The drag coefficient $C_d$ (Eq. (3.3.1)) is approximately constant when the magnetosphere is larger than the ion Larmor radius (approximately 100 km) in both cases as shown in Fig. 3.12. However, the drag coefficient rapidly becomes smaller as the magnetosphere becomes smaller than the ion Larmor radius (<100 km) since ions penetrate into the magnetosphere. Comparing the perpendicular case with the parallel case, the drag coefficient more rapidly becomes smaller around $L_{MHD}=400$ km.

Next, using the Flux-Tube model, the approximate formula for the thrust estimation is obtained and discussed. According to the past research using Hybrid-PIC when the coil axis is perpendicular to the solar wind,

$$C_d = \begin{cases} 
2.9 \times \exp \left[ -0.35 \times \left( \frac{r_{iL}}{L_{MHD}} \right)^2 \right] & r_{iL} < L_{MHD} \\
2.4 / \left( \frac{r_{iL}}{L_{MHD}} \right) \times \exp \left[ -0.17 / \left( \frac{r_{iL}}{L_{MHD}} \right)^2 \right] & r_{iL} > L_{MHD}
\end{cases}$$

for the thrust estimation is obtained and plotted in Fig. 3.12 with the blue line [33]. The formula

---

### Table 3.4: Magnetic sail parameters for various magnetospheric sizes

<table>
<thead>
<tr>
<th>Magnetosphere size [km]</th>
<th>Coil radius [km]</th>
<th>Coil current × Turn [kA turn]</th>
<th>Magnetic moment Wbm</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>8</td>
<td>10</td>
<td>$2.0 \times 10^{12}$</td>
</tr>
<tr>
<td>80</td>
<td>32</td>
<td>45</td>
<td>$1.4 \times 10^{14}$</td>
</tr>
<tr>
<td>150</td>
<td>64</td>
<td>80</td>
<td>$1.0 \times 10^{15}$</td>
</tr>
<tr>
<td>350</td>
<td>175</td>
<td>130</td>
<td>$1.2 \times 10^{16}$</td>
</tr>
<tr>
<td>500</td>
<td>250</td>
<td>200</td>
<td>$3.9 \times 10^{16}$</td>
</tr>
<tr>
<td>650</td>
<td>325</td>
<td>240</td>
<td>$7.8 \times 10^{16}$</td>
</tr>
<tr>
<td>800</td>
<td>400</td>
<td>295</td>
<td>$1.5 \times 10^{17}$</td>
</tr>
</tbody>
</table>
CHAPTER 3. SIMULATION OF MAGNETIC SAIL ON ION INERTIAL SCALE

Figure 3.11: Variation of thrust along with magnetosphere size: $L_{MHD} = 15 \sim 800$ km. Coil axis is parallel to the solar wind in the parallel case and perpendicular in the perpendicular case. (3D Flux-Tube model)

Figure 3.12: Variation drag coefficient along with magnetosphere size. Coil axis is parallel to the solar wind in the parallel case and perpendicular in the perpendicular case.
3.3. SIMULATION RESULTS AND DISCUSSION

is slightly different from the original paper since the definition of the magnetospheric size differs.

Then, approximate formulas for the drag coefficient for the thrust estimation are obtained as

\[
C_d = \begin{cases} 
3.0 \times \exp\left[-0.36 \times \left(\frac{r_i L}{L_{MHD}}\right)^2\right] & r_i L < L_{MHD} \\
2.7/\left(\frac{r_i L}{L_{MHD}}\right) \times \exp\left[-0.26/\left(\frac{r_i L}{L_{MHD}}\right)^2\right] & r_i L > L_{MHD}
\end{cases}
\]

for the parallel case and

\[
C_d = \begin{cases} 
2.8 \times \exp\left[-0.51 \times \left(\frac{r_i L}{L_{MHD}}\right)^2\right] & r_i L < L_{MHD} \\
2.4/\left(\frac{r_i L}{L_{MHD}}\right) \times \exp\left[-0.37/\left(\frac{r_i L}{L_{MHD}}\right)^2\right] & r_i L > L_{MHD}
\end{cases}
\]

for the perpendicular case by the Flux-Tube model. As shown in Fig. 3.12, the drag coefficient calculated by the Flux-Tube model in the perpendicular case differs from the approximate formula. In particular, it is expected that the drag coefficient for \(L_{MHD}=500\) km, 650 km and 800 km in the perpendicular case should be overestimated. This is because some ions are trapped for a long time as an equatorial ring current in the perpendicular case with a large magnetosphere \((L_{MHD}>500\) km) and the ion density is overestimated. In contrast, in the parallel case with any magnetosphere and in the perpendicular case with a smaller magnetosphere, there is no ion that is trapped in the magnetosphere for a long time and it is expected that thrusts and the drag coefficients be calculated correctly. Hence, \(L_{MHD}>10\) km in the parallel case and 10 km<\(L_{MHD}<500\) km in the perpendicular case can be analyzed in the Flux-Tube model.

Thus, the magnetospheric size analyzable by the Flux-Tube model is restricted in comparison with the range where the assumptions on the ion inertial scale are valid. It is also shown that the drag coefficient for the parallel case is larger than that for the perpendicular case obtained by both the Flux-Tube model and Hybrid-PIC simulation in the past research. The drag coefficient calculated by the Flux-Tube model for the perpendicular case is slightly smaller than that calculated by the Hybrid-PIC in the past research. This is because that the Flux-Tube model assumes the magnetic field generated by large coils whereas the Hybrid-PIC simulations in the past research have assumed the magnetic field generated by a magnetic dipole. Hence, the magnetic field around the coil or near the dipole differs. On the ion inertial scale, the ions penetrate into the magnetosphere and reach near the coil. Hence, the difference of the magnetic field in the inner magnetosphere may cause the difference in thrust. If a magnetic dipole is assumed, the magnitude of the magnetic field around the center is stronger and more particles are trapped and reflected by the mirror motion. In the case of a large coil, the magnitude of the magnetic field in the coil center is smaller than that of the magnetic dipole and some particles can slip away. Thus particle motions are affected by the difference of assumptions about the initial magnetic field and the thrust calculated by the Hybrid-PIC with the magnetic dipole is larger than the thrust calculated by the Flux-Tube model with a large coil. However, the Hybrid-PIC in this paper, which uses the finite size coil, resulted
in approximately the same thrust as the thrust calculated using the Flux-Tube model. Therefore, it is expected that the thrust characteristics obtained by the Flux-Tube model should be valid.

Using the approximate formulas for the drag coefficient derived by the Flux-Tube model, the thrust of the magnetic sail with large hoop coil for a desired magnetospheric size can be estimated as long as the typical solar wind parameters are assumed and the magnetospheric size is applicable to the Flux-Tube model. In addition, using the Flux-Tube model, we will be able to obtain the thrust dependency for parameters such as coil radius and solar wind velocity.

3.4 Summary

We performed a new numerical analysis method applicable for a magnetic sail using a Flux-Tube model in order to reveal thrust characteristics of magnetic sail on the ion inertial scale. The Flux-Tube model assumes a steady state for the electromagnetic field and a plasma flow including the finite Larmor effect of ions. The Flux-Tube model achieved quick estimation of the thrust with lower memory capacity compared with earlier study by Hybrid-PIC model as expected. The thrust generated by a magnetic sail with 500 km magnetosphere is calculated to be 1500 N for the magnetic moment parallel to the solar wind and 1640 N for the magnetic moment perpendicular to the solar wind. In the parallel case, the thrust value approximately agrees with the MHD simulation’s result, 1600 N, and the Hybrid-PIC simulation’s result, 1560 N nevertheless the Flux-Tube model neglects unsteady phenomena. Thus the Flux-Tube model is effective in estimating the thrust of the magnetic sail and reveals that unsteady phenomena such as waves do not so much affect the thrust characteristics of the magnetic sail. In addition, approximate formulas for the magnetic sail thrust in both parallel and perpendicular cases are obtained by performing simulations with various sets of parameters. Using these approximate formulas, the thrust level of the magnetic sail for various magnetospheric sizes can be quickly estimated and it is expected that the Flux-Tube model is useful for the design of a magnetic sail.
Chapter 4

Simulation of Magnetic Sail on Intermediate Scale

4.1 Introduction

On the ion inertial scale \( L_{MHD} \sim r_{iL} \), the structure of magnetosphere generated by the artificial magnetic field of magnetic sail and the thrust characteristics are examined by using Flux-Tube model in Chapter 3. In the Flux-Tube model, the ion kinetics are taken into consideration but electron kinetics are neglected by using mass-less fluid approximation. The adaptation range of the formulas is hence restricted to \( L_{MHD} \sim 10 \) km. To simulate smaller magnetosphere \( (L_{MHD} < r_{iL}) \), which is used for the demonstrator spacecraft and can be constructed by the present technology, another simulation model, Full-PIC model is required. On the other hand, Full-PIC model which treat both ions and electrons as particles requires large computational resource. The only several 100 m magnetosphere can be simulated by the realistic conditions because of the limitation of the computational resource.

We hence chose the artificial plasma parameters in order to simulate the moderately large magnetosphere \( (1 \text{ km} \sim 100 \text{ km} \text{ or } L_{MHD}/r_{iL} = 0.01 \sim 1) \). By using the artificial plasma parameters, Full-PIC model requires less than 1/10 of the computational resource compared with the realistic plasma parameters. We first analyzed the dependency of the magnetosphere structure on the simulation scale. The structure of the moderately large magnetosphere can be revealed by the simulation, but the actual thrust level obtained by the magnetosphere can not be revealed by the artificial plasma conditions. We also examined the effects of Interplanetary Magnetic Field (IMF) on the magnetosphere structure. In Flux-Tube model, we didn’t take IMF into consideration since IMF causes the time-depend fluctuation such as the magnetic reconnection, which Flux-Tube model can not deal with. We perform the simulation with IMF by also using the artificial plasma parameters. In the Earth’s magnetosphere, IMF plays a very important role since the magnetic reconnection distorts the structure of the magnetosphere and transfers the momentum.
of plasma through the magneto tail. Only Nishida et al. [38] reflected the influence of IMF in thrust by performing 2D ideal-MHD simulation. However, it is difficult to reproduce the phenomena of a magnetic reconnection correctly by the ideal-MHD simulation including the artificial viscosity and the numerical diffusion. Full-PIC simulation adopted in this study treats both ions and electrons as particles, and it can solve the plasma flow around the magnetosphere including IMF self-consistently. By using Full-PIC model, the unsteady phenomena concerning IMF can be simulated.

### 4.2 Computational Settings

In Full-PIC simulations, we use three-type plasma conditions. One is the realistic solar wind plasma parameter and others are artificial parameters depending on the mass ratio. By using the artificial plasma parameters, the large magnetosphere (~100 km) can be simulated by Full-PIC simulation. Only several 100 m ~ several km magnetosphere can be simulated if the realistic plasma parameters are used by the limitation of the computational resource and the calculation time. The non-dimensional parameters are shown in Table 4.1. The parameters refer to the previous paper about two-dimensional magnetosphere [37]. The parameter was chosen so that especially the ratio of Alfvén velocity $V_A$ and the solar wind velocity $V_{SW}$ might be kept right.

The interaction between the artificial solar wind and an artificial dipole magnetic field is simulated in the three-dimensional space as shown in Fig. 4.1. Only perpendicular case (the magnetic moment is perpendicular to the solar wind flow) is considered. When we assumes the artificial case 1, the ion Larmor radius, electron Larmor radius and Debye length at the magnetopause are calculated as $r_{iL} \sim 1000$ m, $r_{eL} \sim 10$ m and $\lambda_D \sim 10$m, respectively. The simulation parameters are normalized by these scaling lengths. In the largest case, the computational domain has an area of $10r_{iL} \times 10r_{iL} \times 10r_{iL}$ is partitioned into a grid of $512 \times 512 \times 512$ cells ($dx = 2\lambda_D$). The size of the computational domain is decided based on the theoretical magnetosphere size $L_{MHD}$ (Eq. (1.2.3)). The length of the one side is approximately $5L_{MHD}$. The grid spacing $dx$ typically needs to be set such that it is not much larger than the Debye length ($dx/\lambda_D < 3$). The time spacing $dt$ must fill the Courant condition, $dx/dt > 1.73c$ in three-dimension. The solar wind plasma

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Real solar wind</th>
<th>artificial case 1</th>
<th>artificial case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_i/m_e$</td>
<td>1836</td>
<td>100</td>
<td>1000</td>
</tr>
<tr>
<td>$(c/\omega_p)/r_{iL}$</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$c/V_{SW}$</td>
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<td>25</td>
<td>25</td>
</tr>
<tr>
<td>$V_A/V_{SW}$ @magnetosphere</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$V_s/V_{SW}$</td>
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<td>0.25</td>
<td>0.125</td>
</tr>
<tr>
<td>$V_{th}/V_{SW}$</td>
<td>2.6</td>
<td>2.5</td>
<td>3.95</td>
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</tbody>
</table>
is typically represented by 16 super particles associated with each cell and a total of $4.3 \times 10^9$ particles. This is because it was revealed in our previous study that particle number per cell is not influence the precision of the thrust compared with the latitude resolution or the size of calculation domain. 16 particles are hence necessary and sufficient in terms of the calculation cost and the correctness of the simulation. The magnetic field generated by the coil mounted in the spacecraft is approximated by an ideal dipole magnetic field of magnetic moment $M$. The magnetic moment of the coil is arranged in the $y$-direction. The magnetic field is given as the initial condition. Absorbing boundary conditions are used for the electromagnetic field on all outer boundaries. Solar wind particles flow into the computational domain from the inflow boundary at the typical solar wind velocity $V_{SW}$ and thermal distribution. Outgoing particles from the computational domain are eliminated from the calculation. The collision between particles and the spacecraft is not taken into consideration, either, assuming the infinitesimal spacecraft. IMF is considered by introducing the magnetic field $B_{IMF}$ and the electric field $E_{IMF} = -V_{SW} \times B_{IMF}$.

The computational domain is divided into some smaller domain ($4 \times 4 \times 512$ cells in typical case) by the domain decomposition. Each small domain is calculated by one process and bordering physical quantity is exchanged by MPI. As a result, 16384 CPUs are used for the simulation.
4.3 Results and Discussion

4.3.1 Structure of Magnetosphere from Ion Inertial Scale to Electron Inertial Scale

First, we performed the simulation of three typical magnetosphere size: $L_{\text{MHD}}/r_{iL} = 1, 0.1, 0.01$ by using the artificial plasma parameter 1. IMF is neglected in these simulations. The case of $L_{\text{MHD}}/r_{iL} = 1$ is corresponding to the ion inertial scale and others are corresponding to the electron inertial case. The simulation results are shown in Fig. 4.2. (a), (c) and (e) represent the ion density distribution in $yz$-plane and (b), (d) and (f) represent the ion density distribution in $xz$-plane. The horizontal axis and the vertical axis are normalized by the ion Larmor radius $r_{iL}$ and the ion density is normalized by the solar wind plasma density $N_{\text{SW}}$. In the figure of $yz$-plane the magnetic field lines are also included. The dashed line represents the theoretical magnetosphere size obtained by assuming MHD approximation (Eq. (1.2.3)). The magnetosphere of the magnetic sail is symmetric about $xz$-plane and asymmetric about $yz$-plane. The dawn-dusk anomaly is formed by the Hall effect and finite Larmor radius effect even though the dawn-dusk anomaly cannot be observed in the ideal MHD simulation. The comparison between Full-PIC simulation and ideal MHD simulation is performed in Appendix B. The equatorial ring current is also formed around the spacecraft as well as the Earth magnetosphere.

Figure 4.3 shows the trajectories of ion. The initial positions of the ions are arranged at equal intervals in $xz$-plane and the initial velocity is set as the solar wind velocity $V_{\text{SW}}$. Totally the 256 particles are traced. According to the initial positions, lines are colored in different colors. The trajectories of ion in the case of $L_{\text{MHD}}/r_{iL} = 1$ are complicated since not only the dipole magnetic field but also the induced magnetic field by plasma and the induced electric field affect the trajectories of ion. The trajectories of particles not only in the center region (green, blue and purple) but also in the edge region (red and aqua) are bent and the momentum is changed. That is, these particles contribute to the thrust generation. On the other hand, the induced magnetic field and the induced electric field are small on the electron inertial scale and ions are subject to the influence of the artificial dipole magnetic field (Figs. 4.3b and c). Only ions which go through near coil (blue lines) are affected by the strong magnetic field and others go without any momentum change. The trajectories of ion on the electron inertial scale are hence approximately same with the trajectories of test-particle in the dipole magnetic field. However, the charge separation becomes dominant on the electron inertial scale and the electrostatic field also affects the trajectories of ion.

Figure 4.4 shows the energy of each ion along $z$-axis. The energy is normalized by the initial energy of ion. The energy of ion is changed by the magnetopause electric field whose direction is outward of the magnetosphere since the magnetic field does not work. On the ion scale $L_{\text{MHD}}/r_{iL} = 1$, the magnetopause electric field changes the energy of ion a lot. In front of the magnetosphere (-1
4.3. RESULTS AND DISCUSSION

(a) \( L_{MHD}/r_{iL} = 1 \), \( yz \)-plane.

(b) \( L_{MHD}/r_{iL} = 1 \), \( xz \)-plane.

(c) \( L_{MHD}/r_{iL} = 0.1 \), \( yz \)-plane.

(d) \( L_{MHD}/r_{iL} = 0.1 \), \( xz \)-plane.

(e) \( L_{MHD}/r_{iL} = 0.01 \), \( yz \)-plane.

(f) \( L_{MHD}/r_{iL} = 0.01 \), \( xz \)-plane.

Figure 4.2: Structure of typical magnetosphere. Ion density distribution and magnetic field line are represented in \( yz \)-plane and ion density distribution and the theoretical magnetosphere size is represented in \( xz \)-plane. (artificial plasma parameter, 3D Full-PIC model)
(a) $L_{MHD}/r_{iL} = 1$.

(b) $L_{MHD}/r_{iL} = 0.1$.

(c) $L_{MHD}/r_{iL} = 0.01$.

Figure 4.3: Trajectories of ion. (artificial plasma parameter, 3D Full-PIC model)
4.3. RESULTS AND DISCUSSION

ions are slowed down and the energy becomes approximately 0. On the contrary, the energy change on the electron scale $L_{MHD}/r_{iL} = 0.1 (\pm 15\%)$ and 0.01 (±5%) is small compared with the ion inertial scale (±100%) since the induced electric field is small. However, the energy of almost all ions are decreased on the electron inertial scale, different from the ion inertial scale on which the accelerated particles exist. The particles in the west side of magnetosphere (green lines) especially loose the kinetic energy. That is, the momentum of ion is transfer to the spacecraft as the thrust via not only magnetic field but also electric field on the electro inertial scale.

The magnetosphere size obtained by the simulation of $L_{MHD}/r_{iL} = 1$ approximately agrees with the theoretical magnetosphere size (dashed line). Here, the magnetosphere size is estimated by the magnetic flux density distribution along $z$-axis as shown in Fig. 4.5a. The magnitude of the magnetic flux density is normalized by the magnetic field at the magnetopause $B_{MP}$. The current densities $J_x$, $J_{ix}$ and $J_{ex}$ represent the current perpendicular to the $yz$-plane and correspond to the magnetopause current. The current density is normalized by $J_0 = eN_{SW}V_{SW}$. The total current density $J_x$ is calculated as $J_x = J_{ix} - J_{ex}$. In the magnetopause (magnetosphere boundary), the magnetopause current is induced and magnetic field is changed from the dipole magnetic field. It was revealed that the main carrier of the magnetopause current is electron as shown in Fig. 4.5a by comparing the blue line ($J_{ex}$) and the purple line ($J_{ix}$). The change in magnetic field by the interaction with the solar wind is also observed in Fig. 4.2a. The magnetic field is extended at the lower stream side. This region is called as magneto tail. The magnetic neutral point is also observed ($z/r_{iL} = 1.8$).

On the contrary, the magnetosphere size of the electron scale is smaller than the that of the theoretically estimated (Figs. 4.2c for $L_{MHD}/r_{iL} = 0.1$ and 4.2e for $L_{MHD}/r_{iL} = 0.01$, see also section 5.5: theoretical approach in which the plasma kinetics is taken into consideration is presented). This is because that the ion and electron which has larger Larmor radius than the magnetosphere penetrate into magnetic field and reaches around the coil center. The electron Larmor radius becomes comparable with the magnetosphere size ($L_{MHD}/r_{eL} = 1$) in case of $L_{MHD}/r_{iL} = 0.01$ since the mass ratio of ion and electron is set as $m_i/m_e = 100$. In the strong magnetic field around the coil center, the particles are subject to the influence of a magnetic field and its trajectory is bent. Hence, the magnetopause current is induced near coil rather than around the theoretical magnetopause (dashed line). The current densities of the magnetopause current of three cases are almost same. However, the total current by integrating the current density is further larger on the ion inertial scale magnetosphere. The magnetic field is slightly change in electron scale magnetosphere and remains to be the dipole magnetic field (Figs. 4.2c and 4.2e). As shown in Figs. 4.5b and 4.5c, only magnetosphere current and density distribution is observed.

Figure 4.6 also shows one-dimensional cut along $z$-axis about ion density $n_i$, electron density $n_e$, electric field ($z$-direction) $E_z$, total current density ($x$-direction) $J_x$, ion fluid velocity ($z$-direction) $u_{iz}$, electron fluid velocity ($z$-direction) $u_{ez}$ and magnetic field $B$. Each variable is smoothed using low-path filter to reduce the fluctuation caused by plasma waves and particle distribution. When
Figure 4.4: Energy of ion along the trajectories in Fig. 4.3. (artificial plasma parameter, 3D Full-PIC model)
4.3. RESULTS AND DISCUSSION

(a) $L_{MHD}/r_{IL} = 1$.

(b) $L_{MHD}/r_{IL} = 0.1$.

(c) $L_{MHD}/r_{IL} = 0.01$.

Figure 4.5: Magnetic flux density (red line) and current density (total: green, ion: purple and electron: blue) along z-axis (artificial plasma parameter, 3D Full-PIC model)
the solar wind reaches the magnetic field, electron with small mass is first subject to the influence of a magnetic field, and flow velocity falls. The flow velocity of the heavier ion falls following electron. In $L_{\text{MHD}}/r_{iL} = 1$, both ion and electron are strongly magnetized and the flow velocity begins to decrease in the almost same position as shown in Fig. 4.6d ($z/r_{iL} = -2$). Peaks of ion density, electron density and current density are in agreement with the compression region of the magnetic field ($z/r_{iL} = -1$).

In $L_{\text{MHD}}/r_{iL} = 0.1$, electron is still strongly magnetized in spite of ion are decoupled from magnetic field. The current density distribution of $L_{\text{MHD}}/r_{iL} = 0.1$ shown in Fig. 4.5b is characteristic compared with other case. Around the theoretical magnetopause ($z/r_{iL} = 0.1$), the mainly electron induces the current. The electron, which Larmor radius is smaller than that of ion, is affected by the magnetic field, and trajectories is changed to induce the current. However, as shown in Fig. 4.3b, the ions are not affected by the magnetic field around the theoretical magnetopause. Ion and electron moves together to maintain the charge neutrality. The density distribution is hence the dependent on the mainly ion motion. Ion approaches coil center without slowing down (Fig. 4.6e, red line). Therefore, the velocity distribution of ion and an electron differs greatly as shown in Fig. 4.6e. In the domain where the electron velocity of z-direction decreased, the flow of x-direction occurs and current $J_x$ is induced as shown in Fig. 4.6b ($-0.2 < z/r_{iL} < -0.05$). On the other hand, density distributions of ion and electron are mainly affected by the ion motion and, to keep charge neutrality, the electric field is induced around magnetopause ($z/r_{iL} = -0.02$). In Fig. 4.7, the distributions of electron density, current, temperature and pressure are shown. Here, the Maxwell distribution is assumed to obtain the electron temperature. In the current induced region ($-0.2 < z/r_{iL} < -0.05$), the electron density does not increase as above mentioned. However, as shown in Fig. 4.7c, the electron temperature becomes high and consequently the plasma pressure also increases nevertheless the electron density does not increase. In addition, the electron density increases around the magnetosphere ($z/r_{iL} > -0.05$) and the electron pressure also becomes high. The diamagnetic current, or $\nabla p$ drift current, is induced according to the gradient of the electron pressure in Fig. 4.7d. Therefore, the current of dual structure will flow around a magnetosphere.

This electron heating is assumed to be caused by soma plasma instability. When electron is well magnetized ($L_{\text{MHD}}/(c/\omega_{pe}) \sim 1$) and ion is unmagnetized ($L_{\text{MHD}}/(c/\omega_{pi}) \sim 0.1$), electron cyclotron drift instability and modified two stream instability occur depending on plasma conditions. In the diffusion region of the magnetic reconnection, where electron is well magnetized and ion is unmagnetized, the electron cyclotron drift instability and the modified two stream instability are mainly observed on the condition of $v_{\text{beam}}/v_{\text{th,e}} < 1$ and $v_{\text{beam}}/v_{\text{th,e}} > 1$, respectively [59].

In $L_{\text{MHD}}/r_{iL} = 0.01$, both ion and electron are decoupled from the magnetic field and the difference of velocity distribution between ion and electron becomes small. The magnetosphere size becomes similar size to Debye length ($\lambda_D/r_{iL} = 0.01$) and the charge separation is clearly observed as shown in Fig. 4.6c ($-0.008 < z/r_{iL} < -0.002$). Unlike $L_{\text{MHD}}/r_{iL} = 0.1$, the electronic heating was not observed in $L_{\text{MHD}}/r_{iL} = 0.01$. It also becomes impossible to distinguish the
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(a) $L_{MHD}/r_{iL} = 1$, plasma density, electric field and current.
(b) $L_{MHD}/r_{iL} = 0.1$, plasma density, electric field and current.
(c) $L_{MHD}/r_{iL} = 0.01$, plasma density, electric field and current.

(d) $L_{MHD}/r_{iL} = 1$, flow velocity.
(e) $L_{MHD}/r_{iL} = 0.1$, flow velocity.
(f) $L_{MHD}/r_{iL} = 0.01$, flow velocity.

Figure 4.6: Smoothed one-dimensional distribution of ion density, electron density, electric field (z-direction), current density (x-direction), ion fluid velocity (z-direction), electron fluid velocity (z-direction) and magnetic field. (artificial plasma parameter, 3D Full-PIC model)

current induced by drift motion from the current by electron flow and only magnetopause current can be observed.

Thus, the way magnetic field interacts with the solar wind changes depending on the magnetosphere size.

4.3.2 Influence of Interplanetary Magnetic Field

In addition to the simulations neglecting the effects of IMF, we performed the simulation with IMF. Figure 4.8 shows the simulation results of $L_{MHD}/r_{iL} = 1$. The IMF direction is set to southward (Figs. 4.8a and 4.8b) and northward (Figs. 4.8c and 4.8d). The magnetic flux density of IMF is set to quarter of the magnetic flux density at the magnetopause $B_{MP}$. (a) and (c) shows the ion density distribution in $yz$-plane (meridian plane) and magnetic field line. (b) and (d) shows the $xz$-plane (equatorial plane).

The magnetic neutral points are formed in front of the magnetosphere ($z/r_{iL} = -1$) and the magneto tail ($z/r_{iL} = 1.2$) in the southward IMF case. The position of the front neutral point is approximately same with the magnetopause. The point of tail reconnection differs from that
Figure 4.7: Distribution of electron density, current, temperature and pressure in $L_{MHD}/r_{IL} = 0.1$. (yz-plane, artificial plasma parameter, 3D Full-PIC model)
in the no IMF case \((z/r_{iL} = 1.8)\). In Fig. 4.8b, the plasma density of magnetopause region has waved. Around the magnetic neutral point, it is expected that the magnetic reconnection cause the fluctuation of the ion density. On the other hand, the neutral point is formed around cusp region in the northward IMF case. In the front of the magnetosphere in the northward IMF case, the magnetic field is pile-upped and the magnetic flux density becomes high. The plasma density and size of the equatorial ring current grow larger in the northward IMF case.

The current density distribution perpendicular to \(yz\)-plane are shown in Fig. 4.9. Around the magnetic neutral point the current density is high since the drift velocity of ion and electron is large when the magnetic flux density is low. In addition, the finite Larmor motion contributes to the current density. In the case of no IMF, the magnetic flux density outside magnetosphere is approximately zero and the reflected particles by the magnetic field go straight on. On the contrary, when the IMF is considered, the magnetic flux density outside magnetosphere is not zero and the trajectories of the reflected particles are affected by IMF as shown in Fig. 4.9. In the southward IMF case, the direction of ion \((+x)\) which is reflected around the stagnation point is in agreement with the current direction \((+x)\). On the other hand, in the northward IMF case, the direction of ion trajectories \((-x)\) are opposite to the current direction \((+x)\). The current density is thus completely different according to the direction of IMF. The difference of the plasma density of the equatorial ring current is caused by the difference of ion trajectories penetrating into the inner magnetosphere via the magnetopause and the cusp region.

