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Scalar gravitational waves from relativistic stars in scalar-tensor gravity

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Unlike general relativity, scalar gravitational waves can be excited due to the radial oscillations in scalar-tensor gravity. To examine scalar gravitational waves in scalar-tensor gravity, we derive the evolution equations of the radial oscillations of neutron stars and determine the specific oscillation frequencies of the matter oscillations and scalar gravitational waves, where we adopt two different numerical approaches, i.e., the mode analysis and direct time evolution. As a result, we observe the spontaneous scalarization even in the radial oscillations. Depending on the background scalar field and coupling constant, the total energy radiated by the scalar gravitational waves dramatically changes, where the specific oscillation frequencies are completely the same as the matter oscillations. That is, via the direct observations of scalar gravitational waves, one cannot only reveal the gravitational theory but also extract the radial oscillations of neutron stars.

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I. INTRODUCTION

Since general relativity has been proposed, many experiments have been performed to verify the gravitational theory. Most of these attempts are done in the weak gravitational field such as our Solar System, but nothing indicates the failure of general relativity. On the other hand, since the astronomical observations in the strong gravitational field are very poor, the gravitational theory in such a strong-field regime could be still unconstrained. That is, the gravitational theory to describe the phenomena in the strong-field regime might be different from general relativity, and it might be possible for one to probe the gravitational theory via the observations of the deviation from general relativity. In practice, up to now there have been many suggestions to observationally test the gravitational theory in the strong-field regime [1–3]. The technology is developing more and more, which will enable us to accurately observe the phenomena in the strong-field regime. It might be possible to use these new observations as tests of gravitational theory.

So far, many alternative gravitational theories have been proposed. Among them, scalar-tensor theory is the one of the simplest alternative gravitational theories, which must be a natural extension of standard general relativity [4]. One of the motivations to consider scalar-tensor gravity is that this theory can be obtained in the low energy limit of string and/or other gauge theories. In scalar-tensor gravity, the scalar field plays an essential role in addition to the usual tensor field in general relativity, where the matter field is described by using the effective metric $\tilde{g}_{\mu\nu}$ associated with the scalar and gravitational fields, φ and $g_{*\mu\nu}$, via the conformal transformation, i.e., $\tilde{g}_{\mu\nu} = A^2(\varphi) g_{*\mu\nu}$. The Brans-Dicke theory [5] is the simplest version of scalar-tensor gravity, where $A(\varphi)$ is defined as $A(\varphi) = \exp(\alpha\varphi)$. The coupling parameter α can be associated with the Brans-Dicke parameter $\omega_{\rm BD}$ as $\alpha^2 = 1/(2\omega_{\rm BD} + 3)$, which is constrained through the Solar System experiments, i.e., $\omega_{\rm BD} \gtrsim 40{,}000$, which corresponds to $\alpha < 10^{-5}$ [6]. Within this restriction on the coupling parameter, it is almost impossible to predict a large deviation from general relativity in the strong-field regime.

A different functional form of the conformal factor is also suggested by Damour and Esposito-Farèse [7,8], where $A(\varphi) \equiv \exp(\alpha\varphi + \beta\varphi^2/2)$. With this type of coupling, even if α is almost zero, the relativistic stellar models in scalar-tensor gravity can significantly deviate from the predictions in general relativity. Additionally, they found that the stellar models in scalar-tensor gravity suddenly deviate from those in general relativity for the specific values of coupling parameters, which is referred to as *spontaneous scalarization*. With respect to this phenomenon, Harada systematically examined with the technique of catastrophe theory and found that the spontaneous scalarization can happen for $\beta \lesssim -4.35$ [9]. Recently, it was found that spontaneous scalarization was possible for larger values of β in fast rotating relativistic stars [10] and in the neutron star binary system [11–13]. On the other hand, using the observations of a pulsar white dwarf binary, Freire *et al*. set a severe constraint on β, i.e., $\beta \gtrsim -5$ [14]. Additionally, it was reported that β could be constrained to be larger than -4.5, depending on the equation of state [12]. Maybe, although the constraint on β would become severer via the future observations, so we focus on the range of $\beta \gtrsim -5$ in this paper.

The several attempts to observationally distinguish scalar-tensor gravity from general relativity have been done already in the past by using the redshift in the absorption lines of the x and γ rays emitted from the stellar surface [15], the spectrum of the gravitational waves radiated from relativistic stars [16,17], and the rotational effect around compact objects [18]. In this paper, we consider a different approach, i.e., scalar gravitational waves driven by radial oscillations. In fact, gravitational waves cannot be excited due to the radial oscillations in general relativity. This means that the detection of scalar gravitational waves itself becomes proof the existence of the scalar field. From the observational point of view, if the scalar gravitational waves exist, one could, in principle, identify the scalar gravitational waves with more than three gravitational wave detectors, because we have only three degrees of polarizations in scalar-tensor gravity, such as the two usual tensor gravitational waves and a scalar gravitational wave. In fact, we expect that five gravitational wave detectors will be in operation in the future, such as two advanced LIGOs [19] and an advanced Virgo [20], KAGRA [21], and IndIGO [22]. On the other hand, the method to separate and reconstruct an arbitrary number of polarization modes by using the observational data from multiple interferometric gravitational wave detectors is also developing, which is a model-independent approach [23]. It should be noticed that the scalar gravitational waves in the physical frame are proportional to the cosmological value of the scalar field [11,12], whose value must be quite small. That is, if the scalar gravitational waves exist, they might be quite weak and difficult to detect in the current gravitational wave detectors.