Figure 4.10 shows the magnetic flux density and ion density along \(z\)-axis \(((x, y)=(0, 0))\). The magnetosphere size of the southward IMF case becomes slightly smaller than that of the no IMF case since the magnetic field is carried away by the magnetic reconnection. On the contrary, the magnetosphere size of the northward IMF case becomes larger than that of the southward IMF case and no IMF case since the piled-up magnetic field enhance the magnetopause magnetic field. Ion density distribution is shown in Fig.4.10b. In the case of no IMF and the southward IMF case, the magnetopause and equatorial ring current can be distinguished. On the contrary, the boundary of the magnetopause and ring current is ambiguous in the northward IMF.

Figure 4.11 shows the simulation result with various magnetic flux density of IMF. The bold line represents the magnetic flux density along \(z\)-axis. The southward IMF and magnetosphere size \(L_{MHD}/r_{iL} = 0.5\) is assumed. The magnetic flux density of the IMF is set to half, quarter and eighth of the magnetic flux density of the magnetopause. The magnetosphere size represented by dashed-line becomes larger as the magnetic flux density of IMF becomes smaller. These changes of the magnetosphere size may cause the change of the thrust of magnetic sail.

Figure 4.12 shows the simulation results with various IMF direction other than southward and northward. Figures 4.12a and 4.12b shows the case of IMF same direction with solar wind flow\((+z\) direction). Figures 4.12c and 4.12d, and Figs. 4.12e and 4.12f represents the eastward IMF \((+x\) direction) and westward IMF \((-x\) direction), respectively. The ion density in \(yz\)-plane is asymmetrical about \(xz\)-plane in these cases although it is symmetric in the case of no IMF,
Figure 4.8: Structure of magnetosphere with IMF. Ion density distribution, magnetic field line and theoretical magnetosphere size are represented. (artificial plasma parameter, $L_{MHD}/r_{iL} = 1, B_{IMF} = B_{MP}/4$, 3D Full-PIC model)
4.3. RESULTS AND DISCUSSION

(a) Current density without IMF, $yz$-plane.

(b) Ion trajectories without IMF, $xz$-plane.

(c) Current density with southward IMF, $yz$-plane.

(d) Ion trajectories with southward IMF, $xz$-plane.

(e) Current density with northward IMF, $yz$-plane.

(f) Ion trajectories with northward IMF, $xz$-plane.

Figure 4.9: Current density distribution in $yz$-plane and ion trajectories in $xz$-plane on the ion inertial scale with and without IMF. (artificial plasma parameter, $L_{MHD}/r_i = 1$, $B_{IMF} = B_{MP}/4$, 3D Full-PIC model)
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Figure 4.10: a) Magnetic flux density and b) ion density distribution along $z$-axis with various IMF direction. (artificial plasma parameter, $L_{MHD}/r_{iL} = 1$, $B_{IMF} = B_{MP}/4$, 3D Full-PIC model)
4.3. RESULTS AND DISCUSSION

Figure 4.11: Magnetic flux density distribution along z-axis with various IMF magnitude. (artificial plasma parameter, \( L_{MHD}/r_{iL} = 0.5 \), southward IMF, 3D Full-PIC model)

southward IMF and northward IMF. There is no big difference in the size and density distribution of a magnetosphere. Therefore, it is expected that the thrust is also almost equal. However, by the asymmetry of the flow, torque works to the magnetic sail spacecraft. The direction thrust vector is also changed by IMF. Thus, depending on the direction and the magnitude of IMF, the size of magnetosphere and equatorial ring current vary. On the ion scale magnetosphere, the IMF play important role for the thrust characteristics.

Next, we performed simulations of smaller magnetosphere on the electron inertial scale wit IMF. Figures 4.13 and 4.14 show the ion density distribution of \( L_{MHD}/r_{iL} = 0.1 \) and \( L_{MHD}/r_{iL} = 0.01 \) with IMF, respectively. (a) and (b) represent the southward IMF case in \( yz \)-plane and \( xz \)-plane, respectively. (c) and (d) represent the northward IMF case in \( yz \)-plane and \( xz \)-plane, respectively. The plasma flow around the magnetosphere is approximately same with the no IMF case, the southward IMF case and the northward IMF case compared with Fig. 4.2.

The magnetic neutral points are formed in front of the magnetosphere and the magneto tail in the southward IMF case and around the cusp region in the northward IMF case as well as the case of \( L_{MHD}/r_{iL} = 1 \). However, in both \( L_{MHD}/r_{iL} = 0.1 \) and \( L_{MHD}/r_{iL} = 0.01 \), the magnetic neutral point is far away from the magnetopause. Figure 4.15 shows the current density distribution of \( L_{MHD}/r_{iL} = 0.1 \) and \( L_{MHD}/r_{iL} = 0.01 \). Depending on the direction of IMF, the current density distribution is changed on \( L_{MHD}/r_{iL} = 0.1 \). On the contrary, the current density distribution of \( L_{MHD}/r_{iL} = 0.01 \) is approximately same independent of the IMF direction. It is expected that the strength of the frozen-in of electron on the magnetic field cause this difference since the main carrier of the current is electron. Around the magnetic neutral point (bold line), the current is not induced. That is, the magnetic reconnection is not occurred in the magnetic neutral point in the case of the electron inertial scale magnetosphere, and the magnetic field of IMF and the dipole
Figure 4.12: Magnetosphere with various IMF direction: solar wind direction IMF, eastward IMF and westward IMF. Ion density distribution and magnetic field line are represented. (artificial plasma parameter, $L_{MHD}/r_{iL} = 1$, $B_{IMF} = B_{MP}/4$, 3D Full-PIC model)
magnetic field are only connected geometrically. This is because not only ions but also electrons are easily detached from the magnetic field by the finite Larmor radius effect.

As a result, no change in solar wind flow with and without IMF is observed on the electron inertial scale. In a latter half of this study, it is assumed that the thrust of electron scale magnetic sail is not affected by IMF. On the contrary, the torque by electromagnetic force of IMF and asymmetric flow will work on the spacecraft. To reduce the simulation case, we attempt to perform the two-dimensional simulation about torque characteristics of magnetic sail in next chapter.

### 4.3.3 Influence of Artificial Plasma Parameter

Finally, we compared simulation results by artificial plasma parameter case 1 and case 2 (Table 4.1). In these two parameter sets, the mass ratio of ion and electron is changed. In case 1, the electron is 18.36 times heavier than real mass. In case 2, the electron is 1.836 times heavier than real mass. Hence, the stronger electron kinetics effects should be observed in case 1. The simulation results are shown in Fig. 4.16. The magnetosphere size is set as $L_{MHD}/r_{IL} = 0.5$ and the southward IMF is assumed.

In Fig. 4.16c for case 2, more clear shock like structure can be observed. The stronger current also flows in case 2 (Fig. 4.16d). Hence the larger electromagnetic force works on the coil as the repulsive force corresponding to the thrust of magnetic sail. The magnetic flux density distribution along $z$-axis is shown in Fig. 4.17. A rapid change of a magnetic field occurs by the stronger current at the magnetopause. The structure of magnetosphere is thus affected by the mass ratio of ion and electron. However, the magnetosphere size is approximately same in case 1 and case 2. Drag coefficient of case 1 and case 2 are calculated as 5.8 and 4.8, respectively.

The main carrier of the current is electron. Although the strong electron kinetics reduce the current, if the electron kinetics is neglected completely as Hybrid-PIC simulation or Flux-Tube simulation, the current carrier becomes ion and the current decreases. To obtain the exact current density distribution and the thrust level, the realistic mass ratio should be introduced.

### 4.4 Summary

In order to reveal the structure of moderately large magnetosphere ($L_{MHD}/r_{IL}=0.01 \sim 1$), Full-PIC simulations with artificial plasma parameters are performed. As a result, it was revealed that the magnetosphere size obtained by simulation is smaller than that calculated by MHD approximation when the magnetosphere size becomes smaller than the ion Larmor radius ($L_{MHD}/r_{IL} < 1$). The current carrier changes from ion to electrons according to the magnetosphere size. It is expected that the smaller magnetosphere cause smaller thrust level. However, the thrust level of magnetic sail should be estimated by using the realistic plasma parameters. Both two-dimensional simulation and three-dimensional simulation by using the realistic solar wind parameters hence should be
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(a) Southward IMF, $yz$-plane.
(b) Southward IMF, $xz$-plane.
(c) Northward IMF, $yz$-plane.
(d) Northward IMF, $xz$-plane.

Figure 4.13: Structure of magnetosphere with southward IMF northward IMF. Ion density distribution and magnetic field line are represented. (artificial plasma parameter, $L_{MHD}/r_{iL} = 0.1$, 3D Full-PIC model)
4.4. SUMMARY

(a) Southward IMF, $yz$-plane.
(b) Southward IMF, $xz$-plane.
(c) Northward IMF, $yz$-plane.
(d) Northward IMF, $xz$-plane.

Figure 4.14: Structure of magnetosphere with southward IMF and northward IMF. Ion density distribution and magnetic field line are represented. (artificial plasma parameter, $L_{MHD}/r_i = 0.01$, 3D Full-PIC model)
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(a) $L_{MHD}/r_{iL} = 0.1$, southward IMF.

(b) $L_{MHD}/r_{iL} = 0.1$, northward IMF.

(c) $L_{MHD}/r_{iL} = 0.01$, southward IMF.

(d) $L_{MHD}/r_{iL} = 0.01$, northward IMF.

Figure 4.15: Current density distribution perpendicular to $yz$-plane with southward IMF and northward IMF. (artificial plasma parameter, 3D Full-PIC model) The circle represents the magnetic neutral point.
4.4. SUMMARY

(a) Ion density, parameter 1.

(b) Current density, parameter 1.

(c) Ion density, parameter 2.

(d) Current density, parameter 2.

Figure 4.16: Ion density distribution and current density distribution with different artificial plasma parameter. ($L_{MHD}/r_iL = 0.5$, southward IMF, 3D Full-PIC model)
performed to reveal thrust characteristics of the electron inertial scale magnetic sail in next chapter.

The influences of IMF on the structure of magnetosphere are also examined by each magnetosphere sizes. It was revealed that the influence of IMF becomes small as the magnetosphere size become small and can be neglected in smaller magnetosphere which is expected as the demonstrator spacecraft ($L_{MHD}/r_{iL} < 0.01$). The quasi-steady state can be achieved in smaller magnetosphere even if the IMF is taken into consideration. On the contrary, it was revealed that mass ratio of ion and electron ($m_i/m_e$) play important role on the formation of the magnetosphere.

In latter chapters, hence, the realistic plasma parameter is used and IMF is neglected.
Chapter 5

Simulation of Magnetic Sail on Electron Inertial Scale

5.1 Introduction

As mentioned in Chapter 4, in addition to the ion kinetics, the electron kinetics also should be considered on the electron inertial scale. The full kinetic simulation hence required for the small-scale magnetic sail such as the demonstrator spacecraft. The thrust characteristics of the magnetic sail on the electron inertial scale is not clarified at all since the full kinetic simulation requires huge computational cost and spend much simulation time. Only recently, the improvement of high performance computation techniques enables us to perform the full kinetic simulation in three dimensions. We use the K supercomputer for the simulation. In addition, the all spacecraft model proposed by Winglee et al. [7], Cattell et al. [19] and Japan Aerospace Exploration Agency (JAXA) (Fig. 1.7) can be assumed directly, without any assumptions, only in Full-PIC simulation. Finally we perform the parametric study of the thrust of magnetic sail on the electron inertial scale using the realistic space plasma parameters.

The objective of this chapter is to reveal the thrust characteristics of magnetic sail with small magnetospheres from several 100 m to several 1000 m, where the electron kinetics should be considered. First, two-dimensional Full-PIC method was employed since the fundamental physics did not change between two-dimension or three-dimension. The two-dimensional analysis was adopted in order to reduce the computational resource. In the following, we evaluate the finite thrust generation by the electron inertial scale magnetosphere including the electron kinetics in three dimensions. In order to achieve the above objectives, we performed Full-PIC simulations with the realistic mass ratio \( m_i/m_e \sim 1836 \) for hydrogen plasma.
5.2 Computational Settings

5.2.1 Two-dimensional Simulation

The interaction between the solar wind and an artificial dipole magnetic field is simulated in two-dimensional space (xz-plane) and three-dimension by Full-PIC model. The computational domain used by two-dimensional Full-PIC simulations is shown in Fig. 5.1. The computational domain has an area of 15 km × 15 km involving 512 × 512 grids in the typical case. The grid spacing \( \Delta x \) should be chosen not much larger than the Debye length \( (\Delta x / \lambda_D < 3) \) and typically 16 super particles are involved in a grid. In the test simulation below, 4, 16, 64 super particles per cell is tested. Grid spacing and cell number are also evaluated. The time spacing \( dt \) must fill the Courant condition, \( dx/dt > 1.44c \) in three-dimension. The original two-dimensional dipole magnetic field is calculated using the opposite current \( \pm I_{\text{coil}} \). The coil currents are defined as the corresponding current density \( (I_{\text{coil}}/2dx^2) \) at grid center of two source grids \( (\pm R_{\text{coil}}) \).

The solar wind direction is defined as z-axis as shown in Fig. 5.1. The absorbing boundary condition is set for the electromagnetic field in all boundaries. The solar wind particles flow into the computational domain from the inflow boundary with the typical solar wind velocity and the thermal distribution. The outgoing particles from the computational domain are eliminated from the calculation. In addition, as mentioned in Chapter 6, artificial plasma is injected from the injection source grid near the coil for the MPS simulations as shown in Fig. 5.1. The thrust generated by magnetic sail and MPS is calculated using two different methods. In the first method, the electromagnetic force between the induced current by a plasma flow \( J \) and the coil current onboard spacecraft \( I_{\text{coil}} \) is calculated, and the electromagnetic force is designated as \( F_{\text{mag}} \). The second method uses a change in the solar wind momentum passing over a control volume to derive a thrust, \( F_{\text{flow}} \). In the first method, the magnetic field near a coil is calculated based on the Biot-Savart law as in Eq. (2.4.11). In the second method, the thrust is calculated by an arbitrary surface integral of the momentum as in Eq. (2.4.12).

The computational domain is divided into small domain \( (4 \times 512) \) by the domain decomposition. Each small domain is calculated by one process and bordering physical quantity is exchanged by MPI. The domain includes huge number of particles (approximately 0.5 million in typical case). The particles are divided into some groups by the particle decomposition. Each particle group is calculated by one thread by OpenMP. As a result, 512 CPUs (128 processes by MPI and 4 threads by OpenMP) are used for the simulation.

5.2.2 Three-dimensional Simulation

The computational domain used by three-dimensional Full-PIC simulation are shown in Fig. 5.2. In parallel case, the magnetic moment is parallel to the solar wind flow and the mirror point is formed in front of the spacecraft. The computational domain has an area of 3.8 km × 3.8 km × 3.8
5.2. COMPUTATIONAL SETTINGS

km is partitioned into a grid of $256 \times 256 \times 256$ cells ($dx = 15$ m) in the typical case. The original three-dimensional dipole magnetic field is defined by ideal dipole magnetic field ignoring the coil radius. The size of the computational domain is decided based on the theoretical magnetosphere size $L_{MHD}$ (Eq. (1.2.3)). The length of the one side is approximately $5L_{MHD}$. The grid spacing $dx$ typically needs to be set such that it is not much larger than the Debye length ($dx/\lambda_D < 3$). The time spacing $dt$ must fill the Courant condition, $dx/dt > 1.73c$ in three-dimension ($dt=1.5 \times 10^{-8}$ s). The solar wind plasma is typically represented by 16 super particles associated with each cell and a total of $5.4 \times 10^8$ particles. This is because it was revealed in our previous study that particle number per cell is not influence the precision of the thrust compared with the latitude resolution or the size of calculation domain. 16 particles are hence necessary and sufficient in terms of the calculation cost and correctness of the simulation. The magnetic field generated by the coil mounted in the spacecraft is approximated by an ideal dipole magnetic field or a finite size coil magnetic field of magnetic moment $M$. The magnetic moment of the coil is arranged in the $y$-direction (perpendicular to the solar wind, Fig. 5.2a) or $z$-direction (parallel to the solar wind, Fig. 5.2b). The magnetic field is given as the initial condition. Absorbing boundary conditions are used for the electromagnetic field on all outer boundaries. Solar wind particles flow into the computational domain from the inflow boundary at the typical solar wind velocity $V_{SW}$ and thermal distribution. Outgoing particles from the computational domain are eliminated from the calculation. The collision between particles and the spacecraft is not taken into consideration, either, assuming the infinitesimal spacecraft.

The computational domain is divided into some smaller cuboid ($8 \times 8 \times 256$ cells in typical
Figure 5.2: Three-dimensional computational domain: a) perpendicular case and b) parallel case. The solar wind is parallel to $z$-axis.

...by the domain decomposition as shown in Fig. 2.13. Each small domain is calculated by one process and bordering physical quantity is exchanged by MPI. Figure 2.13b shows the one small computational domain. The small domain includes huge number of particles (approximately 0.5 million in typical case). The particles are divided into some groups by the particle decomposition. Each particle group is calculated by one thread by OpenMP. As a result, 4096 CPUs (1024 processes by MPI and 4 threads by OpenMP) are used for the simulation.

5.3 Simulation Results by Two-dimensional Simulation

5.3.1 Structure of Magnetosphere Obtained by Two-dimensional Full-PIC Simulation

Using artificial plasma parameter, the structure of the magnetosphere and influence of the IMF is simulated in Chapter 4. However, the real parameter analysis is required to reveal the thrust characteristics of magnetic sail. First, to demonstrate how the momentum is transferred from the solar wind to the spacecraft with the small-scale magnetosphere affected by the electron kinetics in two-dimension, we performed Full-PIC simulation without a plasma injection. Two-dimensional magnetic sail with the parameters listed in Table 5.1 was simulated in order to clarify the effects of electron’s finite Larmor radius and charge separation on the flow and magnetic field of the small-scale magnetic sail. We consider only an artificial dipole magnetic field without taking into account IMF. From Eq. (1.2.5), theoretical magnetospheric size $L_{MHD}$ is calculated as $L_{MHD}=2200$ m under the conditions in Table 5.1, and hence $L_{MHD}<r_{iL}$ (100 km) and $L_{MHD}>r_{eL}$ (50 m) are satisfied. This case hence corresponds to a magnetosphere on the electron inertial scale. Figure 5.3 shows the schematic illustration of two-dimensional magnetic sail. The spacecraft has long superconductive wire and the two-dimensional dipole magnetic field is generated.

Figure 5.4a shows the spatial distribution of ion density in steady state when the magnetic
Table 5.1: Parameters about the coil of two-dimensional magnetic sail, $L=2.2\text{ km}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current $I_{\text{coil}}$ [A turn]</td>
<td>$4 \times 10^3$</td>
</tr>
<tr>
<td>Radius $R_{\text{coil}}$ [m]</td>
<td>75</td>
</tr>
<tr>
<td>Magnetic moment $M$ [Wb \cdot m]</td>
<td>$6.0 \times 10^5$</td>
</tr>
</tbody>
</table>

Figure 5.3: Concept of two-dimensional magnetic sail.

dipole moment is perpendicular to the solar wind direction ($\alpha=90^\circ$). In Fig. 5.4a, a low density region so-called ‘wake region’ is formed around the origin, $(x, z)=(0, 0)$, in spite of loose coupling between the ions and the magnetosphere ($r_{iL}>L_{MHD}$). Similar but slightly larger wake region also appears in Fig. 5.4b, which shows the electron density distribution.

Figures 5.5a and 5.5b show the schematic illustration of the plasma density distribution and simulation results of the plasma density, the electric field and the current density along $z$-axis ($x=0$ m), respectively. Here, the plasma density represents $n_i-n_e$. At the magnetopause (-1000 m<$x$<900 m), an electron-rich region is formed since electrons cannot penetrate into the magnetic field because of their small Larmor radius $r_{eL}$. On the contrary, ion is abundant in the interior of the magnetosphere ($x>$900 m). By charge separation between ions and electrons at the magnetopause, the outward electric field $E_{MP}$ appears as illustrated in Fig. 5.5a. The trajectories of ions close to the magnetopause are bent by $E_{MP}$ (see also Chapter 4). Consequently, the ion density around spacecraft becomes low nevertheless ions are not magnetized. On the contrary, the magnetized electrons make $E \times B$ drift motions at the magnetopause, thus the magnetopause current $J_{MP}$ that flows in the counter direction of $I_{\text{coil}}$ is induced. The Lorentz force between $J_{MP}$ and the onboard coil current $I_{\text{coil}}$ works as a thrust by magnetic sail as illustrated in Fig. 5.5a.

Dimensions of the wake are determined from the distance between the origin and the magnetopause current peak. From Fig. 5.5b, the magnetospheric size in $z$-direction is calculated as 900 m in the present parameters. Following the same procedure, dimensions of the wake are determined as 900 m (stagnation length) $\times$ 1100 m (cross-sectional length) as shown in Fig.5.4a, and these are quite small compared with the theoretical value $L_{MHD}=2200$ m. This tendency agrees with the results in Chapter 4.

By using Full-PIC simulation, the electron’s finite Larmor effect and the charge separated structure of magnetosphere are revealed for the first time, which are impossible to be analyzed
either by MHD simulation or Hybrid-PIC simulation.  

Finally, the thrust of magnetic sail is obtained as a force per unit length. To check self-consistency of the simulation, thrust is evaluated by two methods: the Lorentz force between induced current by plasma and $I_{\text{coil}}$ (Eq. (2.4.11)) and the momentum change of solar wind particles (Eq. (2.4.12)). The thrust is calculated as $2.0 \times 10^{-6} \pm 0.1 \times 10^{-6}$ N/m.  

To check the dependency of the thrust on particle numbers and grid spacing, simulations with different conditions are also performed: particle number (4 particles / cell, 16 particles / cell, 64 particles / cell), grid number (256×256, 512×512) and grid spacing $dx$ (15 m, 30 m, 60 m). As a result, it was found that the thrust level is overestimated by approximately 20% when the computational domain is smaller than the typical case (512×512, $dx=30$ m) due to the influence of the boundary. When the grid spacing is $dx=60$ m ($dx/\lambda_D > 3$), the thrust is calculated as $2.2 \times 10^{-6} \pm 0.2 \times 10^{-6}$ N/m. The error bar becomes larger by rough grid spacing. On the contrary, no clear difference appears to the thrust by the differences of particle number. As the necessary and sufficient calculation condition, 16 particles / cell is used in the remainder of this study.

We note that thrust characteristics of the magnetic sail cannot be obtained by the simulation using the artificial plasma parameters. However, in order to keep a computational cost low, $m_i/m_e = 10 \sim 100$ is often preferred in Full-PIC simulation [36, 37]. We checked how simulation results are affected by the mass ratio of ion and electron, $m_i/m_e$. In the real mass ratio, ion (proton) is 1836 times heavier than electron. Hence, two simulations with $m_i/m_e = 1836$ and $m_i/m_e = 100$ are performed to evaluate the thrust of the magnetic sail. Here, electrons are 18.36 times heavier than the real mass in the case of $m_i/m_e = 100$. Parameters of the solar wind are listed in Table 1.2 and parameters about the coil of the magnetic sail are listed on Table 5.2. The magnetic moment
5.3. SIMULATION RESULTS BY TWO-DIMENSIONAL SIMULATION

Figure 5.5: a) Schematic illustration of magnetopause and b) one-dimensional distributions of plasma density, electric field and current density along z-axis (x=0 m) in steady state by simulation (magnetic sail, $L_{MHD}=2200$ m, $\alpha=90^\circ$, 2D Full-PIC model).


Table 5.2: Parameters about the coil of magnetic sail, \( L = 1.4 \) km

<table>
<thead>
<tr>
<th>Coil parameters</th>
<th>( L = 1.4 ) km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current ( I_{\text{coil}} ) [( \text{A turn} )]</td>
<td>( 8 \times 10^3 )</td>
</tr>
<tr>
<td>Radius ( R_{\text{coil}} ) [m]</td>
<td>15</td>
</tr>
<tr>
<td>Magnetic moment ( M ) [Wb ( \cdot ) m]</td>
<td>( 2.4 \times 10^5 )</td>
</tr>
</tbody>
</table>

Figure 5.6: Density distribution of ion and streamline calculated by 2D Full-PIC for two simulation cases: a) result for \( m_i/m_e = 1836 \) and b) result for \( m_i/m_e = 100 \).

The results are shown in Fig. 5.6. The magnetospheric size (blocking area size) in the case of \( m_i/m_e = 1836 \), Fig. 5.6a, is approximately 2 times larger than that in the case of \( m_i/m_e = 100 \), Fig. 5.6b. Furthermore, the thrust of the magnetic sail in case of \( m_i/m_e = 1836 \) is calculated as 2.0 \( \mu \text{N/m} \) and that in case of \( m_i/m_e = 100 \) is calculated as 0.6 \( \mu \text{N/m} \). As a result, the mass ratio has a significant influence on the formation of magnetosphere and thrust generation. In addition, checking the detail of each ion and electron motion, ions are scattered by the static electric field due to the charge separation at the boundary of the magnetosphere and electrons are scattered by the magnetic field during their Larmor motions. As the mass ratio becomes smaller, the static electric field due to the charge separation at the boundary of the magnetosphere becomes weaker since more electrons penetrate into the deeper magnetosphere and fewer ions are scattered by the electric field. This difference results in the smaller thrust in case of \( m_i/m_e = 100 \). For this reason, we performed several simulations about magnetic sail with the real mass ratio, \( m_i/m_e = 1836 \) in order to evaluate the thrust characteristics of magnetic sail.

In addition to the simulations neglecting the effects of IMF, we performed the simulation with IMF as shown in Fig. 5.7. As mentioned in Chapter 4, the influence of the IMF on the magnetosphere is small on the electron inertial scale. Although the magnetosphere on MHD scale has the
5.3. SIMULATION RESULTS BY TWO-DIMENSIONAL SIMULATION

Figure 5.7: Ion density distribution and magnetic field line of electron scale magnetosphere without IMF(a) and with IMF(b) (magnetic sail, $L_{MHD}=2200$ m, $\alpha=90^\circ$, 2D Full-PIC model). The magnetic moment is perpendicular to the solar wind direction (z-axis). The absolute value and the direction of IMF are set as 7 nT and 45 deg., respectively.

discontinuity of the magnetic field around the magnetopause current peak, Fig. 5.7a represents that the magnetic discontinuity is not observed in electron inertial scale magnetosphere even if IMF is neglected since the plasma flow can induce the only weaker magnetopause current than the coil current. This is because a larger Larmor radius of ion and electron than the magnetospheric size causes the loose coupling between the plasma flow and the magnetic field. In Fig. 5.7b, The magnetic flux density and the direction of IMF are set as 7 nT and 45 deg., that is, $(B_x, B_z)=(5$ nT, 5 nT) and the similar magnetosphere to Fig. 5.7a is formed as shown in Fig. 5.7b. The thrust per unit length is also calculated as $1.9 \times 10^{-6} \pm 0.1 \times 10^{-6}$ N/m. That is, it was revealed that the thrust of magnetic sail is hardly influenced by IMF on the electron inertial scale as well as MHD scale [38]. Hence, in the remainder of this study, we consider only an artificial dipole magnetic field without taking into account IMF for simplicity.

5.3.2 Effects of Electrostatic Field by Charge Separation in Two-dimensional Magnetosphere

To ensure the effects of the charge separation at the magnetopause, we performed electrostatic plasma simulation and magneto static plasma simulation. In the electrostatic plasma simulation, only static potential is taken into account by solving Poisson equation,

$$\nabla^2 \phi = -\frac{q(n_i - n_e)}{\varepsilon_0}$$

(5.3.1)
CHAPTER 5. SIMULATION OF MAGNETIC SAIL ON ELECTRON INERTIAL SCALE

Figure 5.8: Time history of thrust generation by Full-PIC simulation (EM) and other static simulations: electrostatic (ES), magneto static (MS) and electro magneto static (ESMS) in two dimensions. (magnetic sail, \(L_{MHD} = 1400\) m, \(\alpha = 90^\circ\))

Ion density and electron density is obtained by PIC weighting from the particle analysis. In magneto static simulation only vector potential is taken into consideration by solving vector Poisson equation:

\[
\nabla^2 A = -\mu_0 J
\]  

In addition, the electro magneto static simulation, which solves both scalar potential (Eq. (5.3.1)) and vector potential (Eq. (5.3.2)) is performed. In these static simulations, time variation of electromagnetic field is assumed to be slower than the motion of plasma (\(\sim \omega_p\)). The simulation results are shown in Figs. 5.8 and 5.9. Fig. 5.8 shows the time history of thrust generation. The thrust obtained by electro magneto static simulation (ESMS) is well in agreement with the thrust of Full-PIC simulation (EM). The ion density distribution of Full-PIC simulation and electro magneto static simulation in Fig. 5.9 is also in agreement. Also, the thrust obtained by electrostatic simulation (ES), which considers the electric field of the charge separation, is near value with the thrust of full-PIC simulation (EM). On the contrary, the thrust obtained by the magneto static simulation (MS) is further smaller than the thrust obtained by Full-PIC simulation. It was revealed that the electric field by the charge separation has the dominant influence on the thrust generation of magnetic sail.