The radial oscillations of relativistic stars in general relativity have been examined since early times [24–27] in the context of the stability analysis. Meanwhile, the scalar gravitational waves in scalar-tensor gravity were also examined in the black hole formation due to the dust collapse [28–30] and the test particle around a Kerr black hole [31]. Anyway, this is the first time the scalar gravitational waves driven by the stellar radial oscillations in scalar-tensor gravity suggested by Damour and Esposito-Farèse [7,8] have been calculated. For this purpose, we will derive the perturbation equations of the radial oscillations and make numerical calculations to examine it.

This paper is organized as follows. In the Sec. II, we briefly mention the equilibrium of nonrotating relativistic stars in scalar-tensor gravity. In Sec. III, we derive the perturbation equations describing the radial oscillations of relativistic stars in scalar-tensor gravity. The numerical results are shown in Sec. IV, where the specific frequencies are determined with the mode analysis and the direct time evolution. Then, we conclude in Sec. V. We adopt the geometric units, $c = G_* = 1$, where c and G_* denote the speed of light and the gravitational constant, respectively, and use the metric signature $(-,+,+,+)$.

II. STELLAR MODELS IN SCALAR-TENSOR GRAVITY

In this paper, we consider the neutron star models in scalar-tensor theory of gravity with one scalar field. In fact, this is a natural extension of general relativity, where gravity is mediated not only by a usual tensor field but also by a massless long-range scalar field. To express such a theory, the total action in the Einstein frame is given by Ref. [4],

$$S = \frac{1}{16\pi G_*}\int \sqrt{-g_*}(R_* - 2g_*^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu})d^4x + S_m[\Psi_m, A^2(\varphi)g_{*\mu\nu}], \tag{2.1}$$

where G_* is the bare gravitational constant, R_* is the scalar curvature determined by the Einstein metric $g_{*\mu\nu}$, φ is the scalar field, and Ψ_m represents, collectively, all matter fields. The metric tensor in the Einstein frame $g_{*\mu\nu}$ is related to that in the physical frame (or Jordan-Fierz frame) $\tilde{g}_{\mu\nu}$ as

$$\tilde{g}_{\mu\nu} = A^2(\varphi)g_{*\mu\nu}. \tag{2.2}$$

Hereafter, in order to clarify the frame, the quantities in the physical frame are denoted by a tilde and those in the Einstein frame are denoted by an asterisk. We remark that the field equations are usually formulated in the Einstein frame, but all nongravitational experiments are observed in the physical frame.

Varying the total action S, one can get the field equations in the Einstein frame for the tensor and scalar fields,

$$G_{*\mu\nu} = 8\pi G_* T_{*\mu\nu} + T^{(\varphi)}_{*\mu\nu}, \tag{2.3}$$

$$\Box_*\varphi = -4\pi G_*\alpha(\varphi)T_*, \tag{2.4}$$

where $T_{*\mu\nu}$ is the energy-momentum tensor of the fluid in the Einstein frame, while $T^{(\varphi)}_{*\mu\nu}$ denotes the energy-momentum tensor of the massless scalar field, i.e.,

$$T^{(\varphi)}_{*\mu\nu} \equiv 2\varphi_{,\mu}\varphi_{,\nu} - g_{*\mu\nu}g_*^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta}. \tag{2.5}$$

$T_{*\mu\nu}$ is associated with the energy-momentum tensor in the physical frame $\tilde{T}_{\mu\nu}$ as

$$T_*^{\mu\nu} \equiv \frac{2}{\sqrt{-g_*}}\frac{\delta S_m}{\delta g_{*\mu\nu}} = A^6(\varphi)\tilde{T}^{\mu\nu}. \tag{2.6}$$

In Eq. (2.4), the scalar quantities T_* and $\alpha(\varphi)$ are defined as $T_* \equiv T_*^{\mu\nu}g_{*\mu\nu}$ and $\alpha(\varphi) \equiv d\ln A(\varphi)/d\varphi$. Since $\alpha(\varphi)$ obviously relates a scalar field to matter, the theory with $\alpha(\varphi) = 0$ exactly reduces to general relativity. In addition to the field equations, the law of energy-momentum conservation is given as $\tilde{\nabla}_\nu\tilde{T}_\mu{}^\nu = 0$ in the physical frame, which is transformed into that in the Einstein frame, such as

$$\nabla_{*\nu}T_{*\mu}{}^{\nu} = \alpha(\varphi)T_*\nabla_{*\mu}\varphi. \tag{2.7}$$

In this paper, we adopt the same form of conformal factor $A(\varphi)$ as Damour and Esposito-Farése [7], i.e., $A(\varphi) = \exp(\beta\varphi^2/2)$, where β is a real number. With this