To evaluate the simulation result, the theoretical approach is also attempted in the one-dimensional model. According to the plasma kinetic theory and the coordinate system shown in Fig. 5.10,

\[
v_z \frac{\partial f}{\partial z} - \frac{q}{m} \frac{\partial \phi}{\partial v_z} \frac{\partial f}{\partial v_z} + \frac{q}{m} \frac{\partial A_y}{\partial z} \left( v_z \frac{\partial f}{\partial v_y} - v_y \frac{\partial f}{\partial v_z} \right) = 0
\]  

represents the reduced Boltzmann equation for the one-dimensional problem along the solar wind.
5.3. SIMULATION RESULTS BY TWO-DIMENSIONAL SIMULATION

Figure 5.9: Ion density distribution: a) Full-PIC simulation and b) Electrostatic simulation. (magnetic sail, $L_{MHD}=1400$ m, $\alpha=90^\circ$, 2D model)

![Simulation Results](image)

Figure 5.10: Axis definition used in one-dimensional plasma kinetic theory about magnetosphere boundary.

direction ($z$-axis) [9].

The electric field and the magnetic field acting on the solar wind plasma is represented by the scalar potential $\phi$ and vector potential $A_y$, respectively. The total energy and the generalized canonical moment are conserved as

$$\frac{1}{2} m \left( v_y^2 + v_z^2 \right) + q\phi = \frac{1}{2} m V_{SW}^2$$  \hspace{1cm} (5.3.4)

and

$$mv_y + qA_y = 0$$  \hspace{1cm} (5.3.5)

. The solutions for the ion and electron distribution function are, then, obtained as

$$f_i = N_{SW} V_{SW} \delta \left( v_{iz} + \frac{2e\phi}{m_i} - V_{SW}^2 \right) \delta (v_{iy})$$  \hspace{1cm} (5.3.6)
and
\[ f_e = N_{SW}V_{SW}\delta (v_{ey}^2 + v_{ez}^2 - \frac{2e\phi}{m_e} - V_{SW}^2)\delta (v_{ey} - \frac{eA_y}{m_e}) \] (5.3.7)

, respectively. Here, the vector potential \( A_y \) in the ion momentum equation is neglected since the vector potential is comparable to \( m_e v_{ez} \) but small compared to \( m_i v_{iz} \). This shows \( v_{iz} = 0 \).

\[ n_i = \frac{N_{SW}V_{SW}}{\sqrt{V_{SW}^2 - \frac{2e\phi}{m_i}}} \] (5.3.8)

and
\[ n_e = \frac{N_{SW}V_{SW}}{\sqrt{V_{SW}^2 + \frac{2e\phi}{m_e} - \left( \frac{eA_y}{m_e} \right)^2}} \] (5.3.9)

are obtained by integrating Eqs. (5.3.6) and (5.3.7), respectively. Finally, substituting Eqs. (5.3.8) and (5.3.9) into the Poisson equation about the scalar potential \( \phi \),

\[ \frac{\partial^2 \phi}{\partial z^2} = -\frac{e}{\varepsilon_0} \left( \frac{N_{SW}V_{SW}}{\sqrt{V_{SW}^2 - \frac{2e\phi}{m_i}}} - \frac{N_{SW}V_{SW}}{\sqrt{V_{SW}^2 + \frac{2e\phi}{m_e} - \left( \frac{eA_y}{m_e} \right)^2}} \right) \] (5.3.10)

is obtained. Fig. 5.11a shows the density distribution along \( z \)-axis in two-dimensional Full-PIC simulation result. Fig. 5.11b shows the one-dimensional density distributions of ion and electron obtained by solving Eq. (5.3.10). Here, vector potential \( A_y \) is assumed to be constant since the magnetopause current in the electron inertial scale is very weak. The magnetospheric size obtained by the Full-PIC simulation (\(~900\) m) is smaller than the size obtained by Eq. (1.2.5) (\(~2200\) m) since the finite Larmor effects of both ion and electron is dominant in the electron inertial scale. The magnetospheric size in Fig. 5.11b well agrees with the magnetospheric size in Fig. 5.11a nevertheless the absolute value of the density distribution is different. This is because the structure of the magnetosphere along \( z \)-axis in two-dimensional simulation approximately can be dealt with in one-dimension. The density distribution of ion and electron in the analysis result differs from the simulation results since the thermal distribution of particles is not considered in the theoretical model.

Thus, the magnetopause in the electron inertial scale is formed in the downstream compared with the magnetospheric size \( L_{MHD} \) defined by the fluidic pressure balance (Eqs. (1.2.3) and (1.2.5)).
5.3. SIMULATION RESULTS BY TWO-DIMENSIONAL SIMULATION

5.3.3 Thrust Characteristics Obtained by Two-dimensional Full-PIC Simulation

First, the attitude stability of magnetic sail is analyzed by changing the direction of magnetic momentum with and without IMF. As mentioned in Chapter 4 and above section, IMF does not affect the thrust of electron inertial scale magnetic sail.

On the contrary, the torque characteristics are affected by IMF direction. We considered the two-dimensional magnetic sail with $L_{MHD} = 1400 \text{ m}$ as Table 5.2. The angles of the magnetic momentum and IMF are defined with reference to the solar wind direction ($z$-axis) as shown in Fig. 5.12. The torque around $y$-axis is considered. The counterclockwise torque is defined as plus. In the case of no IMF, the torque is proportional to $\sin (2\alpha)$:

$$M_{mag} \propto \sin (2\alpha) \quad (5.3.11)$$

as shown in Fig. 5.13. The attack angle $\alpha$ varies from 0° to 360° by 30°. For example, in the case of $\alpha = 60^\circ$, the counterclockwise torque works on the coil and the attitude of the spacecraft becomes $\alpha = 90^\circ$. In the case of $\alpha = 120^\circ$, the clockwise torque works on the coil and the attitude of the spacecraft also becomes $\alpha = 90^\circ$. That is, the magnetic sail attitude is stable when the magnetic moment of the onboard coil is perpendicular to the solar wind as $\alpha = 90^\circ$ and $270^\circ$.

In the case with IMF, the torque is proportional to $-\sin (\alpha - \beta)$:

$$M_{mag} \propto -\sin (\alpha - \beta) \quad (5.3.12)$$

as shown in Fig. 5.14. The magnetic flux density of IMF is set as 5 nT. The attack angle $\alpha$ varies from 180° to 270° by 30°. Particularly, we call $\alpha = 180^\circ$ as a parallel case and $\alpha = 270^\circ$ as
a perpendicular case. The direction of IMF $\beta$ varies from $-90^\circ$ to $90^\circ$ by $45^\circ$. The magnitude of the torque with IMF becomes larger than that without IMF. As the attack angle becomes large, the sin curve shift right. The attitude of the spacecraft becomes stable when the magnitude of torque is equal to 0 and the derivative of the torque characteristics is minus. That is, the magnetic sail attitude is stable when the magnetic moment of the onboard coil is parallel to the IMF. The maximum torque per unit length is $9.0 \times 10^{-4}$ Nm/m.

The moment of inertia around $y$-axis is obtained as

$$I_y = 2M_{coil}R_{coil}^2$$

when the distance between the two current wire is larger than the wire radius. Here, $M_{coil}$ is the weight of the superconductive wire. When we assumed $R_{coil}=15$ m, wire length=1 km and $M_{coil}=200$ kg, the attitude of the spacecraft follows

$$\frac{d\alpha(t)}{dt} = -10^{-5} \sin(\alpha(t) - \beta)$$

. The time history of attack angle is shown in Fig. 5.15. Here, the initial attack angle $\alpha(0) = 0$ and the direction IMF $\beta = 30$ deg are assumed. The attitude of the spacecraft changes from $\alpha=0$ deg to $\alpha=30$ deg in less than 10 minutes. The attitude of the spacecraft is not stabilized unless it equips a mechanism which attenuates vibration. In addition, the time scale of change of IMF direction is several hours and the control of the spacecraft attitude is always needed when the torque by IMF works. When designing a magnetic sail spacecraft, the device of shifting the center of a coil and the center of gravity of a satellite is required.

Next, the thrust characteristics are analyzed for various magnetospheric sizes. The theoretical magnetospheric size $L_{MHD}$ was selected from 380 m to 4300 m depending on the coil current.
5.3. SIMULATION RESULTS BY TWO-DIMENSIONAL SIMULATION

Figure 5.13: Calculation results of the torque acting at the magnetic sail for several cases of $\alpha$ without IMF. (magnetic sail, $L_{MHD}=1400$ m, 2D Full-PIC model)

Table 5.3: Simulation parameters for various magnetospheric sizes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid spacing $dx$</td>
<td>30 m</td>
</tr>
<tr>
<td>Time $dt$</td>
<td>$4.0 \times 10^{-8}$ s</td>
</tr>
<tr>
<td>Grid number</td>
<td>$512 \times 512$</td>
</tr>
</tbody>
</table>

$I_{coil}=13$ to $1.6 \times 10^4$ A (electron inertial scale $L_{MHD}<<r_{iL}$). The thrust characteristics of larger magnetospheric size equivalent to MHD scale ($L_{MHD}>>r_{iL}$) and ion inertial scale ($L_{MHD} \sim r_{iL}$) have been already analyzed by MHD simulation [31, 32, 38], Hybrid-PIC simulation [33, 35, 60] and Flux-Tube simulation (Chapter 3) and the thrust characteristics of small magnetospheric size remains to be analyzed. The attack angle is $\alpha=0^\circ$ or $\alpha=90^\circ$. Parameters about magnetic sails and the simulation are listed in Table 5.3 and Table 5.4, respectively.

Results are shown in Fig. 5.16, in which the solid line shows the results of magnetic sail in the parallel case ($\alpha=0^\circ$) and the dotted line shows the results of magnetic sail in the perpendicular

Table 5.4: Magnetic sail design parameters for various magnetospheric sizes

<table>
<thead>
<tr>
<th>Coil distance $R_{coil}$ [m]</th>
<th>75</th>
<th>75</th>
<th>75</th>
<th>75</th>
<th>75</th>
<th>75</th>
<th>75</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current $I_{coil}$ [A]</td>
<td>$1.3 \times 10^1$</td>
<td>$2.5 \times 10^2$</td>
<td>$5.0 \times 10^2$</td>
<td>$1.0 \times 10^3$</td>
<td>$2.0 \times 10^3$</td>
<td>$4.0 \times 10^3$</td>
<td>$8.0 \times 10^3$</td>
<td>$1.6 \times 10^4$</td>
</tr>
<tr>
<td>Magneto-</td>
<td>$3.8 \times 10^2$</td>
<td>$5.4 \times 10^2$</td>
<td>$7.7 \times 10^2$</td>
<td>$1.1 \times 10^3$</td>
<td>$1.5 \times 10^3$</td>
<td>$2.2 \times 10^3$</td>
<td>$3.1 \times 10^3$</td>
<td>$4.3 \times 10^3$</td>
</tr>
</tbody>
</table>
Figure 5.14: Calculation results of the torque acting at the magnetic sail for several cases of $\alpha$ and $\beta$ with IMF: a) coordinated by attack angle $\alpha$ and b) coordinated by $\alpha - \beta$. (magnetic sail, $L_{MHD}=1400$ m, 2D Full-PIC model)
case ($\alpha=90^\circ$). Two important features are found from Fig. 5.16a: 1) rapid increase in thrust as the magnetospheric size becomes larger; and 2) thrust in the parallel case is larger than the thrust in the perpendicular case for the same magnetospheric size. The item 2) indicates that the magnetosphere in parallel case is larger than the magnetosphere in the perpendicular case, and it is supported by Fig. 5.17 which shows the ion density distribution of the parallel case and the perpendicular case for $L=2200$ m. As one can see, the cross-sectional lengths of the magnetosphere along $x$-axis ($z=0$ m) are obtained as 1100 m in the perpendicular case (Fig. 5.17a) and 1300 m in the parallel case (Fig. 5.17b). More particles hence interact with the parallel magnetosphere than the perpendicular magnetosphere due to the size effect, but seeing the upstream region in Fig. 5.17b, it is also found that the mirror magnetic field in the parallel case reflects the particles that approach the coil onboard the spacecraft. In addition to the size effect, the mirror magnet will contribute to produce a larger thrust in the case of parallel magnetosphere.

Non-dimensional force, or the drag coefficient, $C_d$, is defined by

$$C_d = \frac{F}{0.5 m_i N_{SW} V_{SW}^2 (2L_{MHD})}$$

(5.3.14)

. Here cross-sectional area is defined as $2L_{MHD}$ per unit length. $C_d$ is plotted in Fig. 5.16b, and Fig. 5.16c shows the ratio of the cross-sectional length obtained by Full-PIC simulation to the theoretical magnetospheric size $L_{MHD}$. In larger magnetosphere where the dipole approximation is valid ($L_{MHD}>>R_{coil}$), the ratio increases monotonically and asymptotically approaches to unity as $L_{MHD}$ increases. On the contrary, in the case of small magnetosphere ($L_{MHD}<1000$ m or cross-sectional length $<400$ m), the ratio does not monotonically increase since the finite coil size ($R_{coil}=75$ m) cannot be negligible and the magnetic flux density equivalent to the magnetopause ($B_{MP}=50$ nT) of coil magnetic field is extended radially compared with the magnetic flux density.
of an ideal dipole magnetic field. By the extended magnetic field, the cross-sectional length of the smaller magnetosphere becomes larger than that formed by an ideal dipole magnetic field. When the dipole approximation is valid, \( C_d \) in the perpendicular case becomes larger at a constant rate in proportion to the increase in the ratio (cross-sectional length / \( L_{MHD} \)) as represented by the dotted line in Fig. 5.16b since the interactions between the solar wind and the magnetic field are enhanced according to the cross-sectional length rather than the theoretical \( L_{MHD} \). On the contrary, \( C_d \) becomes larger rapidly from \( L_{MHD}=770 \) m to 2200 m in the parallel case represented by the solid line in Fig. 5.16b and the change rate of \( C_d \) from 2200 m to 4300 m becomes smaller. This is because particles are scattered by not only the electric field at the magnetopause but also the magnetic field. As the magnetospheric size becomes larger, it is expected that more particles be scattered by the magnetic field rather than the electric field. Thus, it is assumed that the mechanism how the magnetic sail generates the thrust depends on the magnetospheric size.

In addition, it is expected that the mirror magnetic field cause this feature of \( C_d \) in the parallel case as the biggest difference of the plasma flow from in the perpendicular case. The influence of the mirror magnetic field is considered as follows. The magnetic flux density at the coil center \((\mu_0 I_{coil}/\pi R_{coil})\) is proportional to \( I_{coil} \) and the magnetospheric size \( L_{MHD} \) represented by Eq. (1.2.5) is proportional to square root of \( I_{coil} \). The mirror ratio is approximately assumed as \( B_{MP}/(\mu_0 I_{coil}/\pi R_{coil}) \) using the typical magnetic flux density at magnetopause \( B_{MP} \) (\( \sim 50 \) nT, constant) and the magnetic flux density at the coil center. The mirror ratio hence becomes smaller in inverse proportional to the square of the theoretical \( L_{MHD} \). That is, more particles that approach the coil in the parallel case are reflected, as the theoretical \( L_{MHD} \) becomes larger. These particles are expected to be mainly contributed to the thrust increase in the parallel case from \( L_{MHD}=770 \) m to 2200 m since both ions and electrons slip away the magnetopause because of the large finite Larmor radius \((r_{iL} >> L_{MHD} \text{ and } r_{eL} \sim L_{MHD})\) as same as particles in the perpendicular case without the mirror magnetic field. In the larger magnetosphere from \( L_{MHD}=2200 \) m to 4300 m \((r_{iL} >> L_{MHD} \text{ and } r_{eL} << L_{MHD})\), particles interact with the magnetic field perpendicular to the particle velocity at magnetopause rather than the mirror magnetic field. As a result, \( C_d \) from \( L_{MHD}=2200 \) m to 4300 m in the parallel case becomes larger at a constant rate in proportion to the increase in the ratio of the cross-sectional length of the magnetosphere and the theoretical \( L_{MHD} \) as same as in the perpendicular case. Thrust of magnetic sail, thus, depends on the cross-sectional length of the charge-separated magnetosphere affected by the finite Larmor effect and the mirror magnetic field. MPS is hence expected to be able to increase thrust by expanding the cross-sectional length of the magnetosphere. In addition, by reflecting the solar wind particles by the mirror magnetic field, the thrust increase beyond the increase in the cross-sectional length of the magnetosphere is expected in the parallel case.

In addition, the current distribution affects on the thrust characteristics of the magnetic sail. The main carrier of the magnetopause current is electron and one-dimensional slice of electron current along z-axis is shown in Fig. 5.18. In larger magnetosphere \((L_{MHD}=2000 \) m), the most
5.3. SIMULATION RESULTS BY TWO-DIMENSIONAL SIMULATION

Figure 5.16: Thrust characteristics of magnetic sails about magnetospheric size $L_{MHD}$: a) thrust level $F_{mag}$, b) drag coefficient $C_d$ and c) ratio of the cross-sectional length obtained by the simulations and the theoretical value $L_{MHD}$. The magnetospheric size varied from 380 m to 4300 m by changing the coil current. (magnetic sail, 2D Full-PIC model)
current component is the \( \mathbf{E} \times \mathbf{B} \) drift motion of electron. On the electron inertial scale, the ion hardly moves by the \( \mathbf{E} \times \mathbf{B} \) drift motion and the \( \mathbf{E} \times \mathbf{B} \) drift motion can induce net current. On the contrary, in smaller magnetosphere (\( L_{MHD}=200 \) m), the current induced by the \( \mathbf{E} \times \mathbf{B} \) drift motion of electron is approximately half of the whole current. This is because that finite Larmor effect of electron becomes significant in small magnetosphere and larger current can be induced. Thus, the electron kinetics becomes important in small magnetosphere (\( L_{MHD} \sim r_{eL} \)).

5.4 Simulation Results by Three-dimensional Simulation

5.4.1 Structure of Magnetosphere Obtained by Three-dimensional Full-PIC Simulation

The three-dimensional magnetic sail was simulated by using the typical solar wind parameters at Earth orbit (Table 1.2). For simplicity, direct collisions between plasma particles are neglected in this study. The interplanetary magnetic field (IMF) can be also negligible on the electron inertial scale as revealed by Chapter 4. Table 5.5 lists the design parameters of the three-dimensional magnetic sail; these parameters are expected to give a small-scale magnetic sail (on the order of 100 m) in which electron kinetic effects are significant.

Figure 5.19 shows the three-dimensional ion density distribution of magnetic sail in the steady state when the magnetic moment \( M \) is perpendicular to the solar wind (\( \alpha = 90 \) deg) and parallel to the solar wind (\( \alpha = 0 \) deg), respectively. It takes about four days by using 1024 CPUs to obtain the steady state of the plasma flow and approximately 100 GB of memory was required. In both
5.4. SIMULATION RESULTS BY THREE-DIMENSIONAL SIMULATION

Figure 5.18: Current density distribution along z-axis: a) $L_{MHD}=200$ m and b) $L_{MHD}=2000$ m. (magnetic sail, 2D Full-PIC model)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil radius $R_{coil}$ [m]</td>
<td>$2$</td>
</tr>
<tr>
<td>Coil current $\times$ turn $I_{coil}$ [A]</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Magnetic moment $M$ [Wb-m]</td>
<td>$1.3 \times 10^8$</td>
</tr>
<tr>
<td>Magnetic moment direction</td>
<td>$y$-direction (perpendicular case) and $z$-direction (parallel case)</td>
</tr>
</tbody>
</table>
the perpendicular and parallel cases, a low-density region forms around the spacecraft at \((x, y, z) = (0, 0, 0)\) despite the loose coupling between the ions and magnetic field because of the large Larmor radius of the ions. The magnetosphere is far smaller than that expected by Eq. (1.2.3).

Figure 5.20 shows slices of ion density and electron density in perpendicular case through the \(xz\)-plane (magnetic equatorial plane) and \(yz\)-plane (magnetic meridian plane), respectively. The density distribution of ion and electron is slightly different and the charge neutrality is imperfect. The charge separation hence causes the electric field on the boundary region. The magnetosphere is asymmetric about the \(yz\)-plane and symmetric about the \(xz\)-plane in the perpendicular case since the Lorentz force on the ions is in the same direction for both \(x > 0\) and \(x < 0\) (Fig. 5.21). Clear boundary of density is observed in ion density distribution (Fig. 5.20a and 5.20b) since electrons, which is 1836 times lighter than ion, are affected the thermal velocity and electric field generated by the charge separation severely. At the stagnation point of the magnetosphere, the magnetic flux density is 0.1 mT and this is much larger than the expected value by MHD approximation (50 nT). Ion Larmor radius at the stagnation point is approximately 50 m and ions can interact with the magnetic field. Hence the low density region is formed.

In Fig. 5.22, the one-dimensional cut of ion density in perpendicular case along \(x\)-axis \((y=0\) m and \(z=0\) m, 200 m, 400 m) and \(y\)-axis \((x=0\) m, \(z=0\) m, 200 m, 400 m) are also shown in (a) and (b), respectively. Although the magnetosphere is defined in MHD simulations by the discontinuity in the magnetic field, the small-scale magnetosphere does not exhibit a discontinuity since the plasma can induce a weak magnetopause current. As a result, unlike the magnetosphere obtained by the MHD approximation, the magnetic field is not contained within the magnetosphere in the small-scale magnetosphere. In this study, we therefore define the low-density region in the full-PIC simulations as the magnetosphere instead of the discontinuity in the magnetic field since the
5.4. SIMULATION RESULTS BY THREE-DIMENSIONAL SIMULATION

Figure 5.20: Ion density distribution and electron density distribution in the perpendicular case. ($M = 1.3 \times 10^8$ Wb·m, typical solar wind, 3D Full-PIC model) The streamline represents mass flux flow.
discontinuity in the magnetic field and the low-density region are in good agreement in the MHD simulation.

The cross-sectional area of the magnetosphere $S$ is therefore approximately calculated from the product of the cross-sectional widths along the $x$-axis ($z = 400$ m) and $y$-axis ($z = 400$ m). We referred $z = 400$ m since the streamlines of the solar wind become almost straight at the point as shown in Figs 5.20a and 5.20b. The cross-sectional width is calculated from the region where the ion density becomes below the solar wind density ($5 \times 10^6$ m$^{-3}$) as shown in Figs. 5.22b and 5.22d. The simulation results give $S = \pi \times (300 / 2) \text{ m} \times (350 / 2) \text{ m} = 8.2 \times 10^4 \text{ m}^2$.

In contrast, the magnetosphere in the parallel case is symmetric about the $z$-axis as shown in Fig. 5.23. Electrons penetrate into the low density region deeper than ion by the thermal velocity and electric field of the boundary region as well as the perpendicular case. In same manner with the perpendicular case, the cross-sectional area of the magnetosphere $S$ is approximately calculated from the product of the cross-sectional widths along the $x$-axis ($z = 400$ m) and $y$-axis ($z = 400$ m) shown in Figs. 5.24a and 5.24b. The calculated cross-sectional area is $S = \pi \times (250 / 2) \text{ m}^2 = 4.9 \times 10^4 \text{ m}^2$.

The cross-sectional area $S$ is thus slightly larger in the perpendicular case than in the parallel case. The larger magnetosphere can receive more momentum from the solar wind. The momentum of the solar wind then generates the thrust of the magnetic sail as the repulsive force between the magnetopause current and the coil current onboard the spacecraft. The thrust of the magnetic sail was calculated from the change in momentum of all particles contained in the computational

Figure 5.21: Close-up view of ion density distribution ($M = 1.3 \times 10^8$ Wb-m, typical solar wind, 3D Full-PIC model) and Lorentz force acting on ion in the perpendicular case across the $xz$-plane ($y=0$). The double circle means upward magnetic field perpendicular to the plane.
5.4. SIMULATION RESULTS BY THREE-DIMENSIONAL SIMULATION

Figure 5.22: The one-dimensional cut along a) $x$-axis ($y=0$ m and $z=0$, 200 m, 400 m) and b) $y$-axis ($x=0$ m, $z=0$, 200 m, 400 m). ($M = 1.3 \times 10^8$ Wb·m, typical solar wind, 3D Full-PIC model)
CHAPTER 5. SIMULATION OF MAGNETIC SAIL ON ELECTRON INERTIAL SCALE

Figure 5.23: Ion density distribution and electron density distribution in the parallel case. \((M = 1.3 \times 10^8 \text{ Wb-m}, \text{typical solar wind, 3D Full-PIC model})\) The streamline represents mass flux flow.

domain. As a result, the thrust in the perpendicular case (0.08 ±0.008 mN) is larger than in the parallel case (0.07 ±0.007 mN) because of the larger magnetosphere. Fig. 5.25 represents the time history of thrust generation in the perpendicular case. It is a magnetization period of the coil for the first 0.5 ms. Although the ion generate most thrust, the contribution of electron cannot be ignored, either.

The current density distribution of both the perpendicular case and the parallel case is shown in Fig. 5.26. The magnetopause current induced by the solar wind plasma flows in counter direction to the coil current. The repulsive force between the coil current and the current induced by the solar wind acts on the spacecraft as the thrust force. The current flows around coil in Fig. 5.26a (perpendicular case) represents the equatorial ring current. The carrier of these currents is both ion and electron.

The one-dimensional cut of current density in the perpendicular case and the parallel case along \(y\)-axis \((x=0 \text{ m}, z=0 \text{ m}, 200 \text{ m}, 400 \text{ m})\) are also shown in Figs. 5.27a and 5.27b, respectively. It is expected that most thrust of the magnetic sail is generated in the front side of a magnetosphere since the current is strong at \(z=0 \text{ m}\). Moreover, it is revealed that the position where magnetopause
Figure 5.24: The one-dimensional cut along a) $x$-axis ($y=0$ m and $z=0$ m, 200 m, 400 m) and b) $y$-axis ($x=0$ m, $z=0$ m, 200 m, 400 m). ($M = 1.3 \times 10^8$ Wb-m, typical solar wind, 3D Full-PIC model)
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Figure 5.25: Time history of thrust generation. \((M = 1.3 \times 10^8 \text{ Wb-m}, \text{typical solar wind, perpendicular case, 3D Full-PIC model})\)

Figure 5.26: Current density distribution in the perpendicular case (a) and the parallel case (b) across the \(yz\)-plane \((x = 0)\). The direction of the onboard coil current is also represented. The double circle means upward current perpendicular to the plane and the cross-circle means downward current perpendicular to the plane. \((M = 1.3 \times 10^8 \text{ Wb-m}, \text{typical solar wind, 3D Full-PIC model})\)
5.4. SIMULATION RESULTS BY THREE-DIMENSIONAL SIMULATION

Figure 5.27: The one-dimensional cut along y-axis (z=0 m, z=0 m, 200 m, 400 m): a) perpendicular case and b) parallel case. ($M = 1.3 \times 10^8$ Wb·m, typical solar wind, 3D Full-PIC model)

current flows is mostly in agreement with the position where plasma density becomes low by comparing Fig. 5.22b with 5.27a or 5.24b with 5.27b.

5.4.2 Thrust Characteristics Obtained by Three-dimensional Full-PIC Simulation

The thrust of the magnetic sail was investigated for various different values of the design parameter (the magnetic moment of the onboard coil $M$). Simulations were performed by changing the magnetic moment $M$ from $1.3 \times 10^6$ Wb·m to $1.3 \times 10^{11}$ Wb·m in six steps. Totally twelve simulation cases were carried out by assuming both perpendicular case and parallel case. It takes about four days (1024 CPUs) and one week (4096 CPUs) per case to obtain the steady state of smaller magnetospheres ($M < 1.0 \times 10^{10}$ Wb·m) and larger magnetospheres ($M > 1.0 \times 10^{10}$ Wb·m), respectively, since the larger computational domain ($512 \times 512 \times 512$ cells) is required to surround the whole magnetosphere.
Figure 5.28 shows the results. The vertical axis and horizontal axis represent the magnetic moment $M$ and the thrust of the magnetic sail, respectively. The relationship between the magnetic moment and the thrust was fitted by the least-squares method. This revealed that the thrust of a small-scale magnetic sail is approximately proportional to the magnetic moment of the onboard coil:

$$F \propto M^{1.03} \quad (5.4.1)$$

In addition, the simulation result by Hybrid-PIC simulation is plotted in Fig. 5.28. The thrust obtained by the Full-PIC simulation is two or three times larger than the thrust obtained by Hybrid-PIC simulation, which neglect the electron kinetic effects by assuming massless electron fluid. According to the finite Larmor radius effect of electron that is one of the electron kinetic effects, the drift velocity and the magnetopause current in the non-uniform electromagnetic field around the magnetopause becomes larger. As a result, the thrust of magnetic sail with the small magnetosphere becomes larger by taking the electron kinetic effects into consideration. This shows that electron kinetic effects that are ignored in simulation methods other than Full-PIC simulations become dominant in small-scale magnetospheres and affect the thrust characteristics of small-scale magnetic sails.