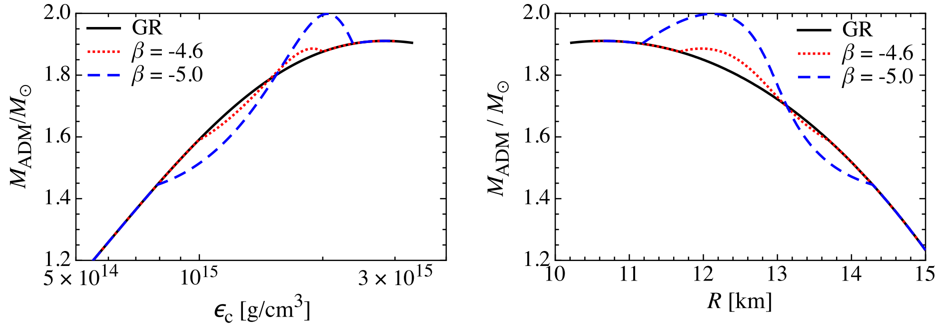

FIG. 1 (color online). Neutron star models in scalar-tensor gravity with $\beta = -5.0$ (broken lines) and $\beta = -4.6$ (dotted lines) and in general relativity with the solid lines. The left and right panels correspond to the ADM masses of neutron stars as functions of the central density and stellar radius, respectively.

conformal factor, the quantity $\alpha(\varphi)$ is expressed as $\alpha(\varphi) = \beta\varphi$, i.e., the theory with $\beta = 0$ agrees with general relativity. At last, we set φ_0 as the cosmological value of the scalar field at infinity. In particular, we adopt $\varphi_0 = 0$ in this paper.

Now, we consider the neutron star models in scalar-tensor theory which are constructed with a perfect fluid of cold degenerate matter. The metric for nonrotating, spherically symmetric neutron star models can be described as

$$\begin{aligned} ds_*^2 &= g_{*\mu\nu}dx^\mu dx^\nu \\ &= -e^{2\Phi}dt^2 + e^{2\Lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \end{aligned} \tag{2.8}$$

where Φ and Λ are functions of r, and $e^{2\Lambda}$ is associated with the mass function $\mu(r)$ as $e^{-2\Lambda} = 1 - 2\mu(r)/r$. With respect to the matter, we assume the perfect fluid

$$\tilde{T}_{\mu\nu} = (\tilde{p} + \tilde{\epsilon})\tilde{u}_\mu\tilde{u}_\nu + \tilde{p}\tilde{g}_{\mu\nu}, \tag{2.9}$$

where $\tilde{u}_\mu$, $\tilde{p}$, and $\tilde{\epsilon}$ are the four-velocity of the fluid, pressure, and total energy density in the physical frame. In particular, the four-velocity of equilibrium neutron star models is given by

$$\tilde{u}^\mu = (A^{-1}e^{-\Phi}, 0, 0, 0). \tag{2.10}$$

Then, the equilibrium models are determined by integrating the Tolman-Oppenheimer-Volkoff equations in scalar-tensor theory [7,9,16], assuming the relation between the pressure and energy density, i.e., equation of state (EOS). In this paper, we adopt the polytrope EOS, $\tilde{p} = K\tilde{\epsilon}^\Gamma$, where we especially fix that $K = 200\ \text{km}^2$ and $\Gamma = 2$. In Fig. 1, we show the neutron star models in scalar-tensor gravity with $\beta = -5.0$ (broken line) and $\beta = -4.6$ (dotted line) together with the case in general relativity (solid line), where the left and right panels correspond to the Arnowitt-Deser-Misner (ADM) masses of neutron stars as functions of the central density $\tilde{\epsilon}_c$ and stellar radius R, respectively. Additionally, the central values of the scalar field are shown in Fig. 2 as a function of the central density. We remark that, as mentioned before, the realistic value of β might be larger than -4.5, and one could have a chance to observe the scalarization even with $\beta = -4.2$ in the rapidly rotating neutron stars and/or in neutron star binaries [10–13]. But, in the case of spherically symmetric stars, one can observe the scalarization only for $\beta - 4.35$ [9], i.e., the stellar models with, for example, $\beta = -4.0$ or -4.2 are completely equivalent to those in general relativity, where no scalar perturbation is induced by the matter motion. So, in this paper, we especially consider the stellar models with $\beta = -4.6$ and -5.0 to examine the stellar oscillations and induced scalar perturbations.

III. RADIAL OSCILLATIONS

In this section, we consider the radial oscillations on the stellar models mentioned in the previous section, adopting the relativistic Cowling approximation. The scalar field is described as

$$\varphi = \varphi^{(\mathrm{B})} + \delta\varphi(t, r), \tag{3.1}$$

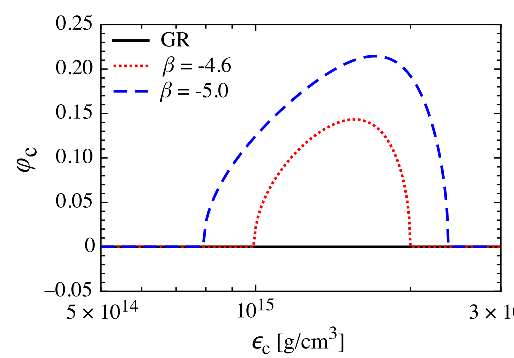

FIG. 2 (color online). Central values of the scalar field φ_c as a function of the central density, where the broken and dotted lines correspond to the neutron star models in scalar-tensor gravity with $\beta = -5.0$ and -4.6, while the solid line corresponds to that in general relativity.