These thrust characteristics (Eq. (5.4.1)) are clearly different from the thrust characteristics of larger magnetic sails as obtained by Hybrid-PIC simulations [33], Flux-Tube simulations (Chapter 4), and MHD simulations [31]:

$$F_{MHD} = C_d \times 0.5m_iN_{SW}V_{SW}^2 \times \pi L_{MHD}^2 \propto M^{\frac{2}{3}} \quad (5.4.2)$$

as shown in Fig. 5.29. The lines which are extrapolated from the full-PIC result and the MHD result cross by $M \sim 1.0 \times 10^{13}$ Wb-m, where the magnetosphere size $L_{MHD} \sim 20$ km is far smaller than the ion Larmor radius (100 km in typical value). It is therefore expected that the kinetic effects of ion also has dominant influence in the thrust characteristics of magnetic sail with small magnetosphere ($L_{MHD} < r_i L$).

Next, we performed nine simulations assuming changes in the solar wind parameters, specifically the number density $N_{SW}$ and plasma velocity $V_{SW}$. No previous study has taken the variation of the solar wind into consideration. However, when assuming a deep space exploration, that becomes important since the solar wind varies spatially and in time along the spacecraft’s trajectory. Density was set to 0.5, 1, and 2 times the typical value (Case 5), and velocity was similarly set to 0.5, 1, and 2 times the typical value (Case 5). The 9 simulation parameters are listed in Table 5.6 were used. The magnetic moment $M$ and other design parameters of the magnetic sail are fixed as listed in Table 5.5. Figure 5.30 shows the results for each set of solar wind parameters. The vertical axis shows the thrust and the horizontal axis shows solar wind velocity. Each legend represents the
Figure 5.28: Thrust characteristics of magnetic sails of various magnetic moments obtained by three-dimensional simulation. (Typical solar wind, $\alpha = 0$ deg and 90 deg, 3D Full-PIC model)

Figure 5.29: Thrust characteristics of small (full-PIC) to large (MHD) magnetic sails.
Table 5.6: Various solar wind parameters

<table>
<thead>
<tr>
<th>Solar wind density $N_{SW}$ [m$^{-3}$]</th>
<th>Case1</th>
<th>Case2</th>
<th>Case3</th>
<th>Case4</th>
<th>Case5</th>
<th>Case6</th>
<th>Case7</th>
<th>Case8</th>
<th>Case9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.5 \times 10^6$</td>
<td>$2.5 \times 10^6$</td>
<td>$2.5 \times 10^6$</td>
<td>$5 \times 10^6$</td>
<td>$5 \times 10^6$</td>
<td>$5 \times 10^6$</td>
<td>$1 \times 10^7$</td>
<td>$1 \times 10^7$</td>
<td>$1 \times 10^7$</td>
<td></td>
</tr>
<tr>
<td>Solar wind velocity $V_{SW}$ [m/s]</td>
<td>$2.5 \times 10^6$</td>
<td>$1 \times 10^6$</td>
<td>$2.5 \times 10^6$</td>
<td>$5 \times 10^6$</td>
<td>$5 \times 10^6$</td>
<td>$2.5 \times 10^5$</td>
<td>$5 \times 10^5$</td>
<td>$1 \times 10^6$</td>
<td></td>
</tr>
</tbody>
</table>

Solar wind density we assumed.

As a result, it is revealed that the thrust of the magnetic sail increases as the velocity and the density of the solar wind increase. This is because the dynamic pressure of the solar wind ($\sim 0.5 m_i N_{SW} V_{SW}^2$) becomes larger according to the increase in the solar wind velocity and the solar wind density. That is, the largest thrust is obtained in Case 9 and smallest thrust is obtained in Case 1.

The relationship between the velocity $V_{SW}$, density $N_{SW}$, and thrust was fitted using the least-squares method. This found that the thrust of the magnetic sail changes approximately in proportion to the change in $V_{SW}$ and $N_{SW}$, which is given by the following relation:

$$F \propto V^{0.92} \times N^{1.15}$$

The thrust is slightly more sensitive to changes in density than velocity. Due to the fast flow velocity, that is, large Larmor radius, the slip of ion from the magnetic field becomes large and the thrust of magnetic sail cannot become large as expected from the increase in the dynamic pressure.

We also noted that the plasma density of the solar wind $N_{SW}$ is assumed to be inversely proportional to the square of the sun–spacecraft distance ($N_{SW} \propto R^{-2}$) and the velocity $V_{SW}$ is assumed to be constant in the interplanetary space. The thrust of the small-scale magnetic sail is therefore inversely proportional to the 2.3 power of the sun–spacecraft distance ($F_{mag} \propto N_{SW}^{1.15} \propto R^{-2.3}$). Magnetic sail differs in the thrust characteristic from the existing propulsion system, solar sail ($F_{solarsail} \propto R^{-2}$). That is, on outer planet orbits, the thrust of magnetic sail becomes smaller value than that of solar sail even if the thrust level is same on Earth orbit. On the contrary, the thrust of the magnetic sail becomes larger on inner planet orbit. The details of this feature are examined in Chapter 7.

Next, simulations were performed at various attack angles $\alpha$ in order to investigate the effect on thrust of the correlation between the solar wind and the design parameters of the magnetic sail. In addition to the dipole magnetic field assumed in previous simulations, the magnetic field is assumed to be generated by a coil of finite size (50 m and 100 m). The magnetic moment is kept constant at $1.3 \times 10^8$ Wb-m. The definition of the attack angle is shown in Fig. 5.31. The attack angle $\alpha$ varies from $0^\circ$ to $90^\circ$ by 5 steps. The torque act on the coil is not taken into consideration.
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Figure 5.30: Thrust characteristics of a magnetic sail under various solar wind parameters (solar wind velocity $V_{SW}$ and density $N_{SW}$) obtained by three-dimensional simulation. $(M = 1.3 \times 10^8 \text{ Wb-m, parallel case, 3D Full-PIC model})$ The thrust is slightly more sensitive to changes in density than velocity since the torque characteristics have already mentioned in two-dimensional simulation.

The results are shown in Figs. 5.32 and 5.35. Figure 5.32 shows the absolute value of thrust at each attack angle $\alpha$ and coil radius $R_{coil}$. As the attack angle $\alpha$ increases, the thrust of the magnetic sail also increases. This is because the cross-sectional area of the magnetosphere changes with attack angle as shown in Figs. 5.20 ($\alpha=90^\circ$) and 5.23 ($\alpha=0^\circ$). These characteristics are in agreement with previous studies on large magnetospheres [38, 60]. This relationship between attack angle $\alpha$ and thrust is found to be common to all sizes of magnetospheres. In addition, the thrust decreases as the coil radius increases as shown in Fig. 5.33. This is because solar wind plasma is able to pass through the center of a coil without undergoing mirror reflection even if the magnetic moment is the same. The density distribution and the streamlines of the solar wind are shown in Fig. 5.34 when the coil axis is parallel to the solar wind direction. When the coil radius is comparable with the magnetosphere size, the interaction between the magnetic field and the solar wind occurs only near the coil and the thrust decreases 40% compared with the thrust of the ideal dipole. The dipole approximation is valid in the range of $R_{\text{coil}} < 20 \text{ m (L/5)}$.

On the contrary, the steering angle $\gamma$ increases as the coil radius increases, peaking at $\alpha = 45$ deg as shown in Fig. 5.35. The steering angle $\gamma$ represents the angle in the $yz$-plane when the attack angle $\alpha$ is in the neighborhood of 0 deg and the angle in the $xz$-plane when the attack angle $\alpha$ is in the neighborhood of 90 deg. Only the angle between the thrust vector and the solar wind direction (anti-sunward direction or orbital-radius direction) affects the orbital change capability of the small-scale magnetic sail which we discuss in Chapter 7. We therefore do not distinguish it from the plane containing the thrust vector.
CHAPTER 5. SIMULATION OF MAGNETIC SAIL ON ELECTRON INERTIAL SCALE

Figure 5.31: Definition of attack angle $\alpha$ and steering angle $\gamma$.

Figure 5.32: Thrust characteristics of a magnetic sail at various attack angles $\alpha$ as obtained by three-dimensional simulation. ($M = 1.3 \times 10^8$ Wb-m, typical solar wind, 3D Full-PIC)

Figure 5.33: Thrust of various coil sizes. ($M = 1.3 \times 10^8$ Wb-m, typical solar wind, $\alpha=0$ deg, 3D Full-PIC model)
5.4 SIMULATION RESULTS BY THREE-DIMENSIONAL SIMULATION

(a) Ideal dipole.

(b) $R_{\text{coil}} = 50$ m.

(c) $R_{\text{coil}} = 100$ m.

Figure 5.34: Ion density distribution and streamlines with various coil sizes. ($M = 1.3 \times 10^8$ Wb·m, parallel case $\alpha = 0$ deg, typical solar wind, 3D Full-PIC model)
5.5 Discussion

As we showed above, we performed large 3D Full-PIC simulations by using parallel computing techniques and huge computational resource. As a result, it was revealed that the thrust of a small-scale magnetic sail is determined by the magnetic moment $M$ of the spacecraft onboard coil (main design parameter of the spacecraft), the solar wind parameters (density $N_{SW}$ and velocity $V_{SW}$), and the attack angle $\alpha$. Combining these results by assuming no cross-correlation between parameters gives empirical formulae for the thrust $F_{PIC}$ and steering angle $\beta$:

$$F_{PIC} = 3.57 \times 10^{-26} N_{SW}^{1.15} V_{SW}^{0.92} M^{1.03} (1.55 \times 10^{-3} \alpha + 1)$$  \hspace{1cm} (5.5.1)

and

$$\gamma = -3.60 \times 10^{-3} (\alpha - 50)^2 + 9.01$$  \hspace{1cm} (5.5.2)

These equations show that the thrust of small-scale magnetic sail is approximately proportional to magnetic moment, solar wind density and solar wind velocity, respectively. In addition, it is revealed that the magnetic sail can generate not only the radial thrust but also the tangential thrust that is effective for accelerating the spacecraft along its trajectory.

This thrust characteristics of magnetic sail are also examined theoretically. By performing test-particle simulation only with the dipole magnetic field, which neglects the time variation of electromagnetic field, it was revealed that the cross-sectional area of the magnetic sail is mainly formed by the interaction between ions and the dipole magnetic field, and electrons hardly influence the structure of the magnetosphere. As shown in Fig. 5.36, the magnetosphere of the almost same structure as that of Full-PIC simulation was formed in the test-particle simulation. This is because
5.5. DISCUSSION

the induced current (both ion and electron are carriers) is very weak to generate the comparable induced magnetic field with original magnetic field. Hence, the formation of the magnetosphere is mainly affected by the interaction between the magnetic field generated by the coil and ions since the mass of ion is 1836 times larger than electron. As a result, the density distribution of ion similar to the self-consistent Full-PIC simulation is obtained by test particle simulation as shown in Fig. 5.36. The momentum change of ion calculated by test particle simulation is approximately in agreement with that calculated by Full-PIC simulation.

However, the density distribution of electron totally differs by the Full-PIC simulation and test-particle simulation. In Full-PIC simulation, electrons move to fill the charge neutrality and induce the current approximately same region with the ion current. On the contrary, in test particle simulation, the electrons move freely independently from ion motion. As a result, the current distribution induced by electron also differs by the Full-PIC simulation and the test particle simulation. Since an electron bears more than half of the whole induced current according to conditions, the electromagnetic force does not correspond with the thrust calculated by momentum change in a test-particle simulation. We summarized the interaction between ion, electron and magnetic field in Fig. 5.37. The interaction between ion and coil magnetic field decide the form of the magnetosphere. The electric field by the charge separation between ion and electron affects the motion of electron. The current distribution which is mainly decided by the electron is also changed (see also Fig. 5.18) since the main carrier of the magnetopause current is electron. Although the momentum change of ion and electron is transferred to the spacecraft via induced current and agrees with the electromagnetic force in Full-PIC, the momentum change in the test particle simulation does not agree with the electromagnetic force acting on the coil.

Figure 5.36: Ion density distribution in yz-plane ($x=0$) obtained by a) Full-PIC simulation and b) Test-particle simulation. ($M = 1.3 \times 10^8$ Wb·m, typical solar wind, perpendicular case, 3D Full-PIC model)
CHAPTER 5. SIMULATION OF MAGNETIC SAIL ON ELECTRON INERTIAL SCALE

As long as considering the momentum change in ion, we could estimate the magnetosphere size theoretically by considering only ion trajectory in the dipole magnetic field. We assumed xz-plane as shown in Fig. 5.38. The vector potential:

\[ (A_x, A_y) = \left( \frac{\mu_0 M_y z}{4\pi r^3}, \frac{\mu_0 M_y x}{4\pi r^3} \right) \]  \hspace{1cm} (5.5.3)

is defined by using the magnetic moment \( M \). Conservation of the generalized momentum of the particle which has the velocity \((0, V_{SW})\) by infinity is used. At the stagnation point \((0, -L_s)\), the particle should have the velocity \((V_{SW}, 0)\) because of the conservation of energy and Larmor motion. The conservation of the generalized momentum along \( x \)-axis is represented as

\[ 0 = m_i V_{SW} - \frac{e\mu_0 M L_s}{4\pi I_s} \]  \hspace{1cm} (5.5.4)

. The equation is solved about \( L_s \) and the magnetosphere size \( L_s \):

\[ L_s = \sqrt{\frac{e\mu_0 M}{4\pi m_i V_{SW}}} \]  \hspace{1cm} (5.5.5)

is obtained.

The magnetosphere size obtained by simulation, \( L_s \) (Eq. (5.5.5)) and \( L_{MHD} \) (Eq. (1.2.3)) that is obtained by MHD approximation are compared in Fig. 5.39. As a result, the magnetosphere size obtained by single particle approximation is well in agreement with the simulation results by Full-PIC and MHD approximation is not valid at all in small-scale magnetosphere. As a result, the magnetosphere size obtained by single particle approximation is well in agreement with the simulation results by Full-PIC \((M > 5 \times 10^7 \text{ Wb-m})\) and MHD approximation is not valid at all in small-scale magnetosphere. In smaller magnetic moment \((M < 5 \times 10^7 \text{ Wb-m})\), the magnetosphere size obtained by Full-PIC becomes larger than the magnetosphere size expected by Eq. (5.5.5)
since the electron kinetics and charge separation become strong when the magnetosphere size, electron Larmor radius and Debye length are comparable. The MHD approximation \( L_{MHD} \) and the theoretical result \( L_s \) agree with \( M=1\times10^{15} \text{ Wb-m} \), which is corresponding to the magnetosphere size \( L=140 \text{ km} \). When the magnetosphere size is larger than ion Larmor radius (100 km in typical case), the MHD approximation is hence expected to be valid as shown in the field of geophysics.

The cross-sectional area of the small-scale magnetosphere is therefore approximately estimated as \( \pi L_s^2 \). Since the magnetic sail catches the solar wind dynamic pressure with this cross-sectional area, the thrust is approximately represented as

\[
F_{mag} = \frac{1}{2} m_i N_{SW} V_{SW}^2 \times \pi \frac{e\mu_0 M}{4\pi m_i V_{SW}} = 2.5 \times 10^{-26} N_{SW} V_{SW} M 
\]

(5.5.6)

. The coefficient and index of Eq. (5.5.6) are mostly in agreement with those of Eq. (5.5.1). The validity of the thrust characteristics based on simulation results is thus confirmed by the theoretical approach.

Similarly, the magnetosphere size in two-dimensional was also calculated by single particle approximation:

\[
(A_x, A_y) = \left( \frac{\mu_0 M_y z}{2\pi r^2}, \frac{\mu_0 M_x x}{2\pi r^2} \right) 
\]

(5.5.7)

\[
0 = m_i V_{SW} - \frac{e\mu_0 M}{2\pi} \frac{L_s}{L_s^2} \quad (5.5.8)
\]

, and

\[
L_s = \frac{e\mu_0 M}{2\pi m_i V_{SW}} 
\]

(5.5.9)

. The results are shown in Fig. 5.40. The magnetosphere size obtained by simulation is greatly
5.6 Summary

We performed two- and three-dimensional Full-PIC simulations with the realistic plasma parameters in order to determine the thrust characteristics of small-scale magnetic sails. It was found by two-dimensional simulations that the influence of the electron kinetics and the charge separation
Figure 5.41: Comparison of ion density distribution in a) two- and b) three-dimensional magnetosphere. ($L_{MHD} = 2200$ m, perpendicular case, typical solar wind)

firstly taken into consideration plays important role on the thrust generation. The IMF, which does not affect the thrust generation, has also dominant influence on the attitude stability of spacecraft. It was also revealed by three-dimensional simulations that the thrust level of the magnetic sail is approximately proportional to the magnetic moment of the onboard coil, solar wind density, and solar wind velocity. This is quite different from large scale magnetic sails since particle motion is the dominant influence on the thrust generation in small-scale magnetic sails. It was also found that the attack angle and coil radius affect the steering angle and that the steering angle increases (up to a maximum of 20 deg) as the coil radius increases.

These thrust characteristics of small-scale magnetic sail is used for mission analysis in Chapter 7. The thrust characteristics of magnetic sail such as the thrust-mass ratio are also examined in Chapter 7 by using the thrust characteristics obtained from the Full-PIC simulations.
Chapter 6

Simulation of Magneto Plasma Sail

6.1 Introduction

The objective of this chapter is to reveal the thrust characteristics of Magneto Plasma Sail with the electron inertial scale magnetospheres (< 100 km), where the electron kinetics should be considered and the spacecraft model feasible with the present technology can be assumed. To determine the maximum thrust gain available by Magneto Plasma Sail, one needs to properly evaluate the thrust characteristics of Magneto Plasma Sail, and for that purpose, fully kinetic simulation including all the related physical phenomena on Magneto Plasma Sail seems the best solution. However, in spite of the necessity for fully kinetic simulation, it has not yet been performed because of the huge computational requirement. The thrust performance of Magneto Plasma Sail also should be revealed based on full kinetic simulation results.

In order to achieve the above objectives, we first perform the two-dimensional Full-PIC simulation with the realistic mass ratio (\(m_i/m_e=1836\)) for hydrogen plasma in order to reveal the mechanism of magnetic inflation. Next, we aim at further improvement in net thrust gain with the various plasma injection parameters in three-dimension.

6.2 Computational Settings

6.2.1 Two-dimensional Simulation

The interaction between the solar wind and an artificial dipole magnetic field is simulated in two-dimensional space (xz-plane). The computational domain used by two-dimensional Full-PIC simulations is shown in Fig. 5.1. The computational domain has an area of 15 km × 15 km involving 512 × 512 grids in the typical case. The grid spacing \(dx\) should be chosen not much larger than the Debye length \((dx/\lambda_D<3)\) and typically 16 super particles are involved in a grid.

The computational domain is divided into small domain (4 × 512) by the domain decomposi-
CHAPTER 6. SIMULATION OF MAGNETO PLASMA SAIL

tion. Each small domain is calculated by one process and bordering physical quantity is exchanged by MPI. The domain includes huge number of particles (approximately 0.5 million in typical case). The particles are divided into some groups by the particle decomposition. Each particle group is calculated by one thread by 7pensfi (s a resultff -4g %6us —4gk processes by 7pens– are used for the simulation.

In addition, artificial plasma is injected from the injection source grid (totally 4 grid point) near the coil for the MPS simulations as shown in Fig. 5.1. By constant rate $\Delta t_{jet}$, plasma particles are added into the calculation domain. The mass flow rate of plasma injection per unit length is hence calculated as

$$\dot{m} = \frac{(m_i + m_e)N_{jet} \times 4(dx)^2}{\Delta t_{jet}}$$

(6.2.1)

. The net thrust by the plasma jet is set to zero by symmetric plasma injection. The thrust by plasma jet $F_{jet}$ is hence the virtual thrust corresponding to the thrust when the plasma jet is injected to one way. By using mass flow rate per unit length of ion and electron, $F_{jet}$ is obtained as

$$F_{jet} = \dot{m}_i V_{jet,i} + \dot{m}_e V_{jet,e} + N_{jet} k_B T_{jet,i} S + N_{inj} k_B T_{jet,e} S$$

(6.2.2)

. The first and second term in Eq. (6.2.2) represent the momentum thrust of ion and electron, respectively. The third and fourth term represent the static pressure thrust of ion and electron, respectively. Here, $S$ is assumed to be $4dx$, the cross section of four injection sources. The thrust by static pressure is too small to be able to neglect.

6.2.2 Three-dimensional Simulation

The interaction between the solar wind and an artificial dipole magnetic field is simulated in three-dimensional space. The computational domain has an area of 3.8 km $\times$ 3.8 km $\times$ 3.8 km is partitioned into a grid of 256 $\times$ 256 $\times$ 256 cells ($dx = 15$ m) in the typical case. The size of the computational domain is decided based on the theoretical magnetosphere size $L_{MHD}$ (Eq. (1.2.3)). The length of the one side is approximately $5L_{MHD}$. The grid spacing $dx$ typically needs to be set such that it is not much larger than the Debye length ($dx/\lambda_D < 3$). The time spacing $dt$ must fill the Courant condition, $dx/dt > 1.73c$ in three-dimension ($dt = 1.5 \times 10^{-8}$ s). The solar wind plasma is typically represented by 16 super particles associated with each cell and a total of $5.4 \times 10^8$ particles. This is because it was revealed in Chapter 5 that particle number per cell is not influence the precision of the thrust compared with the latitude resolution or the size of calculation domain. 16 particles are hence necessary and sufficient in terms of the calculation cost and correctness of the simulation. The magnetic field generated by the coil mounted in the spacecraft is approximated by an ideal dipole magnetic field or a finite size coil magnetic field of magnetic moment $M$. The magnetic moment of the coil is arranged in the $y$-direction (perpendicular to the solar wind, Fig.
5.2a) or the $z$-direction (parallel to the solar wind, Fig. 5.2b). The magnetic field is given as the initial condition. Absorbing boundary conditions are used for the electromagnetic field on all outer boundaries. Solar wind particles flow into the computational domain from the inflow boundary at the typical solar wind velocity $V_{SW}$ and thermal distribution. Outgoing particles from the computational domain are eliminated from the calculation. The collision between particles and the spacecraft is not taken into consideration, either, assuming the infinitesimal spacecraft.

In addition, artificial plasma is injected from the injection source grid near the coil for the MPS simulations. Fig. 6.1 represents the injection source. The plasma particles are added into the calculation domain through the torus volume $Vol_{jet}$ on a concentric circle with $R_{jet}$. The mass flow rate of plasma injection per unit length is hence calculated as

$$\dot{m} = \frac{(m_i + m_e) N_{jet} Vol_{jet}}{\Delta t_{jet}}$$

The net thrust by the plasma jet is set to zero by symmetric plasma injection. The thrust by plasma jet $F_{jet}$ is hence the virtual thrust corresponding to the thrust when the plasma jet is injected to one way. By using mass flow rate of ion and electron, $F_{jet}$ is obtained as Eq. (6.2.2).
6.3 Physics of Magnetic Inflation

6.3.1 Schematic Illustration of Magnetic Inflation

Several simulations of MPS have been previously performed [29, 48, 49]. According to the previous studies, to obtain a large magnetosphere and the increase in thrust, MPS spacecraft can use two methods. One is to expand a magnetosphere by a high-density and high-velocity plasma jet from a magnetic sail spacecraft, and the other is to initiate a ring-current inside a magnetosphere by releasing slow velocity plasma. In the first method which is originally proposed by Winglee et al. [7], when plasma jet is totally magnetized (i.e., the MHD approximation is valid), the plasma flow and the magnetic field move together according to

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u}_{\text{jet}} \times \mathbf{B})
\]

(6.3.1)

whose condition is usually called as ‘frozen-in’. Here, as for the injection plasma, the characteristic length of magnetic field is considered as \(B/\nabla B\) at an injection source. The plasma under the frozen-in condition, hence, satisfies the condition of \(r_{iL}, r_{eL}<<B/\nabla B\). Under the frozen-in condition, the magnetic field is conveyed along with a plasma flow to a far region from spacecraft, resulting in a larger magnetosphere. The frozen-in concept is, however, not suitable for magnetic field inflation if the injected plasma, particularly ion, is not magnetized (i.e., finite Larmor effect appears, \(r_{iL} \sim B/\nabla B\) or \(r_{iL} > B/\nabla B\)), since the plasma flow will escape from the magnetosphere. Instead, the second method for magnetic field inflation relies on a diamagnetic current induced by particle motion (Ring current concept [24]). Figure 6.2 shows the schematic illustration of a magnetized electron motion in a two-dimensional dipole magnetic field. In this case, \(\nabla B\) drift motion of electrons induces a current flow in the same direction as \(I_{\text{coil}}\) and therefore the induced current will increase the magnetic moment, hence thrust will be increased. When a lot of electrons are added, the collective motion of electrons increases the current that flows in the same direction as \(I_{\text{coil}}\), hence thrust by MPS is expected to further increase. Total current induced by the drift motion and the collective motion is called as the diamagnetic current \(I_{\text{plasma}}\).

6.3.2 Theoretical Analysis of Characteristic Parameters of Magnetic Inflation

One-dimensional theoretical approach is also performed to distinguish the characteristic parameters of magnetic inflation. We assumed \(m = m_i = m_e\) for simplicity. The boundary condition for plasma injection is described as \(v_0, n_0, B_0\) and \(A_0\) in \(x<0\) as shown in Fig. 6.3. \(n, v_x, v_y, B\) and \(A\) are function of \(x\).

The energy conservation, the generalized momentum conservation and the equation of continuity are represented as
6.3. PHYSICS OF MAGNETIC INFLATION

Figure 6.2: Schematic illustration of a magnetized electron in the two-dimensional dipole magnetic field. An electron moves along the magnetic field line while making Larmor motion and bounces at the mirror point. In addition, electrons induce the diamagnetic current that flows in the same direction as the coil current.

Figure 6.3: One-dimensional coordinate system for theoretical analysis of plasma injection.
\[ v_x^2 + v_y^2 = v_0^2 \] (6.3.2)

\[ mv_y + qA = qA_0 \] (6.3.3)

and

\[ nv_x = n_0 v_0 \] (6.3.4)

The magnetic field is represented as

\[
\frac{dB}{dx} = -\mu_0 n q v_y = -\frac{\mu_0 n_0 q v_0 v_y}{\sqrt{v_0^2 - v_y^2}}
\] (6.3.5)

using the plasma density and velocity. The derivative of the magnetic field is also described as

\[
\frac{dB}{dx} = \frac{dB}{dA} \frac{dA}{dx} = B \frac{dB}{dA} = \frac{d}{dA} \left( \frac{B^2}{2} \right)
\] (6.3.6)

using vector potential. By substituting \( v_y \) (Eq. (6.3.3)) in Eq. (6.3.5),

\[
\frac{d}{dA} \left( \frac{B^2}{2\mu_0} \right) = \frac{n_0 q^2}{m} \frac{A - A_0}{\sqrt{1 - \left[ \frac{q (A - A_0)}{mv_0} \right]^2}}
\] (6.3.7)

is obtained. By integrating Eq. (6.3.7) on the proper boundary condition,

\[
\frac{B^2}{2\mu_0} = n_0 m v_0^2 \left\{ 1 - \sqrt{1 - \left[ \frac{q (A - A_0)}{mv_0} \right]^2} \right\} + \frac{B_0^2}{2\mu_0}
\] (6.3.8)

is derived. Here, we introduce two variables:

\[
B_{jet} = \sqrt{2\mu_0 n_0 m v_0^2}
\] (6.3.9)

and

\[
\beta_{jet} = \left( \frac{B_{jet}}{B_0} \right)^2
\] (6.3.10)

. We can obtain

\[
\frac{dA}{dx} = B = B_{jet} \sqrt{\frac{1}{\beta_{jet}} - 1 + \sqrt{1 - \left[ \frac{q (A - A_0)}{mv_0} \right]^2}}
\] (6.3.11)
from Eq. (6.3.8). We also introduce two variables:

\[ s = \frac{q (A - A_0)}{mv_0} - \frac{qA_0}{mv_0} < s < 1 \]  
(6.3.12)

and

\[ r_{jet} = \frac{mv_0}{qB_{jet}} \]  
(6.3.13)

Finally, the distribution of plasma and magnetic field along x-axis can be obtained by solving

\[ \frac{dx}{r_{jet}} = \frac{ds}{\sqrt{\frac{1}{\beta_{jet}} - 1 + \sqrt{1 - s^2}}} \]  
(6.3.14)

However, Eq. (6.3.14) cannot be solved analytically and the numerical technique is required. However, it can imagine easily that the solution is dependent on \( B_{jet} \), \( r_{jet} \), \( \beta_{jet} \) and \( \frac{qA_0}{mv_0} = \gamma_{jet} \). \( \beta_{jet} \) and \( \gamma_{jet} \) is non-dimensional quantity. These variables do not change by one dimension, two dimensions, or three dimensions, either.