where $\varphi^{(\mathrm{B})}$ denotes the nonperturbed scalar field, while $\delta\varphi$ corresponds to the perturbations of scalar field. With the Cowling approximation, the metric perturbations in the physical frame are neglected, i.e., $\delta\tilde{h}_{\mu\nu} = 0$. However, the metric perturbations in the Einstein frame $h_{*\mu\nu}$ can be induced by the perturbation of the scalar field even with the Cowling approximation, which is given by $h_{*\mu\nu} = -2g_{*\mu\nu}\delta A/A$. So, the metric perturbations in the Einstein frame are expressed as

$$h_{*tt} = 2\beta\varphi e^{2\Phi}\delta\varphi, \tag{3.2}$$

$$h_{*rr} = -2\beta\varphi e^{2\Lambda}\delta\varphi, \tag{3.3}$$

$$h_{*\theta\theta} = -2\beta\varphi r^2\delta\varphi, \tag{3.4}$$

$$h_{*\phi\phi} = -2\beta\varphi r^2\sin^2\theta\delta\varphi, \tag{3.5}$$

and the other components are zero, where we use $\delta A/A = \beta\varphi\delta\varphi(t,r)$. We remark that the Cowling approximation in general relativity is a well-defined approach where only fluid dynamics is allowed. But, in scalar-tensor gravity, the fluid motion excites the variation of the scalar fields via Eq. (2.4). So, to examine how the scalar fields are affected by the fluid motion, in this paper, we consider only the variations of matter and scalar fields with fixing the physical metric. That is, we assume that the variations of matter and scalar fields hardly affect the physical metric. Of course, one can examine only fluid dynamics by fixing the scalar field and physical metric as in Ref. [16], which may correspond to the standard Cowling approximation. On the other hand, we should consider the full linearized problem including the metric perturbations in the future, as in Ref. [17].

The fluid perturbations are described by the Lagrangian displacement vector

$$\tilde{\xi}^i = (W, 0, 0), \tag{3.6}$$

where W is a function of t and r. Then, the perturbed four-velocity $\delta\tilde{u}^\mu$ has the form

$$\delta\tilde{u}^\mu = \frac{1}{Ae^\Phi}\left(0, \frac{\partial W}{\partial t}, 0, 0\right). \tag{3.7}$$

At last, the pressure and energy density perturbations are described as

$$\tilde{p} = \tilde{p}^{(\mathrm{B})} + \delta\tilde{p}(t,r), \tag{3.8}$$

$$\tilde{\epsilon} = \tilde{\epsilon}^{(\mathrm{B})} + \delta\tilde{\epsilon}(t,r), \tag{3.9}$$

where $\tilde{p}^{(\mathrm{B})}$ and $\tilde{\epsilon}^{(\mathrm{B})}$ denote the pressure and energy density in the equilibrium stellar models, while $\delta\tilde{p}$ and $\delta\tilde{\epsilon}$ denote the pressure and energy density perturbations, respectively.

With the above perturbation variables, the perturbation equations are derived by taking the variation of Eqs. (2.4) and (2.7):

$$\delta(\Box_*\varphi) = -4\pi G_*\delta[\alpha(\varphi)T_*], \tag{3.10}$$

$$\delta(\nabla_{*\nu}T_{*\mu}{}^{\nu}) = \beta\delta[\varphi T_*\nabla_{*\mu}\varphi]. \tag{3.11}$$

From Eq. (3.10), one can get the evolution equation with respect to the scalar field,

$$\begin{aligned}
&-e^{-2\Phi+2\Lambda}\delta\ddot{\varphi}+\delta\varphi''+(\eta-2\beta\varphi\Psi)\delta\varphi' \\
&\quad+2\beta[\varphi\Psi'+\varphi\Psi\eta-\Psi^2-2\pi G_*A^4e^{2\Lambda}(\tilde{\epsilon}-3\tilde{p})(1+4\beta\varphi^2)]\delta\varphi \\
&=4\pi G_*\beta A^4e^{2\Lambda}\varphi(\delta\tilde{\epsilon}-3\delta\tilde{p}),
\end{aligned} \tag{3.12}$$

where the dot and prime denote the partial derivative with respect to t and r, respectively, while $\Psi = \varphi'$ and $\eta = \Phi' - \Lambda' + 2/r$. On the other hand, from Eq. (3.11), one can get the perturbation equations as

$$\ddot{W} = -\frac{1}{\tilde{p}+\tilde{\epsilon}}e^{2\Phi-2\Lambda}[\delta\tilde{p}' + (\Phi' + \beta\varphi\Psi)(\delta\tilde{p} + \delta\tilde{\epsilon})], \tag{3.13}$$