6.4 Simulation Results by Two Dimensions

6.4.1 Two-dimensional Magneto Plasma Sail on Electron Scale

First, the Full-PIC simulation of MPS is performed in two-dimension. The solar wind parameters for MPS simulation are the same as the parameters listed in Table 1.2. The theoretical magnetospheric size obtained by Eq. (1.2.3) is \( L_{MHD} = 2200 \) m, and the value of \( L_{MHD} \) is fixed throughout MPS simulations. In addition to the solar wind parameters, magnetic sail design parameters (for electromagnet) and injection plasma parameters for magnetic field inflation are selected as shown in Tables 6.1 and 6.2, respectively.

When releasing plasma from MPS spacecraft, as shown in Fig. 5.1, four plasma injection source cells located at \( x=\pm 150 \) m (\( R_{plasma} \)) are defined around the coil current, and super particles are added to the injection source cells at a rate of once every 100 steps in the simulation. The plasma is injected toward the magnetic equator (\( \pm x \) direction) in all cases. At the injection source, the magnetic flux density is \( B_0 = 10 \) \( \mu T \). Four kinetic \( \beta_{inj} \) values defined as

\[ \beta_{jet} = \left( \frac{1}{2} N_{jet} m_{i,jet} V_{jet,i}^2 + \frac{1}{2} N_{jet} m_e V_{jet,e}^2 \right) / \left( \frac{B_0^2}{\mu_0} \right) \]

at the plasma injection source are used, and they are high in order of Case 1, Case 2, Case 3 and Case 4. That is, Case 1 indicates the case for the highest kinetic \( \beta_{inj} \) plasma injection and Case 4 indicates the case for the lowest kinetic \( \beta_{jet} \) plasma injection. The static \( \beta \) (the ratio of the plasma pressure to the magnetic pressure at the injection source) is a constant at \( 8.9 \times 10^{-6} \) and
Table 6.1: Magnetic sail design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil distance $R_{coil}$</td>
<td>75 m</td>
</tr>
<tr>
<td>Coil current $I_{coil}$</td>
<td>$4 \times 10^3$ A turn</td>
</tr>
<tr>
<td>Theoretical magnetosphere size $L_{MHD}$</td>
<td>$2.2 \times 10^3$ m</td>
</tr>
</tbody>
</table>

Table 6.2: Magnetro Plasma Sail design parameters

<table>
<thead>
<tr>
<th>Plasma injection parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma density $N_{jet}$ [m$^{-3}$]</td>
<td>$5 \times 10^7$</td>
<td>$5 \times 10^7$</td>
<td>$5 \times 10^7$</td>
<td>$5 \times 10^7$</td>
</tr>
<tr>
<td>Ion velocity $V_{jet,i}$ [m/s]</td>
<td>$5 \times 10^6$</td>
<td>$5 \times 10^4$</td>
<td>$5 \times 10^5$</td>
<td>0</td>
</tr>
<tr>
<td>Ion temperature $T_{jet,i}$ [eV]</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Electron velocity $V_{jet,e}$ [m/s]</td>
<td>$5 \times 10^6$</td>
<td>$5 \times 10^7$</td>
<td>$5 \times 10^5$</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td>Electron temperature $T_{jet,e}$ [eV]</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Kinetic $\beta_{jet}$</td>
<td>$4.1 \times 10^{-2}$</td>
<td>$2.2 \times 10^{-3}$</td>
<td>$4.1 \times 10^{-4}$</td>
<td>$2.2 \times 10^{-5}$</td>
</tr>
<tr>
<td>Static $\beta_{jet}$</td>
<td>$8.9 \times 10^{-6}$</td>
<td>$8.9 \times 10^{-6}$</td>
<td>$8.9 \times 10^{-6}$</td>
<td>$8.9 \times 10^{-6}$</td>
</tr>
<tr>
<td>Mass flow rate $\dot{m}$ [kg/s/m]</td>
<td>$7.5 \times 10^{-11}$</td>
<td>$7.5 \times 10^{-11}$</td>
<td>$7.5 \times 10^{-11}$</td>
<td>$7.5 \times 10^{-11}$</td>
</tr>
<tr>
<td>Equivalent thrust $F_{jet}$ [N/m]</td>
<td>$3.8 \times 10^{-4}$</td>
<td>$5.8 \times 10^{-6}$</td>
<td>$3.8 \times 10^{-5}$</td>
<td>$2.0 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

the thermal $\beta$ is smaller than the kinetic $\beta_{jet}$ in all cases. $\dot{m}$ and $F_{jet}$ is derived from Eqs. (6.2.1) and (6.2.2), respectively.

Simulation results are shown in Fig. 6.4, in which the coil magnetic moment $M$ is set parallel to the solar wind direction ($\alpha=0 ^\circ$). The thrust generated by MPS has a maximum value at $\beta_{jet} \sim 2 \times 10^{-3}$ as shown in Fig. 6.4a. This feature agrees with the past research by MHD simulation performed by Nishida et al. [48], that is, the thrust becomes smaller when the plasma with too large kinetic $\beta_{jet}$ is injected. Despite the fact that the thrust of MPS by MHD simulation becomes approximately zero in the case of high-$\beta$ plasma injection, the thrust of MPS by Full-PIC simulation is larger than the thrust of magnetic sail ($F_{MPS}>F_{mag}$) in all cases. This is because the termination shock, which prevents the momentum of the solar wind from transmitting to the spacecraft, observed in MHD simulation becomes weaker when considering the finite Larmor effect of particles in Full-PIC simulation. In addition, the thrust of MPS by Full-PIC simulation is approximately proportional to the cross-sectional area of magnetosphere when comparing Fig. 6.4a (thrust of MPS) and Fig. 6.4b (cross-sectional length of magnetosphere).

In the following, thrust characteristics of MPS are discussed associated with the flow and field structures in Fig. 6.5. The thrust force of 20 $\mu$N/m is obtained in Case 1, which is 3.4 times larger than thrust without plasma injection, 5.8 $\mu$N/m. On the contrary, thrust obtained in Case 4 is 8.0 $\mu$N/m and an increase of thrust by plasma injection is only 38%.

Figure 6.5 shows the density distribution of ion (a, d, g), the distribution of local kinetic $\beta$ (b, e, h) and the current density distribution (c, f, i) of Case 1, Case 4 and the magnetic sail, respectively. The clear density gap at the magnetopause of MPS becomes ambiguous in Fig. 6.5a in comparison with that at the magnetic sail’s magnetopause in Fig. 6.5g. In contrast, the structure
Figure 6.4: a) thrust of MPS $F_{MPS}$, b) cross-sectional length of the magnetosphere obtained by the various kinetic $\beta_{jet}$ plasma injections ($L_{MHD}=2200$ m, $\alpha=0^\circ$, 2D Full-PIC model). Thrust and cross-sectional length of the magnetic sail are 5.8 $\mu N/m$ and 1300 m, respectively.
of magnetopause current is rather clear and the cross-sectional length of the magnetosphere in Case 1 is expanded from 1300 m (magnetic sail, Fig. 6.5i) to 3300 m (MPS, Fig. 6.5c). Note that 10% error in cross-sectional length is expected because of the broad magnetopause.

From the above discussion, it is indicated that the expanded magnetospheric size greatly contributes to obtain a large thrust increment. Large magnetic field inflation in Case 1 is realized by the condition where electrons are magnetized and ions are not magnetized. Using the plasma parameters of the injected plasma, the Larmor radius of ion \( r_{iL} \sim 50 \text{ km} \) is very large \( (r_{iL}>B/\nabla B) \) and the Larmor radius of electron \( (r_{eL} \sim 2 \text{ m}) \) satisfies the condition of \( r_{eL}<B/\nabla B \). Here, the characteristic length of magnetic field \( B/\nabla B \) at the source is approximately calculated as \( B/\nabla B \sim R_{\text{plasma}}/2 \sim 75 \text{ m} \). The plasma flow toward the injection direction \((\pm x)\) expands the magnetic field since the magnetized electrons move with ions to satisfy the charge neutrality.

On the contrary, in Case 4, only a slight magnetospheric inflation is obtained, that is, from 1300 m (magnetic sail, Fig. 6.5i) to 1400 m (MPS, Fig. 6.5f). The Larmor radius of injected ions \( (r_{iL} \sim 0 \text{ m}) \) and electrons \( (r_{eL} \sim 2 \text{ m}) \) satisfy the conditions of \( r_{iL}<B/\nabla B \) and \( r_{eL}<B/\nabla B \), respectively, and injection plasma is strongly magnetized. The magnetized plasma, especially electron, induces the large diamagnetic current in the same direction of the coil current in the region around the injection source, where the density of plasma is very high, as shown in Fig. 6.5f. The diamagnetic current causes the magnetospheric inflation in Case 4. Here, we note that the contour level of the current density distributions in Fig. 6.5f is 10 times larger than the contour level in Fig. 6.5c. In addition, although there are streamlines that penetrate into the magnetosphere of the magnetic sail (Fig. 6.5g), all streamlines are dammed in Case 4 (Fig. 6.5d) by the mirror magnetic field. As a result, thrust of MPS in Case 4 is also increased by a plasma injection.

Figures 6.5b and 6.5e represent the distribution of the local kinetic \( \beta \) in Case 1 and Case 4, respectively. The local kinetic \( \beta \) is calculated by using the density and the mean velocity of the particles at the local point. At the magnetopause, pressure equilibrium between the solar wind dynamic pressure and the magnetic pressure is approximately established and the local kinetic \( \beta \) approximately becomes unity. In Case 1 and 4, as the magnetosphere is inflated, the boundary of \( \beta=1.0 \) is also inflated from the case of magnetic sail (Fig. 6.5h). Around the plasma injection source of Case 1, the local kinetic \( \beta \) becomes higher than unity; in this case, the injected particles cannot accumulate inside the magnetosphere and the plasma flows out toward \( \pm x \). On the contrary, in Case 4, the local kinetic \( \beta \) is lower than unity around the injection source; in this case the injected plasma is strongly magnetized and the plasma flow toward \( \pm x \) does not occur. As a result, the very high-density region is formed around the injection source and the larger diamagnetic current is induced in Case 4 as the above discussion.

So far, it is successfully shown that thrust of Magneto Plasma Sail can be significantly increased by a plasma injection. However, it is probable that significant amount of thrust can be obtained by collimating a released plasma jet from spacecraft. To evaluate true “thrust gain” by MPS, the
6.4. SIMULATION RESULTS BY TWO DIMENSIONS

(a) Ion density, Case 1 (MPS).
(b) Kinetic $\beta$, Case 1 (MPS).
(c) Current density, Case 1 (MPS).
(d) Ion density, Case 4 (MPS).
(e) Kinetic $\beta$, Case 4 (MPS).
(f) Current density, Case 4 (MPS).
(g) Ion density, magnetic sail.
(h) Kinetic $\beta$, magnetic sail.
(i) Current density, magnetic sail.

Figure 6.5: Distribution of ion density, local kinetic $\beta$ and current density distribution: a, b, c) result for Case 1 (MPS) with the high kinetic $\beta_{jet}$ plasma injection, d, e, f) result for Case 4 (MPS) with the low $\beta_{jet}$ plasma injection and g, h, i) result for the original magnetic sail ($L_{MHD}=2200$ m, $\alpha=0^\circ$, 2D Full-PIC model). The contour lines of $\pm 10 \mu$A/m$^2$ are drawn in the current density distribution to show the magnetopause current.
thrust gain is evaluated based on

\[
\text{Thrust gain} = \frac{F_{\text{MPS}}}{F_{\text{mag}} + F_{\text{jet}}}
\]  

(6.4.1)

In this equation, \(F_{\text{mag}}\) means thrust by magnetic sail and equivalent thrust \(F_{\text{jet}}\) indicates thrust by injected plasma. \(F_{\text{jet}}\) is calculated as Eq. (6.2.2) assuming that injected plasma is collimated to one direction and listed in Table 6.2.

The thrust gain obtained by various kinetic \(\beta_{\text{jet}}\) is shown in Fig. 6.6. In Case 1, the thrust gain is calculated as only 0.05 since the large injection velocity of ions increases \(F_{\text{jet}}\). In Case 4, the thrust gain is calculated as 1.3. Thus, the thrust of MPS can be enhanced significantly by the low kinetic \(\beta_{\text{jet}}\) plasma injection. The inflation by the diamagnetic current (Case 4) expands the magnetic field efficiently than the inflation by the frozen-in of plasma (Case 1). In addition, as shown in Fig. 6.6, the largest thrust gain can be obtained in Case 2. We note that the parameters in Case 2 indicate the moderately low kinetic \(\beta_{\text{jet}}\) plasma injection nevertheless the electron has large injection velocity since the electron mass is 1836 times lighter than ion mass when using the realistic electron-ion mass ratio. The ion velocity and the electron velocity in this case satisfy \(r_{iL} (\sim 50 \text{ m}) \sim B/\nabla B \) and \(r_{eL} (\sim 30 \text{ m}) < B/\nabla B\), respectively. Under this condition, magnetized plasma induces the diamagnetic current, which mainly consists of the drift motion of electron in magnetosphere since the drift velocity is proportional to the square of the injection velocity \((V_{\text{jet},i} \ll V_{\text{jet},e})\). In addition, the ions raise the flow toward the injection direction \((\pm x)\) around the injection source.

Consequently, in Case 2, the strong and widespread diamagnetic current is formed around the coil as shown in Fig. 6.7 by the solid line. The induced current near the coil enhances the original magnetic field and thus the magnetosphere becomes larger.

Figure 6.8 shows the ion density distribution in Case 2 for both the parallel case \((x=0^\circ)\) and the perpendicular case \((x=90^\circ)\). As compared with Fig. 5.17, the magnetosphere is significantly inflated by the plasma injection (the range of axis is different). The magnetic field line is also changed by the diamagnetic current induced by plasma injection. The thrust in the perpendicular case is calculated as 15 \(\mu\text{N/m}\) that is 7.5 times larger than the magnetic sail thrust of 2.0 \(\mu\text{N/m}\). The thrust in the parallel case is calculated as 26 \(\mu\text{N/m}\) that is 4.4 times larger than the magnetic sail thrust, 5.8 \(\mu\text{N/m}\). The thrust of the parallel case is larger than that of the perpendicular case since the magnetosphere size is larger and the particles reflected by the mirror magnetic field in parallel case contribute to the thrust generation. The thrust gains in the above two cases are also calculated as 1.9 and 2.4, respectively.

Time history of the magnetospheric inflation in Case 2 \((x=90^\circ)\) is shown in Fig. 6.9. As the total amount of the injected plasma trapped in the magnetosphere increases, the magnetospheric size becomes larger. The solar wind flow is also bended by the enhanced magnetic field and loses its momentum to produce the larger thrust.
6.4. SIMULATION RESULTS BY TWO DIMENSIONS

Figure 6.6: Thrust gain $F_{MPS}/(F_{mag} + F_{jet})$ obtained by the various kinetic $\beta_{jet}$ plasma injections ($L_{MHD}=2200$ m, $\alpha=0 \, ^\circ$, 2D Full-PIC model). Thrust and cross-sectional length of the magnetic sail are 5.8 $\mu$N/m and 1300 m, respectively.

Figure 6.7: Current density distribution along $x$-axis ($z=0$ m) in Case 1 (dashed), Case 2 (solid), case 4 (dotted). The current widely spreads by the high kinetic $\beta_{jet}$ plasma injection and the current flows locally by the low high kinetic $\beta_{jet}$ plasma injection. The strong and widespread diamagnetic current is formed by Case 2. ($L_{MHD}=2200$ m, $\alpha=0 \, ^\circ$, 2D Full-PIC model)
CHAPTER 6. SIMULATION OF MAGNETO PLASMA SAIL

Figure 6.8: Ion density distribution in Case 2 (MPS, $L_{MHD}=2200$ m, 2D Full-PIC model): a) perpendicular case ($\alpha=90^\circ$) and b) parallel case ($\alpha=0^\circ$).

Figure 6.9: Time history of the magnetospheric inflation in Case 2 ($L_{MHD}=2200$ m, $\alpha=90^\circ$, 2D Full-PIC model). As time passing, the magnetosphere size of MPS becomes larger. The plasma injection starts after $t=7.5$ ms.
6.4 SIMULATION RESULTS BY TWO DIMENSIONS

The one-dimensional profiles of the magnetic flux density and the ion density along z-axis (z=0 m) are shown in Fig. 6.10. The magnetospheric size of the MPS becomes 3 times larger than the original magnetospheric size of the magnetic sail as shown in Fig. 6.10a. The higher density around the source represents the injected plasma trapped by the magnetic field. The diamagnetic current is formed around z=150 m and z=-150 m. The current enhances the magnetic field outside of the current and reduce the magnetic field inside of the current as shown in Fig. 6.10b.

6.4.2 Estimation of Thrust on Two-dimensional MPS

By the plasma injection, the plasma current is formed around the injection source as shown in Figs. 6.5 and 6.7. The plasma current diameter is small compared to the magnetospheric size for the low kinetic $\beta_{jet}$ plasma injection case such as Case 2. Hence, a thin wire current model in Fig. 6.11 is introduced to evaluate an increase in the magnetic moment by a diamagnetic current.

Based on this simple 2D assumption, the diamagnetic plasma current is represented by the line current $I_{plasma}$, which flows in the same direction with the coil current $I_{coil}$. The thin wire approximation is expected to be sufficient since the diameter of the plasma current is small in comparison with the coil distance. The force applied on the plasma current $I_{plasma}$ is simply obtained as

\[ F_{plasma} = \frac{\mu_0}{2\pi} \left( \frac{-I_{coil}I_{plasma}}{R_{plasma} - R_{coil}} + \frac{I_{coil}I_{plasma}}{R_{plasma} + R_{coil}} + \frac{I_{plasma}I_{plasma}}{2R_{coil}} \right) \]  

(6.4.2)

by calculating the magnetic field generated by the each thin wire current. The first term of Eq. (6.4.2) is an attracting force between $I_{coil}$ and $I_{plasma}$. The second and the third term represent the repulsive force acting on $I_{plasma}$ by $I_{coil}$ and $I_{plasma}$, respectively. When the forces are balanced ($F_{plasma}=0$) and the plasma current becomes stable, the ratio of the coil current $I_{coil}$ and the plasma current $I_{plasma}$ is calculated as

\[ \frac{I_{plasma}}{I_{coil}} = \frac{4R_{coil}R_{plasma}}{R_{plasma}^2 - R_{coil}^2} \]  

(6.4.3)

. The total magnetic moment by the coil current and the plasma current is calculated as

\[ M = R_{coil}I_{coil} + R_{plasma}I_{plasma} = R_{coil}I_{coil} \left( 1 + \frac{4R_{plasma}^2}{R_{plasma}^2 - R_{coil}^2} \right) \]  

(6.4.4)

by using Eq. (6.4.3). The figures in parenthesis mean the scaling factor of the magnetic moment. When parameters used in the simulation ($R_{coil}=75$ m and $R_{plasma}=150$ m) are substituted, the factor becomes 6.3. That is, the diamagnetic current is able to enhance the original magnetic moment 6.3 times larger. Then, the magnetospheric size by the MHD approximation is inflated 2.5 times since $L_{MHD}$ is proportional to square root of the magnetic moment as Eq. (1.2.5). When the original magnetospheric size is set as $L_{MHD}=2200$ m, the magnetospheric size by the MHD
Figure 6.10: Density distribution of ion and magnetic flux density along z-axis (x=0 m) for the perpendicular case (z=90°, Case 2). a) represents ion density of magnetic sail (solid line) and MPS (dotted line). b) represents magnetic flux density of magnetic sail (solid line) and MPS (dotted line). (\(L_{MHD}=2200\) m, \(z=90°\), 2D Full-PIC model)
approximation is expected to be expanded up to $L_{MHD}=5500$ m. The thrust levels of 15 $\mu$N/m in the perpendicular case and of 30 $\mu$N/m in the parallel case are expected by linearly extrapolating the thrust characteristics represented in Fig. 5.16a in section 5.3.3 to $L_{MHD}=5500$ m. Compared with the simulation result in section 5.3.3, the magnetospheric size is inflated three times since the cross-sectional length of the magnetosphere and the theoretical $L_{MHD}$ have the relation as shown in Fig. 5.16c and the thrust of 15 $\mu$N/m and 26 $\mu$N/m are obtained in perpendicular case and parallel case, respectively. The thrust of MPS obtained by Full-PIC simulation with the low kinetic $\beta_{jet}$ plasma injection can be approximately estimated by the simple assumptions about the diamagnetic current and the thrust characteristics of magnetic sail.

However, these approximation is only valid in two-dimension since the centrifugal force and hoop force work on the ring shaped plasma current. The simulation in three-dimension is required to model thrust of MPS.

### 6.5 Simulation Results by Three Dimensions

#### 6.5.1 Three-dimensional Magneto Plasma Sail on Electron Scale

In order to improve the thrust generation using the solar wind, Winglee et al. [7] proposed M2P2 based on the frozen-in of plasma to the magnetic field (frozen-in concept). However, the concept is not valid at all in the small-scale magnetosphere since electron kinetics prevents plasma jet from moving along the magnetic field. Instead of the frozen-in concept of Winglee, the use of dipole plasma equilibrium (equatorial ring-current concept) is proposed [24]. It is expected that the plasma injected from a MPS spacecraft remains in the equatorial plane and induces the diamagnetic current. The diamagnetic current flows into the same direction with the coil current to enhance the original magnetic field generated by coil current. We have already demonstrated the ring current concept by two-dimensional simulation and in order to obtain the thrust characteristics of Magneto Plasma Sail, we start three-dimensional simulations. The physics of magnetic inflation
Table 6.3: Plasma injection parameter \((M=1.3 \times 10^8 \text{ Wb} \cdot \text{m})\)

<table>
<thead>
<tr>
<th>Gas species</th>
<th>Typical case</th>
<th>parameter range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection point (R_{jet})</td>
<td>50 m from spacecraft</td>
<td>50, 125, 200, 275 m</td>
</tr>
<tr>
<td>Injection direction (\theta_{jet})</td>
<td>90 deg</td>
<td>-180, -90, 0, 90 deg</td>
</tr>
<tr>
<td>Mass flow rate (\dot{m})</td>
<td>(6.7 \times 10^{-9} \text{ kg/s})</td>
<td>(3.3 \times 10^{-9}, 6.7 \times 10^{-9}, 1.3 \times 10^{-8} \text{ kg/s})</td>
</tr>
<tr>
<td>Ion velocity (V_{jet,i})</td>
<td>(5.0 \times 10^6 \text{ m/s})</td>
<td>const.</td>
</tr>
<tr>
<td>Ion temperature (T_{jet,i})</td>
<td>10 keV</td>
<td>0.4, 0.7, 1.0, 10 keV</td>
</tr>
<tr>
<td>Electron velocity (V_{jet,e})</td>
<td>(1.0 \times 10^7 \text{ m/s})</td>
<td>(1.0 \times 10^6, 1.0 \times 10^7, 1.0 \times 10^8 \text{ m/s})</td>
</tr>
<tr>
<td>Electron temperature (T_{jet,e})</td>
<td>10 keV</td>
<td>0.4, 0.7, 1.0, 10 keV</td>
</tr>
</tbody>
</table>

is already described in above sections and especially we pay our attention to the increase in thrust in three-dimensional simulation.

Parameters for a three-dimensional MPS simulation to demonstrate the equatorial ring-current concept are listed in Table 6.3. Figure 6.12 shows the definition of plasma injection point \(R_{jet}\) and injection direction \(\theta_{jet}\). The plasma jet is injected on a concentric circle with \(R_{jet}\) and the net thrust by the plasma jet is zero. The thrust by plasma jet \(F_{jet}\) is hence the virtual thrust corresponding to the thrust when the plasma jet is injected to one way. Parameters for the three-dimensional Full-PIC simulation of a MPS are restricted of a computational resource. A high-density plasma injection or a high-energy plasma injection result in very small Debye length or very large Larmor radius and that prevent us from simulating the MPS. The magnetic moment of the on-board coil is set parallel to the solar wind \((\alpha = 0 \text{ deg})\) and the plasma is injected toward the magnetic equator. Electron density distribution is shown in Fig. 6.13. Figure 6.13a shows the spatial distribution of electron density by Magnetic Sail in the steady state when the magnetic moment \(M\) is parallel to the solar wind. It takes about four days by using 1024 CPUs to obtain the steady state of the plasma flow. Slices across the \(xz\)-plane (magnetic meridian plane) are shown. The solar wind flows avoiding the magnetic field (Fig. 6.14a) and a low-density region forms around the spacecraft at \((x, y, z) = (0, 0, 0)\) despite the loose coupling between the ions and magnetic field because of the large Larmor radius of the ions \((\sim 100 \text{ km})\). The magnetosphere is symmetric about the \(z\)-axis and slightly charge-separated. The cross-sectional size is 70 m. The thrust of the magnetic sail is calculated as \(F_{mag} = 0.07 \pm 0.01 \text{ mN}\) from the change in momentum of all particles contained in the computational domain. By the plasma injection in Fig. 6.13b, the magnetosphere of MPS inflated from the magnetosphere of magnetic sail (Fig. 6.13a). The cross-sectional size is estimated as 260 m. The thrust of the MPS is calculated as \(F_{MPS} = 0.37 \pm 0.05 \text{ mN}\). The thrust becomes approximately 5 times larger than the thrust of the magnetic sail \((0.07 \text{ mN})\).

Figure 6.14 shows the time history of thrust generation of MPS. The plasma injection start at 0.5 ms and the thrust of MPS represented by red line becomes almost steady state. The small fluctuation that remains in the steady state is caused by the plasma instability. A part of the injected plasma is accelerated in the dipole magnetic field and the thrust of sun-direction occurs
6.5. SIMULATION RESULTS BY THREE DIMENSIONS

Figure 6.12: Definition of injection point $R_{\text{jet}}$ and injection direction $\theta_{\text{jet}}$.

Figure 6.13: Electron density distribution of a) magnetic sail and b) Magneto Plasma Sail across the $xz$-plane ($y=0$). ($M=1.3 \times 10^8$ Wb-m, typical solar wind, 3D Full-PIC model)
as represented by blue line. The thrust of MPS is hence smaller than the momentum change of the solar wind (green line). Here, we note that the magnetic field of MPS is almost same with that of magnetic sail.

Current density distributions of MPS are shown in Fig. 6.15. The diamagnetic current can be observed in Fig. 6.15b. The direction of the diamagnetic current flow is same with the coil current and different from the magnetopause current. The attracting force and the repulsive force by these current works on the coil as the thrust force. This current enhances the magnetic field in outer side and the solar wind strongly interacts with the magnetic field to generate the thrust of MPS. On the other hand, in inner side of the diamagnetic current, the magnetic field is weakened. As a result, the thrust generation by the mirror reflection decreases (see also section 5.3.3). Therefore, the thrust of MPS is not necessarily proportional to the increase in the magnetosphere size.

### 6.5.2 Simulations with Various Plasma Injection Parameters ($M=1.3 \times 10^8$ Wb·m)

Assuming various plasma injection parameters such as injection point $R_{\text{jet}}$, injection direction $\theta_{\text{jet}}$, mass flow rate $\dot{m}$, electron velocity $V_{\text{jet,e}}$ and plasma temperature $T_{\text{jet,e}}$, we performed parametric simulations of MPS. The typical simulation case is shown in Table 6.3. First, the magnetic moment
6.5. SIMULATION RESULTS BY THREE DIMENSIONS

Figure 6.15: Current density distribution of MPS across the $xz$-plane ($y=0$). The contour range differs between a) and b). ($M=1.3 \times 10^8 \text{ Wb}\cdot\text{m}$, typical solar wind, 3D Full-PIC model) The double circle means upward current perpendicular to the plane and the cross-circle means downward current perpendicular to the plane.

is set constant as $M=1.3 \times 10^8 \text{ Wb}\cdot\text{m}$. Ion velocity is set to a constant since it is not a best policy to accelerate ion in order to induce the diamagnetic current. The most portion of the thrust by plasma jet $F_{jet}$ is consist of the electron.

First, the injection point $R_{jet}$ is varied with 50 m, 125 m, 200 m and 275 m. The result is shown in Fig. 6.16. The maximum thrust of MPS is obtained at $R_{jet}=200$ m. Compared with the thrust of magnetic sail, the thrust becomes 9 times larger. Figure 6.17 shows the trajectories of injected particle from $R_{jet}=50$ m and 125m. The particle injected in strong magnetic field (green line) makes small Larmor motion and goes around the spacecraft by $\nabla B$ drift and curvature drift. On the contrary, the particle injected in weak magnetic field (red line) makes large Larmor motion. The red line goes around the spacecraft faster than the green line and can induce larger diamagnetic current. In addition, the area where the trajectory surrounds becomes larger in the red line and hence the additional magnetic moment obtained by the diamagnetic current become larger in the case of $R_{jet}=125$ m.

When the plasma is injected from $R_{jet}=275$ m, the diamagnetic current cannot be induced since the injected plasma is not magnetized any longer. The maximum increase in thrust is therefore achieved by the injection point $R_{jet}=200$ m, where the injected plasma just magnetized.