$$\delta\tilde{\epsilon} = -(\tilde{p}+\tilde{\epsilon})W' - \left[\tilde{\epsilon}' + (\tilde{p}+\tilde{\epsilon})\left(\Lambda' + \frac{2}{r} + 3\beta\varphi\Psi\right)\right]W, \tag{3.14}$$

where we use the Tolman-Oppenheimer-Volkoff equation, i.e., $\tilde{p}' = -(\tilde{p}+\tilde{\epsilon})(\Phi' + \beta\varphi\Psi)$, to derive the above equations. In addition to the evolution equations, one can show the relation between $\delta\tilde{p}$ and $\delta\tilde{\epsilon}$ as $\delta\tilde{p} = c_s^2\delta\tilde{\epsilon}$, where c_s denotes the sound speed defined as $c_s^2 = \partial\tilde{p}/\partial\tilde{\epsilon}$. Consequently, using Eqs. (3.13) and (3.14), one can derive the evolution equation for W:

$$\begin{aligned}
&-e^{-2\Phi+2\Lambda}\ddot{W} + c_s^2W'' \\
&\quad+\left[2c_sc_s' - \Phi' - \beta\varphi\Psi + c_s^2\left(\Lambda' + \frac{2}{r} + 3\beta\varphi\Psi\right)\right]W' \\
&\quad+\left[2c_sc_s'\left(\Lambda' + \frac{2}{r} + 3\beta\varphi\Psi\right) - \Phi'' - \beta\Psi^2 - \beta\varphi\Psi'\right. \\
&\quad\left.+ c_s^2\left(\Lambda'' - \frac{2}{r^2} + 3\beta\Psi^2 + 3\beta\varphi\Psi'\right)\right]W = 0.
\end{aligned} \tag{3.15}$$

In order to calculate the evolutions of the variables $\delta\varphi$ and W, one should impose the appropriate boundary conditions. That is, the scalar gravitational wave $\delta\varphi$ becomes the only outgoing wave at spatial infinity, while the perturbation variables should be regular in the vicinity of the stellar center. The regularity conditions near the center can be written as $W = W_cr$ and $\delta\varphi = \delta\varphi_c$, where W_c and $\delta\varphi_c$ are some constants. Additionally, the Lagrangian perturbation of pressure should vanish at the stellar surface.

This condition leads to the boundary conditions at the stellar surface as

$$W' + \left(\Lambda' + \frac{2}{R} + 3\beta\varphi\Psi\right)W = 0. \tag{3.16}$$

IV. NUMERICAL RESULTS

As shown in the previous section, the matter can oscillate independently of the oscillations of the scalar field, i.e., the fluid oscillations depend only on the background scalar field. So, we consider the fluid oscillations before examining the scalar gravitational waves induced by the matter oscillations in Sec. IV A and then we examine the details of the scalar gravitational waves in Sec. IV B.

A. Fluid oscillations

Assuming a harmonic dependence of time, such as $W(t,r) = W(r)e^{i\omega t}$, the evolution equation for W [Eq. (3.15)] can be reduced to

$$\begin{aligned} c_s^2 W'' &+ \left[2c_s c_s' - \Phi' - \beta\varphi\Psi + c_s^2\left(\Lambda' + \frac{2}{r} + 3\beta\varphi\Psi\right)\right]W' \\ &+ \left[\omega^2 e^{-2\Phi+2\Lambda} + 2c_s c_s'\left(\Lambda' + \frac{2}{r} + 3\beta\varphi\Psi\right) - \Phi'' - \beta\Psi^2 \right. \\ &\left. - \beta\varphi\Psi' + c_s^2\left(\Lambda'' - \frac{2}{r^2} + 3\beta\Psi^2 + 3\beta\varphi\Psi'\right)\right]W = 0. \end{aligned} \tag{4.1}$$

With the boundary conditions at the stellar center and surface together with the normalization condition, the problem to solve becomes the eigenvalue problem with respect to the eigenfrequency ω. In particular, we adopt $W_c = 1$ at the stellar center as the normalization condition. We remark that, in general, one can accurately determine the eigenfrequencies with the mode analysis adopted here, compared with the time domain analysis.

In Figs. 3 and 4, the frequencies of the fundamental and first overtone radial oscillations are shown as functions of the ADM mass and stellar compactness in scalar-tensor gravity with $\beta = -5.0$ (broken lines) and -4.6 (dotted lines) together with the results in general relativity (solid lines). In these figures, the frequencies f_i are determined as $f_i = \omega_i/2\pi$, where i denotes the nodal number of eigenoscillation. From these figures, one can recognize that the frequencies of the radial oscillations in scalar-tensor gravity are affected by the existence of a background scalar field, and one can obviously see the spontaneous scalarization even in the frequencies of the radial oscillations. That is, one might be able to distinguish the gravitational theory via the observation of radial oscillations with the help of the other observations of the ADM mass or stellar compactness. We remark that the stellar compactness could be determined by the observation of the gravitational redshift of the absorption line radiated from the stellar surface. In practice, if the radial oscillations would be excited in the magnetized neutron stars, the hydromagnetic waves could

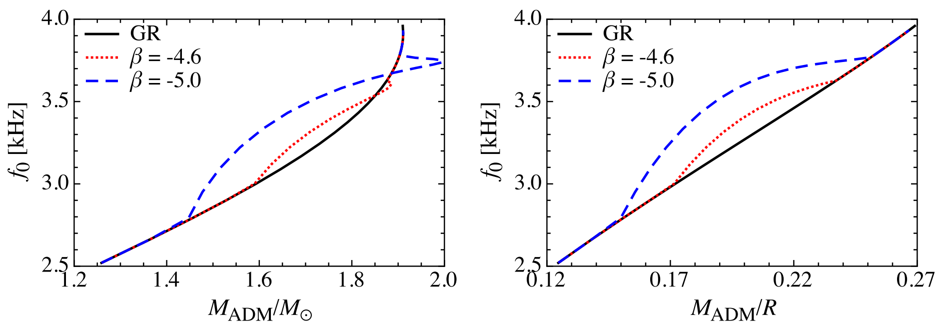

FIG. 3 (color online). Frequencies of fundamental radial oscillations in scalar-tensor gravity with $\beta = -5.0$ (broken lines) and -4.6 (dotted lines), where the left and right panels are functions of ADM mass and stellar compactness, respectively. In addition to the frequencies in scalar-tensor gravity, the results in general relativity are also shown with solid lines.

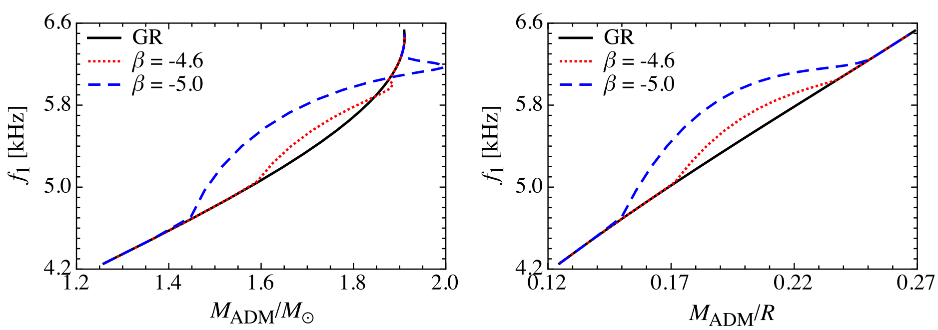

FIG. 4 (color online). Same as in Fig. 3 but for the frequencies of first overtone radial oscillations.

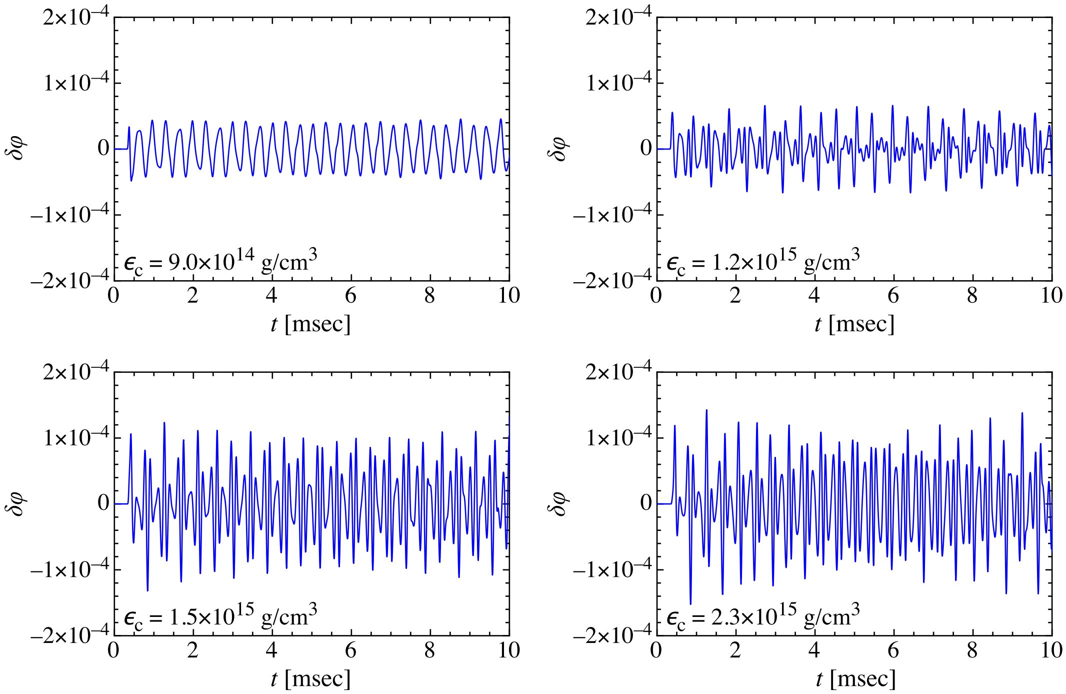

FIG. 5 (color online). Waveforms of scalar gravitational waves for $\beta = -5.0$ emitted from the neutron stars constructed with $\epsilon_c = 9.0 \times 10^{14}$ g/cm^3 (upper left panel), 1.2×10^{15} g/cm^3 (upper right panel), 1.5×10^{15} g/cm^3 (lower left panel), and 2.3×10^{15} g/cm^3 (lower right panel).

be generated also, which might become the source of the radiations of electromagnetic waves [32,33]. If so, it might be possible for one to detect the imprint of radial oscillations with the electromagnetic waves, which could enable us to distinguish the gravitational theory.