Thus, the thrust characteristics of Magneto Plasma Sail is affected by the plasma injection point, that is, the magnetic flux density at plasma injection point. Hence,

$$F_{MPS} = 10^{-5.5} \left( B_0 + 3.2 \times 10^{-5} \right)^{-1.19} \quad (6.5.1)$$

is obtained by using simulation results in Fig. 6.16. Here, $B_0$ is the magnetic flux density at
Figure 6.16: Thrust obtained by various injection points. \( (M=1.3 \times 10^8 \text{ Wb} \cdot \text{m}, \theta_{\text{jet}}=-90^\circ, V_{\text{jet},e} =1.0 \times 10^8 \text{ m/s}, \text{typical solar wind, 3D Full-PIC model}) \)

plasma injection point as we defined in one-dimensional theoretical approach (Fig. 6.3).

The injection direction \( \theta_{\text{jet}} \) is varied with \(-180^\circ, -90^\circ, 0^\circ \) and \(90^\circ \). The injection point is a constant at \( R_{\text{jet}}=50 \) m. The injection direction \(-90^\circ \) represents the same direction with the \( \nabla B \) drift motion of electron. As shown in Fig. 6.18, the largest thrust and the smallest thrust are obtained at \(90^\circ \) and \(-90^\circ \), respectively.

The trajectories of injected electrons by various injection angle is shown in Fig. 6.19. The blue line \( (90^\circ) \) goes around the spacecraft faster than the red line \( (-90^\circ) \) and can induce larger diamagnetic current. Hence, the thrust is affected by the injection direction.

The mass flow rate \( \dot{m} \) and injection velocity \( V_{\text{jet},e} \) are also varied. The results are represented in Figs. 6.20 and 6.21. As mass flow rate becomes larger, the thrust becomes larger. The relation is obtained as

\[
F_{\text{MPS}} = 10^{1.3} \left( \dot{m} - 3.1 \times 10^{-9} \right)^{0.19} \quad (6.5.2)
\]

However, since the specific impulse is proportional to the thrust and inversely proportional to the mass flow rate \( (\propto F_{\text{MPS}}/\dot{m}) \), the specific impulse of MPS becomes small as the mass flow rate become larger.

As the injection velocity becomes larger, the thrust also becomes larger as

\[
F_{\text{MPS}} = 10^{-8.5} \left( V_{\text{jet},e} + 3.0 \times 10^7 \right)^{1.01} \quad (6.5.3)
\]

The energy required to accelerate plasma is approximately proportional to the square of the
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Figure 6.17: Trajectories of particle injected from $R_{\text{jet}}=50$ m and 125 m. ($M=1.3 \times 10^8$ Wb-m, $\theta_{\text{jet}}=-90^\circ$, $V_{\text{jet},e}=1.0 \times 10^8$ m/s, same time period)

Figure 6.18: Thrust obtained by various injection directions. ($M=1.3 \times 10^8$ Wb-m, $R_{\text{jet}}=50$ m, $V_{\text{jet},e}=1.0 \times 10^8$ m/s, typical solar wind, 3D Full-PIC model)
injection velocity ($\propto V_{\text{jet},e}^2$). As shown in Fig. 6.21, even if the injection velocity $V_{\text{jet},e}$ increases 10 times, a thrust has only change by a factor. The thrust-power ratio therefore becomes small as the injection velocity becomes larger. The thrust performance of MPS thus has the trade-off between the increase in thrust, the specific impulse and the thrust-power ratio.

On the contrary, as the injection temperature $T_{\text{jet},e}$ becomes lower, the thrust becomes larger as shown in Fig. 6.22. When the temperature of injected plasma become low, the diffusion across the magnetic field is suppressed. Eq. (6.5.4) represents the diffusion equation in cylindrical coordinate system. Here, $S$ is the source term and $\delta (r)$ represents a delta function. When the diffusion coefficient $D$ and the magnetic field $B$ in the equatorial plane are assumed as Eq. (6.5.5) and Eq. (6.5.6), respectively, the density distribution is derived as Eq. (6.5.7). That is, the plasma density is inversely proportional to the plasma temperature. As shown in Fig. 6.23, the ion density distribution in the equatorial plane is approximately inversely proportional to $r^3$ and in agreement with the analysis result. Therefore, as the injection temperature becomes lower, the total amount of plasma in magnetosphere increases.

When the electron kinetics is dominant, the drift velocity of electron is approximately constant independent on the plasma density. The diamagnetic current hence becomes larger as the injected plasma remains in a magnetosphere, and the thrust of MPS also becomes larger. However, Full-PIC simulation in low injection temperature is very difficult since the Debye length becomes shorter and larger computational resource is required to resolve the motion of injected plasma correctly.

$$\nabla \cdot (D \nabla n) + S \delta (r) = 0$$ (6.5.4)
Figure 6.20: Thrust obtained by various mass flow rates. \( M=1.3 \times 10^8 \) Wb·m, \( R_{\text{jet}}=125 \) m, \( \theta_{\text{jet}}=-90^\circ \), typical solar wind, 3D Full-PIC model.

Figure 6.21: Thrust obtained by various injection velocities. \( M=1.3 \times 10^8 \) Wb·m, \( R_{\text{jet}}=125 \) m, \( \theta_{\text{jet}}=-90^\circ \), typical solar wind, 3D Full-PIC model.
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Figure 6.22: Thrust obtained by various injection temperatures. \((M=1.3 \times 10^8 \text{ Wb-m, } R_{jet}=125 \text{ m, } \theta_{jet}=-90^\circ, \text{ typical solar wind})\)

\[
D = \frac{\eta n(r) (k_B T_i + k_B T_e)}{B^2} \quad \text{or} \quad \frac{k_B T_e}{16 e B} \tag{6.5.5}
\]

\[
B = \frac{B_0}{r^3} \tag{6.5.6}
\]

\[
n(r) = \frac{C}{(T_i + T_e) r^3} \quad \text{or} \quad \frac{C}{T_e r^3} \tag{6.5.7}
\]

According to the above parametric survey, the large thrust increase by Magneto Plasma Sail is achieved by the high mass flow rate, the high injection velocity and low injection temperature since the total amount of plasma in the magnetosphere is increased. Figure 6.24 shows the ion density distribution obtained in the maximum thrust case. The parameters are shown in Table 6.4. Figures 6.24b and d represent the ion density distribution in \(xy\)-slice \((z=400 \text{ m}\)). The magnetosphere size of Magneto Plasma Sail \((1160 \text{ m})\) becomes 4.2 times larger than that of magnetic sail \((260 \text{ m})\). The 14 times larger thrust of Magneto Plasma Sail \((1.0 \text{ mN})\) is obtained.

Figure 6.25 shows the ion density and current density distributions along \(y\)-axis \((x=0, z=0)\). Current is induced where the density gradient is larger. The current density of electron (blue line) is larger than the current density of ion (green line). That is, the carrier of the diamagnetic current is almost electron. Figure 6.25b represents that large charge separation occurs and the electric field to reduce the charge separation is also observed. In addition to the \(\nabla B\) drift and curvature drift, the \(\mathbf{E} \times \mathbf{B}\) drift motion enhance the diamagnetic current and large increase in thrust is obtained.

On the electron inertial scale, IMF hardly affects the thrust of magnetic sail as shown in
Figure 6.23: Ion density distribution in equatorial plane. (typical case in Table 6.3, 3D Full-PIC model)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Gas species</td>
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<tr>
<td>Injection point $R_{jet}$</td>
<td>200 m from spacecraft</td>
</tr>
<tr>
<td>Injection direction $\theta_{jet}$</td>
<td>$-90$ deg</td>
</tr>
<tr>
<td>Mass flow rate $\dot{m}$</td>
<td>$2.7 \times 10^{-8}$ kg/s</td>
</tr>
<tr>
<td>Ion velocity $V_{jet,i}$</td>
<td>$5 \times 10^2$ m/s</td>
</tr>
<tr>
<td>Ion temperature $T_{jet,i}$</td>
<td>100 eV</td>
</tr>
<tr>
<td>Electron velocity $V_{jet,e}$</td>
<td>$1.0 \times 10^8$ m/s</td>
</tr>
<tr>
<td>Electron temperature $T_{jet,e}$</td>
<td>100 eV</td>
</tr>
</tbody>
</table>
Figure 6.24: Ion density distribution in case of the maximum thrust is obtained. \( M=1.3 \times 10^8 \) Wb m, typical solar wind, 3D Full-PIC model
Figure 6.25: Ion and current density distribution along y-axis ($x=0, z=0, M=1.3 \times 10^8$ Wb·m, typical solar wind, 3D Full-PIC model)
Chapter 4. To confirm that the IMF can be neglected in Magneto Plasma Sail, we performed simulation of Magneto Plasma Sail with IMF. The simulation parameters are same with Table 6.4. The northward IMF and the southward IMF are added. The magnetic flux density is set as 100 nT, which is 20 times larger than the typical value of IMF magnitude. The results are shown in Fig. 6.26. From Fig. 6.27, the magnetosphere size of Magneto Plasma Sail with northward IMF becomes slightly larger than that of no IMF and southward IMF. This is because the pile-uped magnetic field in northward IMF traps the electron strongly and diffusion is suppressed. The diamagnetic current by the trapped electron makes the magnetosphere larger. As a result, 1.1 mN (northward IMF) and 0.92 mN (southward IMF) is obtained. The thrust of Magneto Plasma Sail without IMF (1.0 mN) is slightly changed by the strong IMF. However, as long as we consider the typical magnitude of IMF, the effects of IMF on the Magneto Plasma Sail can be neglected.

We summarized the thrust increase and thrust gain in Fig. 6.28 when the magnetic moment is constant in \( M = 1.3 \times 10^8 \text{ Wb}\cdot\text{m} \). In the case where the largest thrust is obtained, the thrust of MPS is 14 times larger than that of magnetic sail. However, the thrust gain is 0.65. That is, a thrust can be obtained efficiently by using magnetic sail and plasma jet separately rather than inflating magnetic field by plasma jet as like MPS. The maximum value of the thrust gain is 1.6. Even if it is the big increase in a thrust, the thrust gain is not necessarily large.

### 6.5.3 Simulation with Various Magnetic Moment

Finally, we performed large parametric survey using the realistic plasma parameters. Table 6.5 shows the parameters used for the parametric survey. No only \( M = 1.3 \times 10^8 \text{ Wb}\cdot\text{m} \) we performed above section, but also various magnetic moments are assumed. These are corresponding to \( 3.3 \times 10^{-3} < L_{MHD}/r_iL < 3.3 \times 10^{-2} \).
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Figure 6.27: One-dimensional ion density distribution along $y$-axis ($z=0$). (MPS, $M=1.3 \times 10^8$ Wb-m, typical solar wind, 3D Full-PIC model)

Figure 6.28: Thrust gain and thrust increase. ($M=1.3 \times 10^8$ Wb-m, typical solar wind, 3D Full-PIC model)

<table>
<thead>
<tr>
<th><strong>Table 6.5:</strong> Parameters used for parametric survey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic moment $M$ [Wb-m]</td>
</tr>
<tr>
<td>Injection point $R_{jet}$ [m]</td>
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<tr>
<td>Electron velocity $V_{jet,e}$ [m/s]</td>
</tr>
<tr>
<td>Ion velocity $V_{jet,i}$ [m/s]</td>
</tr>
<tr>
<td>Temperature $T_{inj}$ [eV]</td>
</tr>
<tr>
<td>Mass flow rate $\dot{m}$ [kg/s]</td>
</tr>
<tr>
<td>Injection direction $\theta_{jet}$ [deg]</td>
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</tbody>
</table>
CHAPTER 6. SIMULATION OF MAGNETO PLASMA SAIL

Simulation results about the thrust increase and the thrust gain are shown in Fig. 6.29. As the magnetic moment, that is, magnetosphere size, becomes larger, thrust increase decrease as shown in Fig. 6.29a. The maximum thrust gain is restricted by the thrust increase. First, the larger thrust gain is obtained by large magnetic moment, and when the magnetic moment is larger, the thrust gain decreases since the thrust increase decreases. In addition, the minimum thrust increase of $M = 1.3 \times 10^{10}$ Wb·m is less than unity. That is, the thrust of MPS is smaller than the original magnetic sail as well as in MHD scale [48].

The maximum thrust increase is $F_{MPS}/F_{mag} = 96$, that is, the thrust of Magneto Plasma Sail is 96 times larger than thrust of magnetic sail in the optical case. However, the thrust gain is only $F_{MPS}/(F_{mag} + F_{jet}) = 0.3$ and thrust generation efficiency is very low. On the other hand, the maximum thrust gain is obtained at $M = 1.3 \times 10^{9}$ Wb·m as $F_{MPS}/(F_{mag} + F_{jet}) = 5.2$. By using the magnetic inflation, Magneto Plasma Sail can achieve the efficient momentum transfer from the solar wind and large thrust is obtained.

We coordinate all simulation cases by $B_{jet}$, $r_{jet}$, $\beta_{jet}$ and $\gamma_{jet}$, which are derived from one-dimensional theoretical analysis of plasma inflation in section 6.3.2. Results are shown in Fig. 6.30. The horizontal axis represents $r_{jet}$ and the contour level represents the thrust gain. The double logarithmic plot is used. As a result, it is found that the large thrust gain is obtained around $(B_{jet}, r_{jet}, \beta_{jet}, \gamma_{jet}) = (10^{-7} \text{ T}, 3000 \text{ m}, 0.01, 1)$. In the design of Magneto Plasma Sail, such conditions of plasma injection are ideal. The result that plasma injection of low $\beta_{jet}$ is effective is in agreement with the result of two-dimensional analysis (Fig. 6.4).

6.5.4 Restriction of Analysis Techniques

The thrust characteristics of Magneto Plasma Sail are simulated by various magnetic moment (the main design parameter of the spacecraft) and various plasma injection parameters. However, the parameters are restricted by the computational techniques and the computational resource. For example, Fig. 6.31 represents the all simulation cases summarized by the magnetic moment $M$ (vertical axis) and the mass flow rate $\dot{m}$ (horizontal axis). In addition to the Full-PIC simulation in above section, the Full-PIC simulation using the artificial plasma parameters (Appendix C), MHD simulation by Nishida and Hybrid-PIC simulation by Kajimura is plotted at the same time. All simulation cases result in the approximately same specific impulse $(100 \text{ s - 10000} \text{ s})$ according to the scaling law on the electron inertial scale $(F \propto M)$ or on MHD scale $(F \propto M^{2/3})$. Very high specific impulse case nor very low specific impulse case is hence not simulated.

When the mass flow rate is large and hence the specific impulse is low, the large amount of injected plasma cause very short Debye length. Requirement of fine grid spacing causes too large requirement of computational resource in the present supercomputing technology. When the magnetic moment is large, Full-PIC simulation cannot be performed because of large computational requirement and other simulation techniques such as Hybrid-PIC model and MHD model are
Figure 6.29: a) thrust increase and b) thrust gain with various magnetic moment \( M \). (typical solar wind, 3D Full-PIC model)
Figure 6.30: Thrust gain summarized by characteristics parameters of magnetic inflation: $r_{jet}$, $B_{jet}$, $\beta_{jet}$ and $\gamma_{jet}$. (typical solar wind, 3D Full-PIC model)
required. However, according to the plasma injection condition, the plasma parameters such as Debye length and Larmor radius of injected plasma are completely different from those of the solar wind. Physics of MPS cannot be represented by a single analysis model.

In order to increase the plasma injection parameters in which the analysis of MPS is possible, the analysis techniques need to be extended. For example, the macro-micro interlocked simulation (Appendix D) is a powerful technique to deal with multi-scale plasma physics. In addition, the Full-PIC model itself should be extended by the introduction of the guiding center approximation for the small Larmor radius and the implicit algorithm for the electromagnetic wave.

### 6.6 Summary

We performed two- and three-dimensional Full-PIC simulations in order to determine the thrust characteristics of small-scale Magneto Plasma Sails. First, we provided the one-dimensional theoretical analysis results about magnetic inflation. As a result, the four characteristic parameters: Larmor radius, plasma injection energy, kinetic $\beta$ and generalized momentum is distinguished. By using two-dimensional Full-PIC simulation, it was revealed that the increase in thrust is certainly affected by the kinetic $\beta$ of injection plasma since the diamagnetic current induced by injected plasma depends on the energy of electron. It was found that the influence of the electron kinetics is dominant in the magnetic inflation on the small-scale magneto plasma sail.

The three-dimensional simulations with various plasma injection parameters and magneto
plasma sail design parameter (magnetic moment) were also performed to reveal the propulsive characteristics. The electron kinetics affects the diamagnetic current as well as the two-dimensional case. The maximum increase (thrust of MPS / thrust of magnetic sail) is 97 but the thrust gain (thrust of MPS / (thrust of magnetic sail + thrust of plasma jet)) is only 0.4 by same condition. On the contrary, the maximum thrust gain 5.2 is obtained around \((B_{inj}, r_{inj}, \beta_{inj}, \gamma_{inj}) = (10^{-7} \text{ T}, 3000 \text{ m}, 0.01, 1)\) and the relation of trade-off between the increase in thrust and the thrust gain is revealed.

These thrust characteristics of Magneto Plasma Sail is used for mission analysis in Chapter 7. The thrust characteristics of MPS such as the thrust-mass ratio, specific impulse and thrust-power ratio are also examined in Chapter 7 by using the thrust characteristics obtained from the Full-PIC simulations.
Chapter 7

Performance of Magnetic Sail and Magneto Plasma Sail

7.1 Introduction

From Chapter 3 to 6, the thrust characteristics of Magnetic Sail and Magneto Plasma Sail were calculated by plasma particle simulations. The thrust characteristics include the magnitude of the thrust, the thrust direction, the attitude stability and the effects of variation of solar wind. In order to reveal the feasibility of Magnetic Sail and Magneto Plasma Sail, we have to assume realistic missions by using magnetic sail or Magneto Plasma Sail. Comparing with the exiting thrust systems. It is revealed what kind of missions are suitable for the magnetic sail and Magneto Plasma Sail.

We assume both low-thrust mission available in the present technology and high-thrust mission which will be achieved in future. The optimal orbit of Magnetic Sail and Magneto Plasma Sail is calculated using Genetic Algorithm. Finally, it is proposed how the feasibility of magnetic sail and Magneto Plasma sail is increased.

7.2 Thrust Model

As we showed above, we performed large 3D full-PIC simulations by using the parallel computing techniques and the huge computational resource. As a result, it was revealed that the thrust of a small-scale magnetic sail is determined by the magnetic moment $M$ of the spacecraft onboard coil (main design parameter of the spacecraft), the solar wind parameters (density $N_{SW}$ and velocity $V_{SW}$), and the attack angle $\alpha$ on the electron inertial scale. Combining these results by assuming no cross-correlation between parameters gives empirical formulae for the thrust $F_{mag}$ and steering angle $\gamma$. These equations show that the thrust of small-scale magnetic sail is approximately proportional to magnetic moment, solar wind density and solar wind velocity, respectively. In
addition, it is revealed that the magnetic sail can generate not only the radial thrust but also the
tangential thrust that is effective for accelerating the spacecraft along its trajectory.

\[ F_{mag} = 3.57 \times 10^{-26} N_{SW}^{1.15} V_{SW}^{0.92} M^{1.03} (1.55 \times 10^{-3} \alpha + 1) \ (1.3 \times 10^{6} < M < 1.3 \times 10^{11}) \] (7.2.1)

Next, using the Flux-Tube model, the approximate formula for the thrust estimation is ob-
tained on the ion inertial scale. Then, approximate formulas for the drag coefficient for the thrust
estimation are obtained as Eq. (7.2.2) for the parallel case and Eq. (7.2.3) for the perpendicular
case. The thrust force is hence calculated as

\[
F_{mag} = \begin{cases} 
3.0 \times \exp \left[ -0.36 \times \left( \frac{r_{IL}}{L_{MHD}} \right)^2 \right] \times 0.5m_i N_{SW} V_{SW}^2 \times \pi L_{MHD}^2 & r_{IL} < L_{MHD} \\
2.7 \times \left( \frac{r_{IL}}{L_{MHD}} \right) \times \exp \left[ -0.26 \times \left( \frac{r_{IL}}{L_{MHD}} \right)^2 \right] \times 0.5m_i N_{SW} V_{SW}^2 \times \pi L_{MHD}^2 & r_{IL} > L_{MHD} 
\end{cases}
\] (7.2.2)

for the parallel case (\( \alpha = 0 \) deg) and

\[
F_{mag} = \begin{cases} 
2.8 \times \exp \left[ -0.51 \times \left( \frac{r_{IL}}{L_{MHD}} \right)^2 \right] \times 0.5m_i N_{SW} V_{SW}^2 \times \pi L_{MHD}^2 & r_{IL} < L_{MHD} \\
2.4 \times \left( \frac{r_{IL}}{L_{MHD}} \right) \times \exp \left[ -0.37 \times \left( \frac{r_{IL}}{L_{MHD}} \right)^2 \right] \times 0.5m_i N_{SW} V_{SW}^2 \times \pi L_{MHD}^2 & r_{IL} > L_{MHD}
\end{cases}
\] (7.2.3)

for perpendicular case (\( \alpha = 90 \) deg). This equation can be rewritten by using magnetic moment as

\[
F_{mag} = \begin{cases} 
1.32 \times 10^{-20} \times \exp \left[ -1.11 \times 10^{10} \left( \frac{V_{SW}}{N_{SW} M} \right)^\frac{1}{3} \right] \times N_{SW} \frac{2}{3} V_{SW} \frac{4}{3} M \frac{2}{3} \ & 4.0 \times 10^{14} < M \\
6.01 \times 10^{-26} \times \exp \left[ -8.42 \times 10^{-12} \left( \frac{V_{SW}}{N_{SW} M} \right)^{\frac{2}{3}} \right] \times N_{SW} V_{SW} M \ & 1.3 \times 10^{11} < M < 4.0 \times 10^{14}
\end{cases}
\] (7.2.4)

for the parallel case (\( \alpha = 0 \) deg) and

\[
F_{mag} = \begin{cases} 
1.26 \times 10^{-20} \times \exp \left[ -1.57 \times 10^{10} \left( \frac{V_{SW}}{N_{SW} M} \right)^\frac{1}{3} \right] \times N_{SW} \frac{2}{3} V_{SW} \frac{4}{3} M \frac{2}{3} \ & 4.0 \times 10^{14} < M \\
5.34 \times 10^{-26} \times \exp \left[ -1.20 \times 10^{-11} \left( \frac{V_{SW}}{N_{SW} M} \right)^{\frac{2}{3}} \right] \times N_{SW} V_{SW} M \ & 1.3 \times 10^{11} < M < 4.0 \times 10^{14}
\end{cases}
\] (7.2.5)

for perpendicular case (\( \alpha = 90 \) deg).

On the MHD scale, the drag coefficient is constant at 2.9 and

\[ F_{mag} = 1.28 \times 10^{-20} N_{SW} \frac{2}{3} V_{SW} \frac{4}{3} M \frac{2}{3} \ & 4.0 \times 10^{14} \ll M \] (7.2.6)
7.2. THRUST MODEL

The thrust direction is obtained by three-dimensional Full-PIC simulation as

\[ \gamma = -3.60 \times 10^{-3} (\alpha - 50)^2 + 9.01 \quad 0 < \alpha < 90 \]  

(7.2.7)

The attitude stability without and with IMF is obtained by two-dimensional Full-PIC simulation as

\[ M_{\text{mag}} \propto \sin (2\alpha) \]  

(7.2.8)

and

\[ M_{\text{mag}} \propto -\sin (\alpha - \beta) \]  

(7.2.9)

It was revealed that the IMF disturbs the attitude of the spacecraft dominantly and the continuous control of attitude is required in Chapter 5.

The effects of finite coil size can be neglected when the coil size is enough smaller than the magnetosphere size \((R_{\text{coil}} < L/5)\). IMF does not influence on the magnetic sail thrust on the electron inertial scale. The torque works on the coil is slightly affected by IMF as shown in Fig. 5.14. However, the magnitude of torque is small, and since IMF changes for a short time, the torque by IMF can be neglected on the average. Hence in this chapter, IMF is neglected.

The thrust increases by plasma injection is controlled by the plasma injection parameters. In two-dimensions, the largest thrust per unit length \((26 \ \mu\text{N/m})\) is obtained. The increase in thrust is corresponding to \(F_{\text{MPS}}/F_{\text{mag}} \sim 4.4\). The thrust gain is 2.4. In three-dimensions, the largest thrust increase \(F_{\text{MPS}}/F_{\text{mag}} \sim 96\) is obtained. The largest thrust gain \(F_{\text{MPS}}/ (F_{\text{mag}} + F_{\text{jet}}) \sim 5.2\) is obtained by \((B_{\text{jet}}, r_{\text{jet}}, \beta_{\text{jet}}, \gamma_{\text{jet}}) = (10^{-7} \ \text{T}, 3000 \ \text{m}, 0.01, 1)\). For simplicity, it is assumed that the thrust of MPS is increased ideally from the thrust of magnetic sail by the largest thrust increase \(F_{\text{MPS}}/F_{\text{mag}} \sim 96\).

We examined the thrust performance of MPS spacecraft based on the above analysis result. The specific impulse is a way to describe the efficiency of rocket. It represents the force with respect to the amount of propellant used per unit time and is defined as

\[ I_{\text{sp}} = \frac{F_{\text{MPS}}}{mg} \]  

(7.2.10)

The thrust-power ratio represents the force with respect to the amount of electric power used per unit time and is defined as

\[ \text{Thrust-power ratio} = \frac{F_{\text{MPS}}}{E_p} \]  

(7.2.11)

In the results of Magneto Plasma Sail, the electric power is assumed to be the energy to accelerate plasma. The energy required to produce plasma is neglected. Here, we note that the specific
CHAPTER 7. PERFORMANCE OF MAGNETIC SAIL AND MAGNETO PLASMA SAIL

Table 7.1: Details of superconducting material [61]

<table>
<thead>
<tr>
<th>Type</th>
<th>BSCCO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width [mm]</td>
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</tr>
<tr>
<td>Thickness [mm]</td>
<td>0.23</td>
</tr>
<tr>
<td>Density [g/cm³]</td>
<td>10</td>
</tr>
<tr>
<td>Critical Current [A]</td>
<td>500</td>
</tr>
</tbody>
</table>

Figure 7.1: Thrust-mass ratio of two-dimensional magnetic sail (parallel case)

The specific impulse and the thrust-power ratio of magnetic sail is infinity since the plasma is not injected from the spacecraft.

There is a thrust-mass ratio:

$$\text{Thrust-mass ratio} = \frac{F_{\text{MPS}}}{m_{\text{mps}}}$$

(7.2.12)

, that is, the acceleration of the spacecraft, as another important performance index of Magneto Plasma Sail. We have to assume the mass of the spacecraft, especially onboard coil, to calculate the thrust-mass ratio.

First, we examined the thrust-mass ratio and specific impulse in two-dimensions. Table 7.1 represents the superconducting material we assumed for the Magneto Plasma Sail onboard coil.

The thrust-mass ratio of two-dimensional magnetic sails with various magnetic moment is shown in Fig. 7.1. The maximum thrust-mass ratio is obtained at $M = 6.0 \times 10^5$ Wb·m/m. Since the increase in coil weight exceeds the increase in the thrust of magnetic sail as the magnetic moment per unit length becomes larger, the thrust-mass ratio declines. The specific impulse of magnetic sail is infinity since the magnetic sail does not require the fuel at all. The thrust-power ratio is also infinity since the superconductive coil is almost loss-less. The thrust performance of two-dimensional magneto plasma sail is thus a very attractive for the deep space exploration. It remains to be analyzed what kind of missions is suitable for MPS with the high specific impulse.
Next, we examined the thrust-mass of two-dimensional Magneto Plasma Sail. We chose the case 2 for the MPS spacecraft model since the largest thrust (26 μN/m) is obtained. The weight of the superconductive coil per unit length is calculated as 0.158 kg/m (4.3 mm × 0.23 mm × 10 g/cm³ × 4 × 10³ ATurn / 500 A × 2) without considering the cooling system and the bus system. As a result, the thrust-mass ratio of the MPS is calculated as 0.16 mN/kg (26 μN/m / 0.158 kg/m). This is similar to the value theoretically predicted by Cattell et al. [19] (0.23 mN/kg, 3-ton superconductive coil and thrust generation of 700 mN). The thrust-mass ratio of the MPS is also competitive with that of other sail type propulsion system such as solar sail (0.07 mN/kg of IKAROS, here only mass of the reflection mirror is considered). If compared with ion engines (0.40 mN/kg of HAYABUSA and 1.9 mN/kg of Deep Space 1, here dry mass of the thrust system without the bus system and the mission equipment), the thrust-mass ratio of MPS is still an order of magnitude smaller. In addition, by considering the mass flow rate of plasma injection, the specific impulse of this MPS is calculated as $I_{SP}=38000$ s. The specific impulse of the MPS is much higher than that of other electric propulsions (3200 s of HAYABUSA and 1000-3000 s of Deep Space 1). This is because MPS converts not the equipped propellant but the momentum of the solar wind to the thrust force. The high specific impulse will make it possible to reduce the wet mass of the spacecraft. The thrust-power ratio is 0.5 mN/kW (26 μN/m / 0.051 kW/m).