B. Scalar gravitational waves

Now, we examine the scalar gravitational waves radiated from the neutron stars by calculating the time evolution of Eq. (3.12) directly. To systematically examine the scalar gravitational waves induced by the matter oscillations, we consider the zero scalar gravitational wave initially and put the initial distribution of matter displacement $W_0(r)$ given by

$$W_0(r) = w\left(\frac{r}{R}\right)\left(\frac{r-R}{R}\right)^2, \tag{4.2}$$

where w is a constant. Then, we determine the value of w in such a way that the initial energy due to the matter oscillations E_0 is fixed especially to $E_0 = 10^{-4} M_\odot$, where E_0 is defined as

$$E_0 = \frac{1}{2}\int \frac{\delta\tilde{p}}{\tilde{\epsilon}}\delta\tilde{\epsilon}^* d^3x. \tag{4.3}$$

The waveforms of scalar gravitational waves driven by the matter oscillations can be calculated as shown in Fig. 5

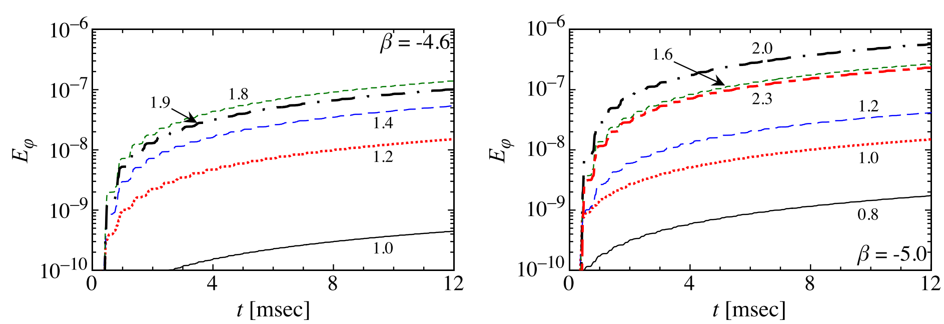

FIG. 6 (color online). Time evolutions of radiated energy of scalar gravitational waves for the different stellar models with $\beta = -4.6$ (left panel) and -5.0 (right panel), where the adopted central densities are shown on each line in units of 10^{15} g/cm^3.

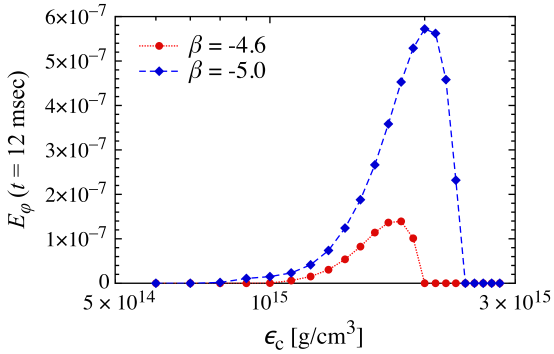

FIG. 7 (color online). The total energy radiated by scalar gravitational waves until $t = 12$ msec as a function of the stellar central density, where the dotted and broken lines correspond to the results for $\beta = -4.6$ and -5.0, respectively.

for $\beta = -5.0$, where the upper left, upper right, lower left, and lower right panels correspond to the waveforms radiated from the neutron stars constructed with $\epsilon_c = 9.0\times10^{14}$, 1.2×10^{15}, 1.5×10^{15}, and $2.3\times10^{15}\,\mathrm{g/cm^3}$, respectively. From these panels, one can see that the scalar gravitational waves are excited if the background scalar field is nonzero, where the amplitudes of the scalar gravitational waves could depend on the strength of the background scalar field.

From an observational point of view, the total radiated energy of scalar gravitational waves is an important property, which can be estimated as

$$E_\varphi(t) \approx \int_0^t |\partial_t \delta\varphi|^2 dt. \tag{4.4}$$

Using the numerical data in the evolution of Eq. (3.12), the total energies radiated from the different stellar models in scalar-tensor gravity are calculated as shown in Fig. 6, where the left and right panels correspond to the time evolutions of the total radiated energies for $\beta = -4.6$ and -5.0. In each panel, the central densities of the adopted stellar models are denoted on each line in units of 10^{15} g/cm^3. From this figure, we find that the total energy radiated by the scalar gravitational waves depends strongly on the stellar models. To clearly see the dependence on stellar models, in Fig. 7, we show the total energies accumulated until $t = 12$ msec as a function of the stellar central density for $\beta = -4.6$ (left panel) and -5.0 (right panel). Comparing this figure to Fig. 2, we find that the central density for the peak of the total radiated energy is shifted to a density region higher than that for the peak of the background scalar field. This means that the massive neutron stars might have a potential to radiate more scalar gravitational waves. Additionally, one observes that the total energy also strongly depends on the coupling parameter β. In practice, the ratio of the total radiated energy for $\beta = -5.0$ to that for $\beta = -4.6$ reaches 4.1, while the ratio of the central value of the background scalar field for $\beta = -5.0$ to that for $\beta = -4.6$ is only 1.5.