We also examined the thrust-mass ratio, thrust-power ratio and specific impulse in three-dimensions. Figure 7.2 shows the thrust-mass ratio of magnetic sail with various magnetic moment. We assume three type of coil size: 2 m, 10 m, and 100 m. The coil of 2 m in radius is the maximum size which can be equipped in the fairing of the rocket. The expansion structure is hence required to adopt the large coil radius. The mass of coil is calculated as

$$m_{mps} = \frac{M}{\pi R_{coil}^2} I_{critical} \times 2\pi R_{coil} \times \text{Width} \times \text{Thickness} \times \text{Density} \propto \frac{M}{R_{coil}}$$  \hspace{1cm} (7.2.13)

. Hence, as the coil radius becomes larger, the coil mass becomes lighter. The thrust of magnetic sail on the electron scale is proportional to $M^{1.03}$ and the thrust-mass ratio becomes higher as the magnetic moment becomes larger. On the contrary, the thrust of magnetic sail on the ion scale and MHD scale is proportional to $M^{2.5}$. The thrust-mass ratio hence decreases as the magnetic moment becomes larger since the increase in coil weight exceeds the increase in the thrust. The thrust-mass ratio feasible with the present technology is approximately $3.3 \times 10^{-5}$ mN/kg. Table 7.2 shows the magnetic sail design parameters feasible in present technology.

Compared with the two-dimensional magnetic sail (from $5.0 \times 10^{-3}$ mN/kg to $3.6 \times 10^{-2}$ mN/kg), the thrust-mass ratio of three-dimensional magnetic sail ($3.3 \times 10^{-5}$ mN/kg) is less than 1/100. To obtain the large acceleration, the two-dimensional magnetic sail is suitable. The specific impulse of magnetic sail is infinity since the magnetic sail does not require the fuel at all. The thrust-power ratio is also infinity since the superconductive coil is almost loss-less.

Figure 7.3 represents the specific impulse and the thrust-power ratio of three-dimensional Mag-
Figure 7.2: Thrust-mass ratio of three-dimensional magnetic sail (electron scale to MHD scale).

Table 7.2: Magnetic sail design parameters

<table>
<thead>
<tr>
<th>$R_{\text{coil}}$</th>
<th>2 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil current</td>
<td>$1.0 \times 10^6$ A turn</td>
</tr>
<tr>
<td>Magnetic moment</td>
<td>$1.3 \times 10^7$ Wb-m</td>
</tr>
<tr>
<td>Coil weight</td>
<td>200 kg</td>
</tr>
<tr>
<td>Thrust</td>
<td>6.6 $\mu$N</td>
</tr>
</tbody>
</table>

neto Plasma Sail by the constant magnetic moment $M=1.3 \times 10^8$ Wb-m. The high specific impulse is obtained since the MPS converts the solar wind momentum to the thrust. In contrast, the thrust-power ratio is low. The thrust-power ratio mainly depends on the injection velocity $V_{\text{jet},e}$ since the electric power required to accelerate plasma jet becomes larger as the injection velocity becomes higher $E_p = 0.5 \dot{m}_i V_{\text{jet},i}^2 + 0.5 \dot{m}_e V_{\text{jet},e}^2$. Thrust efficiency defined as

$$\eta_t = \frac{F^2}{2\dot{m}E_p} \quad (7.2.14)$$

is very low in almost all cases since the ion that account for most of the mass flow rate does not work to inflate the magnetosphere.

The thrust-mass ratio of MPS is also calculated based on the present superconductive technology. The weight of the superconductive coil with $M=1.3 \times 10^7$ Wb-m, $R_{\text{coil}}=2$ m is calculated as 200 kg. The thrust-mass ratio of three-dimensional MPS ($97 \times 3.3 \times 10^{-5} = 3.2 \times 10^{-3}$ mN/kg) is further smaller than that of two-dimensional MPS (0.16 mN/kg) and other existing propulsion system: 0.07 mN/kg of IKAROS (only reflecting mirror), 0.40 mN/kg of HAYABUSA and 1.9 mN/kg of Deep Space 1 (only thrust system without fuel). However, by the improvement of the superconductive technology and the optimal design of the superconductive coil to magnetic sail [16, 17], it is expected that the thrust-mass ratio should be improved by 2~5 times compared with
the assumption of this study. By adopting expansion coil structure, the thrust-mass ratio should be improved up to 0.16 mN/kg with $R_{\text{coil}} = 100$ m.

Figure 7.4 shows the thrust gain of three-dimensional Magneto Plasma Sail with various plasma injection parameters (Table 6.5) summarized by the specific impulse (Eq. (7.2.10)) and the thrust-power ratio (Eq. (7.2.11)). The double logarithmic plot is used. The propulsion efficiency becomes high when both the specific impulse and the thrust-power ratio are high. The high thrust gain is obtained when the thrust efficiency is high. Therefore, the effective thrust of Magneto Plasma Sail is simultaneously obtained by realizing high propulsion efficiency.

On the contrary, the thrust-mass ratio becomes high when the specific impulse and the thrust-power ratio is low as shown in Fig. 7.5. The maximum thrust-mass ratio of three-dimensional Magneto Plasma Sail is $3.2 \times 10^{-3}$ mN/kg. Compared with the two-dimensional magneto plasma sail (0.16 mN/kg), the thrust-mass ratio of three-dimensional magneto plasma sail is smaller. Therefore, the high acceleration and high thrust efficiency can not be achieved at same time.

### 7.3 Genetic Algorithm for Spacecraft Trajectory Optimization

The trajectory of spacecraft can be obtained by integrating the equation of motion with the gravity acceleration and the thrust force by magneto plasma sail:

$$\frac{dr}{dt} = v$$  \hspace{1cm} (7.3.1)
CHAPTER 7. PERFORMANCE OF MAGNETIC SAIL AND MAGNETO PLASMA SAIL

Figure 7.4: Thrust gain of Magneto Plasma Sail. (typical solar wind, 3D Full-PIC model)

Figure 7.5: Thrust-mass ratio of Magneto Plasma Sail. (typical solar wind, 3D Full-PIC model, $R_{coil} = 2m$)
7.3. GENETIC ALGORITHM FOR SPACECRAFT TRAJECTORY OPTIMIZATION

The equation is solved by 4th order Runge-Kutta method. Fig. 7.6 shows the trajectory of the spacecraft only affected by the sun gravity. $1\ AU = 1.5 \times 10^8\ km$ is the average distance between sun and earth. The spacecraft which left at the initial velocity $29.8\ km/s$ = Earth orbital speed describes a circular orbit. It becomes an elliptical orbit, when initial velocity was large, or when small (Fig. 7.6).

When the thrust force according to the above thrust model works on the spacecraft, the trajectory of the spacecraft becomes more complicated. The solar wind density is inversely proportional to square of Sun-Spacecraft distance ($N_{SW} \propto R^{-2}$). The solar wind velocity is approximately constant. In addition, high latitude region differs in the speed of a solar wind from low latitude region. Change of the vector by control of a posture and turning on and off of a propeller also become a control element.

The optimization of such a complicated control element is performed by using Genetic Algorithm. Genetic Algorithm (GA) is a search heuristic that mimics the process of natural evolution:

1. Initialization: Initially many individual solutions are randomly generated to form an initial population.

2. Selection: Selection is the stage of a genetic algorithm in which individual genes are chosen from a population for later breeding. Fitness function is evaluated and individual with high fitness value survives.

3. Genetic operation: Crossover (Fig. 7.8) is a genetic operator used to vary the programming of a chromosome or chromosomes from one generation to the next. We use one-point

\[ M_{\text{spacecraft}} \frac{dv}{dt} = -G \frac{M_{\text{spacecraft}} M_{\text{sun}}}{r^3} \frac{r}{r} + F_{\text{mps}} \]  

(7.3.2)
crossover. Mutation (Fig. 7.9) is a genetic operator used to maintain genetic diversity from one generation of a population of genetic algorithm chromosomes to the next.

4. Termination: This generational process is repeated until a termination condition has been reached.

Figure 7.7 shows the calculation flow of GA. The calculation of fitness function requires large computational efforts since we have to calculate the orbit of a spacecraft about many kinds of genes (10000 individuals in typical case). Hence we use GPGPU (General-purpose computing on graphics processing units) techniques supported by NVIDIA® CUDA tools [62]. Compared with the single CPU (Intel® Core i7 3820 [63]), approximately 40 times faster Selection process is achieved by 2 GPUs (NVIDIA® Tesla C2075).

As the example of trajectory calculation of spacecraft, we performed the orbital change in a circular orbit from a circular orbit. The radial thrust force is constant and the thrust is turned on and turned off. Figure 7.10a shows the trajectory of spacecraft (red line) which change in the large circular orbit (purple line) from the small circular orbit (blue line). The control of turning on and off was performed every 10 days. The green point represents the point where the thrust is turned on. Figures. 7.10b and 7.10c represents the trajectory of spacecraft (red line) which changes in small circular orbit from large circular orbit. The initial velocity differs. It takes 3.1 years to reach small circular orbit when the initial velocity is minus (-3 km/s), that is, initial orbit is small elliptical orbit. On the contrary, it takes 2.8 years when the initial velocity is plus (+3

Figure 7.7: Calculation flow of Genetic Algorithm.
7.3. GENETIC ALGORITHM FOR SPACECRAFT TRAJECTORY OPTIMIZATION

Figure 7.8: Schematic illustration of one-point crossover. Crossover point is randomly selected and genes are rearranged.

Figure 7.9: Schematic illustration of mutation. Two-mutation points are randomly selected and on the other hand, surrounding genes are changed to 0, and, on the other side, surrounding genes are changed to 1.
km/s), that is, the spacecraft once turns around the outside of an initial orbit. Thus, using Genetic Algorithm, the orbital changes by arbitrary thrust can be calculated.

### 7.4 Mission analysis

#### 7.4.1 Comparison of small-scale magnetic sail, large-scale magnetic sail and solar sail

By using these thrust characteristics of small-scale magnetic sails, it is expected to determine the trajectory of magnetic sail spacecraft and the mission analysis is attained. We used these thrust characteristics of small-scale magnetic sails to analyze the thrust-mass ratio of magnetic sail in each point of Mercury, Venus, Earth, Mars and Jupiter. The plasma density of the solar wind $N_{SW}$ is assumed to be inversely proportional to the square of the sun–spacecraft distance ($N_{SW} \propto |R|^{-2}$) and the velocity $V_{SW}$ is assumed to be constant. The thrust of the small-scale magnetic sail (electron inertial scale) is therefore inversely proportional to the 2.3 power of the sun–spacecraft distance ($F \propto |R|^{-2.3}$). On the other hand, the thrust of the large-scale magnetic sail (MHD scale) is inversely proportional to the 4/3 power of the sun-spacecraft distance ($F \propto |R|^{-4/3}$). As the existing thrust system, the thrust of solar sail is inversely proportional to the square of the sun–spacecraft distance ($F \propto |R|^{-2}$). When the thrust-mass ratio on the earth orbit is assumed to be unity, Table 7.3 shows the thrust-mass ratio in each point of Mercury, Venus, Earth, Mars and Jupiter. On the inner planet orbit, the largest thrust-mass ratio is obtained by small-scale magnetic sail because of high density of solar wind plasma. On the contrary, as the sun-spacecraft distance become large, the solar wind plasma density and light pressure become small and the thrust also becomes small. On the outer planet orbit, the large thrust is obtained by the large-scale magnetic sail.

Figure 7.11 shows the optimized trajectory of spacecraft. The objective function is set as the maximizing the sun-spacecraft distance. The three type of thrust system: small scale magnetic sail, large scale magnetic sail and solar sail is assumed. The thrust-mass ratio on the earth orbit and the initial velocity is set as 0.5 mN/kg and 5.0 km, respectively. The control of thrust vector (-30 deg, -10 deg, 10 deg and 30 deg) was performed every 1 day. The objective function is set to maximize

<table>
<thead>
<tr>
<th></th>
<th>Small scale magnetic sail</th>
<th>Large scale magnetic sail</th>
<th>Solar sail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury 0.3AU</td>
<td>15.9</td>
<td>4.98</td>
<td>11.1</td>
</tr>
<tr>
<td>Venus 0.7AU</td>
<td>2.27</td>
<td>1.61</td>
<td>2.04</td>
</tr>
<tr>
<td>Earth 1 AU</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
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<td>0.58</td>
<td>0.44</td>
</tr>
<tr>
<td>Jupiter 5 AU</td>
<td>0.025</td>
<td>0.12</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Figure 7.10: Example of orbital optimization by GA. Orbit change in a circular orbit from a circular orbit. a) small circular orbit to large circular orbit with initial velocity $+3 \text{ km/s}$, b) large circular orbit to small circular orbit with initial velocity $-3 \text{ km/s}$ and c) large circular orbit to small circular orbit with initial velocity $+3 \text{ km/s}$. 
CHAPTER 7. PERFORMANCE OF MAGNETIC SAIL AND MAGNETO PLASMA SAIL

Figure 7.11: Optimized trajectory of small-scale magnetic sail \( \propto |R|^{-2.3} \), large-scale magnetic sail \( \propto |R|^{-4/3} \) and solar sail \( \propto |R|^{-2} \).

the sun-spacecraft distance in 4 years. Therefore, there are \( 4^{14660} \) kinds of patterns in control. Although the thrust mass ratio on the initial point \((X, Y) = (1, 0)\) is same, the optimized trajectory by GA is completely different. The large-scale magnetic sail \( (\propto |R|^{-4/3}) \) arrived farthest, \( R=7.88 \) AU. The large-scale magnetic sail and the solar sail always accelerate to the tangential direction to change to a larger elliptical orbit since the thrust remains large even if the sun-spacecraft distance becomes large. On the contrary, the thrust of small-scale magnetic sail rapidly becomes smaller as the sun-spacecraft distance becomes large. The small-scale magnetic sail goes to a distance, after obtaining big acceleration on an inner planet orbit. As a result, the small-scale magnetic sail reached \( R=5.67 \) AU, which is further than \( R=4.53 \) AU of the solar sail.

7.4.2 Interplanetary Flight by Small-scale Magnetic Sail

In above section, it was revealed that the large-scale magnetic sail \( (\propto |R|^{-4/3}) \) is suitable for the fast interplanetary flight. However, the large-scale magnetic sail \( (L_{MHD}>100 \text{ km}) \) can not be realized by the present technology. The small-scale magnetic sail \( (L_{MHD} \sim 1 \text{ km}) \) is assumed to be the demonstrator spacecraft. Therefore, we optimized trajectories of the small-scale magnetic sail with various thrust-mass ratio and initial velocity. First, the flexibility of thrust vector control was analyzed as 2 (-30 deg, 30 deg) or 4 (-30 deg, -10 deg, 10 deg and 30 deg). The result is shown in Fig. 7.13. The two trajectories are approximately overlapped. Therefore, the flexibility of control seldom influences the optimal orbit.

Next, we changed the control interval, 1 day and 10 days. The results are shown in Fig. 7.13. When the control interval is 1 day, the spacecraft reached \( R=5.67 \) AU. When the control interval
7.4. MISSION ANALYSIS

is 10 days, the spacecraft reached $R=5.62$ AU. Although the optimal orbit is acquired by fine control, the time concerning optimization becomes large.

Finally, we performed trajectory optimization with various thrust-mass ratio and initial velocity. The control flexibility is set as 2 (20 deg and 20 deg). The control interval is 10 days. The initial velocity varies from -5 km to 5 km by 11 steps. The thrust-mass ratio varies in the range of the simulation results from 0.004 mN/kg to 2 mN/kg. The result are shown in Fig. 7.14. When the thrust-mass ratio is large, the spacecraft reached up to $R=39.8$ AU by 4 years. The maximum sun-spacecraft distance becomes smallest when the initial velocity is 0. When the initial velocity is large and the thrust-mass ratio is small, the magnetic sail goes outside directly as well as the large-scale magnetic sail and solar sail. On the contrary, when the initial velocity is small and the thrust-mass ratio is large, the spacecraft accelerates via an inside orbit. Figure 7.15 shows the increase in velocity of spacecraft. The maximum $\Delta V$ is obtained by the trajectory via an inside orbit. As initial velocity is small, since a spacecraft is brought more close to the sun, acceleration becomes larger.

Then, we assumed realistic interplanetary missions. Three kinds of spacecraft, magnetic sails with the solid coil ($R_{coil}=2$ m) and the expansion coil ($R_{coil}=100$ m) and MPS with the expansion coil ($R_{coil}=10$ m and thrust increase is 97 times) have the different thrust characteristics (steering angle and thrust-mass ratio) as shown in Table 7.4, respectively. As for the superconducting coil system, Nagasaki et al. examined the optimization to magnetic sail [16, 17]. Actually, at most 100 m expansion is a limit of the present technology from viewpoints of structure and cooling and the 1000 m coil is a very prospective assumption. The destination of the spacecraft is a flyby of Jupiter (5 AU from sun) or Mercury (0.3 AU from sun) without using Earth swing-by or Venus swing-by. The launch time of the spacecraft is therefore irrelevant. If the spacecraft escapes from

Figure 7.12: Optimized trajectory in case flexibility is 2 and 4.

...
CHAPTER 7. PERFORMANCE OF MAGNETIC SAIL AND MAGNETO PLASMA SAIL

Figure 7.13: Optimized trajectory in case control interval is 1 day and 10 days.

Figure 7.14: Maximum sun-spacecraft distance by various thrust-mass ratio and initial velocity.
the gravitational sphere of Earth with an initial velocity of 3000 m/s at \((X, Y) = (1, 0)\), the spacecraft can fly by only Venus (0.7 AU) or Mars (1.5 AU), with additional acceleration required to reach Mercury or Jupiter. Time frames of less than 10 years are feasible for these missions. The trajectories of the magnetic sail spacecraft from Earth were obtained by numerically integrating the following equations by using the 4th order Runge-Kutta method in a two-dimensionalCartesian coordinate system: Eqs. (7.3.1) and (7.3.2). Here, the mass of the magnetic sail spacecraft \(m_{mps}\) is constant since the specific impulse of the magnetic sail is infinity and no fuels are consumed. Figure 7.16 shows the trajectory of the magnetic sail with the solid coil assuming \(\alpha = 0\) deg and hence steering angle \(\gamma = 0\) deg. The results show that the spacecraft can reach neither Jupiter nor Mercury in 100 years. The trajectories are almost the same as the trajectory of only initial velocity, 3000 m/s and -3000 m/s. The utility of small-scale magnetic sail with the solid coil, hence very low thrust-mass ratio, is very low.

Figure 7.17 shows the trajectory of the small-scale magnetic sail with the expansion coil \((R_{coil} = 1000\) m). The trajectories of the spacecraft are not closed and the elliptical orbit processes gradually because of the force being inversely proportional to the 2.3 power of the sun–spacecraft distance [20, 21]. As shown in Fig. 7.17a, the spacecraft exhibits gradually increasing eccentricity and finally reaches Jupiter after 59 years. For the inner planetary orbit to Mercury, the magnetic sail generates larger thrust since the solar wind density is higher than that in the outer planetary orbit. As a result, the small-scale magnetic sail launched with a small-scale initial velocity is able to reach Mercury in 4.2 years without consuming any fuel (Fig. 7.17b) when the thrust vector is controlled, while the spacecraft can reach neither Jupiter nor Mercury in 100 years without the

Figure 7.15: Increase in velocity of spacecraft by various thrust-mass ratio and initial velocity.
Table 7.4: Magnetic sail design parameters for orbital analysis

<table>
<thead>
<tr>
<th></th>
<th>magnetic sail ($R_{coil} = 2\ \text{m}$)</th>
<th>magnetic sail ($R_{coil} = 100\ \text{m}$)</th>
<th>MPS ($R_{coil} = 10\ \text{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic moment [Wb-m]</td>
<td>$1.0 \times 10^8$</td>
<td>$5.0 \times 10^9$</td>
<td>$5.0 \times 10^{10}$</td>
</tr>
<tr>
<td>Steering angle [deg]</td>
<td>-9 to +9</td>
<td>-20 to 20</td>
<td>-20 to 20</td>
</tr>
<tr>
<td>Spacecraft weight [kg]</td>
<td>500 (200)</td>
<td>500 (200)</td>
<td>500 (200)</td>
</tr>
<tr>
<td>Thrust to mass ratio at Earth orbit [mN/kg]</td>
<td>$1.6 \times 10^{-4}$</td>
<td>$8.0 \times 10^{-3}$</td>
<td>$8.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>Initial velocity [m/s]</td>
<td>-3000, 0, 3000</td>
<td>-3000, 0, 3000</td>
<td>-3000, 0, 3000</td>
</tr>
</tbody>
</table>

Figure 7.16: Trajectories of the small-scale magnetic sail when the spacecraft escapes from the gravitational sphere of Earth at $(X, Y) = (1, 0)$ when (a) bound for Jupiter with an initial velocity of $V_y = 3000\ \text{m/s}$ and (b) bound for Mercury with an initial velocity of $V_y = -3000\ \text{m/s}$. Each line represents the spacecraft trajectory over a period of 100 years.
7.4. MISSION ANALYSIS

Figure 7.17: Trajectories of the small-scale magnetic sail when the spacecraft escapes from the gravitational sphere of Earth at \((X, Y) = (1, 0)\) when (a) bound for Jupiter with an initial velocity of \(V_y = 3000 \text{ m/s}\) and (b) bound for Mercury with an initial velocity of \(V_y = -3000 \text{ m/s}\), and steering angle set to 0 and 20 deg. The dotted line represents the spacecraft trajectory until the flyby.

thrusted vector control. That is, under the assumption of \(\gamma = 20\) deg, the magnetic sail produces tangential acceleration in addition to radial acceleration.

Figure 7.18 shows the change with time in the distance between the Sun and the spacecraft. This result shows that a small-scale magnetic sail can achieve inner planetary missions over a short period of time without consuming fuel even when the initial velocity is smaller than the velocity required for direct approach. However, the initial motivation behind magnetic sails—rapid interplanetary flight for deep space exploration—was not attained. That is, if a magnetic sail spacecraft of the same size is launched from a rocket of the same performance, an inner planet flyby can be achieved in a shorter flight time than an outer planet flyby.

Figure 7.19 shows the correlation between time of flight to achieve an inner planet flyby, thrust-mass ratio and steering angle. The time of flight becomes short in inverse proportion to both the thrust-mass ratio and steering angle. When steering angle is large, the spacecraft can arrive at Mercury even if the initial velocity is 0 km/s.

To shorten the flight time to inner planets by the limitation of coil weight, it is thus necessary to improve the controllable capacity of the thrust vector by using a larger coil as described in the above section and, to improve the thrust level of the magnetic sail, the thrust characteristics and increase in thrust offered by MPS which inflates the original magnetic field using plasma injection from the spacecraft need to be examined in three dimensions. In addition, to achieve deep space exploration by magnetic sails and MPS, it is required not only to improve thrust level but also to
Figure 7.18: Variation in eccentric distance with time (MPS with $R_{coil} = 10\text{m}$).

Figure 7.19: Relation between time of flight to achieve an inner planet flyby, thrust-mass ratio and steering angle.
optimize the spacecraft trajectory by controlling the thrust vector dynamically.

### 7.4.3 Artificial Halo Orbit

The Lagrangian points are the five positions in an orbital configuration where a small object affected only by gravity can theoretically be part of a constant-shape pattern with two larger objects such as a satellite with respect to the Earth and Sun. Figure 7.20 shows the five positions: L1 ~ L5. A halo orbit is a periodic, three-dimensional orbit near the L1, L2, or L3 Lagrange points in the three-body problem of orbital mechanics. Since it is hard to be subject to the influence of gravity, orbital maintenance is easy. A magnetic sail therefore can be used for a maneuvering system to maintain a halo orbit at the Sun-Earth Lagrangian points.

Orbit maintenance requires only low thrust (total $\Delta V$ of approximately a few meters per second per year [64]), and the required thrust generation and control of thrust direction by a small-scale magnetic sail can be achieved even with present technology. When we assumed the thrust-mass ratio: $3.3 \times 10^{-5}$ mN/kg, the total $\Delta V$ is 1.0 m/s/year.

Formation flight in a halo orbit or in interplanetary space is also candidate use for the magnetic sail.

When the thrust of the magnetic sail is always in the anti-sunward, a new periodic orbit appears around a halo orbit (Sun-Earth L1 and L2) [65]. The balance of gravity, centrifugal force, and thrust of magnetic sail on L1, L2 and L3 are represented as
Figure 7.21: Definition of distance $L_0$, $l_1$, $l_2$ and $l_3$.

\[
\begin{align*}
- \frac{G m M_{\text{sun}}}{l_1^2} + \frac{G m M_{\text{earth}}}{(L_0 - l_1)^2} + m \left( \sqrt{\frac{G (M_{\text{sun}} + M_{\text{earth}})}{L_0^3}} \right)^2 \left( l_1 - \frac{M_{\text{earth}}}{M_{\text{sun}} + M_{\text{earth}}} L_0 \right) + F_{\text{mag}} &= 0 \quad (7.4.1) \\
- \frac{G m M_{\text{sun}}}{l_2^2} - \frac{G m M_{\text{earth}}}{(L_0 - l_2)^2} + m \left( \sqrt{\frac{G (M_{\text{sun}} + M_{\text{earth}})}{L_0^3}} \right)^2 \left( l_1 - \frac{M_{\text{earth}}}{M_{\text{sun}} + M_{\text{earth}}} L_0 \right) + F_{\text{mag}} &= 0 \quad (7.4.2) \\
\frac{G m M_{\text{sun}}}{l_3^2} + \frac{G m M_{\text{earth}}}{(L_0 + l_3)^2} - m \left( \sqrt{\frac{G (M_{\text{sun}} + M_{\text{earth}})}{L_0^3}} \right)^2 \left( l_1 - \frac{M_{\text{earth}}}{M_{\text{sun}} + M_{\text{earth}}} L_0 \right) - F_{\text{mag}} &= 0 \quad (7.4.3)
\end{align*}
\]

respectively. The distance between Sun and Earth, L1, L2, L3 are defined as Fig. 7.21.

For the magnetic sail feasible with the present technology ($3.3 \times 10^{-5}$ mN/kg thrust-mass ratio), the interval between the original halo orbit and the artificial halo orbit is approximately 100 km. Table 7.5 shows calculation results of Eqs. (7.4.1), (7.4.2) and (7.4.3). Since the directions of the thrust of magnetic sail are always same direction with the centrifugal force, positions of equilibrium are moved to the side where centrifugal force becomes small, that is, near side of sun. The large-scale structure at a Lagrangian point such as a space telescope etc. can consist of formation flights using the spacecraft which carries superconductive coil and the spacecraft which is not carried superconductive coil without consuming fuel.

### 7.4.4 Missions by Future Magneto Plasma Sail

In the above section, we assumed magnetic sail feasible with the present technology. That is, 200 kg superconductive coil with the magnetic moment $M = 1.3 \times 10^7$ Wb·m. The thrust-mass ratio is only
7.5. SUMMARY

We first summarized the thrust characteristics of magnetic sail and Magneto Plasma Sail in both two-dimensions and three-dimensions obtained by Full-PIC simulation and Flux-Tube simulations. The thrust of magnetic sail is approximately proportional to the magnetic moment on the electron inertial scale and proportional to the 2/3 power of magnetic moment on larger magnetosphere. As a result, the thrust-mass ratio of magnetic sail, that is, the acceleration of the spacecraft peaks at small magnetosphere size. The dependence of the sun-spacecraft distance also differs between the electron inertial scale \((R^{-2.3})\) and MHD scale \((R^{-1.3})\). The optimized trajectory calculated by Genetic Algorithm is hence characteristic, respectively: small-scale magnetic sail once goes inside earth orbit to obtain large \(\Delta V\) and large-scale magnetic sail directly goes to outer solar system.

We also examined the realistic missions using magnetic sail and magneto plasma sail feasible with the present technology: interplanetary flight and Lagrange point mission. It was revealed
that the large-scale magnetic sail is suitable for the interplanetary missions. To adopt the small-scale magnetic sail for the interplanetary missions, the small initial velocity is required to obtain large $\Delta V$. The capability of the thrust control is also important to shorten the trip time. On the contrary, on the Lagrange points, small thrust level of magnetic sail enable to form artificial halo orbit and formation flight using magnetic sail make it possible to construct large structure on space.

For more effective thrust generating or directional control, an expansion coil needs to be inquired.
Chapter 8

Concluding Remarks

8.1 Summary and Conclusions

In the present thesis, the propulsive characteristics of Magnetic Sail and Magneto Plasma Sail were reported. The thrust system utilizing the interplanetary environment, especially the solar wind, is expected to achieve the high propulsive characteristics. However, the large computational requirement of simulation prevent from revealing the propulsive characteristics.

In chapter 1, the history of the interplanetary flight by the spacecraft and the previous researches about Magnetic Sail and Magneto Plasma Sail are introduced. The subject left behind by the previous researches, that is, the subject which this study works on, was also described.