Furthermore, in order to see the specific oscillation frequencies of scalar gravitational waves, we calculate the fast Fourier transform (FFT) for the stellar models with $\epsilon_c = 1.5\times10^{15}$ g/cm^3 and show it in Fig. 8, where the left and right panels correspond to the results for $\beta = -4.6$ and -5.0, respectively. In both panels, we also denote the eigenfrequencies of the matter radial oscillations calculated with the mode analysis shown in Sec. IV A with the broken vertical lines. From this figure, one can find that the scalar gravitational waves driven by the matter radial oscillations could oscillate with the same frequencies as those of the matter oscillations. That is, in scalar-tensor gravity, one has a chance to extract the frequencies of the radial oscillations of neutron stars via the observations of scalar gravitational waves, which can be written as functions of the stellar mass and/or stellar compactness as in Figs. 3 and 4. This is an advantage in scalar-tensor gravity, because it is impossible to observe the radial oscillations of neutron stars via the radiated gravitational waves in general relativity, where the gravitational waves cannot be excited due to the radial oscillations of neutron stars.

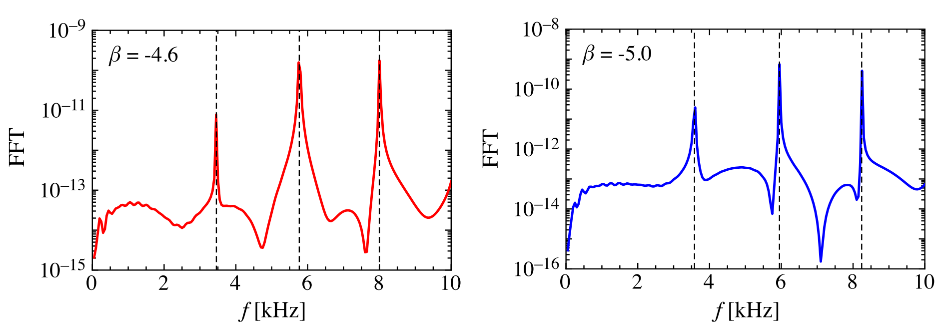

FIG. 8 (color online). FFT calculated from the waveforms of scalar gravitational waves radiated from the neutron stars with $\epsilon_c = 1.5\times10^{15}$ g/cm^3 in scalar-tensor gravity with $\beta = -4.6$ (left panel) and -5.0 (right panel). In both panels, the vertical broken lines denote the frequencies of the matter oscillations calculated with the eigenvalue problem as in Sec. IV A.

V. CONCLUSION

Neutron stars are one of the best candidates to probe the gravitational theory in the strong-field regime. In this paper, we especially focus on the radial oscillations of neutron stars in scalar-tensor gravity to examine the scalar gravitational waves driven by the matter oscillations. In fact, the gravitational waves are not excited due to the radial stellar oscillations in general relativity, while one can expect to observe the scalar gravitational waves due to such oscillations in scalar-tensor gravity. For the calculations of the scalar gravitational waves, we first derive the perturbation equations for radial oscillations in scalar-tensor gravity. From the equation of the system of radial oscillations, we find that the matter oscillations depend only on the background scalar field, independent of the scalar gravitational waves. On the other hand, the wave equation of scalar gravitational waves has a source term composed of the matter oscillations. Due to such a specific coupling, we can determine the frequencies of matter radial oscillations by the mode analysis. As a result, we show that the spontaneous scalarization can be observed even in the radial oscillations, which might enable us to find the imprint of gravitational theory with the help of the other observations such as stellar mass and/or compactness.

Additionally, to examine the scalar gravitational waves driven by the matter radial oscillations, we directly make a numerical simulation of the evolution equations, where we fix the initial energy of matter oscillations to be $10^{-4}M_{\odot}$. Then, we find that the scalar gravitational waves can be excited if the background scalar field exists. We also find that the total energy radiated by the scalar gravitational waves depends strongly on the background scalar field and the coupling constant β, where the massive star has a potential to radiate more energy of scalar gravitational waves. Furthermore, we make the fast Fourier transform to see the specific oscillation frequencies of radiated scalar gravitational waves, which are exactly the same as the frequencies of matter oscillations. That is, via the observations of scalar gravitational waves, one can extract the frequencies of stellar radial oscillations. This is an advantage in scalar-tensor gravity, because the radial gravitational waves cannot be excited in general relativity, as mentioned before. We remark that one might have another chance to observe an imprint of radial oscillation in the gravitational waves, if radial oscillations are strongly excited, for example, in core-collapse supernovae, and those oscillations make nonlinear coupling with nonradial oscillations, where the oscillations with combination frequencies could be excited [34]. If so, it might be possible for one to measure the background scalar field via such nonlinear coupling. In this paper, as a first step, we neglect the effects of the solid crust layer, magnetic fields, and the exotic matter inside the star, which are also important properties of neutron stars. Such effects could bring us additional information about the stellar properties [35–38], which might make the observational constraints in the gravitational theory stronger.

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