In chapter 2, the simulation scale of magnetic sail and magneto plasma sail varies from MHD scale to electron scale and various simulation models valid in each scale are required. We developed three simulation codes adopting ideal-MHD model, Flux-Tube model and Full-PIC model. The ideal-MHD model performs the single fluid approximation and not suitable for the simulation on the ion inertial scale and the electron inertial scale. The Flux-Tube method is newly proposed by this study in order to achieve the fast and exact thrust estimation on the ion inertial scale. The validity of newly developed Flux-Tube code is confirmed by the comparison of one-dimensional simulation of magnetopause structure with theoretical analysis results. As a result, it was revealed that two structures are well in agreement. Full-PIC model, which requires large computational resource, is developed by using the parallelization techniques. As a result, the high scalability and the high efficiency simulation are achieved. The validity of the three simulation codes is also described by performing the test simulations: wave propagation and energy conservation.

In chapter 3, we have proposed a new numerical analysis method applicable for a magnetic sail using a Flux-Tube model in order to reveal thrust characteristics. The Flux-Tube model assumes a steady state for the electromagnetic field and a plasma flow including the finite Larmor effect of ions. The Flux-Tube model does not have to compute nether the time evolution of the electromagnetic field nor the particle motion. It is hence expected that the Flux-Tube model will
be able to achieve quick estimation of the thrust with lower memory capacity. As a result, the thrust generated by a magnetic sail with 500 km magnetosphere is calculated to be 1500 N for the magnetic moment parallel to the solar wind and 1640 N for the magnetic moment perpendicular to the solar wind. In the parallel case, the thrust value approximately agrees with the MHD simulation’s result, 1600 N, and the Hybrid-PIC simulation’s result, 1560 N nevertheless the Flux-Tube model neglects unsteady phenomena. Thus the Flux-Tube model is effective in estimating the thrust of the magnetic sail and reveals that unsteady phenomena such as waves do not so much affect the thrust characteristics of the magnetic sail. In addition, approximate formulas for the magnetic sail thrust in both parallel and perpendicular cases are obtained by performing simulations with various sets of parameters. Using these approximate formulas, the thrust level of the magnetic sail for various magnetospheric sizes can be quickly estimated and it is expected that the Flux-Tube model is useful for the design of a magnetic sail.

In chapter 4, in order to reveal the structure of moderately large magnetosphere \((L_{MHD}/r_iL=0.01 \sim 1)\), the Full-PIC simulation with artificial plasma parameters were performed. As a result, it was revealed that magnetosphere size obtained simulation is smaller than that calculated by MHD approximation when the magnetosphere size becomes smaller than the ion Larmor radius \((L_{MHD}/r_iL< 0.1)\). The current carrier changes from ion to electrons according to the magnetosphere size. It is expected that the smaller magnetosphere cause smaller thrust level. However, the thrust level of magnetic sail should be estimated by using the realistic plasma parameters. Both two-dimensional simulation and three-dimensional simulation by using the realistic solar wind parameters hence performed to reveal thrust characteristics of the electron inertial scale magnetic sail in next chapter. The influences of IMF on the structure of magnetosphere were also examined by each magnetosphere sizes. It was revealed that the influence of IMF becomes small as the magnetosphere size become small and can be neglected in smaller magnetosphere which is expected as the demonstrator spacecraft \((L_{MHD}/r_iL< 0.01)\). The quasi-steady state can be achieved in smaller magnetosphere even if the IMF is taken into consideration. On the contrary, it was revealed that mass ratio of ion and electron \((m_i/m_e)\) play important role on the formation of the magnetosphere.

In chapter 5, we performed three-dimensional Full-PIC simulations with the realistic plasma parameters in order to determine the thrust characteristics of small-scale magnetic sails. It was found that the thrust level of the magnetic sail is approximately proportional to the magnetic moment of the onboard coil, solar wind density, and solar wind velocity. This is quite different from large magnetic sails since particle motion is the dominant influence on the thrust generation in small-scale magnetic sails. It was also found that the attack angle and coil radius affect the steering angle and that the steering angle increases (up to a maximum of 20 deg) as the coil radius increases. Finally, we evaluated the thrust-mass ratio of the small-scale magnetic sail by using the thrust characteristics obtained from the Full-PIC simulations. The results showed that the thrust of a small-scale magnetic sail is insufficient for deep space exploration. However, the use of thrust improvement techniques such as plasma injection remains to be analyzed. In order to
achieve fast interplanetary flight for deep space exploration, increased thrust using MPS is needed, and optimization of the magnetic sail orbit through the ability to turn the coil on and off and to control the steering angle is also needed.

In chapter 6, we also performed two- and three-dimensional Full-PIC simulations in order to determine the thrust characteristics of small-scale Magneto Plasma Sails. First, we provided the one-dimensional theoretical analysis results about magnetic inflation. As a result, the four characteristic parameters: Larmor radius, plasma injection energy, kinetic $\beta$ and generalized momentum is distinguished. By using two-dimensional Full-PIC simulation, it was revealed that the increase in thrust is certainly affected by the kinetic $\beta$ of injection plasma since the diamagnetic current induced by injected plasma depends on the energy of electron. It was found that the influence of the electron kinetics is dominant in the magnetic inflation on the small-scale magneto plasma sail.

The three-dimensional simulations with various plasma injection parameters and magneto plasma sail design parameter (magnetic moment) were also performed to reveal the propulsive characteristics. The electron kinetics affects the diamagnetic current as well as the two-dimensional case. The maximum increase (thrust of MPS / thrust of magnetic sail) is 97 but the thrust gain (thrust of MPS / (thrust of magnetic sail + thrust of plasma jet)) is only 0.4 by same condition. On the contrary, the maximum thrust gain 5.2 is obtained around $(B_{jet}, r_{jet}, \beta_{jet}, \gamma_{jet}) = (10^{-7} \, \text{T}, 3000 \, \text{m}, 0.01, 1)$ and the relation of trade-off between the increase in thrust and the thrust gain is revealed.

In chapter 7, we summarized the thrust characteristics of magnetic sail and Magneto Plasma Sail in both two-dimensions and three-dimensions obtained by Full-PIC simulation and Flux-Tube simulations. The thrust of magnetic sail is approximately proportional to the magnetic moment on the electron inertial scale and proportional to the $2/3$ power of magnetic moment on larger magnetosphere. As a result, the thrust-mass ratio of magnetic sail, that is, the acceleration of the spacecraft peaks at small magnetosphere size. The dependence of the sun-spacecraft distance also differs between the electron inertial scale ($R^{2.3}$) and MHD scale ($R^{-1.3}$). The optimized trajectory calculated by Genetic Algorithm is hence characteristic, respectively: small-scale magnetic sail once goes inside earth orbit to obtain large $\Delta V$ and large-scale magnetic sail directly goes to outer solar system. We also examined the realistic missions using magnetic sail and magneto plasma sail feasible with the present technology: interplanetary flight and Lagrange point mission. It was revealed that the large-scale magnetic sail is suitable for the interplanetary missions. To adopt the small-scale magnetic sail for the interplanetary missions, the small initial velocity is required to obtain large $\Delta V$. The capability of the thrust control is also important to shorten the trip time. On the contrary, on the Lagrange points, small thrust level of magnetic sail enable to form artificial halo orbit and formation flight using magnetic sail make it possible to construct large structure on space. For more effective thrust generating or directional control, an expansion coil needs to be inquired.

In conclusion, the thrust generation of Magnetic Sail and Magneto Plasma Sail were successively revealed. Using plasma particle simulation models both on the electron inertial scale and on the
ion inertial scale, the mechanism of the thrust generation and the structure of the small-scale magnetosphere affected by the plasma kinetics were also revealed. These simulations, which become available by the improvement of the performance of supercomputer and the parallel computation techniques, enable us to obtain the propulsive characteristics such as the specific impulse, thrust-mass ratio and thrust-power ratio by assuming the spacecraft model feasible with the present technology. The mission design using the demonstrator spacecraft, by which only small thrust is available, was proposed based on the propulsive characteristics revealed in this study. Through the analysis on various scales, it was revealed that the further improvement of the propulsive characteristics is expected by technical innovation on the field of the superconductivity and the spacecraft structure. The possibility of the further research on the development of Magnetic Sail and Magneto Plasma Sail was shown by this work.

8.2 Suggestions for Future Work

Several suggestions for further improvement of the propulsive characteristics of Magnetic Sail and Magneto Plasma Sail can be made. Although the Magneto Plasma Sail achieved the high specific impulse, the thrust-mass ratio feasible with the present technology is still low compared with the existing propulsion system. It is easily improbable by making a coil into deployment structure. The improvement of superconductive technology and the examination of the cooling method of the superconductive coil are also needed. The low thrust-power ratio must be solved to realize the efficient propulsive system. The low-power ratio is mainly caused by the high injection velocity of electron. Almost same conditions we revealed as the optimal plasma injection: \((B_{jet}, r_{jet}, \beta_{jet}, \gamma_{jet}) = (10^{-7} \ T, 3000 \ m, 0.01, 1)\) can be achieved by using high plasma density inserted of the high injection velocity of electron and the higher thrust-power ratio is expected since the power of plasma injection is proportional to plasma density and square of the injection velocity. However, the shorter Debye length makes it difficult to confirm the thrust increase by a plasma injection of high deity. The introductions of Adaptive Mesh Refinement (AMR) or Macro-Micro Interlocked Simulation techniques have to be promoted.

It is hoped that the present study will contribute to further improvement of space exploration and the demining of the above suggestions.
Bibliography


Publication List

Major Publications


Presentations in International Meeting


Awards

1. 54th space science and technology conference second prize at student session, “Numerical Model for Pure Magnetic Sail,” 2010.

Appendix A

Comparison between Full-PIC simulation and Hybrid-PIC simulation

Hybrid-PIC simulation considers ion as particle and electron as mass-less fluid. The charge-neutrality is also assumed. When the magnetosphere with magnetosphere size $L_{MHD}/r_{iL} < 1$ is simulated, the magnetosphere structure does not agree with the structure obtained by Full-PIC simulation. The simulation results by Hybrid-PIC simulation (MHD scale and ion inertial scale) and Full-PIC simulation (electron inertial scale) are shown in Fig.A.1. The red dashed line represents the magnetosphere size calculated by the MHD approximation (Eq. (1.2.3)) and the black circle represents the magnetopause current, that is, the magnetosphere size obtained by simulation. On MHD scale, the magnetosphere size by simulation is in good agreement with the theoretical value. On ion inertial scale and electron inertial scale, it is expected that the magnetosphere size by simulation should become smaller than the theoretical values because of the ion kinetics. In Full-PIC simulation, this is right. However, as shown in Fig. A.1b, the magnetosphere size obtained by Hybrid-PIC simulation becomes larger than the theoretical value.

This is because the electron kinetics is neglected in Hybrid-PIC simulation. Then, we performed a theoretical analysis by assuming the charge neutrality and the mass-less electron as same as Hybrid-PIC simulation.

The momentum equation of mass-less electron is

$$\rho_e E + J_e \times B - \nabla p_e = 0$$  \hspace{1cm} (A.0.1)

Here, electron charge density $\rho_e$ and electron current density $J_e$ are

$$\rho_e = -en_e$$  \hspace{1cm} (A.0.2)

and

$$J_e = -en_e U$$  \hspace{1cm} (A.0.3)
Figure A.1: Ion density distribution and magnetic flux density distribution by Hybrid-PIC simulation: a) MHD scale and b) ion inertial scale) and Full-PIC simulation: c) electron inertial scale.
The charge neutrality \( n = n_i = n_e \) is also assumed.

The one-dimensional space along \( x \)-axis is considered. The ampere’s law is

\[
\frac{\partial B_y}{\partial x} = \mu_0 J_{ez} \tag{A.0.4}
\]

Here, it is assumed that electron mainly induces the plasma current and ion does not contribute to the plasma current on the ion inertial scale. The momentum equation (Eq. (A.0.1)) is also written as

\[
\rho_e E_x - J_{ez} B_y - \frac{\partial p_e}{\partial x} = 0 \tag{A.0.5}
\]

\[
- \rho_e \frac{\partial \phi}{\partial x} - \frac{1}{\mu_0} B_y \frac{\partial B_y}{\partial x} - \frac{\partial p_e}{\partial x} = 0 \tag{A.0.6}
\]

\[
\frac{\partial}{\partial x} \left( \rho_e \phi + \frac{B_y^2}{2\mu_0} + p_e \right) = 0 \tag{A.0.7}
\]

That is,

\[
\rho_e \phi + \frac{B_y^2}{2\mu_0} + p_e = \text{const.} \tag{A.0.8}
\]

is obtained. \( \phi \approx 0 \) is assumed since the charge neutrality is ensured. In far point from magnetosphere, magnetic flux density is \( B_y \to 0 \). Then,

\[
\frac{B_y^2(x)}{2\mu_0} + p_e(x) = p_{SW} \tag{A.0.9}
\]

is obtained. Here, \( p_{SW} \) represents the thermal pressure of solar wind. Therefore, the magnetosphere boundary is defined as the point where

\[
B_y = \sqrt{2\mu_0 p_{SW}} \tag{A.0.10}
\]

is filled when the charge neutrality and the mass-less electron are assumed. On MHD scale, the current carrier is ion and above assumption is not valid. As a result, the magnetosphere size under the same assumption with Hybrid-PIC simulation is theoretically calculated as

\[
L_{Hybrid} = \left( \frac{\mu_0 M^2}{32\pi^2 p_{SW}} \right)^{\frac{1}{8}} \tag{A.0.11}
\]

The ratio of \( L_{MHD} \) obtained MHD approximation and \( L_{Hybrid} \) is

\[
L_{Hybrid}/L_{MHD} = \left( \frac{N_{SW} m_i V_{SW}^2}{2N_{SW} k_B T_e} \right)^{\frac{1}{8}} \approx 2.3 \tag{A.0.12}
\]

in typical solar wind parameters. This result well agrees with the magnetosphere size obtained by Hybrid-PIC simulation (Fig. A.1).
As a result, it was revealed the electron kinetic should be taken into consideration in the condition of $L_{\text{MHD}}/r_iL < 1$. 
Appendix B

Comparison between Full-PIC Simulation and MHD Simulation

MHD model assumes the plasma flow as the single fluid by neglecting both ion and electron kinetics as mentioned in Chapter 2. The Hall effect is also neglected in Ideal-MHD model we developed. We examined how the magnetosphere formation is affected by these assumptions. Table B.1 shows the simulation conditions of Full-PIC model and Ideal-MHD model.

Figure B.1 shows the ion density distribution calculated by Full-PIC model and Ideal-MHD model using the artificial plasma parameter in Chapter 4 (Table 4.1). The magnetosphere size is set as $L_{MHD}/r_{iL} = 1$. The magnetosphere size obtained by Full-PIC simulation is approximately in agreement with that obtained by MHD simulation and theoretical approach (Eq. (1.2.3)). However, in MHD simulation, the Bow shock is clearly observed although the shock wave should not be able to be done when the finite Larmor motion is taken into consideration. The dawn-dusk asymmetry is observed only in Full-PIC simulation (Fig. B.1b) since the Hall-effect is neglected in Ideal-MHD model (Fig. B.1d). The equatorial ring current is also observed only in Full-PIC simulation. It is expected that the ideal frozen-in condition in MHD simulation prevents the solar wind plasma from approaching the coil center, where the magnetic flux density is high, across the magnetic field line.

The one-dimensional slice of the ion density (bold line) and the magnetic flux density (dashed line) along $z$-axis are plotted in Fig. B.2. The plasma density at bow shock of MHD simulation (-1.4$z/r_{iL}$<-1.0) is twice higher that of Full-PIC simulation. The magnetic field of MHD simu-

Table B.1: Comparison of simulation conditions between Full-PIC model and Ideal-MHD model

<table>
<thead>
<tr>
<th>Grid spacing $dx$</th>
<th>0.03$r_{iL}$</th>
<th>0.02$r_{iL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid number</td>
<td>256</td>
<td>512</td>
</tr>
<tr>
<td>Time $dt$</td>
<td>0.4$\omega_{pe}$</td>
<td>0.02$\omega_{pe}$</td>
</tr>
</tbody>
</table>
Figure B.1: Comparison of ion density distribution. \( L_{MHD}/r_{iL} = 1 \)
Figure B.2: One-dimensional slice of magnetic flux density and ion density along \( z \)-axis. \( (L_{MHD}/r_{iL} = 1) \)

Equation is compressed rapidly compared with that of Full-PIC simulation and, the magnetosphere boundary is made in the position as theoretical \( (L_{MHD}/r_{iL} = 1) \) (Eq. (1.2.3)). The position of the magnetic neutral point slightly differs between Full-PIC model \( (z/r_{iL} = 1.7) \) and Ideal-MHD model \( (z/r_{iL} = 2.0) \).

As a result, it was revealed that the finite Larmor radius of the solar wind plasma, which is taken into consideration in Full-PIC model, plays very important role on the formation of the magnetosphere even on the ion inertial scale \( (L_{MHD}/r_{iL} = 1) \). The shock formation and the induced magnetic field are especially affected by the plasma kinetics.
Appendix C

Simulation of Magneto Plasma Sail on Intermediate Scale

In the analysis in Chapter 5, the value of the magnetic moment was small \((M=1.3 \times 10^7 \sim 1.3 \times 10^{10} \text{ Wb}\cdot\text{m})\). The magnetosphere is hence corresponding to the electron inertial scale. The dependence of the increase in a thrust to the plasma injection parameters has not generalizable on the ion inertial scale yet. Therefore, we performed 3D full-PIC simulation about Magneto Plasma Sail on the ion inertial scale to reveal the scaling law of the thrust increase and the thrust gain.

We performed simulations with the artificial plasma parameters (Table 4.1) to enable Full-PIC simulation on the ion inertial scale. The magnetosphere size defined by Eq. (1.2.3) is set to \(L_{\text{MHD}}/r_{iL} = 1\). The simulation result are shown in Fig. C.1. Simulation conditions are described in Table C.1. Distributions of ion and electron are mostly in agreement, and the charge neutrality is maintained.

Figure C.2 shows the magnetic flux density along \(z\)-axis (solar wind direction). By the plasma injection, the magnetic flux density dynamically changes by the diamagnetic current. The magnetosphere size of MPS (1.8\(r_{iL}\)) is 1.6 times larger than the magnetosphere size of magnetic sail (1.1\(r_{iL}\)). Here, we defined the magnetosphere size by the point where the magnetic flux density is equal to the magnetic flux density of magnetopause \(B_{MP}\), that is \(B/B_{MP} = 1\). The thrust level is not calculated since the thrust obtained by the artificial plasma parameter is not suitable for the mission design. However, the thrust is approximately proportional to the square of the magnetosphere size. And the thrust increase by 2.6 times is expected. The plasma inflation on the ion inertial scale is demonstrated by three-dimensional Full-PIC simulation.

Next, we performed simulations with various plasma injection parameters. In order to check the validity of \(\beta_{\text{jet}}\) as the scaling parameter (see also Chapter 5), we changes the ratio of \(\beta\). In typical case of above table (Table C.1), the ratio of thermal \(\beta\) of ion and electron, and kinetic \(\beta\) of ion and electron is 1 : 1 : 0.04 : 4. We change the ratio of \(\beta\) while keeping the total \(\beta\), that is, total energy of plasma injection, constant. The result is shown in Fig. C.3. The magnetosphere size
Figure C.1: a) ion density distribution and b) electron density distribution of Magneto Plasma Sail. ($L_{MHD}/r_{iL} = 1$, artificial plasma parameters, 3D Full-PIC)

Table C.1: Normalized simulation condition of Magneto Plasma Sail on the ion inertial scale

<table>
<thead>
<tr>
<th>Solar wind</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>attack angle $\alpha$</td>
<td>90 deg</td>
</tr>
<tr>
<td>IMF $B_{MP}/4$</td>
<td></td>
</tr>
<tr>
<td>Plasma injection</td>
<td></td>
</tr>
<tr>
<td>Density $N_{jet}$</td>
<td>$150 N_{SW}$</td>
</tr>
<tr>
<td>Ion velocity $V_{jet,i}$</td>
<td>$0.08 V_{SW}$</td>
</tr>
<tr>
<td>Electron velocity $V_{jet,e}$</td>
<td>$8.3 V_{SW}$</td>
</tr>
<tr>
<td>Plasma temperature $T_{jet}$</td>
<td>$T_{SW}$</td>
</tr>
<tr>
<td>Position $R_{jet}$</td>
<td>$0.3r_{iL}$</td>
</tr>
<tr>
<td>Direction $\theta_{jet}$</td>
<td>0 deg</td>
</tr>
<tr>
<td>Computational domain</td>
<td></td>
</tr>
<tr>
<td>grid spacing $dx$</td>
<td>$0.02r_{iL}$</td>
</tr>
<tr>
<td>grid number</td>
<td>512</td>
</tr>
</tbody>
</table>
is approximately same independing on the ratio of $\beta$. That is, it is expected that approximately same thrust is obtained if the total energy of plasma injection is same.

Figure C.4 shows the simulation result with various plasma injection density. As the plasma density becomes higher, the total energy of plasma injection becomes larger. The injection point and velocity is same with the typical case. In the maximum case ($N_{jet}/N_{SW} = 600$), the magnetosphere size ($4r_{iL}$) is 3.6 times larger than that of the magnetic sail. In the minimum case ($N_{jet}/N_{SW} = 37.5$), the magnetosphere size ($2.5r_{iL}$) is 2.3 times larger than that of the magnetic sail. This is larger than the magnetosphere size of the typical case ($1.8r_{iL}$). As shown in Fig. C.5a, the magnetosphere size is not a monotone change to the plasma injection density. The plasma injection density is approximately proportional to the mass flow rate. Hence, the plasma inflation on the ion inertial scale differs from the plasma inflation on the electron inertial scale (Fig. 6.20). Or restrictions of the parameter which can be set in Full-PIC simulation may be the cause. In three-dimensional Full-PIC simulation with realistic plasma parameter, the high plasma injection density cannot be used by the restriction of the Debye length. Hence, the plasma density on the electron scale is lower than that of ion inertial scale as shown in Fig. C.5b.

Plasma injection point is also varied from $R_{jet}/r_{iL} = 0.1$ to 0.5 by 5 steps. The simulation results are shown in Fig. C.6. Since the total energy of plasma injection is set constant, the plasma density is becomes lower as the plasma injection point becomes far from the center. In the maximum case ($R_{jet}/r_{iL} = 0.5$), the magnetosphere size ($2r_{iL}$) is 1.8 times larger than that of the magnetic sail. In the minimum case ($R_{jet}/r_{iL} = 0.1$), the magnetosphere size ($3.2r_{iL}$) is 2.9 times larger than that of the magnetic sail. As shown in Fig. C.7a, the magnetosphere size
Figure C.3: One-dimensional distribution of magnetic flux density with various plasma $\beta$. ($L_{MHD}/r_{iL} = 1$, artificial plasma parameters, 3D Full-PIC)

Figure C.4: One-dimensional distribution of magnetic flux density with various plasma injection density. ($L_{MHD}/r_{iL} = 1$, artificial plasma parameters, 3D Full-PIC)
Figure C.5: Magnetosphere size with various plasma injection density: a) ion inertial scale and b) electron inertial scale to ion scale.
APPENDIX C. SIMULATION OF MAGNETO PLASMA SAIL ON INTERMEDIATE SCALE

Figure C.6: One-dimensional distribution of magnetic flux density with plasma injection point. \( (L_{MHD}/r_{iL} = 1, \text{artificial plasma parameters, 3D Full-PIC}) \)

becomes minimum at \( R_{jet}/r_{iL} = 0.3 \). Figure C.7b represents simulation results of both ion inertial scale and electron inertial scale. The horizontal axis is changed from \( R_{jet}/r_{iL} \) to \( R_{jet}/L_{mag} \). On the ion inertial scale and the electron inertial scale, the increase in magnetosphere size has a reverse tendency. It is expected that the change of parameters other than the injection point influence the increase in the magnetosphere size.

The magnetosphere size of original magnetic sail is then varied from \( L_{MHD}/r_{iL} = 0.01 \) to 1. The plasma injection point is constant in \( R_{jet}/L_{mag} = 0.3 \). Hence, the magnetic flux density at the injection point is same in all cases. The plasma injection velocity is also set constant. Therefore, the Larmor radius of injected plasma at injection point is same. The plasma density is also same by \( N_{jet}/N_{SW} = 75 \). The simulation result is shown in Fig. C.8. The horizontal axis and the vertical axis represent the magnetosphere size \( L_{MHD}/r_{iL} \) and, the thrust increase and the thrust gain expected by the increase in the magnetosphere. As shown in Fig. C.8a, the increase in thrust is large in smaller magnetosphere size. This is because the larger diamagnetic current can be induced in smaller magnetosphere by the larger finite Larmor radius effect. Since the magnetic flux density and the injection velocity are same, the ratio of the Larmor radius of injected plasma and the magnetosphere size is becomes larger as the magnetosphere size becomes smaller.

On the contrary, the thrust gain is large in large magnetosphere as shown in Fig. C.8b. This is because the thrust corresponding to the plasma injection becomes larger in smaller magnetosphere. The thrust of the magnetic sail is proportional to the magnetic moment \( M \) as revealed in Chapter 5 on the electron inertial scale. From Eq. (1.2.3), the magnetic moment is proportional to the 3rd power of the magnetosphere size. As a result, the thrust of the magnetic sail is also proportional
Figure C.7: Magnetosphere size with various plasma injection point: a) ion inertial scale and b) electron inertial scale to ion scale.
Figure C.8: a) Thrust increase and b) thrust gain with various magnetosphere size. (artificial plasma parameters, 3D Full-PIC)

to the 3rd power of the magnetosphere size:

\[ F_{mag} \propto M \propto L^3 \]  \hspace{1cm} (C.0.1)

On the other hand, the thrust of the plasma injection is proportional to the square of the magnetosphere size:

\[ F_{jet} \propto L^2 \]  \hspace{1cm} (C.0.2)

since the mass flow rate is calculated as Eq. (6.2.3). Here, \( N_{jet} = \text{const}, Vol_{inj} \propto L^3, \Delta t \propto L \). Hence, the thrust of plasma injection becomes dominant in small magnetosphere and thrust gain becomes smaller in spite of the large increase in thrust.

Thus, we examined the magnetic inflation on the ion inertial scale and obtained results approximately agreed with the results on the electron inertial scale. The trade-off relation between
the thrust increase and the thrust gain is also in agreement by simulations in Chapter 6. The optimized design of MPS is required about both the plasma injection parameters and the magnetic moment of onboard coil.
Appendix D

Development of Macro-Micro Interlocked Simulation

The plasma density of injected plasma for a magnetic inflation is much higher than the typical solar wind density. The shorter Debye length and more particles in Magneto Plasma Sail simulation require larger computational resource than that in Magnetic Sail simulation. To reduce the computational resource, the introduction of the nested grid and Adaptive Mesh Refinement (AMR) techniques to the Magneto Plasma Sail simulation have been examined. On the other hand, the part of the injected plasma (small Larmor radius and small Debye length) can be assumed to be a fluid. We hence developed simulation model connecting the Macro model (MHD model) and the Micro model (Full-PIC model).

The way coupling MHD model and Full-PIC model can be classified as shown in Table D.1. Using time dependent two-way coupling model, the self consistent simulation with the high density plasma injection can be performed while reducing the computational effort. However, we first developed the time independent one-way coupling model from MHD model to Full-PIC model since there is little correction of the existing codes.

The calculation flow of one-way coupling model is shown in Fig. D.1. On the time independent one-way coupling model, the solar wind flow is first calculated by MHD model. The number

<table>
<thead>
<tr>
<th>Table D.1: Summary of Macro-Micro interlocked models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time independ</td>
</tr>
<tr>
<td>One-way (MHD → Full-PIC)</td>
</tr>
<tr>
<td>One-way (Full-PIC → MHD)</td>
</tr>
<tr>
<td>Two-way (MHD ↔ Full-PIC)</td>
</tr>
</tbody>
</table>
of particles generated in the computational domain depends on the plasma density obtained by MHD simulation. The velocity of each particles are determined by the static pressure and velocity obtained by MHD simulation. Here, we assumed that bulk velocity of ion and electron is same and the thermal distribution follows Maxwell distribution.

First, we simulated the magnetosphere with $L_{MHD}/r_{iL} = 1$ using artificial plasma parameter in Chapter 4 (Table 4.1). Figure D.2 shows the results of plasma particle generation from the MHD model. The density distribution in Full-PIC model becomes noisy compared with the MHD model since the initial position of particles is determined by the random number.

Figure D.3 represents the comparisons between Full-PIC model and one-way coupling model. The ion density, stream lines and current density of one-way coupling model are in agreement with those of Full-PIC model, respectively. The thrust levels calculated by Full-PIC model and one-way coupling model are also in agreement. Thus, the one-way coupling model can simulate the plasma flow around magnetic sail correctly. The calculation time spend for the calculation of the steady state is almost same since the high plasma density such as plasma injection is not consider in this
Figure D.2: Plasma density distribution: a) MHD model (initial condition) and b) Full-PIC model (after particle generation). ($L_{MHD}/r_L = 1$, artificial plasma parameter)

test simulation.

Thus, the initial test of the one-way coupling model is successfully performed and the magnetosphere formation is reproduced. As next step, the thrust generation of MPS should be simulated by one-way coupling model to increase the calculation parameters in which analysis of MPS is possible.
Figure D.3: Comparisons between Full-PIC model and one-way coupling model.